Journal of Statistics Applications & Probability
An International Journal

http://dx.doi.org/10.18576/jsap/120338

Efficient Circular Repeated Measurements Designs Nearly Strongly Balanced for Carryover Effects

Muhammad Shahbaz, Muhammad Ejaz Malik, Muhammad Daniyal, H. M. Kashif Rasheed, Zahid Bashir and Rashid Ahmed[*]

Department of Statistics, The Islamia University of Bahawalpur, Bahawalpur 61300, Pakistan

Received: 3 Apr. 2022, Revised: 21 Sep. 2022, Accepted: 23 Oct. 2022
Published online: 1 Sep. 2023

Abstract: Repeated measurements designs (RMDs) are always economical but with the use of these designs, there may arise carryover effects. Minimal strongly balanced RMDs are well known to estimate the treatment effects and carryover effects independently. Where these minimal designs cannot be constructed, minimal nearly strongly balanced RMDs are used to balance the carryover effects. In this article, efficient circular nearly strongly balanced RMDs are constructed in periods of two and three different sizes with the smallest of size four.

Keywords: Carry over effects; Residual effects; RMDs; Minimal designs.

1 Introduction

Repeated measurements designs (RMDs) are very useful in the fields of medicine, pharmacology, animal sciences and psychology, but with the use of RMDs, carryover effects may arise. Effect which a treatment has during its period of application (direct effect) may persist into the preceding period. Such effect is called carryover effect or residual effect. Balanced or strongly balanced RMDs are useful to balance these effects. RMD is minimal strongly balanced (SBRMD) with respect to the carryover effects if each treatment is immediately preceded once by each other treatment (including itself). Using method of cyclic shifts, these designs can be constructed through its Rule I for some specific cases of v and p, where v is number of treatments and p is period size. For most of the remaining cases, Rule II provides construction of nearly SBRMDs. RMD is minimal circular nearly strongly balanced (MCNSBRMD) if each treatment is immediately preceded once with all other treatments (including itself) except $v-1$ which is not preceded with itself. [1,2] constructed minimal balanced RMDs for v even with $p = v$. [3] used RMDs in experiments of biology. [4] mentioned the importance of RMDs with unequal periods sizes in industrial and agricultural experiments. [5] introduced the idea of a circular balanced RMD. [6] gave the complete solution of cyclic minimal balanced in linear periods for every possible $p < v$. [7] constructed uniform circular balanced RMDs for v (i) prime, and (ii) even. [8] presented a simple method for the construction of uniform circular balanced RMDs for v odd. [9] gave an easy method to construct minimal balanced and strongly balanced RMDs with unequal period sizes. Using method of cyclic shifts, [10] constructed (i) efficient RMDs with equal as well unequal period sizes, (ii) SBRMDs for periods of two different sizes for $5 \le v \le 10$, $3 \le p_1 \le 6$ and $3 \le p_2 \le 10$. [11] gave a general procedure to construct balanced RMDs for v odd. [12]] constructed some minimal circular SBRMDs (MCSBRMDs) in periods of equal sizes. [13] constructed some minimal SBRMDs with first residuals. [14] constructed some circular first-and second-order balanced RMDs through method of cyclic shifts. [15] constructed universally optimal weakly balanced RMDs for $p = v$. [16,17] developed some generators to obtain circular balanced RMDs in periods of equal sizes. [18] developed generators to obtain MCSBRMDs in periods of three different sizes for p_1 even, $3 \le p_2 \le 10$ and $2 \le p_2 \le 9$. [19] developed some generators for general construction of MCSBRMDs in periods of unequal sizes. [20] developed some generators to obtain MCBRMDs in periods of unequal sizes. [21] constructed MCBRMDs in periods of three different sizes. [22] developed some generators to obtain minimal circular weakly balanced RMDs in periods of equal sizes. [23] developed some infinite series to obtain minimal circular strongly partially balanced RMDs in periods of equal sizes. Using method of cyclic shifts (Rule I), recently, [24] constructed MCNSBRMDs in

