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Asset pricing and Modern portfolio theory: An Application to portfolio optimization of different Moroccan and Multinational assets using Excel and Python programming analysis

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Asset pricing and Modern portfolio theory: An Application to portfolio optimization of different Moroccan and Multinational assets using Excel and Python programming analysis

تسعير الأصول ونظرية المحفظة الحديثة: تطبيق لتحسين المحفظة لمختلف أصول مغربية و متعددة الجنسيات باستخدام تحليل برمجة Excel و Python

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Abstract

One of the problems considered in financial mathematics is finding portfolios of given financial assets that minimize risk for targeted returns. The set of such portfolios is called the envelope of the assets. Traditionally, this problem is solved as a calculus minimization problem involving partial derivatives and Lagrange multipliers. The mean-variance framework is the preferred method for picking investments for many retail and institutional investors. Meanwhile, big data and the real-time economy have created new asset allocation challenges and opportunities. These changes have been thoroughly recorded, and portfolio management firms constantly incorporate the data into complex models. In this study, we tackle this issue by engineering practical tools for asset allocation and implementing them in the Python programming language. With its clear syntax, efficient development, and usability, Python programming provides an ideal framework for this research study. We turn to convex optimization to formulate specific portfolio optimization problems and incorporate different investment constraints. We give code samples from the associated procedures and useful graphics that illustrate the input data and the produced results. We discover that most optimization issues can be written in convex form and hence quickly implemented and solved using Python modules to generate portfolios from real-world data. In this research work, we presented the results of three case studies that we have carried out on Moroccan Assets and multinational companies. We used and analyzed data from Yahoo Finance and Investing.com for the period that covers ten years, between 2012 and 2022. For our third case, we have used the Pandas-Datareader library in Python, and we have computed all the parameters for portfolio optimization, and we discussed the outcome profitability
Keywords: Assets, Investment, Portfolio Theory, Python programming, Excel, Digital management.

المستخلص

تتمثل إحدى المشكلات التي يتم أخذها في الاعتبار في الرياضيات المالية في العثور على محافظ لأصول مالية معينة تقلل من مخاطر العوائد المستهدفة. تسمى مجموعة هذه المحافظ مغلف الأصول. تقليديا يتم حل هذه المشكلة كمسألة تقليل التفاضل والتكامل تتضمن المشتقات الجزئية ومضاعفات Lagrange. حتى يومنا هذا ، يعد إطار متوسط التباين هو الطريقة المفضلة لاختيار الاستثمارات للعديد من المستثمرين الأفراد والمؤسسات. وفي الوقت نفسه ، خلقت البيانات الضخمة والاقتصاد في الوقت الفعلي تحديات وفرصًا جديدة في توزيع الأصول. تم تسجيل هذه التغييرات بدقة ، وتقوم شركات إدارة المحافظ باستمرار بدمج البيانات في نماذج معقدة. في هذه الدراسة ، نتعامل مع هذه المشكلة من خلال هندسة أدوات عملية لتخصيص الأصول وتنفيذها بلغة برمجة Python. من خلال بناء الجملة الواضح ، والتطوير الفعال ، وسهولة الاستخدام ، توفر برمجة Python إطارًا مثاليًا لهذه الدراسة البحثية. ننتقل إلى التحسين المحدب لصياغة مشاكل محددة لتحسين المحفظة ودمج قيود الاستثمار المختلفة. نقدم عينات من التعليمات البرمجية من الإجراءات ذات الصلة ، بالإضافة إلى رسومات مفيدة توضح بيانات الإدخال والنتائج المنتجة. نكتشف أن معظم مشكلات التحسين يمكن كتابتها في شكل محدب وبالتالي يتم تنفيذها وحلها بسرعة من خلال استخدام وحدات Python لإنشاء حافظات من بيانات العالم الحقيقي. في هذا العمل البحثي ، قدمنا نتائج ثلاث دراسات حالة أجريناها على الأصول المغربية وكذلك الشركات متعددة الجنسيات. قمنا باستخدام وتحليل البيانات من Yahoo Finance و Investing.com للفترة التي تغطي عشر سنوات ، بين 2012 و 2022. بالنسبة لحالتنا الثالثة، استخدمنا مكتبة Pandas-data reader في Python وقمنا بحساب جميع المعلمات لتحسين المحفظة و ناقشنا ربحية النتيجة
الكلمات المفتاحية : ا لأصول ، الاستثمار ، نظرية المحفظة ، برمجة Python، Excel ، الإدارة الرقمية.

