Markov Models on Share Price Movements in Nigeria Stock Market Capitalization

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Abstract: The stock market’s trends can impact companies in different ways. The rise and fall of share price values affect a company’s market capitalization and therefore its market value, and are exposed to market risk. This study assumed share volatility as a stochastic process with Markov property. Thus proposed a first order, time homogenous Markov chain model for trend prediction of two banks; that is Fidelity bank and Access bank closing share prices from 1-4-2016 to 23-03-2022. The prediction was done by establishing three states that exist in stock price change which are share prices increase, decline (decrease) or steady. The transition matrix was generated. The powers of transition matrices and probability vectors were also generated for some years and equilibrium was attained.

Keywords: Markov model, Transition Matrix, Markov chain, Stock market, Initial probability matrix

1 Introduction

A stock market is a type of financial market where shares of publicly owned companies can be bought and sold. In a study, \cite{1} defined a stock market as an established legitimate structure encompassing the trading of shares of many companies or organizations.

Similarly, in another study, \cite{2} pointed out that a Stock market constitutes exchanges and markets where different financial instruments like equity, stocks, bonds etc. are provided and traded. The financial markets are exposed to market risk, involving up and down movements of price, with different frequency and magnitude.

Furthermore, Volatility is widely identified as a strong representative for risk. Owing to the fact that accurate volatility prediction plays an important role in financial study, \cite{3} on the other hand, in a study of volatility forecasting revisited Markov switching with time-varying probability transition. In the same study markov switching heterogenous Auto-regressive (MS-HAR) model with jump- driven time- varying transition probabilities were used to predict the future volatility in Chinese stock market and the result showed that MS-HAR models for in-sample results are more powerful than HAR- RV-type models.

Different studies applied the Markov chain model, just as in the study of stock price prediction using Markov chain model: a study for TCS share values, \cite{4}, applied a Markov chain model in analyzing the stock market movement and forecasting its share prices. The result showed that the Markov chain model is an ideal statistical method of prediction to analyze and predict the future behaviour of stock market through initial state probability vector. In another study on the use of Markov chain model in predicting the daily trend of various global stock indices \cite{5} investigated the daily trend of various stock indices and compared the results with that of traditional forecasting methods such the Moving average and the Trend projection methods. The result showed that the Markov model out-performed the traditional models used in the study during the three time periods.

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Also [6] examined the application of Markov chain model to a historical share price data set of a banking company HBL, the result showed that the data set behaved with Markovian property and exhibited periodicity validated by convergence of the transition probability matrix to a steady state distribution thereby proving that Markov chain model can be applied to shares traded on Pakistan stock exchange.

Finally, having looked into several studies on volatility and the models applied in forecasting it, this study focuses on forecasting volatility and stock prices via Markov chain model for trend prediction of two banks; that is the Fidelity Bank and the Access Bank closing share prices from 1-4-2016 to 23-03-2022

2 Methods

To understand the Markov chain we discuss the stochastic process. Thus, a stochastic process is a system where there are observations at certain times and the outcomes, which are the observed values at each time, are random variables. A stochastic process is one where a random variable evolves over time. There are two ideas of time, the discrete and the continuous. Discrete time is countable, whilst the continuous time is not. The discrete time stochastic process is a stochastic process in which the state of the system can be observed at discrete instants in time. On the contrary the states of a continuous-time stochastic process can be observed at any instant in time. For simplification purposes, we will utilize the power of a discrete-time stochastic process for our Markov chain model.

Definition 2.1: Let \((\Omega, \mathcal{F}, P)\) be a probability space and let \(T\) be an arbitrary set (called the index set). Any collection of random variables \(X = \{X_t : t \in T\}\) defined on \((\Omega, \mathcal{F}, P)\) is called a stochastic process with index set \(T\).

Definition 2.2: A stochastic process \(X = (X_n, n \in N)\) in a countable space is a discrete time Markov chain if for all \(n \geq 0, X_n \in S\), for all \(n \geq 1\) and for all \(i_0, i_{n-1}, i_n \in S\) one has \(\{X_{i_0} = i_0 | X_{i_1} = i_1, ..., X_0 = i_0\} = P\{X_n = i_0 | X_{i_1} = i_1\}\).