[*] Corresponding author e-mail: rashid701@hotmail.com

periods of three different sizes with smallest of size two. In this article, MCNSBRMDs are constructed in periods of (i) two different sizes for $v \leq 100, 5 \leq p_1$ (odd) ≤ 9 and $p_2 = 4$, and (ii) three different sizes for $v \leq 100, 6 \leq p_1$ (odd) $\leq 9, 5 \leq p_2 \leq 8$ and $p_3 = 4$. These designs are highly efficient to estimate the treatment effects and carryover effects independently. Rest of the paper is organized as follows: Procedure to find the efficiency of Separability is described in Section 2. In Section 3, MCNSBRMDs are constructed through Rule II, in periods of two different sizes with $p_3 = 4$. In Section 4, MCNSBRMDs are constructed through Rule II, in periods of three different sizes with $p_3 = 4$. Efficiencies of these designs are also presented.

2 Efficiency of Separability

[25] derived the following formula for efficiency of Separability (Es) which is also useful for our proposed MCNSBRMDs.

$$Es = [1 - \{\frac{(l_1 + 4l_2)v - (l_1 + 2l_2)^2}{(v-1)(l_1 + 2l_2)^2}\}^{\frac{1}{2}}] \times 100\% \tag{1}$$

Where,
l_1 : No. of treatments preceded once by other treatment.
l_2 : No. of treatments preceded twice by other treatment.
Our proposed designs possess at least 90% efficiency of Separability and are therefore, highly efficient according to this criterion.

3 Construction of Efficient MCNSBRMDs in Periods of Two Different Sizes

In this Section, using method of cyclic shifts (Rule II) introduced by [26], MCNSBRMDs are constructed from the following sets of shifts in periods of two different sizes $p_1 = r$ (odd) and $p_3 = 4$ for $v = ri + 3$. In these designs only one pair $(v-1, v-1)$ do not appear while all others appear once.

$S_u = [q_{u1}, q_{u2}, \ldots, q_{u(r-1)}]$; $u = 1, 2, \ldots, i$.
$S_{u+1} = [q_{(u+1)1}, q_{(u+1)2}]$t
Where
- Each element of sets lies between 0 and $v-2$.
- S^* contains each of $0, 1, 2, \ldots, v-2$ exactly once.
- S^* contains (i) each element of all sets, (ii) $(v-1)$-(Sum of elements in each set S_u).

Example 3.1: $S_1 = [1, 3, 6, 4]$ and $S_2 = [2, 5]$t provide following MCNSBRMD for $v = 8$, $p_1 = 5$ and $p_2 = 4$.

Here S^* = $[1, 3, 6, 4, 0, 2, 5]$ contains each of $0, 1, \ldots, 6$exactly once. Hence given sets of shifts provide MCNSBRMD. Using Rule II, complete design is obtained from the given sets of shifts in the following manners.

Take $v-1$ subjects for S_1. Consider $0, 1, \ldots, v-2$ as 1st unit values of all subjects, respectively. To get the 2nd unit values, add 1(mod 7) to all 1st unit values. Then add 3(mod 7) to 2nd unit values, and so on, see table 1.

Table 1: Design obtained from S_1

Subject	1	2	3	4	5	6	7
1	0	1	2	3	4	5	6
2	1	2	3	4	5	6	0
3	4	5	6	0	1	2	3
4	3	4	5	6	0	1	2
5	0	1	2	3	4	5	6

Take $v-1$ more subjects for S_2. Consider $0, 1, \ldots, v-2$ as 1st unit values of all subjects, respectively. To get the 2nd unit values, add 2(mod 7) to all 1st unit values. Then add 5(mod 7) to 2nd unit values. Consider $v-1$ which is 7 here, as the 4th unit value for each subject, see table 2.

Table 2: Design obtained from S_2

Subject	8	9	10	11	12	13	14
1	0	1	2	3	4	5	6
2	2	3	4	5	6	0	1
3	0	1	2	3	4	5	6
4	7	7	7	7	7	7	7

• **Efficient MCNSBRMDs are presented in table 3 for** $5 \leq p_1 \leq 9, p_2 = 4$ **and** $v \leq 100$.

