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Minimal Circular Nearly Strongly-Balanced Neighbor Designs When Left and Right Neighbor Effects are Equal

Abdul Salam[1], *Rashid Ahmed*[1], *Muhammad Daniyal*[1,*] *and Berihan R. Elemary* [2]

[1]Department of Statistics, The Islamia University of Bahawalpur, Pakistan
[2]Department of Applied, Mathematical and Actuarial Statistics, Damietta University and NUB-Nahda University, Beni Suef, Egypt

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Abstract: In many agricultural and allied research, neighboring treatment effects become the major source of bias which may be reduced if neighbor balanced or strongly neighbor balanced designs are used. Strongly balanced neighbor designs play an important role to incorporate the neighbor effects. Method of cyclic shifts (Rule I) provides minimal strongly balanced neighbor designs for some combinations of v and k. For the remaining cases, the method of cyclic shifts (Rule II) provides their better alternates which are minimal nearly strongly balanced neighbor designs that are highly efficient to estimate the treatment effects and neighbor effects separately. In this article, some new construction procedures are developed to obtain minimal nearly strongly balanced neighbor designs in blocks of (i) equal sizes, and (ii) two different sizes.

Keywords: Method of cyclic shifts; Left neighbor effects; Right neighbor effects; Neighbor balanced designs.

1 Introduction

Neighbor balanced designs (NBDs) are used in the experiments where the response of a treatment applied on a given unit is affected by the treatments applied to its neighboring units. Minimal NBDs are economical, therefore, these are used to balance the neighbor effects. The bias due to neighbor effects is minimized with the use of NBDs, [1] and [2]. Minimal strongly NBDs (SBNDs) are used to estimate the treatment direct effect and neighbor effects independently. A design is said to be minimal neighbor balanced if each treatment has all other treatments once as its neighbors. If each treatment also appears as its own neighbor, then the design is known as SBND. SBNDs play an important role to incorporate the neighbor effects, therefore, these designs have an edge over neighbor balanced designs. For some combinations of v and k, minimal SBNDs can be constructed through the method of cyclic shifts (Rule I). These designs should also be constructed for the remaining cases. This problem can be solved through the method of cyclic shifts (Rule II) which provides the nearly strongly balanced neighbor designs (NSBNDs) which are the best alternates to the SBNDs and are highly efficient to estimate the treatment effects and neighbor effects separately.

(i) The designs in which each treatment appears an equal number of times with all other treatments (including itself except treatment labeled as "v-1" which does not appear with itself) as a neighbor are called NSBNDs.

(ii) The designs in which each treatment appears exactly once with all other treatments (including itself) except $v-1$ which does not appear with itself as a neighbor, are called minimal NSBNDs.

These designs are not available in the literature, therefore, this study deals with the construction of minimal circular NSBNDs (CNSBNDs). Minimal circular SBNDs and minimal CNSBNDs are applied in the biometrics, agriculture and plant breeding experiments. Nearest neighbor balanced designs in linear blocks for the first-order autoregressive model has been suggested by [3]. A catalogue of neighbor balanced designs using border plots was presented by [4]. Neighbor designs were used in virus research as a technique by [5]. A brief review on one-dimensional neighbor designs since 1967 was given by [6], therefore, readers are referred to it for detailed literature discussion $(1967-2011)$. The designs

* Corresponding author e-mail: muhammad.daniyal@iub.edu.pk

which are balanced to estimate the direct and interference effects of treatments have been derived by [7]. To find left and right neighbors of treatment from a balanced incomplete block design, a method that found left and right neighbor designs possessing circular property from a series has been suggested by [8]. [9] constructed the one-sided right neighbor designs and concluded that the neighbor treatments followed the circularity property of the same order. [10] described some methods to construct CNBDs and CPNBDs to estimate treatment direct and neighbor effects. They also proposed a class of CNBDs in blocks of unequal sizes. In this article, some new construction procedures are developed to obtain minimal CNSBNDs in blocks of (i) equal sizes, and (ii) two different sizes, when left- and right- neighbor effects are considered to be the same. The rest of the paper is organized as follows. In Section 2, the method of cyclic shifts (Rule II) is explained for the construction of CNSBNDs. In Section 3, minimal CNSBNDs are constructed.

2 Method of cyclic shifts

[11] developed this method which is described here for the construction of CNSBNDs.

- Following is a design obtained from $S_1 = [2,4,5]$ and $S_2 = [1,3]$t for $v = 12$ and $k = 4$, using Rule II.

