Soft Ideal Theory Soft Local Function and Generated Soft Topological Spaces

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Abstract: The purpose of this paper is to introduce the notion of soft ideal in soft set theory. The concept of soft local function is also introduced. These concepts are discussed with a view to find new soft topologies from the original one. The basic structure, especially a basis for such generated soft topologies also studied here. Finally, the notion of compatibility of soft ideals with soft topologies is introduced and some equivalent conditions concerning this topic are established here.

1 Introduction

The concept of soft sets was first introduced by Molodtsov [19] in 1999 as a general mathematical tool for dealing with uncertain objects. In [19,18], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [16], the properties and applications of soft set theory have been studied increasingly [3,13,18,22]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1,2,6,14, 15,16,17,18,20,27]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [7].

Recently, in 2011, Shabir and Naz [23] initiated the study of soft topological spaces. They defined soft topology τ on the collection of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established many of their properties. Hussain and Ahmad [9] investigated the properties of open (closed) soft, soft nbd and soft closure. They also defined and discussed the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and will strengthen the foundations of the theory of soft topological spaces. Kandil et al. [11] introduced a unification of some types of different kinds of subsets of soft topological spaces using the notion of γ-operation. The purpose of this paper is to introduce the notion of soft ideal in soft set theory. The concept of soft local function is also introduced. These concepts are discussed with a view to find new soft topologies from the original one. The basic structure, especially a basis for such generated soft topologies also studied here. Finally, the notion of

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compatibility of soft ideals with soft topologies is introduced and some equivalent conditions concerning this topic are established here.

2 Preliminaries

Definition 2.1. [19] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) denoted by F_A is called a soft set over X, where F is a mapping given by $F : A \rightarrow P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F,A) and if $e \notin A$, then $F(e) = \phi$ i.e
$F_A = \{F(e) : e \in A \subseteq E, F : A \rightarrow P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition 2.2.[16] Let F_A, $G_B \in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if

(1)$A \subseteq B$, and
(2)$F(e) \subseteq G(e)$, $\forall e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition 2.3.[16] Two soft subset F_A and G_B over a common universe set X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 2.4.[3] The complement of a soft set (F,A), denoted by $(F,A)'$, is defined by $(F,A)' = (F',A)$, $F' : A \rightarrow P(X)$ is a mapping given by $F'(e) = X - F(e)$, $\forall e \in A$ and F' is called the soft complement function of F.
Clearly $(F')'$ is the same as F and $((F,A)')' = (F,A)$.

Definition 2.5.[23] The difference of two soft sets (F,E) and (G,E) over the common universe X, denoted by $(F,E) - (G,E)$ is the soft set (H,E) where for all $e \in E$, $H(e) = F(e) - G(e)$.

Definition 2.6.[23] Let (F,E) be a soft set over X and $x \in X$. We say that $x \in (F,E)$ read as x belongs to the soft set (F,E) whenever $x \in F(e)$ for all $e \in E$.

Definition 2.7.[16] A soft set (F,A) over X is said to be a NULL soft set denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A$, $F(e) = \phi$ (null set).

Definition 2.8.[16] A soft set (F,A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A$, $F(e) = X$. Clearly we have $X'_A = \phi_A$ and $\phi'_A = X_A$.

Definition 2.9.[16] The union of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C),

where $C = A \cup B$ and for all $e \in C$,
$$H(e) = \begin{cases} F(e), & e \in A - B, \\ G(e), & e \in B - A, \\ F(e) \cup G(e), & e \in A \cap B \, . \end{cases}$$

Definition 2.10.[16] The intersection of two soft sets (F,A) and (G,B) over the common universe X is the soft set (H,C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F,E) over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition 2.11.[28] Let I be an arbitrary indexed set and $L = \{(F_i, E), i \in I\}$ be a subfamily of $SS(X)_E$.

(1)The union of L is the soft set (H,E), where $H(e) = \bigcup_{i \in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcup}_{i \in I}(F_i, E) = (H,E)$.
(2)The intersection of L is the soft set (M,E), where $M(e) = \bigcap_{i \in I} F_i(e)$ for each $e \in E$. We write $\tilde{\bigcap}_{i \in I}(F_i, E) = (M,E)$.

Definition 2.12.[23] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if

(1)$\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$,
(2)the union of any number of soft sets in τ belongs to τ,
(3)the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.13.[9] Let (X, τ, E) be a soft topological space. A soft set (F,A) over X is said to be closed soft set in X, if its relative complement $(F,A)'$ is an open soft set.

Definition 2.14.[9] Let(X, τ, E) be a soft topological space. The members of τ are said to be open soft sets in X. We denote the set of all open soft sets over X by $OS(X, \tau, E)$, or $OS(X)$ and the set of all closed soft sets by $CS(X, \tau, E)$, or $CS(X)$.

Definition 2.15.[23] Let (X, τ, E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft closure of (F,E), denoted by $cl(F,E)$ is the intersection of all closed soft super sets of (F,E) i.e
$cl(F,E) = \tilde{\bigcap}\{(H,E) : (H,E) \text{ is closed soft set and } (F,E) \tilde{\subseteq} (H,E)\})$.

Definition 2.16.[28] Let (X, τ, E) be a soft topological space and $(F,E) \in SS(X)_E$. The soft interior of (G,E), denoted by $int(G,E)$ is the union of all open soft subsets of (G,E) i.e $int(G,E) = \tilde{\bigcup}\{(H,E) : (H,E) \text{ is an open soft set and } (H,E) \tilde{\subseteq} (G,E)\})$.

Definition 2.17.[28] The soft set $(F,E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$, and the soft point (F,E) is denoted by x_e.

Proposition 2.1.[24] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it.

Definition 2.18.[28] The soft point x_e is said to be belonging to the soft set (G,A), denoted by $x_e \tilde{\in} (G,A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

Definition 2.19.[28] A soft set (G,E) in a soft topological space (X,τ,E) is called a soft neighborhood (briefly: nbd) of the soft point $x_e \tilde{\in} X_E$ if there exists an open soft set (H,E) such that $x_e \tilde{\in} (H,E) \tilde{\subseteq} (G,E)$.