Table 3: MCNSBRMDs for $5 \leq p_1 \leq 9, p_2 = 4$ and $v \leq 100$

v	p_1	P_2	**Sets of shifts**	**Es**
8	5	4	[1,6,3,4]+[5,2]t	0.90
18	5	4	[1,16,3,14]+[4,6,7,8]+[10,11,12,5]+[15,2]t	0.95
28	5	4	[20,1,26,3]+[5,6,7,9]+[10,11,12,13]+[15,16,17,14]+ [18,21,22,24]+[25,2]t	0.97
38	5	4	[30,1,36,3]+[5,14,8,10]+[9,11,12,13]+[15,16,17,7]+ [20,21,22,23]+[24,26,27,28]+[18,31,32,34]+[35,2]t	0.97
48	5	4	[37,46,3,8]+[9,6,7,4]+[10,11,12,13]+[15,16,27,19]+ [20,5,22,23]+[25,26,33,28]+[30,31,32,14]+[35,36,40,38]+ [18,41,42,44]+[45,2]t	0.98
58	5	4	[50,1,56,3]+[5,6,7,39]+[10,11,9,13]+[15,16,17,19]+ [20,21,22,23]+[8,26,27,24]+[30,31,32,44]+[35,36,37,38]+ [12,41,42,43]+[45,46,40,48]+[18,51,52,54]+[55,2]t	0.98
68	5	4	[60,1,66,3]+[5,6,7,9]+[15,11,12,13]+[19,37,42,36]+ [20,45,22,23]+[46,44,43,39]+[38,21,17,28]+[35,10,33,31]+ [8,41,32,27]+[14,34,47,57]+[58,51,52,53]+[55,56,48,50]+ [18,61,62,64]+[65,2]t	0.99
78	5	4	[70,1,76,3]+[17,6,62,50]+[11,12,13,14]+[15,5,10,9]+ [21,22,23,24]+[25,26,28,29]+[7,31,32,33]+[35,36,37,16]+ [40,41,63,43]+[45,42,47,48]+[66,58,53,54]+[55,8,57,52]+ [60,61,56,34]+[65,39,67,68]+[18,71,72,74]+[75,2]t	0.99 0.99
88	5	4	[80,1,86,3]+[5,6,7,8]+[11,12,13,14]+[49,16,20,23]+ [42,34,19,57]+[25,26,27,67]+[30,31,32,33]+[28,59,10,38]+ [77,41,56,43]+[45,46,47,21]+[50,51,52,53]+[54,9,17,58]+ [72,62,63,64]+[65,24,35,68]+[70,71,60,73]+[75,76,40,78]+ [18,81,82,84]+[85,2]t	0.99
98	5	4	[90,1,96,3]+[5,6,7,9]+[10,11,12,14]+[15,16,17,19]+ [8,22,23,24]+[25,26,27,29]+[31,32,33,34]+[35,36,37,38]+ [78,71,42,43]+[13,46,47,39]+[81,51,52,53]+[55,56,44,61]+ [60,58,62,66]+[21,63,67,68]+[85,79,77,73]+[65,76,69,40]+ [80,59,82,83]+[86,28,88,89]+[18,91,92,94]+[95,2]t	0.99
10	7	4	[3,2,7,4,5,6]+[1,8]t	0.91
24	7	4	[1,22,8,4,5,6]+[7,3,9,14,11,12]+[15,10,16,17,18,19]+[21,2]t	0.96
38	7	4	[19,1,36,3,4,5]+[27,8,9,10,32,12]+[14,15,16,17,18,11]+ [21,22,23,24,25,26]+[28,29,30,31,33,34]+[35,2]t	0.97
52	7	4	[33,1,50,3,4,5]+[7,40,9,10,11,12]+[14,15,16,17,18,22]+ [48,21,23,24,25,36]+[29,28,30,31,32,34]+ [35,26,37,38,19,8]+[42,43,44,45,46,47]+[49,2]t	0.98
66	7	4	[47,1,64,3,4,5]+[7,8,54,10,11,13]+[14,26,16,17,18,19]+ [21,22,24,22,25,15]+[28,29,30,31,32,33]+ [50,36,37,38,39,51]+[43,42,44,45,46,57]+ [49,35,40,52,53,41]+[56,34,58,59,60,61]+[63,2]t	0.97
80	7	4	[61,1,78,3,4,5]+[7,8,9,10,20,12]+[14,16,15,27,18,19]+ [21,22,23,24,25,26]+[41,40,30,36,31,70]+ [35,31,37,38,39,29]+[52,34,44,45,46,47]+ [11,50,51,42,53,54]+[56,57,58,59,60,43]+ [63,64,65,66,68,69]+[33,71,72,73,74,75]+[77,2]t	0.97

