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Dynamic of Scaled Atomic Phase Entropy of a Single Two-Level Atom Interaction with SU(1,1) Quantum System

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Abstract: In this paper, we study the dynamics of the atomic inversion, scaled atomic Wehrl entropy and marginal atomic Q-function for a single two-level atom interacting with SU(1,1) quantum system. We obtain the wave function and system density matrix using specific initial conditions. We examine the effects of different parameters on the scaled atomic Wehrl entropy, atomic Q-function and their marginal distribution. We observe an interesting monotonic relation between the different physical quantities for different values of the initial atomic position and detuning parameter

Keywords: Scaled atomic Wehrl entropy, atomic Q-function, atomic inversion

1 Introduction

The most important problems in quantum optics are the studies of different systems interaction such as field-atom, atom-atom and the field-field interaction. These problems have considered the subject of great deal of research works during the last decades. In this way, there are numerous papers on these problems. For example the atom-field interaction has been considered in[1]-[11], but field-field interaction [12]-[24], while atom-atom interaction [25]-[37]. These interactions has been classified from the point of view of Lie algebra depending on the nature of the interaction. For example, the Hamiltonian which represents the interaction between two fields is described in the form of the parametric frequency converter is of SU(2) Lie algebra type. While the Hamiltonian which represents the non-degenerate parametric amplifier is of SU(1,1) Lie algebra type. On the other hand, the degenerate parametric amplifier, which contains in its interaction term the second harmonic generation, is of SU(1,1) Lie algebra type . In this context a system which describes the interaction between SU(2) and SU(1,1) Lie algebra has been considered [38], in which a Hamiltonian of the following from was treated

$$H = \hbar\left\{\omega k_z + \frac{\omega_0}{2}\sigma_z + \lambda\left(k_-\sigma_+ + k_+\sigma_-\right)\right\},$$

where ω is the frequency of the system, $\hbar\omega_0$ is the energy difference between the atomic levels and λ is a coupling constant.

Recently, much attention has been focused on information entropies as a measure or quantifier the entanglement in quantum information [39]. In this way the von Neumann entropy [40], linear entropy, and Shannon information entropy [41] have been frequently used in entanglement-discussions concerning a variety of quantum systems. Some problem appear with some of these measures such as the SE involves only the diagonal elements of the density matrix so in can gives information similar to that obtained from the NE. On the other hand, there is an additional entropic quantity, namely, the semiclassical, atomic phase-space atomic Wehrl entropy (AWE) [42]. This measure has been successfully applied as entanglement quantifier in the JCM. For example, AWE of the modes are initially prepared in a finite dimensional trio-coherent state (FTCS) has discussed [43]. Also, the dynamical properties of the AWE for a single two-level trapped ion interacting with a laser field has been investigated [44]. It is shown that the AWE gives quantitative (qualitative) information on the entanglement of the bipartite system.

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In this article, we consider the extension of the problem by considering which is called the scaled atomic Wehrl entropy associated with the reduced atomic density operator as an entanglement quantifier between SU(1,1) and SU(2) quantum system. We focus on the effect of the excitation number, initial atomic state and detuning parameter on the evolution of the atomic inversion, scaled atomic Wehrl entropy and marginal atomic Q-function.

The paper is organized as follows: In Sec. 2, the system Hamiltonian of the interaction between SU(1,1) and SU(2) is introduced, followed by a discussion of the method to calculate the scaled atomic Wehrl entropy and marginal atomic Q-function in Sec. 3. Numerical results of the calculated scaled atomic Wehrl entropy are presented and compared with the marginal atomic Q-function in Sec. 4, We conclude in Sec. 5, with a summary and an outlook.

2 The System Hamiltonian

The Hamiltonian which describe the interaction between a single two-level atom and SU(1,1) quantum system take the following form

$$H = \omega k_z + \Omega_1 S_{11} + \Omega_2 S_{22} + \lambda\left(k_- S_{12} + k_+ S_{21}\right), \quad (1)$$

where ω is the frequency of the system, Ω_i is the energy and S_{ij} are elements of the $SU(1,1)$ group obeying the following commutation relation

$$[S_{ij}, S_{kl}] = S_{il}\delta_{kj} - S_{kj}\delta_{il}, \quad (2)$$

while $k_\pm$ and k_z satisfy the following commutation relation

$$[k_z, k_\pm] = \pm k_\pm \ , \ [k_-, k_+] = 2k_z \text{ and } [S_{ij}, k_{\pm,z}] = 0. \quad (3)$$