Take "v-1" blocks for $[2,4,5]$. Assign $0,1,...,v-2$ to first unit of each block. To get the 2nd elements of all blocks, add 2 $(mod\,(v-1))$ of $[2,4,5]$ to the first element of all blocks. Add 4 $(mod\,11)$ to the 2nd element of all blocks. Then add 5 $(mod\,11)$ to the 3rd element of all blocks.

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Blocks

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
0	1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	0	1
6	7	8	9	10	0	1	2	3	4	5
0	1	2	3	4	5	6	7	8	9	10

Take $v-1$ more blocks for $[1,3]$t. Assign $0,1,...,v-2$ to first unit of each block. To get the 2nd elements of all blocks, add 1 $(mod\,11)$ to the first unit elements. Add 3 $(mod\,11)$ to the 2nd element of all blocks. Assign 11 as the 4th element of all blocks.

Blocks

12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
0	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	0
4	5	6	7	8	9	10	0	1	2	3
11	11	11	11	11	11	11	11	11	11	11

- How minimal CNSBNDs are constructed, using Rule II.

Let $S_j = [q_{j1}, q_{j2}, ..., q_{j(k-1)}]$ and $S_i = [q_{i1}, q_{i2}, ..., q_{i(k-2)}]$t, where $0 \le q_{ji} \le v-2$.
If each element $0,1,2,...,v-2$ appears once in S^*, where
$S^* = [q_{j1}, q_{j2}, ..., q_{j(k-1)}, (q_{j1}+q_{j2}+...+q_{j(k-1)}), (v-1)-q_{j1}, (v-1)-q_{j2}, ..., (v-1)-q_{j(k-1)}, (v-1)-(q_{j1}+q_{j2}+ ...+q_{j(k-1)}), q_{i1}, q_{i2}, ..., q_{i(k-2)}, (v-1)-q_{i1}, (v-1)-q_{i2}, ..., (v-1)-q_{i(k-2)}]$
then it will be minimal CNSBND.

Example 2.1: $S_1 = [2,4,5]$ and $S_2 = [1,3]$t produce minimal CNSBND for $v = 12$ and $k = 4$. Here $q_{11} = 2, q_{12} = 4, q_{13} = 5, (q_{11}+q_{12}+q_{13})\,mod\,(11) = 0, v-1-q_{11} = 9, v-1-q_{12} = 7, v-1-q_{13} = 6, v-1-(q_{11}+q_{12}+q_{13})\,mod\,(v-1) = 0, q_{21} = 1, q_{22} = 3, v-1-q_{21} = 10, v-1-q_{22} = 8. Now\,S^* = [2,4,5,0,9,7,6,0,1,3,10,8]$ in which each element from $0,1,2,...,10$ appears exactly once. Hence $[2,4,5]$ and $[1,3]$t produce minimal CNSBND for $v = 12$ and $k = 4$.

3 Efficiency of Separability

Neighbor balanced designs must be able to measure the direct and neighbor effects separately, therefore, a measure of Separability called efficiency of Separability (E_s) is discussed here. To elaborate the concept of Es using the method of chi-square explained by [12], we consider our proposed design obtained through $S_1 = [2,4,5], S_2 = [1,3]$t for $v = 12$ and $k = 4$. The first step is to calculate the contingency table containing the number of observations having treatment in the unit j (direct effect) related to the treatment applied in its neighboring units $j \pm 1$ (neighbor effect).

Neighbor effect

Direct effect	No	0	1	2	3	4	5	6	7	8	9	10	11	Total
0	0	2	1	1	1	1	1	1	1	1	1	1	2	14
1	0	1	2	1	1	1	1	1	1	1	1	1	2	14
2	0	1	1	2	1	1	1	1	1	1	1	1	2	14
3	0	1	1	1	2	1	1	1	1	1	1	1	2	14
4	0	1	1	1	1	2	1	1	1	1	1	1	2	14
5	0	1	1	1	1	1	2	1	1	1	1	1	2	14
6	0	1	1	1	1	1	1	2	1	1	1	1	2	14
7	0	1	1	1	1	1	1	1	2	1	1	1	2	14
8	0	1	1	1	1	1	1	1	1	2	1	1	2	14
9	0	1	1	1	1	1	1	1	1	1	2	1	2	14
10	0	1	1	1	1	1	1	1	1	1	1	2	2	14
11	0	2	2	2	2	2	2	2	2	2	2	2	0	22
Total	0	14	14	14	14	14	14	14	14	14	14	14	22	176

A column of "No" in the contingency table exhibits the observations which do not have a neighbor effect. The expected frequencies under the assumption of the independent model which shows that the probability of observation that falls into the specific column is not the function of the row and where the observation occurs are mentioned below.