A soft set (G,E) in a soft topological space (X,τ,E) is called a soft neighborhood of the soft (F,E) if there exists an open soft set (H,E) such that $(F,E) \tilde{\in} (H,E) \tilde{\subseteq} (G,E)$. The neighborhood system of a soft point x_e, denoted by $N_\tau(x_e)$, is the family of all its neighborhoods.

Theorem 2.1.[25] Let (X,τ,E) be a soft topological space. A soft point $e_x \tilde{\in} cl(F,E)$ if and only if each soft neighborhood of e_x intersects (F,E).

Definition 2.20.[21] Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X)_E$. Define $\tau_{(F,E)} = \{(G,E) \tilde{\cap} (F,E) : (G,E) \in \tau\}$, which is a soft topology on (F,E). This soft topology is called soft relative topology of τ on (F,E), and $[(F,E), \tau_{(F,E)}]$ is called soft subspace of (X,τ,E).

Definition 2.21.[28] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u : X \to Y$ and $p : A \to B$ be mappings. Then the mapping $f_{pu} : SS(X)_A \to SS(Y)_B$ is defined as:

(1)If $(F,A) \in SS(X)_A$. Then the image of (F,A) under f_{pu}, written as $f_{pu}(F,A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that
$$f_{pu}(F)(b) = \begin{cases} \cup_{x \in p^{-1}(b) \cap A} u(F(a)), & p^{-1}(b) \cap A \neq \phi, \\ \phi, & otherwise. \end{cases}$$
for all $b \in B$.

(2)If $(G,B) \in SS(Y)_B$. Then the inverse image of (G,B) under f_{pu}, written as $f_{pu}^{-1}(G,B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that
$$f_{pu}^{-1}(G)(a) = \begin{cases} u^{-1}(G(p(a))), & p(a) \in B, \\ \phi, & otherwise. \end{cases}$$
for all $a \in A$.

Definition 2.22.[28] Let (X,τ,A) and (Y,τ^*,B) be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then

(1)The function f_{pu} is called continuous soft (cts-soft) if $f_{pu}^{-1}(G,B) \in \tau \, \forall \, (G,B) \in \tau^*$.
(2)The function f_{pu} is called open soft if $f_{pu}(G,A) \in \tau^* \forall \, (G,A) \in \tau$.

Definition 2.23.[5] Let (X,τ,E) be a soft topological space and $x,y \in X$ such that $x \neq y$. Then (X,τ,E) is called soft Hausdorff space or soft T_2 space if there exist open soft sets (F,E) and (G,E) such that $x \in (F,E)$, $y \in (G,E)$ and $(F,E) \tilde{\cap} (G,E) = \tilde{\phi}$

Definition 2.24.[10]. A non-empty collection I of subsets of a set X is called an ideal on X, if it satisfies the following conditions

(1)$A \in I$ and $B \in I \Rightarrow A \cup B \in I$,
(2)$A \in I$ and $B \subseteq A \Rightarrow B \in I$,
    i.e. I is closed under finite unions and subsets.

3 Soft ideal, soft local function and generated soft topology

In this section We generate a soft topological space finer than the given soft topological space (X,τ,E) on the same set X with a fixed set of parameters E by using the soft ideal notion. We denote $(X,\tau,E,\tilde{I})$ as a soft topological space with soft ideal $\tilde{I}$ on X.

Definition 3.1. Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with the same set of parameters E. Then $\tilde{I} \subseteq SS(X)_E$ is called a soft ideal on X with the same set E if

(1)$(F,E) \in \tilde{I}$ and $(G,E) \in \tilde{I} \Rightarrow (F,E) \tilde{\cup} (G,E) \in \tilde{I}$,
(2)$(F,E) \in \tilde{I}$ and $(G,E) \tilde{\subseteq} (F,E) \Rightarrow (G,E) \in \tilde{I}$,
    i.e. $\tilde{I}$ is closed under finite soft unions and soft subsets.

Example 3.1. Let X be a universe set. Then each of the following families is a soft ideal over X with the same set of parameters E,

(1)$\tilde{I} = \{\tilde{\phi}\}$,
(2)$\tilde{I} = SS(X)_E = \{(F,E) : (F,E) \, is \, a \, soft \, set \, over \, X \, with \, the \, fixed \, set \, of \, parameters \, E\}$,
(3)$\tilde{I}_f = \{(F,E) \in SS(X)_E : (F,E) \, is \, a \, finite \, soft \, set\}$, called soft ideal of finite soft subsets of $\tilde{X}$,
(4)$\tilde{I}_c = \{(F,E) \in SS(X)_E : (F,E) \, is \, a \, countable \, soft \, set\}$, called soft ideal of countable soft subsets of $\tilde{X}$,
(5)$\tilde{I}_{(F,E)} = \{(G,E) \in SS(X)_E : (G,E) \tilde{\subseteq} (F,E)\}$.
(6)$\tilde{I}_n = \{(G,E) \in SS(X)_E : int(cl(G,E)) = \tilde{\phi}\}$, called soft ideal of nowhere dense soft sets in (X,τ,E).

Theorem 3.1. Let $\tilde{I}$ be a soft ideal over a universe X. Then The collection $\tilde{I}_e = \{F(e) : (F,E) \in \tilde{I}\}$ defines an ideal on X for each $e \in E$.

Proof. Let $F(e), G(e) \in \tilde{I}_e$. Then $\exists\, (F,E), (G,E) \in \tilde{I}$. Since $(F,E)\tilde{\cup}(G,E) \in \tilde{I}$ from Definition 3. Then $F(e) \cup G(e) \in \tilde{I}_e$. Now, let $F(e) \in \tilde{I}_e$ and $G(e) \subseteq X$ such that $G(e) \subseteq F(e)$. Then $\exists\, (F,E), (G,E) \in \tilde{I}$ such that $(G,E)\tilde{\subseteq}(F,E)$. It follows that $(G,E) \in \tilde{I}$. Thus $G(e) \in \tilde{I}_e$. This means that $\tilde{I}_e = \{F(e) : (F,E) \in \tilde{I}\}$ defines an ideal on X for each $e \in E$.