The Heisenberg equation of motion for any operator O is given by

$$i\frac{dO}{dt} = [O, H], \quad (\hbar = 1), \quad (4)$$

thus, the equations of motion for S_{ij} and k_z are given by

$$\begin{aligned} i\frac{dS_{11}}{dt} &= [S_{11}, H] \\ &= \lambda\left(k_- S_{12} - k_+ S_{21}\right), \end{aligned} \quad (5)$$

$$\begin{aligned} i\frac{dS_{22}}{dt} &= [S_{22}, H] \\ &= -\lambda\left(k_- S_{12} - k_+ S_{21}\right), \end{aligned} \quad (6)$$

$$\begin{aligned} i\frac{dk_z}{dt} &= [k_z, H] \\ &= -\lambda\left(k_- S_{12} - k_+ S_{21}\right), \end{aligned} \quad (7)$$

$$2i\frac{dk_z}{dt} = -2\lambda\left(k_- S_{12} - k_+ S_{21}\right)$$

$$\begin{aligned} i\frac{dS_{11}}{dt} - i\frac{dS_{22}}{dt} + 2i\frac{dk_z}{dt} &= 0 \\ \frac{1}{2}\left(S_{11} - S_{22}\right) + k_z &= \text{constant of motion} \end{aligned} \quad (8)$$

from the above equation, we can see that $N = \frac{1}{2}(S_{11} - S_{22}) + k_z$ is constant of motion, therefore, the Hamiltonian takes the following form

$$H = \omega N + C, \quad (9)$$

where $C = \frac{\Delta}{2}(S_{11} - S_{22}) + \lambda\left(k_-\sigma_+ + k_+\sigma_-\right)$ with $\Delta = \Omega_1 - \Omega_2$. We note that $[N, C] = 0$, therefore $[N, H] = [H, C] = 0$, i.e. N and C are the constants of motion, where the time evolution operator is defined as

$$U(t) = \exp(-iHt), \quad (10)$$

thus

$$U(t) = \exp(-i\omega N t)\exp(-iCt), \quad (11)$$

where

$$\exp(-i\omega N t) = \begin{bmatrix} \exp\left[-i\omega\left(k_z + \frac{1}{2}\right)t\right] & 0 \\ 0 & \exp\left[-i\omega\left(k_z - \frac{1}{2}\right)t\right] \end{bmatrix}, \quad (12)$$

$$C = \begin{bmatrix} \frac{\Delta}{2} & \lambda k_- \\ \lambda k_+ & \frac{-\Delta}{2} \end{bmatrix}, C^2 = \begin{bmatrix} \mu_1^2 & 0 \\ 0 & \mu_2^2 \end{bmatrix}, \quad (13)$$

where

$$\mu_j^2 = \frac{\Delta^2}{4} + \nu_j, \quad j = 1, 2, \ \ \nu_1 = \lambda^2 k_- k_+ \text{ and } \nu_2 = \lambda^2 k_+ k_- \quad (14)$$

we note that

$$k_- \mu_2^2 = \mu_1^2 k_- \quad (15)$$

also

$$k_+ \mu_1^2 = \mu_2^2 k_+ \quad (16)$$

$$C^3 = \begin{bmatrix} \frac{\Delta}{2}\mu_1^2 & \mu_1^2 \lambda k_- \\ \mu_2^2 \lambda k_+ & \frac{-\Delta}{2}\mu_2^2 \end{bmatrix}, \ C^4 = \begin{bmatrix} \mu_1^4 & 0 \\ 0 & \mu_2^4 \end{bmatrix} \quad (17)$$

$$\begin{aligned} \exp(-iCt) &= I + \frac{-iCt}{1!} + \frac{(-iCt)^2}{2!} + \frac{(-iCt)^3}{3!} + \ldots. \\ &= I - iCt - i\frac{C^2 t^2}{2!} + i\frac{C^3 t^3}{3!} + \ldots. \end{aligned} \quad (18)$$