Carry-over effect

Direct effect	No	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1.75	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.75
1	0	1.11	1.75	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.75
2	0	1.11	1.11	1.75	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.75
3	0	1.11	1.11	1.11	1.75	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.75
4	0	1.11	1.11	1.11	1.11	1.75	1.11	1.11	1.11	1.11	1.11	1.11	1.75
5	0	1.11	1.11	1.11	1.11	1.11	1.75	1.11	1.11	1.11	1.11	1.11	1.75
6	0	1.11	1.11	1.11	1.11	1.11	1.11	1.75	1.11	1.11	1.11	1.11	1.75
7	0	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.75	1.11	1.11	1.11	1.75
8	0	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.75	1.11	1.11	1.75
9	0	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	0.01	1.75	1.11	1.75
10	0	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.11	1.75	1.75
11	0	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	2.75

The Pearson chi-square can be calculated as

$$\chi^2 = \Sigma\Sigma\left(\frac{(O_{ij}-E_{ij})^2}{E_{ij}}\right) = 5.115$$

The chi-square value is then manipulated with the following relation called Vc $(Cramers\ V)$.

$$Vc = \left[\frac{\frac{\chi^2}{Total\ number\ of\ incidences}}{min(number\ of\ rows-1, number\ of\ columns-1)}\right]^{1/2}$$

$$= \left[\frac{\frac{5.115}{176}}{min(11,12)}\right]^{\frac{1}{2}}$$

$$= 0.079$$

$$E_s = (1-Vc) = (1-0.08) = 0.921$$

The high value of Es indicates the suitability of neighbor designs for the estimation of direct effects and neighbor effects separately. All the designs proposed in this article are highly efficient in this regard.

4 Construction of Minimal CNSBNDs

In this Section, minimal CNSBNDs are constructed through the method of cyclic shifts (Rule II).

4.1 Minimal CNSBNDs in equal blocks sizes

Using Rule II, minimal CNSBNDs are constructed here in k sizes.

4.1.1 Construction

Minimal CNSBNDs can be obtained for $v = 8i+4$, i integer and $k = 4$ from $(i+1)$ sets of shifts which are generated as:

• Consider $B = [0,1,2,3,...,m]$, where $m = (v-2)/2$.
• Discard 1 and $3i$ from the elements of B.
• Divide remaining elements of B into i classes of k_1 size such that their sum is multiple of $(v-1)$.
• From each class, discard one value (any), resulting are i sets.
• Consider $[1,3i]$t as the $(i+1)th$ set.

Example 4.1.1: Minimal CNSBND can be constructed through 4.1.1 for $v = 20$ and $k = 4$ as: Here $i = 2$, $m = 9$ then $B = [0,1,2,3,4,5,6,7,8,9]$. Discarding 1 and 6, remaining are divided into two groups $(0,2,8,9)$ and $(3,4,5,7)$. Hence $S_1 = [2,8,9], S_2 = [4,5,7], S_3 = [1,6]$t produce required design for $v = 20$ and $k = 4$ with $E_s = 0.90$.

4.1.2 Construction

Minimal CNSBNDs can be constructed for $v = 10i+6$, i integer and $k = 5$ from $(i+1)$ sets of shifts which are generated as:

• Consider $B = [0,1,2,...,m-1,m]$, where $m = (v-2)/2$.
• Let $b = [(m(m+1))/2]\ mod(v-1)$. If $b \geq 3$ then discard three values q_1, q_2 and q_3 from B, where $q_1+q_2+q_3 = b$, otherwise replace one or more values with their complements such that $b \geq 3$ then discard q_1, q_2 and q_3 from B. Complement of a is $(v-1)-a$ in Rule I.
• Divide remaining elements of B into i classes of k_1 size such that their sum is multiple of $(v-1)$.
• From each class, discard one value (any), resulting are i sets.

• Consider $[q_1, q_2, q_3]$t as $(i+1)th$ set of shifts.