Definition 2.3 [7]: Let \((\Omega, \mathcal{F}, P)\) be a probability space. Let \(\{\mathcal{F}_k\}_{k=0}^\infty\) be a filtration under \(\mathcal{F}\). Let \(\{X_k\}_{k=0}^\infty\) be a stochastic process on \((\Omega, \mathcal{F}, P)\). This process is said to be Markov if:

(i) The stochastic process \(\{X_k\}\) is adapted to the filtration \(\{\mathcal{F}_k\}\), and

(ii) (The Markov Property). For each \(k = 0, 1, ..., n-1\), the distribution of \(X_{k+1}\) conditioned on \(\mathcal{F}_k\) which is the same thing as \(X_{k+1}\) conditioned on \(X_k\)

Theorem 2.1:
Let
\[
d\xi(t) = f(\xi(t), t)dt + \tau(\xi(t), t)dB(t) \quad t \geq 0
\]
be Itô equation whose coefficients satisfy the conditions of the existence and uniqueness theorem, and by making use of the notation;
\[
P(\xi(t) \in A | \xi(s) = x) = P(x, s; A, t), \quad \text{let } \xi(t) = X_0 \text{ be a solution of (1). Then } \xi(t) \text{ is a Markov process whose transition probability is defined by}
\[
P(x, s; A, t) = P\{\xi(t) \in A\},
\]
where \(\xi_{s,t}(x)\) is the solution of the equation
\[
\xi_{s,t}(x) = x + \int_s^t f(\xi_{s,r}(x), r)dr + \int_s^t \tau(\xi_{s,r}(x), r)dB(r) \quad t \geq s.
\]

Proof.
Let \(h(x, w)\) be a scalar bounded measurable random function of \(x\) independent of \(\mathcal{F}_s\). Let \(\xi\) be an \(\mathcal{F}_t\) - measurable random variable. Then
\[
E(h(\xi, w)|\mathcal{F}_s) = H(\xi)
\]
where \(H(x) = Eh(x, w)\). Now assume that \(h(x, w)\) has the following simple form
\[
h(x, w) = \sum_{i=1}^k u_i(x)v_i(w)
\]
with \( u_i(x) \)'s as bounded deterministic functions of \( x \) and \( v_i(w) \)'s are bounded random variables independent of \( \mathcal{F}_s \). Clearly,

\[
H(x) = \sum_{i=1}^{k} u_i(x) E V_i(w).
\]

Moreover, for any set \( G \in \mathcal{F}_s \), we compute

\[
E[h(\xi, w)IG] = E \left( \sum_{i=1}^{k} u_i(\xi) v_i(w) IG \right) = \sum_{i=1}^{k} E[u_i(\xi)IG] E V_i(w) \]

\[
= E \left( \sum_{i=1}^{k} u_i(\xi) E V_i(w) IG \right) = E[H(\xi)IG].
\]

By definition, this means that (4) holds if \( h(x, w) \) has the form of (5). Since any bounded measurable random function \( h(x, w) \) can be approximated by functions of form (5), the general result of the lemma follows immediately.

Again let \( G_s = \sigma\{B(r) - B(s) : r \geq s\} \). Clearly, \( G_s \) is independent of \( \mathcal{F}_s \). Moreover, the value of \( \xi_i(s, t) \) depends completely on the increments \( B(r) - B(s) \) for \( r \geq s \) and so is \( G_s \) measurable. Hence, \( \xi_i(s, t) \) is independent of \( \mathcal{F}_s \). On the other hand, note that \( \xi_i(t) = \xi_{i,s}(s, t) \) on \( t \geq s \), since both \( \xi(t) \) and \( \xi_{i,s}(s, t) \) satisfy the equation

\[
\xi_i(t) = \xi_i(s) + \int_{s}^{t} f(\xi(r), r) dr + \int_{s}^{t} g(\xi(r), r) dB(r)
\]

whose solution is unique. For any \( A \in B^d \), with \( h(x, w) = I_A(\xi_{i,s}(s, t)) \) to compute that

\[
P(\xi_i(t) \in A|\mathcal{F}_s) = E[I_A(\xi_i(t))|\mathcal{F}_s] = E[I_A(\xi_{i,s}(s, t))|\mathcal{F}_s]
\]

\[
= E[I_A(\xi_{i,s}(s, t))|_{x=\xi_i(s)}]
\]

\[
= P(x, s; A, t) = P(\xi_i(s), s; A, t)
\]

if \( P(x, s; A, t) \) is defined by (2).