94	7	4	[75,1,83,3,4,5]+[7,8,9,10,11,12]+[14,6,16,0,18,19]+ [21,40,23,24,25,26]+[58,30,31,32,46,34]+ [35,13,37,38,90,22]+[42,43,41,45,33,47]+ [49,50,51,52,53,54]+[56,57,17,59,60,61]+ [55,64,65,66,67,68]+[70,71,72,73,74,29]+ [77,78,79,69,81,82]+[84,85,86,87,88,89]+[91,2]t	0.99
12	9	4	[10,5,1,3,4,6,8,7]+[9,2]t	0.92
30	9	4	[24,1,28,3,5,4,6,7]+[10,8,11,12,13,14,15,16]+ [19,18,20,21,22,23,25,26]+[27,2]t	0.97
48	9	4	[24,1,28,3,5,4,6,7]+[10,8,11,12,13,14,15,16,]+ [19,18,20,21,22,23,25,26]+[27,2]t	0.98
66	9	4	[32,1,64,3,4,5,6,7]+[9,10,11,12,36,14,15,16]+ [44,19,20,21,31,23,24,25]+[28,27,29,39,22,13,33,34]+ [36,37,38,30,40,41,42,43]+[45,46,47,48,49,50,54,51]+ [52,55,56,57,58,59,60,61]+[63,2]t	0.97
84	9	4	[50,82,3,4,5,6,7,9]+[8,10,11,12,63,14,15,16]+ [18,19,20,21,22,23,75,25]+[27,28,30,29,32,33,34,35]+ [37,36,38,39,40,13,43,42]+[45,47,46,48,49,24,51,52]+ [54,31,56,57,58,59,60,61]+[41,64,65,66,67,68,69,70]+ [72,73,74,55,76,77,78,79]+[81,2]t	0.99

4 Construction of Efficient MCNSBRMDs in Periods of Three Different Sizes

In this Section, MCNSBRMDs are constructed from the following sets of shifts in periods of three different sizes, $p_1 = r$ (odd),$p_2 = s > 4$, and $p_3 = 4$ for$v = ri + s + 3$. In these designs only one pair $(v-1, v-1)$ do not appear while all others appear once.

$S_u = [q_{u1}, q_{u2}, ..., q_{u(r-1)}]$; $u = 1, 2, ..., i$.
$S_{u+1} = [q_{(u+1)1}, q_{(u+1)2}, ..., q_{(u+1)(s-1)}]$, $S_{u+2} = [q_{(u+2)1}, q_{(u+2)2}]$t
Where
• Each element of sets lies between 0 and $v-2$.
• S^* contains each of $0, 1, 2, ..., v-2$ exactly once.
• S^* contains (i) each element of all sets, (ii) $(v-1)$-(Sum of elements in each set S_u).

Efficient MCNSBRMDs are presented in table 4 for $v \leq 100, 7 \leq p_1$ (odd)$\leq 9, 5 \leq p_2 \leq 8$ and $p_3 = 4$, where $p_1 > p_2$.