Example 4.1.2: Minimal CNSBND can be constructed through 4.1.2 for $v = 26$ and $k = 5$ as: Here $i = 2, m = 12, B = [0,1,2,3,4,5,6,7,8,9,10,11,12]$ and $b = 3$. Discarding 0,1 and 2, remaining are divided into two groups $(3,4,5,6,7)$ and $(8,9,10,11.12)$. Hence $S_1 = [4,5,6,7], S_2 = [9,10,11,12], S_3 = [0,1,2]$t produce required designs with $E_s = 0.90$.

4.1.3 Construction

Minimal CNSBNDs can be constructed for $v = 12i+8$, i integer and $k = 6$ from $(i+1)$ sets of shifts which are generated as:

• Consider $B = [0,1,2,...,m-1,m]$, where $m = (v-2)/2$.

• Let $b = [(m(m+1))/2]\ mod(v-1)$. If $b \geq 6$ then discard four values q_1, q_2, q_3 and q_4 from B, where $q_1+q_2+q_3+q_4 = b$, otherwise replace one or more values with their complements such that $6 \leq b \leq m-2$ then discard q_1, q_2, q_3 and q_4 from B.

• Divide remaining elements of B into i classes of k_1 size such that their sum is multiple of $(v-1)$.

• From each class, discard one value (any), resulting are i sets.

• Consider $[q_1, q_2, q_3, q_4]$t as $(i+1)th$ set.

Example 4.1.3: Minimal CNSBND can be constructed through 4.1.3 for $v = 20$ and $k = 6$ as: Here $i = 1, m = 9, B = [0,1,2,3,4,5,6,7,8,9]$ and $b = 7$. Discarding 0,1,2 and 4, remaining are $(3,5,6,7,8,9)$. Hence $S_1 = [5,6,7,8,9], S_2 = [0,1,2,4]$t produce required designs with $E_s = 0.91$.

4.2 Minimal CNSBNDs in k_1 and k_2

Using Rule II, minimal CNSBNDs are constructed here in k_1 and k_2 sizes.

4.2.1 Construction

Minimal CNSBNDs can be constructed for $v = 2ik_1+2$, i integer, $k_1 = 4j$, j integer, and $k_2 = 3$ from $(i+1)$ sets of shifts which are generated as:

• Consider $B = [0,1,2,3,...,m]$, where $m = (v-2)/2$.

• Discard c from B, where $c = ij$.

• Divide remaining elements of B into i classes of k_1 size such that their sum is multiple of $(v-1)$.

• From each class, discard one value (any), resulting are i sets.

• Consider $[c]$t as $(i+1)th$ set.

Example 4.2.1: Minimal CNSBND is obtained through 4.2.1 for $v = 18, k_1 = 8$ and $k_2 = 3$ as: Here $i = 1, j = 2, m = 8, B = [0,1,2,3,4,5,6,7,8]$. Discarding 2, remaining are $(0,1,3,4,5,6,7,8)$. Hence $S_1 = [1,3,4,5,6,7,8], S_2 = [2]$t produce required with $E_s = 0.95$.

4.2.2 Construction

Minimal CNSBNDs can be constructed for $v = 2ik_1+4, i$ integer, $k_1 = 4j, j\ (integer) > 1$ and $k_2 = 4$ from $(i+1)$ sets of shifts which are generated as:

• Consider $B = [0,1,2,3,...,m]$, where $m = (v-2)/2$.

• Discard 1 and $i \times (k_1 - j)$ from the elements of B.

• Divide remaining elements of B into i classes of k_1 size such that their sum is multiple of $(v-1)$.

• From each class, discard one value (any), resulting are i sets.

• Consider$[1, i \times (k_1 - j)]$t as $(i+1)th$ set.

Example 4.2.2: Minimal CNSBND is obtained through 4.2.2 for $v = 36, k_1 = 8$ and $k_2 = 4$ as: Here $i = 2, j = 2, m = 17, B = [0,1,2,...,17]$. Discarding 1 and 12, remaining are divided into two groups $(0,2,3,4,13,15,16,17)$ and $(5,6,7,8,9,10,11,14)$. Hence $S_1 = [2,3,4,13,15,16,17], S_2 = [5,6,7,8,9,10,11], S_3 = [1,12]$t produce required designs with $E_s = 0.94$.

4.2.3 Construction

Minimal CNSBNDs can be constructed for $v = 2ik_1 + 2, i\ integer, k_1 > 3$ and $k_2 = 3$ from $(i+1)$ sets of shifts which are generated as:

• Consider $B = [0, 1, 2, ..., m-1, m]$, where $m = (v-2)/2$.