This completes the proof.

**Definition 2.4:** A Markov model is a stochastic method for randomly changing systems that possess the Markov property. This means that, at any given time, the next state is only dependent on the current state and is independent of anything in the past.

There are different Markov models, namely: the Markov chain, hidden Markov model, Markov decision process, partially observable Markov decision process, Markov random field, hierarchical Markov model and the tolerant Markov model.

**Definition 2.5:** Any random process that satisfies Markov property is called a Markov process. A Markov property can be stated as: the state at time \( t + 1 \) of the process wholly depends on its immediate past state that is on the state at time \( t \).

Mathematically, if \( \{X_t, t \geq 0\} \) is a sequence of events, then

\[
P[X_{t+1} = s|X_t = s, X_{t-1} = s, ..., X_1 = s_1, X_0 = s_0]
\]

\[
= P[X_{t-1} = s|X_t = s] = p_{ij} \geq 0,
\]

where \( \sum_{j=1}^{d} p_{ij} = 1 \). Such type of the random process is termed as a discrete time Markov chain, where \( s_0, s_1, s_2, ..., s_d \) are the states of the Markov chain in the state space \( S \) and the probability \( P_{ij} \) is called transition or Markov chain probability.

The conditional probability that \( X_{t+1} = j \) given the current state \( X_t = i \); the probability of moving from state \( i \) to state \( j \) related to the conditional expectation \( E(X|G) \). That is if \( G \) is the \( \sigma \) - algebra generated by a random variable \( Y \), i.e. \( G = \sigma\{Y\} \), we write \( E(X|G) = E(X|Y) \). If \( X \) is the indicator function of set \( A \), we write \( E(I_A(G)) = P(A|G) \).

A transition probability matrix for our three states will be written as

\[
p(P_{ij}) = \begin{bmatrix}
p_{11} & p_{12} & p_{13} \\
p_{21} & p_{22} & p_{23} \\
p_{31} & p_{32} & p_{33}
\end{bmatrix}
\]
The three state transition probability matrix with state space \(E(D, U, I)\), the following table shows the transitions that each probability \(P_{ij}\) in the aforementioned transition probability matrix shows, for example \(P_{11}\) is a transition from a decrease \(I\) share price to a decrease in share price the next day.

This study covered two banks listed on the Nigerian Stock exchange. The data on share price of the two banks namely Fidelity Bank plc (FBplc), and Access Bank plc (ABplc), were collected from the daily list published by the Nigeria Stock Exchange from 2016 to 2022. The transition from one state to another (that is the share price trend, which could be that a decline in price can be followed by another decline or a decline is followed by steady or a decline followed by an increase etc) was observed from the data collected and the result for each bank for the period (7 years).

2.1 Construction of Markov Chain Model

To check accuracy of the Markov chain model as a model for prediction, a Markov chain model needs to be constructed for predicting the movement of the share prices. Constructing of a Markov chain Model will include: holding and doing initial analysis of the data set for the particular period, movement of the closing share prices occurs and is presumed to be in three states: Share prices Increase = \(I\), Share prices Decline = \(D\), and Share prices remain steady = \(S\)

\(D\) = Bank share price declines  
\(S\) = Bank share price remains the same  
\(I\) = Bank share price increases.

The state space of the Markov model can be written as \((D, S, I)\). As earlier mentioned, initial state vector, also known as initial state distribution can be defined as:

\[
\pi_0 = [\pi_1(i_1), \pi_2(i_2), \pi_3(i_3)].
\]

Where \(\pi_1(i_1), \pi_2(i_2), \pi_3(i_3)\), provide the Increase \((I)\), Decline \((D)\), and steady \((S)\) respectively in the probability of the closing share prices for the two banks.