Corollary 3.1. A soft ideal over a universe X with the same set of parameters E is a parameterized collection of crisp ideals on X.

Proof. Obvious from Theorem 3.

Remark 3.1. The converse of theorem 3 is not true in general, as shown in the following example.

Example 3.2. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tilde{I} = \{\tilde{\phi}, (F_1,E), (F_2,E), (F_3,E), (F_4,E)\}$ where $(F_1,E), (F_2,E), (F_3,E), (F_4,E)$ are soft sets over X defined as follows:
$F_1(e_1) = \{h_2\}$, $F_1(e_2) = \{h_1\}$,
$F_2(e_1) = \{h_1, h_2\}$, $F_2(e_2) = \{h_3\}$,
$F_3(e_1) = \{h_2\}$, $F_3(e_2) = \{h_1, h_3\}$,
$F_4(e_1) = \{h_1\}$, $F_4(e_2) = \{h_3\}$. Then $\tilde{I}_{e_1} = \{\phi, \{h_1\}, \{h_2\}, \{h_1, h_2\}\}$ and $\tilde{I}_{e_2} = \{\phi, \{h_1\}, \{h_3\}, \{h_1, h_3\}\}$ are ideals on X. However, $\tilde{I}$ is not soft ideal on X because $(F_1,E)\tilde{\cup}(F_2,E) = (G,E)$, where $G(e_1) = \{h_1, h_2\}$, $G(e_2) = \{h_1, h_3\}$ and $(G,E) \notin \tilde{I}$.

Definition 3.2. Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. Then

$$(F,E)^*(\tilde{I}, \tau)\ (or F_E^*) = \tilde{\cup}\{x_e \in \varepsilon : O_{x_e}\tilde{\cap}(F,E) \tilde{\notin} \tilde{I} \,\forall\, O_{x_e} \in \tau\}$$

is called the soft local function of (F,E) with respect to $\tilde{I}$ and τ, where O_{x_e} is a τ-open soft set containing x_e.

Theorem 3.2. Let $\tilde{I}$ and $\tilde{J}$ be any two soft ideals with the same set of parameters E on a soft topological space (X, τ, E). Let $(F,E), (G,E) \in SS(X)_E$. Then

(1) $(\tilde{\phi})^* = \tilde{\phi}$,
(2) $(F,E)\tilde{\subseteq}(G,E) \Rightarrow (F,E)^* \subseteq (G,E)^*$,
(3) $\tilde{I} \subseteq \tilde{J} \Rightarrow (F,E)^*(\tilde{J}) \subseteq (F,E)^*(\tilde{I})$,
(4) $(F,E)^* \tilde{\subseteq} cl(F,E)$, where cl is the soft closure w.r.t. τ,
(5) $(F,E)^*$ is τ-closed soft set.
(6) $((F,E)^*)^* \tilde{\subseteq} (F,E)^*$,
(7) $((F,E)\tilde{\cup}(G,E))^* = (F,E)^* \cup (G,E)^*$,
(8) $\tilde{\cup}_j (F,E)^* = (\tilde{\cup}_j (F,E))^*$,
(9) $((F,E)\tilde{\cap}(G,E))^* \subseteq (F,E)^* \cap (G,E)^*$,
(10) $(F,E)^* - (G,E)^* = ((F,E) - (G,E))^* - (G,E)^* \tilde{\subseteq} ((F,E) - (G,E))^*$,
(11) $(G,E) \in \tau \Rightarrow (G,E)\tilde{\cap}(F,E)^* = (G,E)\tilde{\cap}((G,E) \cap (F,E))^* \tilde{\subseteq} ((G,E)\tilde{\cap}(F,E))^*$,

(12) $(H,E) \in \tilde{I} \Rightarrow ((F,E)\tilde{\cup}(H,E))^* = (F,E)^* = ((F,E) - (H,E))^*$.

Proof.