then, one can write the time evolution operator as

$$U(t) = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}, \tag{19}$$

where

$$\begin{aligned}
F_{11} &= \exp\left[-i\omega\left(k_z + \frac{1}{2}\right)t\right]\left(\cos\mu_1 t - \frac{i\Delta}{2}\frac{\sin\mu_1 t}{\mu_1}\right), \\
F_{12} &= -i\lambda\exp\left[-i\omega\left(k_z + \frac{1}{2}\right)t\right]\frac{\sin\mu_1 t}{\mu_1}k_-, \\
F_{21} &= -i\lambda\exp\left[-i\omega\left(k_z - \frac{1}{2}\right)t\right]\frac{\sin\mu_2 t}{\mu_2}k_+, \\
F_{22} &= \exp\left[-i\omega\left(k_z - \frac{1}{2}\right)t\right]\left(\cos\mu_2 t + \frac{i\Delta}{2}\frac{\sin\mu_2 t}{\mu_2}\right). \tag{20}
\end{aligned}$$

The time evolution for the expectation value of any operator can be calculated through the following relation

$$\begin{aligned}
\langle O(t)\rangle &= \langle\Psi(t)|O(t)|\Psi(t)\rangle \\
&= \langle\Psi(0)|U^+(t)O(0)U(t)|\Psi(0)\rangle. \tag{21}
\end{aligned}$$

Let us assume the initial state of the system can be written as

$$\begin{aligned}
|\Psi(0)\rangle &= |\Psi(0)\rangle_{SU(2)}|\Psi(0)\rangle_{S(1,1)} \\
&= (\cos\frac{\theta}{2}|e\rangle + \sin\frac{\theta}{2}|g\rangle)|m,k\rangle, \tag{22}
\end{aligned}$$

where

$$\begin{aligned}
k_z|m,k\rangle &= (m+k)|m,k\rangle, \\
k_+|m,k\rangle &= \sqrt{(m+1)(m+2k)}|m+1,k\rangle, \\
k_-|m,k\rangle &= \sqrt{m(m+2k-1)}|m-1,k\rangle. \tag{23}
\end{aligned}$$

$$\begin{aligned}
|\Psi(t)\rangle &= U(t)|\Psi(0)\rangle \\
&= \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}\begin{bmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{bmatrix}|m,k\rangle \\
&= (F_{11}\cos\frac{\theta}{2} + F_{12}\sin\frac{\theta}{2})|m,k\rangle|e\rangle \\
&\quad + (F_{21}\cos\frac{\theta}{2} + F_{22}\sin\frac{\theta}{2})|m,k\rangle|g\rangle. \tag{24}
\end{aligned}$$

Substituting from Eqs. (20) in Eq.(24), then the final form of the wave function can be written as

$$\begin{aligned}
|\Psi(t)\rangle &= e^{-i\omega\left(k_z+\frac{1}{2}\right)t}\left\{\left(\cos\mu_1 t - \frac{i\Delta}{2}\frac{\sin\mu_1 t}{\mu_1}\right)\cos\frac{\theta}{2}\right. \\
&\quad \left. - i\lambda\frac{\sin\mu_1 t}{\mu_1}k_-\sin\frac{\theta}{2}\right\}|m,k,e\rangle \\
&\quad + e^{-i\omega\left(k_z-\frac{1}{2}\right)t}\left\{\left(\cos\mu_2 t + \frac{i\Delta}{2}\frac{\sin\mu_2 t}{\mu_2}\right)\sin\frac{\theta}{2}\right. \\
&\quad \left. - i\lambda\frac{\sin\mu_2 t}{\mu_2}k_+\cos\frac{\theta}{2}\right\}|m,k,g\rangle. \tag{25}
\end{aligned}$$

Then, the wave function can be written in the form

$$|\Psi(t)\rangle = A(t)|m,k\rangle|e\rangle + B(t)|m,k\rangle|g\rangle, \tag{26}$$

and consequently the density matrix $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$ becomes

$$\begin{aligned}
\rho(t) &= A(t)|m,k\rangle|e\rangle\langle e|\langle k,m|A^*(t) + B(t)|m,k\rangle|g\rangle \\
&\quad \otimes\langle g|\langle k,m|B^*(t) + B(t)|m,k\rangle|g\rangle\langle e|\langle k,m|A^*(t) \\
&\quad + A(t)|m,k\rangle|e\rangle\langle g|\langle k,m|B^*(t), \tag{27}
\end{aligned}$$