3 Deriving the State Probability Matrix

Based on the Markov chain model, the state probabilities is solved by multiplying initial probability matrix with the transition probability matrix. More so, the initial probability matrix detects where the bank share price begins. And this is written as

\[
\pi(1) = \pi(0)p \\
\vdots \\
\vdots \\
\pi(n+1) = \pi(n)p
\]

\(n\) -step Transition Matrix.

The absolute probabilities at any level where \(n\) is more than unity, was decided by using the \(n\) - step transition probabilities. This is a higher order transition probability \(P_{ij}^{(n)}\) of the transition matrix \(P_{ij}\). The \(n\) - step matrix shows the behavior of prices \(n\) - steps later. The components of this matrix constitute the probabilities that an object in a given state will be in the next state \(n\) - steps later. These frequent transitions were used to evaluate whether the transition probabilities converge over repeated iterations that is \(\lim_{n \to \infty} P_{ij}^{(n)}\), in matrix terms \([1]\).Let \(p\) be the transition matrix of the Markov chain, then

\[
p^1 = pp^{(0)} \text{ (for } n = 1\text{)} \text{ and } \quad p^2 = pp^{(1)} = pp^{(0)} = p^2p^{(0)} \text{ (for } n = 2)\]
Generally, $p^n = p^n p^{(0)}$.

Matrix of transition probabilities brings about exact explanation of the behavior of a Markov chain. Every element in the matrix stands for the probability of the transition from a particular state to the next state. The transition probabilities are usually empirically considered, that is to say, it is entirely based on experiment and observation rather than theory. On the other hand, it depends on practical experience without reference to scientific principles. Historical data collected can be translated to probability that make up the Markov matrix of probabilities. To compute the probability matrix for Markov process with three states, one can construct a table as shown below.

### Table 1. Transition Matrix

<table>
<thead>
<tr>
<th>State</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Sum of row</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_{11}$</td>
<td>$p_{12}$</td>
<td>$p_{13}$</td>
<td>$T_1$</td>
</tr>
<tr>
<td>2</td>
<td>$p_{21}$</td>
<td>$p_{22}$</td>
<td>$p_{23}$</td>
<td>$T_2$</td>
</tr>
<tr>
<td>3</td>
<td>$p_{31}$</td>
<td>$p_{32}$</td>
<td>$p_{33}$</td>
<td>$T_3$</td>
</tr>
</tbody>
</table>

Every entry $p_{ij}$ in the table 1 above indicates the number of times a transition has taken place from state $i$ to state $j$. The transition matrix is constituted by dividing every element in each row by the sum of each row.

This study covered two banks listed on the Nigerian Stock Exchange. The data on share price of the two banks namely Fidelity Bank PLC, and Access Bank PLC, were collected from the daily list published by the Nigeria Stock Exchange from 2016 to 2022.

The transition from one state to another state can be seen, for example, when there is a price decline it can be followed by another decline or decline is followed by steady or decline followed by an increase and so on. The transition observed from the data collected, with the result from each bank is as follows (as in tables 2, 3 and 4 below):

### Table 2. Transition Matrix of share price of Fidelity Bank PLC from 2016 - 2022

<table>
<thead>
<tr>
<th>Decline in share price $(D)$</th>
<th>Steady share price $(S)$</th>
<th>Increase in share price $(I)$</th>
<th>Sum of rows</th>
</tr>
</thead>
<tbody>
<tr>
<td>415</td>
<td>62</td>
<td>138</td>
<td>615</td>
</tr>
<tr>
<td>Steady share price $(S)$</td>
<td>61</td>
<td>121</td>
<td>263</td>
</tr>
<tr>
<td>Increase in share price $(I)$</td>
<td>139</td>
<td>80</td>
<td>603</td>
</tr>
</tbody>
</table>

### Table 3. Transition Matrix of share price of Access Bank PLC from 2016 - 2022

<table>
<thead>
<tr>
<th>Decline in share price $(D)$</th>
<th>Steady share price $(S)$</th>
<th>Increase in share price $(I)$</th>
<th>Sum of rows</th>
</tr>
</thead>
<tbody>
<tr>
<td>410</td>
<td>80</td>
<td>126</td>
<td>616</td>
</tr>
<tr>
<td>Steady share price $(S)$</td>
<td>79</td>
<td>98</td>
<td>269</td>
</tr>
<tr>
<td>Increase in share price $(I)$</td>
<td>127</td>
<td>91</td>
<td>596</td>
</tr>
</tbody>
</table>
Table 4. Transition Matrix of share price of the two banks merged from 2016 -2022