Table 4: MCNSBRMDs for $v \leq 100$, $7 \leq p_1(odd) \leq 9$, $5 \leq p_2 \leq 8$ and $p_3 = 4$, where $p_1 > p_2$

v	p_1	p_2	p_3	Sets of Shifts	Es
15	7	5	4	[5,6,7,8,9,10]+[2,1,12,13]+[3,4]t	0.98
29	7	5	4	[4,12,6,7,8,9]+[13,14,15,16,17,18]+[20,21,22,23,24,25]+ [2,1,26,27]+[3,11]t	0.99
43	7	5	4	[5,6,7,8,9,10]+[13,24,14,15,16,25]+[20,21,22,23,12,17]+ [27,28,30,29,31,32]+[26,34,35,36,38,37]+ [40,41,2,1]+[3,18]t	0.99
57	7	5	4	[5,6,14,8,9,10]+[13,12,28,15,16,17]+[40,20,21,22,23,24]+ [27,47,29,30,31,34]+[32,35,36,37,38,39]+ [41,42,43,44,45,46]+[48,49,50,51,52,53]+ [54,55,2,1]+[3,25]t	0.99
71	7	5	4	[4,6,7,8,9,10]+[11,13,14,15,16,17]+[12,20,21,22,23,24]+ [40,27,28,29,30,31]+[34,35,36,37,38,39]+ [41,42,43,44,45,46]+[48,49,50,51,52,53]+ [55,56,57,58,59,60]+[62,63,64,65,66,67]+ [68,69,2,1]+[3,32]t	0.99