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1. Introduction

Undoubtedly, finance has played a major role in the strategic governance of companies. Finance has changed significantly over the past few decades, moving from a minor and subordinate position to a major role in corporate governance. Today, finance is increasingly able to influence corporate strategies, and it is not uncommon for it to serve as the starting point for new strategies and business models.

The position and contribution of finance to the survival and development of companies are becoming increasingly essential over time. Furthermore, finance has also developed economic concepts that are used in decision-making involving the distribution of funds in uncertain situations. Moreover, it is clear that portfolio management has undergone a spectacular development since the mid-20th century. Indeed, it was the work of Markowitz in the 1950s that marked the starting point of modern theoretical developments in the management of investments in financial assets and the functioning of financial markets.

Although the notion of diversification was known well before, it was Markowitz's work that made it possible to build the concepts and mathematical models to determine the optimal proportions to invest in different financial assets according to the choice of the investor or asset manager. Subsequently, the works of Sharpe, Lintner, and Mossin conducted in 1960 and those of Fama have allowed the development and refinement of the analysis of the conditions of balance and efficiency of financial markets related to portfolio management. Similarly, the work of William F. Sharpe (1964) clarified the conditions of equilibrium of an "efficient market" of financial assets, while those carried out by Samuelson [1965] and Mossin [1966] deepened the notion of Sharpe. This is how the so-called modern financial theory (MFT) was born. So, investors seek the best combination to meet their needs in an uncertain environment. Uncertainty must be estimated, analyzed, modeled, and managed to determine the optimal allocation. As this task is not simple, the investor must rely on quantity and function forecasts.

One of the most well-known optimization models in finance is the portfolio selection model developed by Harry Markowitz, which is the foundation of modern portfolio theory. Markowitz's mean-variance approach led to important developments in financial economics, such as Tobin's mutual fund separation theorem and Sharpe's capital asset pricing model (CAPM). Markowitz received the Nobel Prize in Economics in 1990 for his significant impact on theory and practice. Therefore, several developments and adjustments to the original concept have occurred in recent decades. In addition, asset allocation has become increasingly important in the current investment landscape due to the prevalence of low-interest rates and high market volatility. This has led to the development of new approaches to asset allocation, such as factor investing and risk parity, which aim to optimize risk-adjusted returns.

In practice, in the world of financial speculation and securities transactions, every investor, whether a trader, a mutual fund, or an individual investor, must make a decision on how to use their limited resources. The idea is to identify a combination of securities that complement each other rather than a single appropriate asset. The practice of allocating among different investments to reduce risk is known as diversification.

1.2 The aim

The objective of this research work is to design and develop effective asset allocation tools in Python. These tools should be simple enough for interested practitioners to understand the underlying theory, while providing enough numerical solutions. This leads us to formulate the main problem of this study as follows:
How can we effectively and comprehensibly apply the principles of portfolio theory and convex optimization mathematics in practical asset allocation? That is, how can we be good managers of our resources? Since most portfolios include multiple high-yield options, the main problem is deciding which of these attractive projects to pursue and which to reject. Different questions arise:

- How can we choose projects that will take us furthest and fastest in the desired direction?
- What could be a concrete example of sectors and instruments in a financial market context for this portfolio optimization problem?
- How can we optimize a portfolio in Python and Excel with weight constraints?
- What is the optimal diversification strategy for a portfolio when only risky assets are available on the markets?

To achieve these objectives, this project was conducted on three case studies conducted on assets of multinational companies and Moroccan companies. These studies were carried out using Excel and programming in Python.

