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Estimation of the Weibull Distribution Parameters and Reliability Using Kernel and Bayes Approaches

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Abstract: A new estimation technique based on the non-parametric kernel density estimation is introduced as an alternative and reliable technique for estimation in life testing models. This technique estimates the density functions of the parameters and reliability directly from the data without any prior assumptions about the underlying distribution parameters. The efficiency of this technique has been studied comparing to the Bayesian estimation of the parameters and reliability of the Weibull distribution based on the non-informative, informative and the informative conjugate priors, via Monte Carlo simulations, which indicated the robustness of the proposed method than the Bayesian approach. Finally, a numerical example is given to illustrate the densities and the inferential methods developed in this paper.

Keywords: Kernel density estimation; Bayes estimation; Non informative and informative conjugate priors.

1 Introduction

In statistical inference, it is quite rare to have only one plausible model for representing some experimental data set, instead there are several to choose from, thus in such situations one can be chosen on the grounds of goodness-of-fit tests and convenience. This problem became the most important challenging the statisticians in the last two decades. Moreover, in physical science problems, the data base is usually very small and that often a given data set is censored, in these cases the statistical conclusion on model parameters is nevertheless desired.

To overcome on this difficulty, Bayesian analysis advocated, which enables the utilization of prior information, that is, information other than data, such as past experience with a related phenomenon or conjectures about possible values of parameters or the physical meaning of such parameters. However, in most scientific applications, unfortunately no physical meaning can be usefully exploited in formulating prior knowledge on the parameters and exactly known objective prior distributions are rarely available, in this cases Bayes estimation could be performed by using non-informative priors. Many authors used the non-informative prior, among of them [1,2]. [3] derived an informative conjugate prior by assuming both parameters have gamma distribution.Recently, some authors used the technical information about the real systems or the reliability level at a given time and converted into degree of belief about the model parameters that improved the estimations, see [4,5]. But, actually even though the disadvantage of Bayesian inference is that, different priors when combined with the same likelihood produced different posterior distributions,in such situations different decisions can result from different choices of prior distributions and that has worried some statisticians.

In this paper, efforts have been devoted in finding a suitable estimation technique using the kernel function that improved the estimation comparing to the Bayesian estimation even with the informative conjugate priors. To clarify that, we employed the Weibull distribution as the most popular in life testing models and played a central role in analysis of lifetime or survival data. Moreover, Weibull distribution is one of the models, which has different priors for the parameters that have been studied extensively in the literature.

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The purpose of this paper is deriving the point and interval estimates of the parameters based on these different priors and compare the results with the kernel estimates when the underlying distribution is the two-parameter Weibull which has the probability density function (pdf) given by:

$$f(x) = \alpha\beta^{-\alpha}x^{\alpha-1}exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right),\ x>0 \tag{1}$$

where α>0 and β>0 are the shape and scale parameters respectively.

2 Kernel Function

2.1 Kernel Estimators

In this section, we introduce the kernel estimator based on a random sample $x_1,x_2,x_3,\ldots,x_n$ of size n from the random variable X with unknown probability density function $f(x)$ and support on $(0,\infty)$is given by

$$\widehat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x-x_i}{h}\right), \tag{2}$$

where h is called the bandwidth or smoothing parameter which chosen such that $h\to 0$ and $nh\to\infty$.as $n\to\infty$. The role of the bandwidth h is to scale our kernels, if h is large the density estimate could be too smooth and if it is small the estimate could be too variable. Unfortunately, the choice of h is the main problem of the kernel, where the optimal h is not known in general.However, the optimal choice for h that minimizes the mean squared errors is given by $h = 1.06Sn^{-0.2}$, where S is the sample standard deviation. To utilize the kernel function for estimating the pdf $\hat{f}(\theta)$, we can summarize the method in the following algorithm:

1.Generate a random sample $X = (x_1,x_2,x_3,\ldots,x_n)$ from the parent distribution $f(x;\theta)$ with given specified value for the parameter θ.
2. Bootstrapping with replacement n samples $X_1^*,X_2^*,X_3^*,\ldots,X_n^*$ where $X_i^* = (x_1^*,x_2^*,x_3^*,\ldots,x_n^*)$ for $i = 1,2,\ldots,n$ from the given random sample in step 1.
3. For each sample in step 2, calculate any equivariant estimator such as the maximum likelihood estimators (MLEs) or the best linear unbiased estimators (BLUEs) for θ, thus we have an objective and informative random sample $\theta_1,\theta_2,,\ldots,\theta_n$ of size n from the MLEs, (say) of the parameter θ which has an unknown density function $\hat{f}(\theta)$, and support $(0,\infty)$.
4. Thus, based on the informative sample in step 3 we can use the kernel estimator (2) for estimating $\hat{f}(\theta)$, at any given value for θ.
5. Based on the kernel estimator $\hat{f}(\theta)$, we can derive the point and interval estimates for the unknown parameterθ or a function of θ.
6. For the simulations replicate the steps 1 to 5.