85	7	5	4	[4,6,7,8,9,10]+[69,13,14,15,16,17]+[18,19,20,21,22,23]+ [27,26,28,29,30,32]+[31,33,12,35,36,37]+ [5,41,42,43,44,52]+[48,49,50,51,11,53]+ [55,56,58,57,59,62]+[34,63,64,65,66,67]+ [46,60,70,71,72,54]+[76,77,78,79,38,81]+ [82,83,2,1]+[3,39]t	0.99
99	7	5	4	[5,6,7,8,9,10]+[12,13,14,15,16,17]+[19,20,21,22,23,24]+ [26,27,28,29,30,31]+[82,34,35,36,37,38]+ [40,41,42,43,44,45]+[48,49,50,18,58,89]+ [55,56,57,51,59,60]+[61,62,63,64,65,73]+ [69,70,71,72,66,74]+[76,77,78,79,47,81]+ [83,84,85,86,87,88]+[90,91,92,93,94,95]+ [96,97,2,1]+[3,46]t	0.99
17	9	5	4	[4,6,7,8,9,10,11,12]+[14,15,2,1]+[3,5]t	0.98
35	9	5	4	[16,7,6,8,9,10,11,12]+[15,5,17,4,19,20,21,22]+ [24,25,26,28,27,29,30,31]+[32,33,2,1]+[3,14]t	0.99
53	9	5	4	[5,6,7,8,9,10,11,12]+[14,15,16,17,18,19,20,24]+ [22,25,26,27,28,29,30,31]+[33,34,35,41,37,21,39,40]+ [38,43,44,45,46,47,48,49]+[50,51,2,1]+[3,23]t	0.99
71	9	5	4	[18,6,7,8,9,10,11,12]+[14,15,16,17,5,19,20,21]+ [33,24,26,25,27,28,29,30]+[23,62,35,36,37,38,39,49]+ [42,43,44,45,46,47,48,64]+[50,51,4,53,54,55,56,57]+ [60,61,34,63,22,65,66,67]+[68,69,2,1]+[3,32]t	0.99
89	9	5	4	[5,6,7,8,9,10,11,12]+[13,14,15,16,17,18,19,43]+ [23,24,25,26,27,28,29,30]+[32,33,34,67,4,37,38,39]+ [42,79,44,45,46,47,48,49]+[50,51,22,53,31,55,56,57]+ [59,60,61,62,63,64,58,66]+[69,70,71,72,73,74,75,76]+ [78,54,80,81,82,83,84,85]+[86,87,2,1]+[3,41]t	0.99
23	7	6	4	[3,4,5,9,7,16]+[17,11,12,13,14,15]+[20,21,2]+[8,10,18,19]t	0.99
37	7	6	4	[3,4,5,6,7,11]+[10,16,12,13,14,15]+[17,18,19,20,21,22]+ [24,26,25,8,9,29]+[34,35,2]+[30,31,32,33]t	0.99
51	7	6	4	[3,4,5,6,7,8]+[11,10,12,13,14,15]+[18,34,19,20,21,22]+ [24,43,26,27,28,29]+[0,31,32,33,38,36]+ [44,35,39,40,41,42]+[48,49,2]+[37,45,46,47]t	0.99
65	7	6	4	[3,4,5,6,7,8]+[10,11,12,13,14,15]+[29,17,19,20,21,22]+ [23,24,25,26,50,28]+[56,32,33,34,35,36]+ [38,39,40,41,18,43]+[44,45,46,61,48,49]+ [52,9,54,55,42,57]+[62,63,2]+[58,59,60,47]t	0.99
79	7	6	4	[3,4,5,6,7,8]+[9,10,34,12,58,13]+[16,17,18,19,15,43]+ [25,24,26,27,22,32]+[31,29,33,37,35,39]+ [14,38,36,40,41,42]+[11,46,47,65,49,50]+ [52,53,54,55,56,48]+[59,60,61,62,63,64]+ [66,67,68,69,70,71]+[76,77,2]+[51,73,74,75]t	0.99
93	7	6	4	[3,18,5,6,7,86]+[10,11,12,13,14,15]+[30,4,19,20,21,22]+ [24,25,26,27,28,31]+[32,16,33,34,49,36]+ [38,35,40,41,42,43]+[45,46,57,48,39,82]+ [9,53,72,55,56,79]+[8,60,61,62,63,64]+ [66,67,52,69,70,71]+[73,74,29,75,77,78]+ [47,80,81,83,84,85]+[90,91,2]+[58,87,88,89]t	0.99
27	9	6	4	[3,5,21,10,14,24,20,22]+[8,9,13,14,15,17,18,19,25]+ [2,4,7,16,23]+[1,12]t	0.99
45	9	6	4	[3,4,5,6,7,20,9,10]+[12,13,14,15,16,17,18,19]+ [21,22,23,28,25,26,27,37]+[29,30,31,32,33,38,35,36]+ [42,43,2]+[34,39,40,41]t	0.99
63	9	6	4	[3,20,5,6,7,8,9,10]+[12,14,13,15,16,17,18,19]+ [21,22,23,24,11,26,27,28]+[30,31,32,33,34,35,36,50]+ [39,40,41,42,37,44,45,46]+[48,49,25,51,52,53,54,55]+ [60,61,2]+[43,57,58,59]t	0.99