(1) Obvious from Definition 3.
(2) Let $x_e\tilde{\in}(F,E)^*$. Then $O_{x_e}\tilde{\cap}(F,E) \notin \tilde{I} \,\forall\, O_{x_e} \in \tau$. Since $O_{x_e}\tilde{\cap}(F,E)\tilde{\subseteq}O_{x_e}\tilde{\cap}(G,E)$ and $O_{x_e}\tilde{\cap}(F,E) \notin \tilde{I}$. Then $O_{x_e}\tilde{\cap}(G,E) \notin \tilde{I} \,\forall\, O_{x_e} \in \tau$ from Definition 3.1. Hence $x_e\tilde{\in}(G,E)^*$. Thus $(F,E)^*\tilde{\subseteq}(G,E)^*$.
(3) Let $x_e\tilde{\in}(F,E)^*(\tilde{J})$. Then $O_{x_e}\tilde{\cap}(F,E) \notin \tilde{J} \,\forall\, O_{x_e} \in \tau$. Since $\tilde{I} \subseteq \tilde{J}$. Then $O_{x_e}\tilde{\cap}(F,E) \notin \tilde{I} \,\forall\, O_{x_e} \in \tau$. Hence $x_e\tilde{\in}(F,E)^*(\tilde{I})$. Thus $(F,E)^*(\tilde{J})\tilde{\subseteq}(F,E)^*(\tilde{I})$.
(4) Assume that $x_e\tilde{\notin}cl(F,E)$. Then $\exists\, O_{x_e} \in \tau$ such that $O_{x_e}\tilde{\cap}(F,E) = \tilde{\phi} \in \tilde{I}$. Hence $x_e\tilde{\notin}(F,E)^*$. Thus $(F,E)^*\tilde{\subseteq}cl(F,E)$.
(5) Clearly $(F,E)^*\tilde{\subseteq}cl(F,E)^*$. So let $x_e\tilde{\in}cl((F,E)^*)$. Then $O_{x_e}\tilde{\cap}(F,E)^*) \neq \tilde{\phi} \,\forall\, O_{x_e} \in \tau$ from Theorem 2.1. Hence $\exists\, y_{e'}\tilde{\in}O_{x_e}\tilde{\cap}(F,E)^*$. Thus $y_{e'} \in O_{x_e}$ and $y_{e'} \in (E,E)^*$. It follows that $O_{y_{e'}}\tilde{\cap}(F,E) \notin \tilde{I} \,\forall\, O_{y_{e'}} \in \tau$. This implies that $O_{x_e}\tilde{\cap}(F,E) \notin \tilde{I} \,\forall\, O_{x_e} \in \tau$. So $x_e\tilde{\in}(F,E)^*$. This means that $(F,E)^* = cl(F,E)^*$ and consequently $cl(F,E)^* = (F,E)^*$.
(6) Since $((F,E)^*)^*\tilde{\subseteq}cl(F,E)^*$ from (4). Then $((F,E)^*)^*\tilde{\subseteq}cl(F,E)^* = (F,E)^*$ from (5).
(7) Let $x_e\tilde{\in}((F,E)\tilde{\cup}(G,E))^*$. Then $O_{x_e}\tilde{\cap}((F,E)\tilde{\cup}(G,E)) = (O_{x_e}\tilde{\cap}(F,E))\tilde{\cup}(O_{x_e}\tilde{\cap}(G,E)) \notin \tilde{I} \,\forall\, O_{x_e} \in \tau$. Hence either $O_{x_e}\tilde{\cap}(F,E) \notin \tilde{I}$ or $O_{x_e}\tilde{\cap}(G,E) \notin \tilde{I} \,\forall\, O_{x_e} \in \tau$ from Definition 3.1. This means that either $x_e\tilde{\in}(F,E)^*$ or $x_e\tilde{\in}(G,E)^*$. Thus $x_e\tilde{\in}(F,E)^*\tilde{\cup}(G,E)^*$. It follows that $((F,E)\tilde{\cup}(G,E))^*\tilde{\subseteq}(F,E)^* \cup (G,E)^*$. For the reverse inclusion, since $(F,E), (G,E)\tilde{\subseteq}((F,E)\tilde{\cup}(G,E))$. Then $(F,E)^*\tilde{\subseteq}((F,E)\tilde{\cup}(G,E))^*$ and $(G,E)^*\tilde{\subseteq}((F,E)\tilde{\cup}(G,E))^*$ from (2). Hence $(F,E)^* \cup (G,E)^*\tilde{\subseteq}((F,E)\tilde{\cup}(G,E))^*$ and it is implies that $((F,E)\tilde{\cup}(G,E))^* = (F,E)^* \cup (G,E)^*$.
(8) Obvious from (7).
(9) Since $((F,E)\tilde{\cap}(G,E))\tilde{\subseteq}(F,E), (G,E)$. Then $((F,E)\tilde{\cap}(G,E))^*\tilde{\subseteq}(F,E)^*$ and $((F,E)\tilde{\cap}(G,E))\tilde{\subseteq}(G,E)^*$ from (2). Hence $((F,E)\tilde{\cap}(G,E))^*\tilde{\subseteq}(F,E)^*\tilde{\cap}(G,E)^*$.
(10) We first prove that $(F,E)^* - (G,E)^* = ((F,E) - (G,E))^* - (G,E)^*$. Since $(F,E) - (G,E)\tilde{\subseteq}(F,E)$. Then $((F,E) - (G,E))^*\tilde{\subseteq}(F,E)^*$. Hence $((F,E) - (G,E))^* - (G,E)^*\tilde{\subseteq}(F,E)^* - (G,E)^*$. For the reverse inclusion, since $(F,E) = [(F,E) - (G,E)]\tilde{\cup}[(F,E)\tilde{\cap}(G,E)]$. Then $(F,E)^* = [[(F,E) - (G,E)]\tilde{\cup}[(F,E)\tilde{\cap}(G,E)]]^*$. It follows that

$(F,E)^* = [[(F,E) - (G,E)]\tilde{\cup}[(F,E)\tilde{\cap}(G,E)]]^* = [(F,E) - (G,E)]^*\tilde{\cup}[(F,E)\tilde{\cap}(G,E)]^*\tilde{\subseteq}[(F,E) - (G,E)]^*\tilde{\cup}(G,E)^*$ from (7). Hence $(F,E)^* - (G,E)^*\tilde{\subseteq}[[(F,E) - (G,E)]^*\tilde{\cup}(G,E)^*] - (G,E)^* = [[(F,E) - (G,E)]^*\tilde{\cup}(G,E)^*]\tilde{\cap}(G,E)^{*'} = [(F,E) - (G,E)]^* - (G,E)^*$. Thus $(F,E)^* - (G,E)^* = ((F,E) - (G,E))^* - (G,E)^*$. Also, if $x_e\tilde{\in}((F,E) - (G,E))^* - (G,E)^*$. Then $x_e\tilde{\in}((F,E) - (G,E))^*$ and consequently $(F,E)^* - (G,E)^* = ((F,E) - (G,E))^* - (G,E)^*\tilde{\subseteq}((F,E) - (G,E))^*$.