• Let $b = [(m(m+1))/2]\ mod(v-1)$. If $1 \leq b \leq m$ then discard b from B, otherwise replace one or more values with their complements such that $1 \leq b \leq m$ then discard b from B.

• Divide remaining elements of B into i classes of k_1 size such that their sum is multiple of $(v-1)$.

• From each class, discard one value (any), resulting are i sets.

• Consider $[b]$t as $(i+1)$th set.

Example 4.2.3: $S_1 = [1, 2, 4, 6, 12], S_2 = [7, 8, 9, 10, 11], S_3 = [3]$t produce minimal CNSBND through 4.2.3 for $v = 26, k_1 = 6$ and $k_2 = 3$ with $E_s = 0.96$.

4.2.4 Construction

Minimal CNSBNDs can be constructed for $v = 2ik_1 + 4$, i integer, $k_1 > 4$ and $k_2 = 4$ from $(i+1)$ sets of shifts which are generated as:

•Consider $B = [0, 1, 2, ..., m-1, m]$, where $m = (v-2)/2$.

• Let $b = [(m(m+1))/2]\ mod(v-1)$. If $1 \leq b \leq m$ then discard two values q_1 and q_2 from B, where $q_1 + q_2 = b$, otherwise replace one or more values with their complements such that $1 \leq b \leq m$ then discard q_1 and q_2 from B.

• Divide remaining elements of B into i classes of k_1 size such that their sum is multiple of $(v-1)$.

• From each class, discard one value (any), resulting are i sets.

• Consider $[q_1, q_2]$t as $(i+1)$th set.

Example 4.2.4: $S_1 = [2, 3, 4, 5, 7], S_2 = [0, 1]$t produce minimal CNSBND through 4.2.4 for $v = 16, k_1 = 6$ and $k_2 = 4$ with $E_s = 0.94$.

4.2.5 Construction

Minimal CNSBNDs can be constructed for $v = 2ik_1 + 6, i\ integer, k_1 > 5$ and $k_2 = 5$ from $(i+1)$ sets of shifts which are generated as:

• Consider $B = [0, 1, 2, ..., m-1, m]$, where $m = (v-2)/2$.

• Let $b = [(m(m+1))/2]\ mod(v-1)$. If $3 \leq b \leq m-1$ then discard three values q_1, q_2 and q_3 from B, where $q_1 + q_2 + q_3 = b$, otherwise replace one or more values with their complements such that $3 \leq b \leq m-1$ then discard q_1, q_2 and q_3 from B.

• Divide remaining elements of B into i classes of k_1 size such that their sum is multiple of $(v-1)$.

• From each class, discard one value (any), resulting are i sets.

• Consider $[q_1, q_2, q_3]$t as $(i+1)th$ sets.

Example 4.2.5: $S_1 = [4, 5, 10, 12, 13], S_2 = [2, 6, 8, 9, 11], S_3 = [0, 1, 3]$t produce minimal CNSBND through 4.2.5 for $v = 30, k_1 = 6$ and $k_2 = 5$ with $E_s = 0.97$.

5 Concluding Remarks

Neighbor balanced designs have been extensively used in the fields of life sciences especially in agricultural and botanical experimentation where there exist neighboring effects on the plots from their adjacent plots. Well known method of cyclic shifts has been utilized to propose new generators and construction procedures of these important designs. Method of cyclic shifts (Rule I) provides us minimal CSBNDs while Rule II provides minimal CNSBNDs in which one pair of $(v-1, v-1)$ is missing. Both the proposed structure of designs proved to be efficient based on the efficiency of separability. Therefore, researchers and experimenters have a wide range of choices for the designs where there are neighboring effects from the adjacent plots. In the appendix, there are catalogs of proposed designs for equal and two different period sizes to create a choice for the researchers to select designs of their interest.

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Conflict of interest

The authors declare that they have no conflict of interest.