2. Mathematical models:

2.1 Contributions of Markowitz:

- The interest of diversification is not based on the absence of correlation between returns, but on their imperfect correlation.
- "Spread your eggs in imperfectly correlated baskets rather than putting them in perfectly positively correlated baskets."
- The risk reduction allowed by diversification is limited by the degree of correlation between assets.

Diversifying a portfolio allows for a decrease in "risk" without necessarily decreasing the average return.

Example: Portfolio P consists of two securities, in proportions α and $(1-\alpha)$.

Expected return:

$$\mu_P = \alpha\mu_1 + (1-\alpha)\mu_2$$

Variance of return:

$$\sigma_P^2 = \alpha^2\sigma_1^2 + (1-\alpha)^2\sigma_2^2 + 2\alpha(1-\alpha)\sigma_{1,2}$$

$$\sigma_P^2 = \alpha^2\sigma_1^2 + (1-\alpha)^2\sigma_2^2 + 2\alpha(1-\alpha)\sigma_1\sigma_2\rho_{1,2}$$

In the theory of Markowitz, the essential characteristics of an asset or portfolio are its "return" (average) and its "risk".

2.2 Efficient Frontier

If all available "risky" securities are combined in all possible ways, the set of possible portfolios is obtained, characterized by an average rate of return of μ and standard deviation σ.

An efficient portfolio is a portfolio:

- Whose average return is maximized for a given level of risk,
- Or whose risk is minimized for a given return.

In the presence of a risk-free asset (Tobin 1958)

The risk-free asset pays a fixed real rate of return, without the risk of default.

In a portfolio comprising

- A risky security (or portfolio), (σ_R, μ_R), in proportion α,
- And a risk-free asset, $0, r_f)$, in proportion (1 – α),

Expected return and risk combine linearly:

$$R_P = \alpha R_R + (1-\alpha)r_f \Rightarrow \mu_P = \alpha\mu_R + (1-\alpha)r_f \quad and \quad \sigma_P = \alpha\sigma_R$$

Hence:

$$\mu_P = r_f + \frac{\mu_R - r_f}{\sigma_R}\sigma_P$$

$\frac{\mu_R - r_f}{\sigma_R}$: This is called the "Sharpe ratio" of the risky asset.

$\frac{\mu_P - r_f}{\sigma_R}$: This is called the "Sharpe ratio" of the portfolio.

Thus: $\frac{\mu_R - r_f}{\sigma_R} = \frac{\mu_P - r_f}{\sigma_R}$

The portfolio has the same Sharpe ratio as the risky asset it contains.

$\mu_P - r_f \rightarrow$ measures the average excess return (compensation for risk)

$\sigma_R \rightarrow$ measures the "amount" of risk

$\frac{\mu_P - r_f}{\sigma_R} \rightarrow$ can be interpreted as the unit compensation for risk.

2.3 The concept of optimal portfolio selection:

Markowitz's work in 1954 was the first attempt to theorize the financial management of portfolios, and his model suggests a procedure for selecting multiple stocks based on statistical criteria in order to obtain optimal portfolios. Specifically, Markowitz showed that investors seek to optimize their choices by considering not only the expected return on their investments but also the risk of their portfolio, which he mathematically defines as the variance of the return. Thus, the "efficient portfolio" is the most profitable portfolio for a given level of risk.

Step 1:

We set a target expected return and find all the minimum variance portfolios that satisfy the return objective. This gives us a set of minimum variance portfolios.

Step 2:

From these portfolios, we select the one that has the highest return for a given variance. By doing this for multiple expected return values, we end up with one or more efficient portfolios. Thus, between two portfolios (sets of assets) characterized by their (assumed random) return, we make the following assumptions:

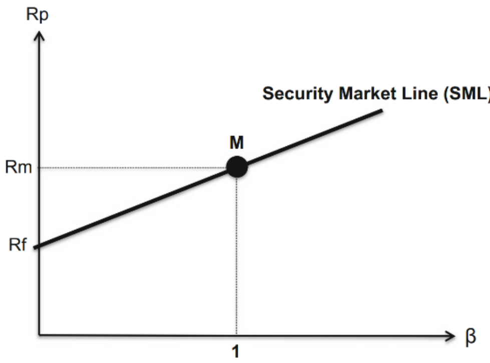
Assumption 1:

At the same level of risk, we choose the portfolio with the highest expected return (maximum gain).