81	9	6	4	[3,4,5,6,7,8,9,10]+[12,14,13,15,16,17,18,19]+ [20,21,22,23,24,25,26,27]+[30,29,31,32,33,34,35,59]+ [39,40,41,42,43,44,45,46]+[48,49,50,64,38,53,54,77]+ [56,57,58,72,61,62,63,51]+[66,67,68,69,70,71,11,73]+ [78,79,2]+[74,75,76,55]t	0.99
99	9	6	4	[3,4,5,6,7,8,9,10]+[12,13,14,15,16,17,18,19]+ [21,22,23,24,25,27,26,28]+[52,30,31,32,33,34,35,36]+ [39,40,41,42,43,44,60,45]+[48,49,50,51,83,53,54,55]+ [57,58,59,77,92,62,29,64]+[66,67,68,69,70,71,37,75]+ [73,76,63,78,79,80,81,82]+[84,85,86,87,88,89,56,91]+ [96,97,2]+[61,93,94,95]t	0.99
19	9	7	4	[3,5,4,7,8,6,10,11]+[16,17,2]+[9,12,13,14,15]t	0.98
37	9	7	4	[12,4,5,6,7,8,9,10]+[3,13,14,15,16,17,18]+ [20,21,23,22,24,25,26,27]+[34,35,2]+[0,30,31,32,33]t	0.96 0.96
55	9	7	4	[30,40,5,6,7,8,9,10]+[12,13,14,16,17,15,18,19]+ [21,22,23,24,25,26,27,28]+[3,31,32,33,35,34,36,37]+ [11,39,41,4,42,43,44,46]+[52,53,2]+[45,48,49,50,51]t	0.99
73	9	7	4	[23,4,5,6,7,8,9,10]+[12,13,14,15,16,17,18,28]+ [56,21,22,30,24,25,26,65]+[3,31,32,33,34,35,36,38]+ [37,39,40,41,42,43,44,45]+[48,50,49,51,52,53,27,55]+ [57,58,59,60,61,62,63,64]+[70,71,2]+[54,66,67,68,69]t	0.99
91	9	7	4	[3,4,5,6,7,8,9,10]+[11,13,12,15,16,17,18,21]+ [19,22,23,24,25,26,27,30]+[40,28,31,32,33,34,35,37]+ [20,14,39,41,42,43,44,45]+[48,49,50,51,52,53,56,55]+ [47,59,58,60,61,62,46,64]+[66,65,67,68,69,70,71,81]+ [75,76,77,79,78,80,29,82]+[88,89,2]+[63,84,85,86,87]t	0.99
29	9	8	4	[3,4,5,6,7,12,10,9]+[13,8,14,15,16,17,18,19]+[26,27,2]+ [11,21,22,23,24,25]t	0.99
47	9	8	4	[21,11,5,7,6,8,31,9]+[12,13,28,15,16,17,18,19]+ [20,3,22,23,25,24,26,27]+[30,10,32,33,34,35,37,36]+ [44,45,2]+[38,39,4,41,42,43]t	0.99
65	9	8	4	[3,4,5,6,7,8,9,10]+[11,13,14,15,16,17,18,24]+ [21,22,23,61,25,26,27,31]+[30,54,32,33,34,35,37,36]+ [38,40,41,42,28,44,45,46]+[48,49,50,51,52,53,43,55]+ [62,63,2]+[56,57,58,59,39,19]t	0.99
83	9	8	4	[3,4,5,6,7,8,9,10]+[12,42,14,15,16,17,18,19]+ [21,22,23,24,25,26,27,28]+[29,31,32,33,34,35,36,37]+ [39,40,41,74,43,44,45,46]+[47,49,67,51,52,53,54,55]+ [57,58,60,75,13,62,63,48]+[66,20,68,69,70,71,72,73]+ [80,81,2]+[0,59,76,77,78,79]t	0.99

Acknowledgement

Authors are thankful to the reviewers for their valuable suggestions and corrections.

References

[1] E. R. Williams, Experimental designs balanced for the estimation of residual effects of treatments. Australian Journal of Science Research, A2, 149-168 (1949).
[2] E. R. Williams, Experimental designs balanced for pairs of residual effects. Australian Journal of Science Research, A3, 351-363 (1950).
[3] S. C. Pearce, Experimenting with blocks of natural size. Biometrics, **18**, 699-706 (1964).
[4] S. Kageyman, Construction of balanced block designs. Utilitas Mathematica, **9**, 209-229 (1976).
[5] C. G. Magda, Circular balanced repeated measurements designs. Communications in Statistics-Theory and Methods, **9**, 1901-1918 (1980).
[6] G. Constantine, and A. Hedayat, A construction of repeated measurements designs balance for residual effects. Journal of Statistical Planning and Inference, 6, 153-164 (1982).