where

$$\begin{aligned}
A(t) &= e^{-i\omega\left(k_z+\frac{1}{2}\right)t}\left\{\left(\cos\mu_1 t - \frac{i\Delta}{2}\frac{\sin\mu_1 t}{\mu_1}\right)\cos\frac{\theta}{2}\right. \\
&\quad \left. - i\lambda\frac{\sin\mu_1 t}{\mu_1}k_-\sin\frac{\theta}{2}\right\}, \tag{28} \\
B(t) &= e^{-i\omega\left(k_z-\frac{1}{2}\right)t}\left\{\left(\cos\mu_2 t + \frac{i\Delta}{2}\frac{\sin\mu_2 t}{\mu_2}\right)\sin\frac{\theta}{2}\right. \\
&\quad \left. - i\lambda\frac{\sin\mu_2 t}{\mu_2}k_+\cos\frac{\theta}{2}\right\}, \tag{29}
\end{aligned}$$

one can easily cheek that

$$|A(t)|^2 + |B(t)|^2 = 1 \tag{30}$$

Thus, the expectation value for any operator can be calculated through the following equation

$$\langle O(t)\rangle = \langle\Psi(0)|O(t)|\Psi(0)\rangle = \langle\Psi(t)|O(0)|\Psi(t)\rangle, \tag{31}$$

where $|\Psi(0)\rangle$ and $|\Psi(t)\rangle$ are defined by Eqs. (22) and (26).

Therefore, the expectation values of the atomic operators σ_x and σ_y can be obtained as follows

$$\begin{aligned}
\langle\sigma_x(t)\rangle &= \tfrac{1}{2}\left\{\left(\cos(\mu_1 t)\cos(\mu_2 t) - \tfrac{\Delta^2}{4}\frac{\sin(\mu_1 t)\sin(\mu_2 t)}{\mu_1\mu_2}\right)\right. \\
&\quad \left.\times\cos\omega t - \tfrac{\Delta}{2}\left(\frac{\sin(\mu_1 t)\cos(\mu_2 t)}{\mu_1} + \frac{\cos(\mu_1 t)\sin(\mu_2 t)}{\mu_2}\right)\sin\omega t\right\}\sin\theta, \\
\langle\sigma_y(t)\rangle &= \tfrac{1}{2}\left\{\left(\cos(\mu_1 t)\cos(\mu_2 t) - \tfrac{\Delta^2}{4}\frac{\sin(\mu_1 t)\sin(\mu_2 t)}{\mu_1\mu_2}\right)\right. \\
&\quad \left.\times\sin\omega t + \tfrac{\Delta}{2}\left(\frac{\sin(\mu_1 t)\cos(\mu_2 t)}{\mu_1} + \frac{\cos(\mu_1 t)\sin(\mu_2 t)}{\mu_2}\right)\cos\omega t\right\}\sin\theta, \tag{32}
\end{aligned}$$

Now, we close this section by presenting the concept of the atomic population inversion $\rho_z(t)$ which is the simplest important quantity to be calculated. It is related to the difference between the probabilities of finding the atom in the upper and lower state.

$$\begin{aligned}
\rho_z(t) &= \frac{\cos\theta}{2} + \lambda^2\left\{m(m+2k-1)\frac{\sin^2\mu_2 t}{\mu_2^2}\sin^2\frac{\theta}{2}\right. \\
&\quad \left. - (m+1)(m+2k)\frac{\sin^2\mu_1 t}{\mu_1^2}\cos^2\frac{\theta}{2}\right\}, \tag{33}
\end{aligned}$$

where

$$\mu_1 = \sqrt{\frac{\Delta^2}{4} + \lambda^2 (m+1)(m+2k)},$$
$$\mu_2 = \sqrt{\frac{\Delta^2}{4} + \lambda^2 m(m+2k-1)}. \quad (34)$$

Now, we are in a position to use the results obtained in this section to discuss the dynamical behavior of the atomic inversion, marginal atomic Q-function and scaled AWE in the following sections.