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Appendix

Table1: Designs for $v = 8i+4$, i integer, $k = 4$ and $v \leq 100$.

v	k	$sets$
12	4	[2,4,5]+[1,3]t
20	4	[2,8,9]+[4,5,7]+[1,6]t
28	4	[2,12,13]+[3,5,8]+[4,6,7]+[1,9]t
36	4	[2,16,17]+[3,4,13]+[5,6,10]+[7,8,9]+[1,12]t
44	4	[2,20,21]+[3,4,17]+[5,6,14]+[7,8,12]+[9,10,11]+[1,15]t

Table 2: Designs for $v = 10i+6$, i integer, $k = 5$ and $v \leq 100$.

v	k	$sets$
16	5	[3,4,6,7]+[0,1,2]t
26	5	[4,5,6,7]+[8,9,10,11]+[0,1,2]t
36	5	[3,4,13,15]+[5,6,7,8]+[11,12,14,16]+[1,2,10]t
46	5	[3,4,8,10]+[5,6,7,9]+[11,12,14,21]+[15,16,17,19]+[0,1,2]t

Table 3: Designs for $v = 12i+6$, i integer, $k = 6$ and $v \leq 100$.

v	k	$sets$
20	6	[5,6,7,8,9]+[0,1,2,4]t
32	6	[4,5,10,13,14]+[6,7,8,9,12]+[0,1,2,3]t
44	6	[4,5,6,9,19]+[7,8,12,18,20]+[11,13,14,15,16]+[1,2,3,10]t
56	6	[5,6,7,8,9]+[10,15,16,22,23]+[12,13,14,18,26]+[4,11,17,19,25]+[0,1,2,3]t

Table 4: Designs for $v = 2ik_1+2$, i integer, $k_1 = 4j, 1 \leq j\,(integer) \leq 4$ and $k_2 = 3$.

v	k_1	k_2	$sets$
12	4	3	[2,3,4]+[1]t
20	4	3	[3,6,8]+[4,5,7]+[2]t
28	4	3	[2,11,12]+[6,8,10]+[5,7,9]+[3]t
36	4	3	[2,15,16]+[5,13,14]+[7,11,12]+[8,9,10]+[4]t
44	4	3	[2,19,20]+[7,15,18]+[3,8,13]+[4,9,12]+[6,10,11]+[5]t
50	4	3	[2,23,24]+[5,21,22]+[3,7,19]+[4,10,17]+[8,11,14]+[9,12,13]+[6]t

Table 5: Designs for $v = 2ik_1+4$, i integer, $k_1 = 4j, 2 \leq j\ (integer) \leq 5, k_2 = 4$ and $v \leq 100$.

v	k_1	k_2	$sets$
20	8	4	[2,3,4,5,7,8,9]+[1,6]t
36	8	4	[2,3,4,13,15,16,17]+[5,6,7,8,9,10,11]+[1,12]t
22	8	4	[2,3,4,21,23,24,25]+[5,6,7,8,15,19,20]+[9,10,11,12,13,14,16]+[1,18]t

Table 6: Designs for $v = 2ik_1+2$, i integer, $6 \leq k_1 \leq 15, k_2 = 3$.

v	k_1	k_2	$sets$
14	6	3	[2,3,5,7,9]+[1]t
26	6	3	[1,2,4,6,12]+[7,8,9,10,11]+[3]t
38	6	3	[2,3,4,10,18]+[5,7,8,12,16]+[9,13,14,15,17]+[1]t
50	6	3	[3,4,5,14,23]+[1,2,7,11,13]+[9,10,12,21,22]+[16,17,18,19,20]+[6]t

Table 7: Designs for $v = 2ik_1 + 4$, i integer, $6 \leq k_1 \leq 16, k_2 = 4$.

v	k_1	k_2	*sets*
16	6	4	[2,3,4,5,7]+[0,1]t
28	6	4	[2,3,4,6,12]+[5,7,8,10,11]+[1,9]t
40	6	4	[1,3,4,5,8]+[7,9,11,15,17]+[6,10,12,13,14]+[0,2]t
52	6	4	[2,3,5,17,24]+[6,7,8,9,10]+[13,14,15,16,21]+[12,19,20,22,25]+[1,18]t

Table 8: Designs for $v = 2ik_1 + 6$, i integer, $6 \leq k_1 \leq 15, k_2 = 5$.

v	k_1	k_2	*sets*
18	6	5	[3,4,5,6,7]+[0,1,2]t
30	6	5	[4,5,10,12,13]+[2,6,8,9,11]+[0,1,3]t
42	6	5	[6,7,8,15]+[9,11,18,19,20]+[12,13,14,16,17]+[0,2,3]t
54	6	5	[4,5,6,7,8]+[11,12,13,19,25]+[14,16,17,18,20]+[3,9,10,22,24]+[0,1,2]t