[7] K. Afsarinejad, Circular balanced uniform repeated measurements designs. Statistics and Probability Letters, **7**, 187-189 (1989).
[8] K. Afsarinejad, Circular balanced uniform repeated measurements designs, II. Statistics and Probability Letters, **9**, 141-143 (1990).
[9] K. Afsarinejad, Repeated measurements designs with unequal periods sizes. Journal of the Italian Statistical Society, **2**, 161-168 (1994).
[10] I. Iqbal, and B. Jones, Efficient repeated measurements designs with equal and unequal period sizes. Journal of Statistical Planning and Inference, **42**, 79-88 (1994).
[11] V. K. Sharma, S. Jaggi, and C. Varghese, Minimal balanced repeated measurements designs. Journal of Applied Statistics, **30**, 867-872 (2003).
[12] I. Iqbal, and M. H. Tahir, Circular strongly balanced repeated measurements designs. Communications in Statistics-Theory and Methods, **38**, 3686-3696 (2009).
[13] V. K. Sharma, Y. Gharde, and C. Varghese, (Minimal strongly balanced changeover designs with first residuals. African Journal of Mathematics and Computer Science Research, **3**(9), 195-198 2010).
[14] I. Iqbal, M. H. Tahir, and S. S. A. Ghazali, Circular first-and second-order balanced repeated measurements designs. Communications in Statistics-Theory and Methods, **39**, 228-240 (2010).
[15] R. A. Bailey, P. J. Cemron, K. Fillipiak, J. Kunert, and A. Markiewicz, On optimality and construction of circular repeated measurements designs. Statistica Sinica, **27**, 1-22 (2017).
[16] Z. Bashir, R. Ahmed, M. H. Tahir, S. S. A. Ghazali, and F. Shehzad, Some extensions of circular balanced and circular strongly balanced repeated measurements designs. Communications in Statistics - Theory and Methods, **47**(9), 2183-2194 (2018).
[17] M. Rajab, R. Ahmed, F. Shehzad, and M. H. Tahir, Some new constructions of circular balanced repeated measurements designs. Communications in Statistics -Theory and Methods, **47**(17), 4142-4152 (2018).
[18] U. Rasheed, H. M. K. Rasheed, M. Rasheed, and R. Ahmed, Minimal circular strongly balanced repeated measurements designs in periods of three different sizes. Communications in Statistics - Theory and Methods, **47**(16), 4088-4094 (2018).
[19] M. Daniyal, R. Ahmed, F. Shehzad, M. H. Tahir, and Z. Iqbal, Construction of repeated measurements designs strongly balanced for residual effects. Communications in Statistics-Theory and Methods, **49**(17), 4288-4297 (2020).
[20] R. Ahmed, F. Shehzad, M. Rajab, M. Daniyal, and M. H. Tahir, Minimal circular balanced repeated measurements designs in periods of unequal sizes. Communications in Statistics-Theory and Methods, **48**(21), 5223-5232 (2019).
[21] H. M. K. Rasheed, M. Rasul, R. Ahmed, M. Batool, M. H. Tahir, and F. Shehzad, Circular balanced repeated measurement designs in periods of three different sizes. Communications in Statistics-Simulation and Computation, **48**(10), 3022-3030 (2019).
[22] A. Khan, R. Ahmed, F. Shehzad, M. H. Tahir, and S. S. Ghazali, A. Construction of circular partially-balanced repeated measurement designs using cyclic shifts. Communications in Statistics-Simulation and Computation, **48**(2), 506-515 (2019).
[23] R. Jabeen, H. M. K. Rasheed, R. Ahmed, and F. Shehzad, Construction of circular strongly partially-balanced repeated measurements designs. Journal of King Saud University- Science, **31**, 345-351 (2019).
[24] H. M. K. Rasheed, H. Khan, R. Ahmed, F. Jamal, Minimal circular nearly strongly balanced repeated measurements designs in unequal period sizes. Proceedings of the Pakistan Academy of Sciences: A: Physical and Computational Sciences, **58**(4), 59-65 (2022).
[25] J. Divecha, and J. Gondaliya, Construction of minimal balanced crossover designs having good efficiency of separability. Electronic Journal of Statistics, **8**, 2923-2936 (2014).
[26] I. Iqbal, Construction of experimental designs using cyclic shifts. Unpublished Ph.d Thesis. U.K: University of Kent at Canterbury (1991).