<table>
<thead>
<tr>
<th></th>
<th>Decline in share price (D)</th>
<th>Steady share price (S)</th>
<th>Increase in share price (I)</th>
<th>Sum of rows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decline in share price (D)</td>
<td>825</td>
<td>142</td>
<td>264</td>
<td>1231</td>
</tr>
<tr>
<td>Steady share price (S)</td>
<td>140</td>
<td>219</td>
<td>173</td>
<td>532</td>
</tr>
<tr>
<td>Increase in share price (I)</td>
<td>266</td>
<td>171</td>
<td>762</td>
<td>1199</td>
</tr>
</tbody>
</table>

Similarly, from the share price movement merged and the transition probabilities calculated, the transition matrix for each bank is shown below as:

**Transition Matrix for Access Bank**

\[
P = \begin{pmatrix}
410 & 80 & 126 \\
816 & 616 & 816 \\
79 & 88 & 92 \\
127 & 91 & 378 \\
596 & 596 & 596 \\
\end{pmatrix} = \begin{pmatrix}
0.6656 & 0.1299 & 0.2045 \\
0.2937 & 0.3643 & 0.3420 \\
0.2131 & 0.1527 & 0.6342 \\
\end{pmatrix}
\]

With values for every movement of the vector of Access bank as follows

\[D = 0.6656\]
\[S = 0.1299\]
\[I = 0.2045\]

This indicates that, with a probability of:

(i) 0.6656, Bank’s share price that declines or decreases will still decrease
(ii) 0.1299, Bank’s share price that declines will still remain steady or same.
(iii) 0.2045, Bank’s share price that declines will still increase.

\[S = 0.2937\]
\[S = 0.3643\]
\[I = 0.3420\]

This indicates that, with a probability of:

(i) 0.2937, Bank’s share price that is steady will decline.
(ii) 0.3643, Bank’s share price that is steady will remain steady or same.
(iii) 0.3420, Bank’s share price that is steady will still increase.

This indicates that, with a probability of:

(i) 0.2131, Bank’s share price that increases will still decline.
(ii) 0.1527, Bank’s share price that increases will remain steady.
(iii) 0.6342, Bank’s share price that increases still increases.

**Transition Matrix for Fidelity Bank**

\[
P = \begin{pmatrix}
0.615 & 0.615 & 0.1231 \\
0.1231 & 0.1231 & 0.264 \\
0.264 & 0.264 & 0.613
\end{pmatrix}
= \begin{pmatrix}
0.6748 & 0.1008 & 0.2244 \\
0.2319 & 0.4601 & 0.3080 \\
0.2305 & 0.1327 & 0.6368
\end{pmatrix}
\]

With values for every movement of the vector of Fidelity Bank as follows:

This indicates that, with a probability of:

(i) 0.6748, Bank’s share price that declines or decreases will still decrease
(ii) 0.1008, Bank’s share price that declines will still remain steady or same.
(iii) 0.2244, Bank’s share price that declines will still increase.

Only the description of decline is presented here for Fidelity Bank, to reduce space.

**Transition Matrix of the two Banks merged**

\[
P = \begin{pmatrix}
0.615 & 0.615 & 0.1231 & 0.1231 & 0.264 \\
0.1231 & 0.1231 & 0.264 & 0.264 & 0.613 \\
0.264 & 0.264 & 0.613 & 0.615 & 0.615 \\
0.1231 & 0.1231 & 0.264 & 0.264 & 0.613 \\
0.615 & 0.615 & 0.1231 & 0.1231 & 0.264
\end{pmatrix}
= \begin{pmatrix}
0.6702 & 0.1154 & 0.2145 & 0.0263 & 0.4117 & 0.3252 \\
0.2632 & 0.4117 & 0.3252 & 0.2632 & 0.4117 & 0.3252 \\
0.2219 & 0.1426 & 0.6355 & 0.2219 & 0.1426 & 0.6355
\end{pmatrix}
\]