(11) Let $x_e\tilde{\in}(G,E)\tilde{\cap}(F,E)^*$. Then $x_e\tilde{\in}(G,E)$ and $x_e\tilde{\in}(F,E)^*$. Hence $(O_{x_e}\tilde{\cap}(F,E)) \notin \tilde{I} \,\forall\, O_{x_e} \in \tau$. Since (G,E) is τ- open soft set containing x_e. Then $(F,E)\tilde{\cap}(G,E)) \notin \tilde{I}$. Hence $(O_{x_e}\tilde{\cap}(G,E))\tilde{\cap}(F,E) \notin \tilde{I}$, as $O_{x_e}\tilde{\cap}(G,E) \in \tau$. It follows that $O_{x_e}\tilde{\cap}((G,E)\tilde{\cap}(F,E)) \notin \tilde{I} \,\forall\, O_{x_e} \in \tau$. Thus $x_e\tilde{\in}((G,E)\tilde{\cap}(F,E))^*\tilde{\cap}(F,E)^*$ This implies that $(G,E)\tilde{\cap}(F,E)^*\tilde{\subseteq}(G,E)\tilde{\cap}((G,E) \cap (F,E))^*$. For the reverse inclusion, since $((F,E)\tilde{\cap}(G,E))\tilde{\subseteq}(F,E)$. Then $((F,E)\tilde{\cap}(G,E))^*\tilde{\subseteq}(F,E)^*$ from (2). Hence $(G,E)\tilde{\cap}((F,E)\tilde{\cap}(G,E))^*\tilde{\subseteq}G,E)\tilde{\cap}(F,E)^*$. Thus $(G,E)\tilde{\cap}(F,E)^* = (G,E)\tilde{\cap}((G,E) \cap (F,E))^*$. Also, if $x_e\tilde{\in}((G,E)\tilde{\cap}(F,E))^*\tilde{\cap}(F,E)$. Then $x_e\tilde{\in}((G,E)\tilde{\cap}(F,E))^*$.

(12) We first prove that $((F,E)\tilde{\cup}(H,E))^* = (F,E)^*$. So let $x_e\tilde{\in}((H,E)\tilde{\cup}(F,E))^*$. Then $O_{x_e}\tilde{\cap}((H,E)\tilde{\cup}(F,E)) \notin \tilde{I}\forall\, O_{x_e} \in \tau$. Hence $(O_{x_e}\tilde{\cap}(H,E))\tilde{\cup}(O_{x_e}\tilde{\cap}(F,E)) \notin \tilde{I}$. It follows that either $O_{x_e}\tilde{\cap}(H,E) \notin \tilde{I}$ or $O_{x_e}\tilde{\cap}(F,E) \notin \tilde{I}$. But $O_{x_e}\tilde{\cap}(H,E) \notin \tilde{I}$ gives $(H,E) \in \tilde{I}$ which is a contradiction. Thus $O_{x_e}\tilde{\cap}(F,E) \notin \tilde{I} \,\forall\, O_{x_e} \in \tau$. So $x_e\tilde{\in}(F,E)^*$. For the reverse inclusion, since $(F,E)\tilde{\subseteq}((H,E)\tilde{\cup}(F,E))$. Then $(F,E)^*\tilde{\subseteq}((H,E)\tilde{\cup}(F,E))^*$ from (2).
Now we prove that $(F,E)^* = ((F,E) - (H,E))^*$. Since $((F,E) - (H,E))\tilde{\subseteq}(F,E)$. Then $((F,E) - (H,E))^*\tilde{\subseteq}(F,E)^*$ from (2). For the reverse inclusion, let $x_e\tilde{\notin}((F,E) - (H,E))^*$. Then $\exists O_{x_e} \in \tau$ such that $O_{x_e}\tilde{\cap}((F,E) - (H,E)) \in \tilde{I}$. Since $(H,E) \in \tilde{I}$. Then $(H,E)\tilde{\cup}(O_{x_e}\tilde{\cap}((F,E) - (H,E))) \in \tilde{I}$. Hence $(H,E)\tilde{\cup}(O_{x_e}\tilde{\cap}(F,E)) \in \tilde{I}$. Thus $O_{x_e}\tilde{\cap}(F,E)) \in \tilde{I}$ for some $O_{x_e} \in \tau$. It follows that $x_e\tilde{\notin}(F,E)^*$. This means that $(F,E)^*\tilde{\subseteq}((F,E) - (H,E))^*$. This completes the proof.

Theorem 3.3. Let (X,τ,E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. Then the operator $cl^* : SS(X)_E \to SS(X)_E$ defined by:

$$cl^*(F,E) = (F,E)\tilde{\cup}(F,E)^*. \tag{1}$$

is a soft closure operator.

Proof. $cl^*(\tilde{\phi}) = \tilde{\phi}\tilde{\cup}(\tilde{\phi})^* = \tilde{\phi}\tilde{\cup}\tilde{\phi} = \tilde{\phi}$ from Theorem 3.2 (1), and obviously $(F,E)\tilde{\subseteq}cl^*(F,E)$ $\forall(F,E) \in SS(X)_E$. Now $cl^*((F,E)\tilde{\cup}(G,E)) = ((F,E)\tilde{\cup}(G,E))\tilde{\cup}((F,E)\tilde{\cup}(G,E))^* = ((F,E)\tilde{\cup}(G,E))\tilde{\cup}((F,E)^*\tilde{\cup}(G,E)^*) = ((F,E)\tilde{\cup}(F,E)^*)\tilde{\cup}((G,E)\tilde{\cup}(G,E)^*) = cl^*(F,E)\tilde{\cup}cl^*(G,E)$ from Theorem 3.2 (7). Also, for any $(F,E) \in SS(X)_E$, $cl^*(cl^*(F,E)) = cl^*((F,E)\tilde{\cup}(F,E)^*) = ((F,E)\tilde{\cup}(F,E)^*)\tilde{\cup}((F,E)\tilde{\cup}(F,E)^*)^* = ((F,E)\tilde{\cup}(F,E)^*)\tilde{\cup}((F,E)^* \cup ((F,E)^*)^*)\tilde{\subseteq}((F,E)\tilde{\cup}(F,E)^*))\tilde{\cup} ((F,E)^* \cup ((F,E)^*)) = cl^*(F,E)$ from Theorem 3.2 (6).