2.1 Scaled atomic Wehrl entropy, marginal distribution and entanglement quantifiers

In this section: we investigate the marginal atomic Q-function and atomic Wehrl entropy AWE. We start our investigation by defining the atomic Q-function as [1]

$$Q_A(\Theta, \Phi, t) = \frac{1}{2\pi} \langle \Theta, \Phi \,| \hat{\rho}_{11}(t) | \, \Theta, \Phi \rangle \,, \quad (35)$$

where $\hat{\rho}$(t) is the density matrix which is given in equation (27) and $|\Theta, \Phi\rangle$ is the atomic coherent state expressed as

$$|\Theta, \Phi\rangle = \cos(\Theta/2)\,|e\rangle + \sin(\Theta/2)\, e^{i\Phi}\,|g\rangle \,, \quad (36)$$

where $0 \leq \Theta \leq \pi, 0 \leq \Phi \leq 2\pi$. the definition (35) means that two different spin coherent states overlap unless they directed into two antipodal points on the sphere [1].

The scaled atomic Wehrl entropy can be written in terms of the atomic Q- function as [1]:

$$SS_{AW}(t) = \frac{2\ln(2)}{1-2\ln(2)} \{\ln(2\pi\sqrt{e}) + \int_0^{2\pi}\int_0^{\pi} Q_A(\Theta, \Phi, t) \ln Q_A(\Theta, \Phi, t) \sin\Theta d\Theta d\Phi\} \quad (37)$$

One can easily check that the Q_A is normalized. By integrating the atomic Q-function Q_A over the atomic variable Φ, we obtain the marginal atomic Q-function as follows

$$Q_\Phi = \int_0^{\pi} Q_A \sin\Theta d\Theta. \quad (38)$$

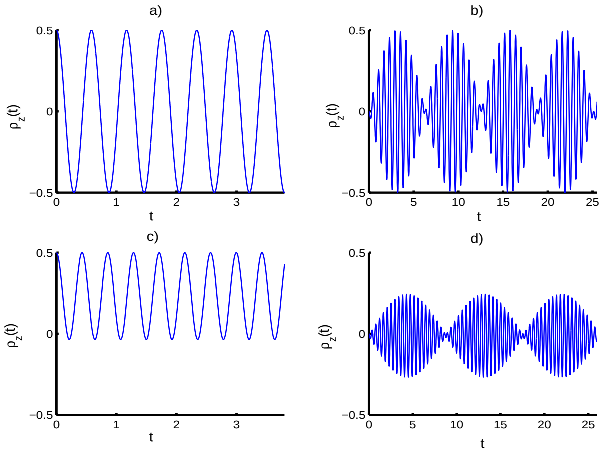

Fig. 1: Time evolution of the atomic inversion $\rho_z(t)$, for $\Delta = 0, \lambda = 0.5$, $k = \frac{1}{4}$and with different values of the excitation number m and initial atomic position θ and relative phase $\phi = \frac{\theta}{2}$ where: Fig. (a) $(m, \theta) = (10, 0)$, Fig. (b) $(m, \theta) = \left(10, \frac{\pi}{2}\right)$, Fig. (c) $(m, \theta) = (20, 0)$ and Fig. (d) $(m, \theta) = \left(20, \frac{\pi}{2}\right)$.

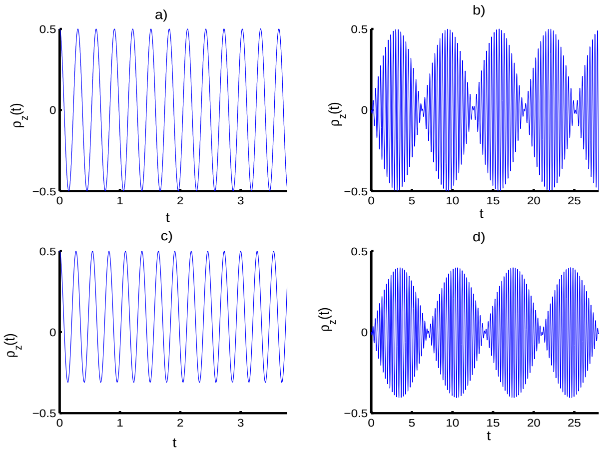

Fig. 2: Time evolution of the atomic inversion $\rho_z(t)$, for $\Delta = 20, \lambda = 0.5$, $k = \frac{1}{4}$ and with different values of the excitation number m and initial atomic position θ and relative phase $\phi = \frac{\theta}{2}$ where: Fig. (a) $(m, \theta) = (10, 0)$, Fig. (b) $(m, \theta) = \left(10, \frac{\pi}{2}\right)$, Fig. (c) $(m, \theta) = (20, 0)$ and Fig. (d) $(m, \theta) = \left(20, \frac{\pi}{2}\right)$.