Assumption 2:

At the same expected return, we choose the portfolio with the lowest risk (risk aversion).

This principle leads to eliminating a certain number of portfolios that are less efficient than others.



2.4 CAPM:

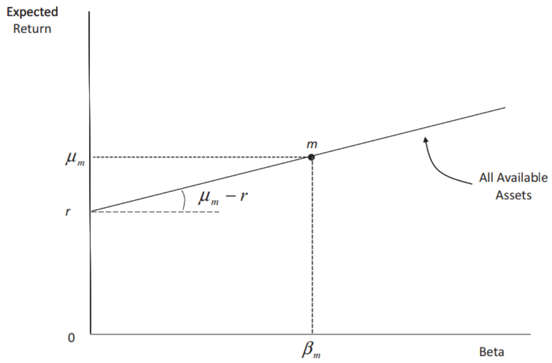
a) ***The expected return of a security does not depend on its specific risk.***

$$\mu_i = r_f + \beta_i(\mu_M - r_f)$$

The return of a security depends on the market risk premium and the beta of the security.

b) Beta indicates the portion of non-diversifiable risk.

At equilibrium, all portfolios and all assets are on the "CAPM line" (SML = Security Market Line).



- The beta of a portfolio is equal to the weighted average of the betas of the securities it contains.
- The beta of the market portfolio is equal to 1.
- An efficient portfolio is composed of risk-free securities and the market portfolio (two-fund separation theorem).

Only non-diversifiable risk "deserves" a higher return than the risk-free rate.

- A stock a located above the SML is "undervalued": its expected return is higher than that of an efficient portfolio with the same beta, the demand for this stock should increase, as well as its price (so that its expected return decreases).
  - On the other hand, a stock B located below the SML is "overvalued" (its current price is higher than the equilibrium price, its current return is lower than its equilibrium return).