Definition 3.3. Let (X,τ,E) be a soft topological space, $\tilde{I}$ be a soft ideal over X with the same set of parameters E and $cl^* : SS(X)_E \to SS(X)_E$ be the soft closure operator. Then there exists a unique soft topology over X with the same set of parameters E, finer than τ, called the $*$-soft topology, denoted by $\tau^*(\tilde{I})$ or τ^*, given by

$$\tau^*(\tilde{I}) = \{(F,E) \in SS(X)_E : cl^*(F,E)' = (F,E)'\}. \tag{2}$$

Example 3.2.

(1) If $\tilde{I} = \{\tilde{\phi}\}$, then $(F,E)^*(\tilde{I},\tau) = cl(F,E) \,\forall(F,E) \in SS(X)_E$. Hence $Cl^*(F,E) = cl(F,E)$ and $\tau^* = \tau$.

(2) If $\tilde{I} = SS(X)_E$, then $(F,E)^*(\tilde{I},\tau) = \tilde{\phi} \,\forall(F,E) \in SS(X)_E$. Hence $Cl^*(F,E) = (F,E)$ and $\tau^* = SS(X)_E$ (the soft discrete topology).

(3) If $\tilde{I}\tilde{\subseteq}\tilde{J}$, then $(F,E)^*(\tilde{J},\tau)\tilde{\subseteq}(F,E)^*(\tilde{I},\tau)$. Hence the $\star$-soft topological space $(X,\tau^*(\tilde{J}),E)$ is finer than the $\star$-soft topological space $(X,\tau^*(\tilde{I}),E)$.

(4) Let $X = \{h_1,h_2,h_3\}$, $E = \{e\}$ and $\tau = \{\tilde{X},\tilde{\phi},(F,E)\}$ where (F,E) is a soft set over X defined by $F(e) = \{h_2\}$. Then τ defines a soft topology on X. Let $\tilde{I} = \{\tilde{\phi},(G,E)\}$ be a soft ideal over X where (G,E) is a soft set over X defined by $G(e) = \{h_2\}$. Then $\tau^* = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E)\}$ where $(F_1,E),(F_2,E)$ are soft sets over X where $F_1(e) = \{h_2\}$ and $F_2(e) = \{h_1,h_3\}$.

(5) Let $X = \{h_1,h_2,h_3\}$, $E = \{e\}$ and $\tau = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E)\}$ where $(F_1,E),(F_2,E)$ are soft sets over X defined defined as follows: $F_1(e) = \{h_2\}$, $F_2(e) = \{h_1,h_2\}$. Then τ defines a soft topology on X. Let $\tilde{I} = \{\tilde{\phi},(G_1,E),(G_2,E),(G_3,E)\}$ be a soft ideal over X where $(G_1,E),(G_2,E),(G_3,E)$

are soft sets over X defined by $G_1(e) = \{h_1\}$, $G_2(e) = \{h_2\}$ and $G_3(e) = \{h_1, h_2\}$. Then $\tau^* = SS(X)_E$.

(6) Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$ where $(F_1, E), (F_2, E)$ are soft sets over X defined defined as follows:

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_2\}$,

$F_2(e_1) = X$, $F_2(e_2) = \{h_2\}$. Then τ defines a soft topology on X. Let $\tilde{I} = \{\tilde{\phi}, (G_1, E), (G_2, E), (G_3, E)\}$ be a soft ideal over X where $(G_1, E), (G_2, E), (G_3, E)$ are soft sets over X defined by $G_1(e_1) = \phi$, $G_1(e_2) = \{h_1\}$,

$G_2(e_1) = \{h_1\}$, $G_2(e_2) = \phi$,

and $G_3(e_1) = \{h_1\}$, $G_3(e_2) = \{h_1\}$. Then $\tau^* = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E), (F_{11}, E), (F_{12}, E)\}$ is a soft topology finer than τ, where $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E), (F_{11}, E)$ are soft sets over X defined defined as follows:

$F_1(e_1) = \{h_2\}$, $F_1(e_2) = X$,

$F_2(e_1) = \{h_2\}$, $F_2(e_2) = \{h_2\}$,

$F_3(e_1) = \{h_1\}$, $F_3(e_2) = X$,

$F_4(e_1) = \{h_1\}$, $F_4(e_2) = \{h_2\}$,

$F_5(e_1) = X$, $F_5(e_2) = \{h_2\}$,

$F_6(e_1) = X$, $F_6(e_2) = \phi$,

$F_7(e_1) = \phi$, $F_7(e_2) = X$,

$F_8(e_1) = \{h_1\}$, $F_8(e_2) = \phi$,

$F_9(e_1) = \phi$, $F_9(e_2) = \{h_2\}$,

$F_{10}(e_1) = \phi$, $F_{10}(e_2) = \phi$,

$F_{11}(e_1) = X$, $F_{11}(e_2) = X$.

Theorem 3.4. Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. Then

$$\beta(\tilde{I}, \tau) = \{(F, E) - (G, E) : (F, E) \in \tau, (G, E) \in \tilde{I}\}$$

is a soft basis for the soft topology $\tau^*(\tilde{I})$.

Proof. Since $\tilde{X} \in \tau, \tilde{\phi} \in \tilde{I}$. Then $\tilde{X} - \tilde{\phi} \in \beta$. Hence $\tilde{X} \in \beta$ and $\tilde{\bigcup}_{j \in J}((F_j, E) - (G_j, E)) = \tilde{X}$. Also, let $((F_1, E) - (G_1, E)), ((F_2, E) - (G_2, E)) \in \beta$ such that $x_e \tilde{\in} ((F_1, E) - (G_1, E)) \tilde{\cap} ((F_2, E) - (G_2, E))$. Then $x_e \tilde{\in} ((F_1, E) - (G_1, E)) \tilde{\cap} ((F_2, E) - (G_2, E)) = ((F_1, E) \tilde{\cap} (F_2, E)) - ((G_1, E) \tilde{\cup} (G_2, E)) \in \beta(\tilde{I}, \tau)$. Thus β is a soft basis of τ^*.

Corollary 3.2. Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. Then $\tau \subseteq \beta(\tilde{I}, \tau) \subseteq \tau^*(\tilde{I})$.