To throw more light on the price movement of these banks the transition diagraph of the banks are drawn and shown below:
**4 Behavior of Share Price Trend**

The higher power transition probability $p_{ij}^{(n)}$ of the transition matrix $p_{ij}$ of every bank was computed so that the behavior of share price is observed and the results were obtained using R statistical software. These are shown below.
4.1 Powers of the Transition Matrix for Access Share Price

\[ p^2 = \begin{bmatrix} 0.525 & 0.165 & 0.310 \\ 0.375 & 0.223 & 0.402 \\ 0.322 & 0.180 & 0.498 \end{bmatrix} \]

\[ p^5 = \begin{bmatrix} 0.425 & 0.181 & 0.394 \\ 0.413 & 0.182 & 0.404 \\ 0.407 & 0.182 & 0.410 \end{bmatrix} \]

\[ p^{10} = \begin{bmatrix} 0.416 & 0.182 & 0.402 \\ 0.416 & 0.182 & 0.402 \\ 0.322 & 0.182 & 0.403 \end{bmatrix} \]

\[ p^{15} = \begin{bmatrix} 0.416 & 0.182 & 0.402 \\ 0.416 & 0.182 & 0.402 \\ 0.416 & 0.182 & 0.402 \end{bmatrix} \]

\[ p^{20} = \begin{bmatrix} 0.416 & 0.182 & 0.402 \\ 0.416 & 0.182 & 0.402 \\ 0.416 & 0.182 & 0.402 \end{bmatrix} \]

\[ p^{25} = \begin{bmatrix} 0.416 & 0.182 & 0.402 \\ 0.416 & 0.182 & 0.402 \\ 0.416 & 0.182 & 0.402 \end{bmatrix} \]

4.2 Powers of the Transition Matrix for Fidelity Share Price

\[ p^2 = \begin{bmatrix} 0.530 & 0.144 & 0.325 \\ 0.334 & 0.276 & 0.390 \\ 0.333 & 0.169 & 0.498 \end{bmatrix} \]

\[ p^5 = \begin{bmatrix} 0.425 & 0.175 & 0.340 \\ 0.408 & 0.182 & 0.410 \\ 0.408 & 0.178 & 0.413 \end{bmatrix} \]

\[ p^{10} = \begin{bmatrix} 0.415 & 0.178 & 0.407 \\ 0.415 & 0.178 & 0.407 \\ 0.415 & 0.178 & 0.407 \end{bmatrix} \]

\[ p^{15} = \begin{bmatrix} 0.415 & 0.178 & 0.407 \\ 0.415 & 0.178 & 0.407 \\ 0.415 & 0.178 & 0.407 \end{bmatrix} \]

\[ p^{20} = \begin{bmatrix} 0.415 & 0.178 & 0.407 \\ 0.415 & 0.178 & 0.407 \\ 0.415 & 0.178 & 0.407 \end{bmatrix} \]

\[ p^{25} = \begin{bmatrix} 0.415 & 0.178 & 0.407 \\ 0.415 & 0.178 & 0.407 \\ 0.415 & 0.178 & 0.407 \end{bmatrix} \]
### 4.3 Powers of the Transition Matrix for Two Banks Merged

$$
p^2 = \begin{bmatrix}
0.527 & 0.155 & 0.317 \\
0.357 & 0.246 & 0.397 \\
0.327 & 0.175 & 0.498
\end{bmatrix}
$$

$$
p^5 = \begin{bmatrix}
0.425 & 0.178 & 0.397 \\
0.412 & 0.182 & 0.407 \\
0.408 & 0.181 & 0.412
\end{bmatrix}
$$

$$
p^{10} = \begin{bmatrix}
0.416 & 0.180 & 0.405 \\
0.416 & 0.180 & 0.405 \\
0.416 & 0.180 & 0.405
\end{bmatrix}
$$

$$
p^{15} = \begin{bmatrix}
0.416 & 0.180 & 0.405 \\
0.416 & 0.180 & 0.405 \\
0.416 & 0.180 & 0.405
\end{bmatrix}
$$

$$
p^{20} = \begin{bmatrix}
0.416 & 0.180 & 0.405 \\
0.416 & 0.180 & 0.405 \\
0.416 & 0.180 & 0.405
\end{bmatrix}
$$

$$
p^{25} = \begin{bmatrix}
0.416 & 0.180 & 0.405 \\
0.416 & 0.180 & 0.405 \\
0.416 & 0.180 & 0.405
\end{bmatrix}
$$

From the result in 4.3, it is observed that after period ten (10) years equilibrium is reached.