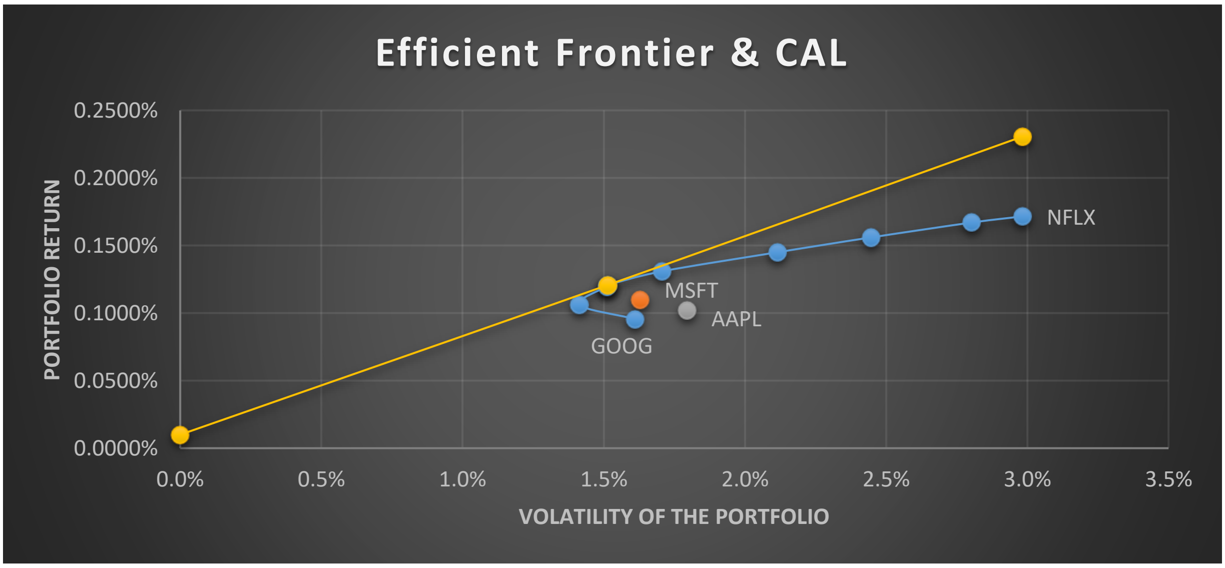
3. Result and discussion

3.1 Case studies on Excel:

3.1.1 First case study on Excel:

Let's imagine that we want to invest in four risky assets (Apple, Google, Microsoft, and Netflix). The first main problem we need to solve is to find the best weight distribution that will give us the best combination of higher returns with lower risk, in other words, the highest Sharpe ratio.

The curve that allows us to establish the efficient frontier is represented in the figure below. This curve crosses different points that correspond to different weight combinations in this portfolio. This model can be represented on a scatter plot, with risk on the X-axis and return on the Y-axis.



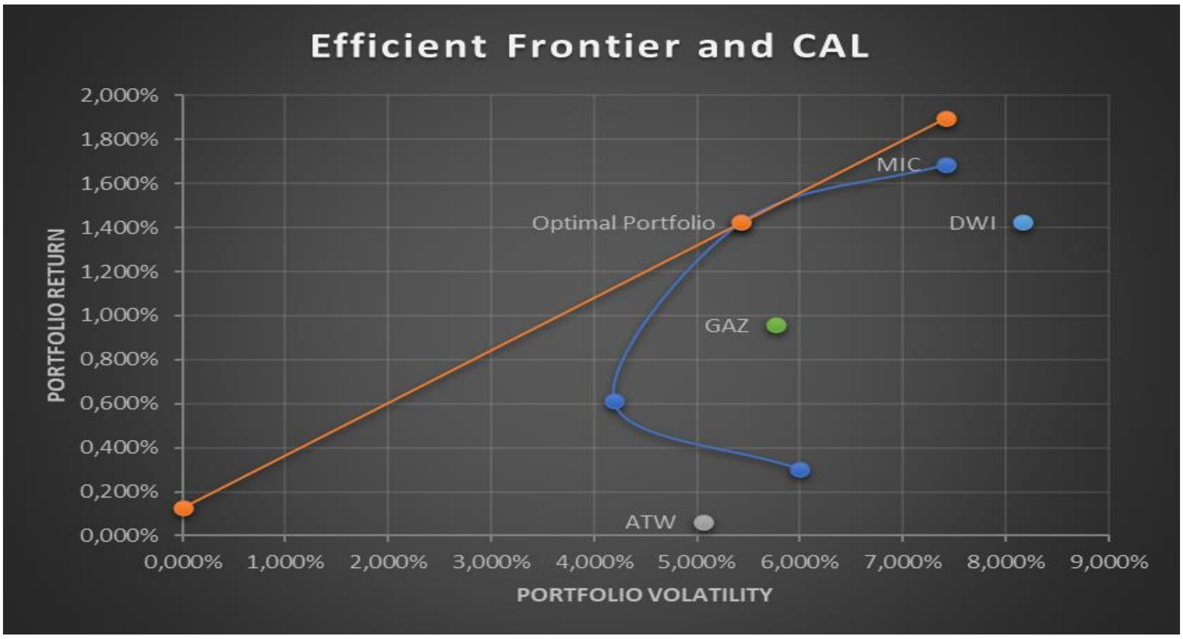
Analyzing this figure allows us to find the tangency portfolio, which corresponds to the point of intersection between the efficient frontier and the capital allocation line that represents the highest Sharpe ratio. This point represents the optimal portfolio and corresponds to the best-expected return/risk combination.

The results obtained for the Optimal Portfolio weight distribution show that MSFT has the highest rate of 41%, followed by NLFX with 24%, then AAPL with 18%, and GOOG with only 17%. The estimated values of the Sharpe ratio and the standard deviation for this portfolio are 1.1879 and 1.541%, respectively.