Proof. Immediate from Theorem 3.2 (3) and Theorem 3.4. **Theorem 3.5.** Let (X, τ, E) be a soft topological

space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. If $(F, E)^d, (F, E)^{d^*}$ are the derived soft sets of (F, E) in (X, τ, E) and $(X, \tau^*(\tilde{I}), E)$ respectively, then $(F, E)^{d^*} \tilde{\subseteq} (F, E)^d$.

Proof.

Let $x_e \tilde{\in} (F, E)^{d^*}$ if and only if $x_e \tilde{\in} cl^*((F, E) - x_e)$ if and only if $x_e \tilde{\in} [((F, E) - x_e)] \tilde{\cup} [((F, E) - x_e)]^*$ if and only if $(O_{x_e} - x_e) \tilde{\cap} (F, E) \notin \tilde{I} \forall O_{x_e} \in \tau$. Then $x_e \tilde{\in} (F, E)^d$.

4 Compatibility of soft ideals with soft topology

Definition 4.1. Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. We say that the soft topology τ is compatible with the soft ideal $\tilde{I}$, denoted by $\tau \sim \tilde{I}$, if the following holds for every $(F, E) \in SS(X)_E$:
if for every soft point x_e, $x_e \in (F, E)$ there exists O_{x_e} such that $O_{x_e} \tilde{\cap} (F, E) \in \tilde{I}$, then $(F, E) \in \tilde{I}$.

Theorem 4.1. Let (X, τ, E) be a soft topological space, $\tilde{I}$ be a soft ideal over X with the same set of parameters E and $\tau \sim \tilde{I}$. Then the following are equivalent:

(1) For every $(F, E) \in SS(X)_E$, $(F, E) \tilde{\cap} (F, E)^* = \tilde{\phi}$, then $(F, E) = \tilde{\phi}$.

(2) For every $(F, E) \in SS(X)_E$, $((F, E) - (F, E)^*)^* = \tilde{\phi}$.

(3) For every $(F, E) \in SS(X)_E$, $((F, E) \tilde{\cap} (F, E)^*)^* = (F, E)^*$.

Proof.

(1) $\implies$ (2) Let $(F, E) \in SS(X)_E$. Since $((F, E) - (F, E)^*) \tilde{\cap} ((F, E) - (F, E)^*)^* = \tilde{\phi}$. Then $((F, E) - (F, E)^*)^* = \tilde{\phi}$ by (1).

(2) $\implies$ (3) Let $(F, E) \in SS(X)_E$. Since $(F, E) = ((F, E) - ((F, E) \tilde{\cap} (F, E)^*)) \tilde{\cup} ((F, E) \tilde{\cap} (F, E)^*)$. Then $(F, E)^* = [(F, E) - ((F, E) \tilde{\cap} (F, E)^*) \tilde{\cup} ((F, E) \tilde{\cap} (F, E)^*)]^* = [(F, E) - ((F, E) \tilde{\cap} (F, E)^*)]^* \tilde{\cup} [(F, E) \tilde{\cap} (F, E)^*]^* = \tilde{\phi} \tilde{\cup} [(F, E) \tilde{\cap} (F, E)^*]^* = [(F, E) \tilde{\cap} (F, E)^*]^*$ by (2).

(3) $\implies$ (1) Let $(F, E) \in SS(X)_E$ and $(F, E) \tilde{\cap} (F, E)^* = \tilde{\phi}$. Then $(F, E)^* = [(F, E) \tilde{\cap} (F, E)^*]^* = (\tilde{\phi})^* = \tilde{\phi}$. Hence $(F, E) \in \tilde{I}$.

Corollary 4.1. Let (X, τ, E) be a soft topological space, $\tilde{I}$ be a soft ideal over X with the same set of parameters E, $(F, E) \in SS(X)_E$ and $\tau \sim \tilde{I}$. Then $((F, E)^*)^* = (F, E)^*$.

Proof. Let $(F, E) \in SS(X)_E$. Since $(F, E)^* = ((F, E) \tilde{\cap} (F, E)^*)^* \tilde{\subseteq} (F, E)^*)^*$ from Theorem 4.1

(3). But we have $((F,E)^*)^* \widetilde{\subseteq} (F,E)^*$ from Theorem 3.2 (6). Thus $((F,E)^*)^* = (F,E)^*$.

Theorem 4.2. Let (X, τ, E) be a soft topological space and $\tilde{I}$ be a soft ideal over X with the same set of parameters E. Then the following are equivalent:

(1) $\tau \sim \tilde{I}$.
(2) For every $(F,E) \in SS(X)_E$ such that $(F,E) \widetilde{\cap} (F,E)^* = \tilde{\phi}$, $(F,E) \in \tilde{I}$.
(3) For every $(F,E) \in SS(X)_E$, $(F,E) - (F,E)^* \in \tilde{I}$.
(4) For every τ^*-closed soft subset (F,E), $(F,E) - (F,E)^* \in \tilde{I}$.
(5) For every $(F,E) \in SS(X)_E$, if (F,E) contains no non-null soft set (G,E) with $(G,E) \widetilde{\subseteq} (G,E)^*$, then $(F,E) \in \tilde{I}$.

Proof.

(1) $\Longrightarrow$ (2) Let $(F,E) \in SS(X)_E$ such that $(F,E) \widetilde{\cap} (F,E)^* = \tilde{\phi}$. Then $\forall \ x_e \widetilde{\in} (F,E)$ and $x_e \widetilde{\notin} (F,E)^*$, we have $O_{x_e} \widetilde{\cap} (F,E) \in \tilde{I}$ for some $O_{x_e} \in \tau$. Thus $(F,E) \in \tilde{I}$ by (1).