3 Numerical results

The population inversion of the atom is one of the important atomic dynamic variables of the system. This in fact would give us information about the behavior of the atom state during interaction time. In figure (1), we have plotted the dynamical behavior for different values of the involved parameters. We concentrate on the variation of the initial atomic position θ from the excited state i.e. $\theta = 0$ to the superposition state i.e. $\theta = \pi/2$ as well as on the excitation number m, which is in analogy with the usual Jaynes–Cummings model, corresponding to the number of photons. Firstly, we consider that the system is initially in the excited state $\theta = 0$ and the absence of the detuning parameter $\Delta = 0$. It is observed that the atomic population inversion has a regular and periodic oscillation

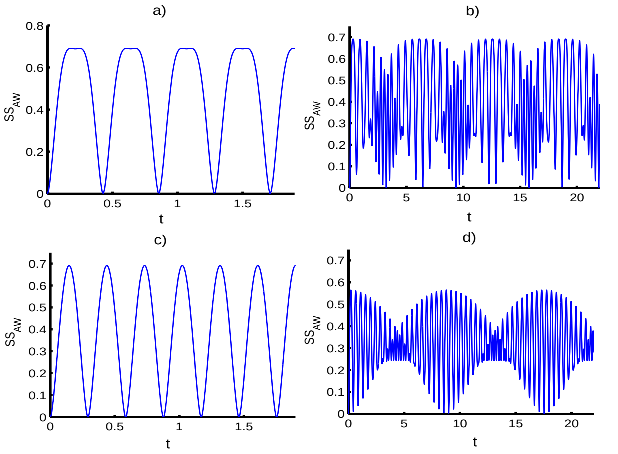

Fig. 3: Time evolution of the scaled atomic Wehrl entropy $SS_{AW}(t)$, for $\Delta = 0, \lambda = 0.5$, $k = \frac{1}{4}$ and for different values of the excitation number m and initial atomic position θ and relative phase $\phi = \dfrac{\theta}{2}$ where: Fig. (a) $(m,\theta) = (10,0)$, Fig. (b) $(m,\theta) = \left(10, \dfrac{\pi}{2}\right)$, Fig. (c) $(m,\theta) = (20,0)$ and Fig. (d) $(m,\theta) = \left(20, \dfrac{\pi}{2}\right)$.

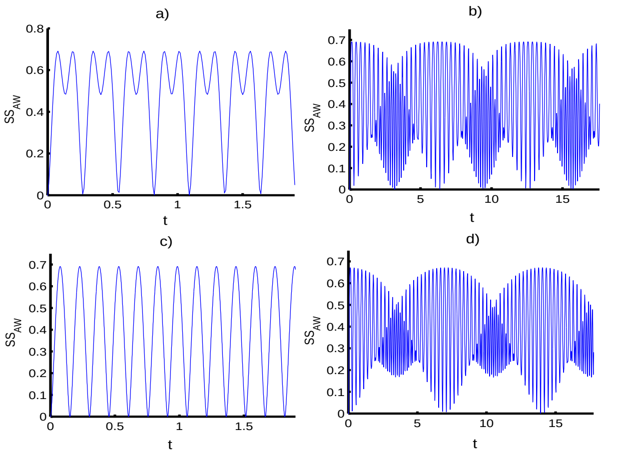

Fig. 4: Time evolution of the scaled atomic Wehrl entropy $SS_{AW}(t)$, for $\Delta = 20, \lambda = 0.5$, $k = \frac{1}{4}$ and for different values of the excitation number m and initial atomic position θ and relative phase $\phi = \dfrac{\theta}{2}$ where: Fig. (a) $(m,\theta) = (10,0)$, Fig. (b) $(m,\theta) = \left(10, \dfrac{\pi}{2}\right)$, Fig. (c) $(m,\theta) = (20,0)$ and Fig. (d) $(m,\theta) = \left(20, \dfrac{\pi}{2}\right)$.

where the amplitude of oscillation is decrease by increasing the number of photon excitation. The structure of the atomic inversion oscillations is changed when the atom is initially in the superposition state see Fig. 1(b,d). The number of oscillation is increased when the effect of the atomic inversion is taken into account (see Fig. 2).