The following statements can thus be obtained.

(i) The share price of a bank that will rise after initially falling is 0.405. Or a bank’s share price will rise by 0.405 after initially falling.

(ii) The share price of a bank that will not change despite the initial increase is 0.180. Or Despite the initial increase, a bank’s share price will remain unchanged at 0.180.

(iii) The share price of a bank that will fall if it initially remains same is 0.416. Or 0.416 is the decline in a bank’s share price if it initially stays the same.

The above equilibrium state can be used to calculate additional probabilities; when there is movement from one state to another and when there is no movement at all. Assuming one does not know where the bank share price begins, one can bring in an initial probability vector.

$$(P_1^{(0)}, P_2^{(0)}, P_3^{(0)})$$

This gives $n$ levels of transition probabilities [2].

When a bank’s stock price starts with a certain state probability, $P_1^{(0)} (0.334, 0.334, 0.334)$, then the probability of the share price of the bank increasing after 20 years is seen below

$$(0.334, 0.334, 0.334) \begin{bmatrix}
0.416 & 0.180 & 0.405 \\
0.416 & 0.180 & 0.405 \\
0.416 & 0.180 & 0.405
\end{bmatrix}$$

In other words, if the bank’s share price started with a probability of 0.334, the probability of the share price going up after 10 years is 0.405. Though we have an estimated result, however, one can observe that the probability of the share price trend off the two banks goes on to increase till equilibrium, thereafter it became consistent. More so, if any of the bank share price begins in a particular state with $P_1^{(0)} (0.334, 0.334, 0.334)$, then the probability of their share prices rising, falling and going steady at a given time can be computed by multiplying the state vector by the higher probability at a time. For example, the probabilities of Access bank share price increasing, declining and remaining steady after 2 years is

$$(0.334, 0.334, 0.334) \begin{bmatrix}
0.525 & 0.165 & 0.310 \\
0.375 & 0.223 & 0.402 \\
0.322 & 0.180 & 0.498
\end{bmatrix} = (0.408, 0.190, 0.404)$$
That is to say the probability of Access bank share price increasing after 2 years is 0.404, the probability of the bank share price declining is 0.408 and the probability of the bank share price remaining steady is 0.190. The probabilities of two banks in Nigeria stock market rising were calculated for selected years from now and the result is shown in the tables 5, 6, 7 and 8 below.

Table 5: The Probabilities of Access bank and Fidelity Bank in the Nigeria stock Market increasing

<table>
<thead>
<tr>
<th>Years from now (period)</th>
<th>Access</th>
<th>Fidelity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.4041</td>
<td>0.4051</td>
</tr>
<tr>
<td>5</td>
<td>0.4035</td>
<td>0.3884</td>
</tr>
<tr>
<td>10</td>
<td>0.4031</td>
<td>0.4078</td>
</tr>
<tr>
<td>15</td>
<td>0.4028</td>
<td>0.4078</td>
</tr>
<tr>
<td>20</td>
<td>0.4028</td>
<td>0.4078</td>
</tr>
<tr>
<td>25</td>
<td>0.4028</td>
<td>0.4078</td>
</tr>
</tbody>
</table>

Table 6: The Probabilities of Access bank and Fidelity Bank in the Nigeria stock Market Remaining Steady

<table>
<thead>
<tr>
<th>Selected Years from now (period)</th>
<th>Access</th>
<th>Fidelity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.1897</td>
<td>0.1967</td>
</tr>
<tr>
<td>5</td>
<td>0.1820</td>
<td>0.1787</td>
</tr>
<tr>
<td>10</td>
<td>0.3928</td>
<td>0.1784</td>
</tr>
<tr>
<td>15</td>
<td>0.3928</td>
<td>0.1784</td>
</tr>
<tr>
<td>20</td>
<td>0.3928</td>
<td>0.1784</td>
</tr>
<tr>
<td>25</td>
<td>0.3928</td>
<td>0.1784</td>
</tr>
</tbody>
</table>