3.1.2 2nd case study on Excel:

In this second case study, we will deal with five Moroccan companies listed on the stock market: Micro Data SA, Afriquia Gaz, Disway SA, Alliances Développement Immobilier SA, and Attijariwafa Bank. To carry out this study, we used the historical data of these five assets from the Investing.com website. Mathematical analysis was performed on data from the period between 2012 and 2022 on a monthly basis. We used monthly closing prices from the previous year and calculated the month-to-month return percentage.

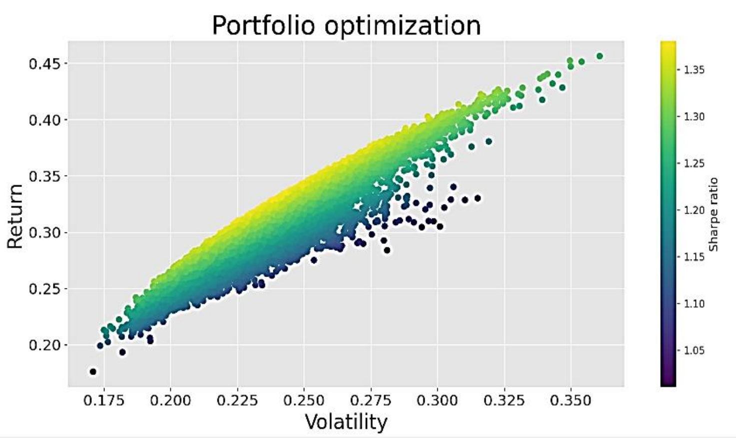
The mathematical procedures for calculating the proportions of the different assets in the final composition of the portfolio are similar to those used in the first case study. The final result and estimates obtained for the different parameters are presented in the figure below. The following figure illustrates the risk-return profile of different portfolios generated by combining the five assets in different weights.



At the end of this analysis, we can conclude that the best distribution of these five assets is that of the optimal portfolio. Therefore, we clearly see that the "distribution of the optimal portfolio weights" for these five different assets is the one that assigns the highest weight to MIC at 48%, followed by GAZ with a proportion of 27%, then DWY with 25%, while ATW and ADI display a weight of 0% for each of them.

3.2 Case study on Python:

This case study concerns seven stocks on the Python programming language:



As illustrated in the figure above, on the X-axis, we have volatility or risk measured by the standard deviation, and on the Y-axis, we have the associated return. In principle, we are looking for a portfolio or one of these points that is as far to the left as possible, meaning with less risk, and as far up as possible, meaning with higher returns. As we can see, the lighter the color, the higher the Sharpe ratio, and the darker the color, the lower the Sharpe ratio.

The overall analysis of these numerous random distributions of portfolios generated by different iterations allows us to find the best combination of return and risk, which corresponds to the weight distribution of the Optimal Portfolio.

```
Portfolio optimization ×
C:\Users\Amine\AppData\Local\Microsoft\WindowsApps\python3.10.exe "C:
Lowest risk
Returns                                              0.176126
Risk                                                 0.170889
Sharpe                                               0.913609
Weights    [0.059, 0.03, 0.311, 0.009, 0.553, 0.037, 0.002]
Name: 5845, dtype: object
['AAPL', 'TSLA', 'GOOG', 'MSFT', 'KO', 'NFLX', 'AMZN']

Highest return
Returns                                              0.456541
Risk                                                 0.360749
Sharpe                                               1.210095
Weights    [0.153, 0.524, 0.125, 0.04, 0.009, 0.113, 0.035]
Name: 15286, dtype: object
['AAPL', 'TSLA', 'GOOG', 'MSFT', 'KO', 'NFLX', 'AMZN']

Highest Sharpe
Returns                                               0.356658
Risk                                                  0.259052
Sharpe                                                1.299576
Weights    [0.152, 0.261, 0.009, 0.276, 0.106, 0.091, 0.106]
```

The above figure shows that the weight distribution of the different assets that generate the Optimal Portfolio corresponds to the one that assigns MSFT the first place with 27.6%, followed by TSLA with a proportion of 26.1%, then by AAPL with 15.2%, followed by AMZN and KO with the same proportions and equal to 10.9%; while NFLX has a proportion of 9.1% and GOOG is last with less than 1%.