The scaled atomic phase space entropy as a quantifier of the entanglement between two-level atom and SU(1,1) quantum field is plotted in Fig. 3,4. As seen from Fig. 3 $SS_{AW}(t)$ has a periodic behavior and regular oscillation. The system returns to its separable state ($SS_{AW}(t) = 0$) at $t_s = 0.45m$ where $m = 0,1,2,\ldots$ On the other hand the system is maximally entangled state ($SS_{AW}(t) = \ln(2)$) at the middle of the time interval $0 < t < t_s$. Fig. 3 (d), depicts that the entanglement is gradually decreases by increases the number of photon excitation when the atom is initially in the superposition state.

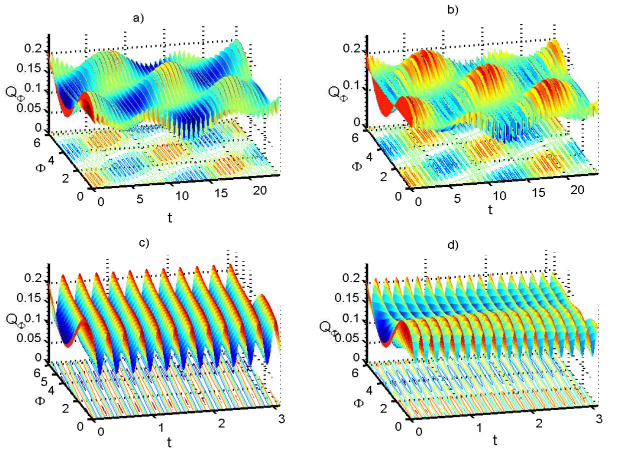

Fig. 5: The surface plot of the marginal atomic Q-function $Q_{\Phi}(t)$ versus the time t and the phase space parameter Φ for $\Delta = 0, \lambda = 0.5$, $k = \frac{1}{4}$and with different values of the excitation number m and initial atomic position θ and relative phase $\phi = \dfrac{\theta}{2}$ where: Fig. (a) $(m,\theta) = (10,0)$, Fig. (b) $(m,\theta) = \left(10, \dfrac{\pi}{2}\right)$, Fig. (c) $(m,\theta) = (20,0)$ and Fig. (d) $(m,\theta) = \left(20, \dfrac{\pi}{2}\right)$.

Now, we are going to answer the question "What is the impact of the detuning parameter on the atom-SU(1,1) field entanglement for different values of the number of photon excitation and initial atomic position?" As presented in Fig. 4, where the $SS_{AW}(t)$ was plotted as a function of the time when the atom is initially in the excited and superposition state. It is interesting to note the high amount of the quantum entanglement can be obtained in the presence of the detuning parameter during the time evolution.

In Figure (5) depicts the evolution of Q_{Φ}(t) as a function of the time and atomic phase space parameter Φ for different modes of excitations. It is interesting to mention here that the behavior of $Q_{\Phi}(t)$ for different values of the non-fluctuating components of Rabi frequency. It is observed that $Q_{\Phi}(t)$ oscillates between minimum and maximum peaks. The distribution of the marginal atomic Q-function peaks in depending the initial state setting of the two-level atom. On the other hand the number of peaks in increased by increasing the atomic Q-function peaks.

4 Conclusion

It is well known that the study of the physical properties of the atom–field interaction is a central and important problem in the field of quantum information. In this paper, we have discussed the problem of the interaction between two-level atom and SU(1,1) quantum system. The model was considered when the two-level atom is initially in superposition state and the wave function is obtained analytically. Using the scaled atomic phase space entropy the system entanglement have been investigated. The analysis herein has been carried out at two distinct considerations of the detuning parameter and number of photon excitation. Firstly, when the deteuning parameter was neglected, the scaled atomic phase space entropy has a regular oscillations between 0 and $\ln(2)$ during the time evolution. There is some monotonic correlation between the behavior of the atomic inversion, scaled atomic phase space entropy and the marginal atomic Q-function.

Quantum entanglement is a key resource which distinguishes quantum information theory from classical one. It plays a central role in quantum information and computation and communication. In the present work, our results show that the SU(1,1) quantum field–atom interaction considering the effect of the number of photons excitation and detuning parameter have much richer structure.

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