Table 7: The Probabilities of Access bank and Fidelity Bank in the Nigeria stock Market Declining

<table>
<thead>
<tr>
<th>Selected Years from now (period)</th>
<th>Access</th>
<th>Fidelity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.4081</td>
<td>0.3998</td>
</tr>
<tr>
<td>5</td>
<td>0.4158</td>
<td>0.4145</td>
</tr>
<tr>
<td>10</td>
<td>0.4168</td>
<td>0.4158</td>
</tr>
<tr>
<td>15</td>
<td>0.4168</td>
<td>0.4158</td>
</tr>
<tr>
<td>20</td>
<td>0.4168</td>
<td>0.4158</td>
</tr>
<tr>
<td>25</td>
<td>0.4168</td>
<td>0.4158</td>
</tr>
</tbody>
</table>

Table 8: The Representation of the two Banks merged

<table>
<thead>
<tr>
<th>Selected years from now (period)</th>
<th>Probabilities of banks' share price increasing</th>
<th>Probabilities of banks' share price remaining steady</th>
<th>Probabilities of banks' share price declining</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.4048</td>
<td>0.1924</td>
<td>0.4045</td>
</tr>
<tr>
<td>5</td>
<td>0.4061</td>
<td>0.1807</td>
<td>0.4158</td>
</tr>
<tr>
<td>10</td>
<td>0.4058</td>
<td>0.1804</td>
<td>0.4168</td>
</tr>
<tr>
<td>15</td>
<td>0.4058</td>
<td>0.1804</td>
<td>0.4168</td>
</tr>
<tr>
<td>20</td>
<td>0.4058</td>
<td>0.1804</td>
<td>0.4168</td>
</tr>
<tr>
<td>25</td>
<td>0.4058</td>
<td>0.1804</td>
<td>0.4168</td>
</tr>
</tbody>
</table>

5 Discussion and Conclusion

Following the obtained Matrices for each bank and the merged banks one can predict the probability of going from one given state to another for a transition. Despite the current share price, one can forecast that eventually the share price will decline with a probability of 0.4168, remain steady with a probability of 0.1804 then increases with probability of 0.4058. Or it shows that in spite of the stocks’ most recent changes in closing price, approximately 41.68% of the time they will drop in value tomorrow, 18.04% will maintain the value and 40.58% will have increases in price.
In Table 8, one observed that the probability of share price of two banks in the Nigerian Share Rising (NSR) had an increase and then dropped, then after 10 years it became constant. However, despite the bank current price today that is either of the aforementioned banks, the probability of its shares price increasing in the next 10 to 15 years is approximately 0.4. The probability of a bank’s share declining is still approximately 0.4 and this indicates that any investor buying a share today has equal chances of the share price increasing and declining in the next 10-15 years.

Similarly, the Table 5 shows that the probability of each bank share price rising for the Fidelity bank, dropped after 2 years and increased afterwards till equilibrium was attained after 10 years, then became constant. While Access bank had a decrease then equilibrium was reached after 15 years it became constant.

On the other hand, for a share price of a company to remain steady for a long period of time shows a bad sign and affects the company’s performance. Looking at the probability of the banks, we would see that the share price was not steady, however became steady for the remaining years. Furthermore, the probability of fidelity share price being steady is 0.174 compared to that of Access, indicating that fidelity bank alternates more than the Access bank.

The Markov property means that given a Markov process, the past and future are independent when the present is known, showing that the share price process is Markov.

Furthermore, a Markov transition matrix is a square matrix describing the probabilities of moving from one state to another in a dynamic system. In each row are the probabilities of moving from the state represented by that row, to the other states. Thus the rows of a Markov transition matrix each sum to one.

Figure 1, 2 and 3 show the state transition diagram for the Markov chain. In the diagrams there are three possible states Decline, Steady and Increase and the arrows from each state to other states show the transition probabilities $P_{ij}$. When there is no arrow from state $i$ to state $j$ it means that $P_{ij} = 0$. That is to say, that all states communicate with each other because there exists a non-zero probability to go from each state to another. Hence, transition probability shows that the Markov chain is communicate with each other.

References