It's worth noting that the weight distribution of assets in the optimal portfolio is not fixed and can change over time, depending on various factors such as market conditions, asset performance, and investor risk appetite. Furthermore, constructing an optimal portfolio is not a one-time activity, and investors should regularly rebalance their portfolios to maintain the desired asset allocation and risk level. Rebalancing involves selling or buying assets to bring the portfolio back to its target allocation. Overall, constructing an optimal portfolio is a complex and dynamic process that requires careful consideration of various factors. Investors should seek the advice of financial professionals and conduct thorough research before making investment decisions.

4. Conclusion:

Modern portfolio theory (MPT) is a financial theory that covers topics like diversification and risk management mathematically. The MPT provides the investor with a collection of tools for constructing a diversified portfolio whose return is maximized for a given degree of risk. The standard deviation is widely used to calculate risk.

The Efficient Frontier assists us in identifying and visualizing a portfolio of assets with the best expected return for any given risk level. Portfolios that are on the curve are the most efficient. Other collections either offer lower expected returns for the same level of risk or have higher risk levels for the same level of expected returns. The Capital Allocation Line (CAL) is a graphical representation of the risk-reward profile of risky assets that may be used to locate the optimal portfolio. The Sharpe ratio, or reward-to-risk ratio, is the slope of the line. The Sharpe ratio calculates the increase in expected return per unit of extra standard deviation.

In order to apply these theoretical concepts related to the construction of the best Portfolio, three different case studies were carried out;

The first case study was conducted on four multinational Assets (Google, Microsoft, Apple and Netflix). The results obtained for the Weight distribution of the Optimal Portfolio shows that MSFT has the highest rate of 41%, followed by NLFX with 24%, then AAPL with 18%, and GOOG with only 17%. The estimated values of the Sharpe Ratio and the Standard deviation for this Portfolio were equal to 1.1879 and 1.541% respectively

Moreover, the Expected Return to be generated by the Optimal Portfolio established was estimated to a value of 1.21%, whereas that of the Minimal risk Portfolio was evaluated to 0.106% only.

The second Case study was conducted on a set of Five Moroccan Assets (MIC, GAZ, DWY, ATW and ADI). The obtained Weight distribution values for the Optimal Portfolio shows that MIC comes first with 48%, followed by GAZ with a proportion of 27%, then DWY with 25%, whereas both ATW and ADI showed a weight of a value equal to 0 %. The estimated values of the Sharpe Ratio and the Standard deviation for this Portfolio were equal to 0.84 and 5.43% respectively

On the other hand, the Expected Daily Return of the Optimal Portfolio was estimated to a value of 1.42%%. This value is more than two times that estimated Expected Daily return of the Minimal Risk Portfolio which was evaluated to 0.61% only.

Furthermore, for these two case studies, we have completed the analysis by establishing the curve that represents the Expected Return for a given Risk level. The analysis of these figures allows us to find the tangency portfolio, which corresponds to the point of intersection between the Efficient Frontier and the Capital Allocation Line with that of the highest Sharpe Ratio.

In the third case study we used encoding by Python to calculate the best distribution of resource allocations over seven different Assets. These include Google, Apple, Netflix, Tesla, Amazon, Microsoft and Coke. The outcome of our analysis revealed that the weight distribution of the different assets that generate the Optimal Portfolio corresponds to the one that assigns MSFT the first place with 27.6%, followed by TSLA with a proportion of 26.1%, then AAPL with 15.2%, followed by AMZN and KO with the same proportions and equal to 10.9%; while NFLX has a proportion of 9.1% and GOOG is last with less than 1%. The Sharpe ratio for this portfolio was estimated to 1.21, with an expected return of 0.46 and a corresponding risk level of 0.36.

5. Bibliography

Bernstein, W. J. (2001). *The Intelligent Asset Allocator How to Build Your Portfolio to Maximize Returns and Minimize Risk.*

Bertsekas, D. P. (1974). Necessary and sufficient conditions for existence of an optimal portfolio. 248.