(2) $\Longrightarrow$ (3) Let $(F,E) \in SS(X)_E$. Since $((F,E) - (F,E)^*) \widetilde{\cap} ((F,E) - (F,E)^*)^* = ((F,E) \widetilde{\cap} (F,E)^{*'}) \widetilde{\cap} ((F,E) - (F,E)^*)^* \widetilde{\subseteq} ((F,E) \widetilde{\cap} (F,E)^{*'}) \widetilde{\cap} (F,E)^* = \tilde{\phi}$. Then $(F,E) - (F,E)^* \in \tilde{I}$ by (2).

(3) $\Longrightarrow$ (4) Let (F,E) be a τ^*-closed soft subset. Then $(F,E) \in SS(X)_E$. Hence $(F,E) - (F,E)^* \in \tilde{I}$ by (3).

(4) $\Longrightarrow$ (1) Let $(F,E) \in SS(X)_E$ and assume that for every $x_e \widetilde{\in} (F,E)$ there exists O_{x_e} such that $O_{x_e} \widetilde{\cap} (F,E) \in \tilde{I}$. Then $x_e \widetilde{\notin} (F,E)^*$. Hence $((F,E) \widetilde{\cap} (F,E)^*) = \tilde{\phi}$ and since $(F,E) \widetilde{\cup} (F,E)^*$ is τ^*-closed soft set, we have $((F,E) \widetilde{\cup} (F,E)^*) - ((F,E) \widetilde{\cup} (F,E)^*)^* \in \tilde{I}$ by (4). Hence $((F,E) \widetilde{\cup} (F,E)^*) - ((F,E)^* \widetilde{\cup} ((F,E)^*)^*) = ((F,E) \widetilde{\cup} (F,E)^*) - ((F,E)^*) = (F,E) \in \tilde{I}$ by Theorem 3.2 (6,7). Thus $\tau \sim \tilde{I}$.

(3) $\Longrightarrow$ (5) Let $(F,E) \in SS(X)_E$ such that (F,E) contains no non-null soft set (G,E) with $(G,E) \widetilde{\subseteq} (G,E)^*$. Since $(F,E) \widetilde{\cap} (F,E)^* \widetilde{\subseteq} (F,E)^* = ((F,E) \widetilde{\cap} (F,E)^*)^*$ from Theorem 4.1 (3). It follows that $(F,E) \widetilde{\cap} (F,E)^* \widetilde{\subseteq} ((F,E) \widetilde{\cap} (F,E)^*)^*$. By assumption, $(F,E) \widetilde{\cap} (F,E)^* = \tilde{\phi}$. Thus $(F,E) = (F,E) - (F,E)^* \in \tilde{I}$ by (3).

(5) $\Longrightarrow$ (3) Let $(F,E) \in SS(X)_E$. Since $((F,E) - (F,E)^*) \widetilde{\cap} ((F,E) - (F,E)^*)^* = \tilde{\phi}$ and $(F,E) - (F,E)^*)$ contains no non-null soft set (G,E) with $(G,E) \widetilde{\subseteq} (G,E)^*$. Hence $(F,E) - (F,E)^* \in \tilde{I}$ by (5).

Theorem 4.3. If (X, τ, E) is a soft topological space, $\tilde{I}$ be a soft ideal over X with the same set of parameters E and

compatible with τ. Then a soft set is τ^*-closed if and only if it is the union of a τ-closed soft set and a soft set in $\tilde{I}$.

Proof.

Let (F,E) be a τ^*-closed soft set. Then $cl^*(F,E) = (F,E)$ and $(F,E) \widetilde{\cup} (F,E)^* = (F,E)$. Hence $(F,E)^* \widetilde{\subseteq} (F,E)$. Thus $(F,E) = ((F,E) - (F,E)^*) \widetilde{\cup} (F,E)^*$, $(F,E) - (F,E)^* \in \tilde{I}$ from Theorem 4.2 and $(F,E)^*$ is τ-closed soft set from Theorem 3.2 (5). Conversely, let $(F,E) = (G,E) \widetilde{\cup} (I,E)$, where (G,E) is τ-closed soft set and $(I,E) \in \tilde{I}$. Then $(F,E)^* = ((G,E) - (I,E))^* = (G,E)^* \widetilde{\subseteq} cl(G,E) = (G,E) \widetilde{\subseteq} (F,E)$ from Theorem 3.2 (4,12). Hence $(F,E) \widetilde{\cup} (F,E)^* = (F,E)$. Thus $cl^*(F,E) = (F,E)$. It follows that (F,E) is a τ^*-closed soft set.

Theorem 4.4. Let (X, τ, E) be a soft topological space, $\tilde{I}$ be a soft ideal over X with the same set of parameters E. If $\tau \sim \tilde{I}$, then $\beta(\tilde{I}, \tau)$ is a soft topology and hence $\beta = \tau^*$.

Proof. Let $(G,E) \in \tau^*$. Then $(G,E)' = (F,E) \widetilde{\cup} (I,E)$, where (F,E) is τ-closed soft set and $(I,E) \in \tilde{I}$. Hence $(G,E) = (X,E) - ((F,E) \widetilde{\cup} (I,E)) = ((X,E) - (F,E)) \widetilde{\cap} ((X,E) - (I,E)) = ((X,E) - (F,E)) - (I,E)$, where $(F,E) \in \tau$ and $(I,E) \in \tilde{I}$. Thus $(G,E) = ((X,E) - (F,E)) - (I,E) \in \beta(\tilde{I}, \tau)$. This means that $\tau^* \subseteq \beta(\tilde{I}, \tau)$. But $\beta(\tilde{I}, \tau) \subseteq \tau^*(\tilde{I})$ from Corollary 3.2. It follows that $\beta = \tau^*$.

5 Acknowledgements

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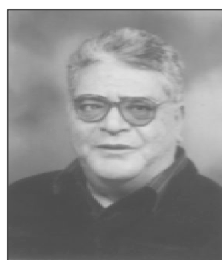
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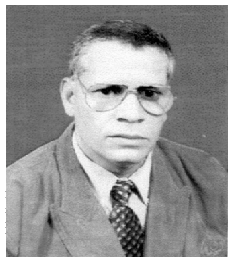
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