Blay, H. M. (2013). *Risk-Return Analysis: The Theory and Practice of Rational Investing (Volume One).* McGraw-Hill.

Brief Overview of Python : Informatics Practices . (2022).

Edwin J. Elton, M. J. (2006). Improved Estimates of Correlation Coefficients and their Impact on Optimum Portfolios.

Fama, E. F. (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy*.

Levy, H. (2012). *The Capital Asset Pricing Model in the 21st Century: Analytical, Empirical, and Behavioral Perspectives* . Cambridge.

Levy, H. (2014). Risk-Return Analysis: The Theory and Practice of Rational Investing. *Quantitative Finance*.

Luca, P. D. (2018). *Analytical Corporate Valuation ; Fundamental Analysis, Asset Pricing, and Company Valuation* .

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*.

Matatko, J. (1980). Estimation Risk and Optimal Portfolio Choice.by V. S. Bawa; S. J. Brown; R. Klein. *Journal of the Royal Statistical Society. Series A (General)*.

Michele Cagan, C. (2016). *Stock Market 101 From Bull and Bear Markets to Dividends, Shares, and Margins—Your Essential Guide to the Stock Market.*

Bernstein, W. J. (2001). *The Intelligent Asset Allocator How to Build Your Portfolio to Maximize Returns and Minimize Risk.*

Bertsekas, D. P. (1974). Necessary and sufficient conditions for existence of an optimal portfolio. 248.

Blay, H. M. (2013). *Risk-Return Analysis: The Theory and Practice of Rational Investing (Volume One).* McGraw-Hill.

Brief Overview of Python : Informatics Practices . (2022).

Edwin J. Elton, M. J. (2006). Improved Estimates of Correlation Coefficients and their Impact on Optimum Portfolios.

Fama, E. F. (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy*.

Levy, H. (2012). *The Capital Asset Pricing Model in the 21st Century: Analytical, Empirical, and Behavioral Perspectives* . Cambridge.

Levy, H. (2014). Risk-Return Analysis: The Theory and Practice of Rational Investing. *Quantitative Finance*.

Luca, P. D. (2018). *Analytical Corporate Valuation ; Fundamental Analysis, Asset Pricing, and Company Valuation* .

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*.

Matatko, J. (1980). Estimation Risk and Optimal Portfolio Choice.by V. S. Bawa; S. J. Brown; R. Klein. *Journal of the Royal Statistical Society. Series A (General)*.

Michele Cagan, C. (2016). *Stock Market 101 From Bull and Bear Markets to Dividends, Shares, and Margins—Your Essential Guide to the Stock Market.*

Miller, M. H. (1960). Portfolio Selection: Efficient Diversification of Investments.by Harry M. Markowitz (reviewed by Merton Miller). *The Journal of Business*.

Norton, R. W. (n.d.). *Introduction to Finance : Markets, Investments, and Financial Management* .

PARAMESWARAN, S. (2011). *Fundamentals of Financial Instruments. An Introduction to Stocks, Bonds, Foreign Exchange, and Derivatives* .

RICHARD A. FERRI, C. (2010). *All About ASSET ALLOCATION.*

RÖMAN, J. R. (2017). *Analytical Finance Volume I ; The Mathematics of Equity Derivatives, Markets, Risk and Valuation.*

Sanford J. Grossman, R. J. (1982). Consumption correlatedness and risk measurement in economies with non-traded assets and heterogeneous information.

Sharpe, W. F. (1971). Mean-Absolute-Deviation Characteristic Lines for Securities and Portfolios. *Management Science*.

Sharpe, W. F. (1994, January). The Sharpe Ratio. *The Journal of Portfolio Management*. doi:10.3905/jpm.1994.409501

Simon Benninga, T. M. (2018). *Principles of Finance with Excel.* Oxford Universiy Press.

Statman, M. (1987). How Many Stocks Make a Diversified Portfolio? *The Journal of Financial and Quantitative Analysis*.

Willial Kinlaw, M. P. (2017). *A Practitioner's Guide to Asset Allocation.* Canada: John Wiley & Sons, Inc., Hoboken, New Jersey.