For the point kernel estimate, we found the expected value of θ, which is the sample mean that has minimum mean squared error (MSE) comparing to the Bayes estimate as indicated from the simulations. Thus, if the MSE is accepted as an index of precision, we found the kernel mean is more efficient than their Bayesian counterparts For the 100% probability interval, say$[\theta_L,\theta_U]$, for θcan be derived as the following:

$$P(\theta\le t) = \int_{-\infty}^{t}\hat{f}(\theta)d\theta = \frac{1}{nh}\sum_{i=1}^{n}\int_{-\infty}^{t} K\left(\frac{\theta-\theta_i}{h}\right)d\theta = \tau$$

thus,

$$\sum_{i=1}^{n}\Pi\left(\frac{t-\theta_i}{h}\right) = n\tau, \tag{3}$$

where $\pi(x) = \int_{-\infty}^{x}K(y)\,dy$. The lower and upper 100% γ confidence bounds for θ are the values of t satisfying the equation (3) with $\tau = \gamma$ and $1-\gamma$ and can be named as θ_L and θ_U respectively. Thus, for deriving the value of the quantile estimator θ_L (say), equation(3) can be solved recurrently as the limit of the sequence $\left\{\widetilde{\theta}_1,\widetilde{\theta}_2,\widetilde{\theta}_3,\ldots\ldots\right\}$ that defined by the formulas

$$\widetilde{\theta}_{r+1} = \widetilde{\theta}_r + C\left[n\gamma - \sum_{i=1}^{n}\Pi\left(\frac{\widetilde{\theta}_r-\theta_i}{h}\right)\right], \widetilde{\theta}_1 = \frac{1}{n}\sum_{i=1}^{n}\theta_i, \tag{4}$$

for $r = 1,2,\ldots$ The convergence of (4) is guaranteed by the condition $0 < C \leq \frac{2h}{nL_1}$, where $L_1 = K(0)$, see [6].
The central rule for applying this technique is deriving the MLEs of the Weibull parameters $\theta = (\alpha, \beta)$ based on the complete sample, which are the solutions of the two equations:

$$\beta = \left(\frac{\sum_{i=1}^{n} x_i{}^{\alpha}}{n} \right)^{\frac{1}{\alpha}}, \tag{5}$$

$$\frac{n}{\alpha} + \sum_{i=1}^{n} ln(x_i) - \frac{n \sum_{i=1}^{n} x_i{}^{\alpha} ln(x_i)}{\sum_{i=1}^{n} x_i{}^{\alpha}} = 0. \tag{6}$$

3 Bayes Estimation

Suppose n units are put on the test then the likelihood function can be written as:

$$L(data|\alpha,\beta) \propto \alpha^n \beta^{-n\alpha} \prod_{i=1}^{n} x_i^{\alpha-1} \exp(-\beta^{-\alpha} \sum_{i=1}^{n} x_i{}^{\alpha}). \tag{7}$$

Let $h(\alpha,\beta)$ be the joint prior density on the distribution parameters, which measures the uncertainty about the values of α and β. Then the joint posterior density is

$$g(\alpha,\beta|data) = Ch(\alpha,\beta)L(data|\alpha,\beta), \tag{8}$$

where C is the normalizing constant and does not depend on α and β.

3.1 Informative Conjugate Prior

[3] derived an informative conjugate prior by assuming both parameters have gamma distribution, which can be transformed to our parameters as

$$h_1(\alpha,\beta) \propto \beta^{-\alpha(a+\frac{h}{\varphi}+1)-1} \alpha^{2a+1} \exp(-\alpha g) \exp(-b\psi\beta^{-\alpha}), \tag{9}$$

where
$a > -1$; $g, h, b > 0$; $\alpha, \beta > 0$ and $\varphi(\alpha), \psi(\alpha)$ are increasing function of α.
If $a = -1; g = h = b = 0$, we get the non-informative prior

$$h(\alpha,\beta) \propto \frac{1}{\alpha\beta}.$$

Using (7) and (9) in (8) the joint posterior density for α and β can be written as

$$g(\alpha,\beta|data) = C_1 \alpha^{n+2a+1} \beta^{-\alpha(n+a+h/\varphi+1)-1} \prod_{i=1}^{n} x_i^{\alpha-1} exp(-\alpha g) exp(-\beta^{-\alpha}(b\psi + \sum_{i=1}^{n} x_i{}^{\alpha})). \tag{10}$$

The marginal posteriors pdf of α and β can be given respectively as

$$g_1^*(\alpha|data) = C_1 \Gamma(n+a+h/\varphi+1) \alpha^{n+2a} exp(-\alpha g) \prod_{i=1}^{n} x_i{}^{\alpha-1} Z_1{}^{-\left(n+a+\frac{h}{\varphi}+1\right)}, \tag{11}$$

$$g_2^*(\beta|data) = C_1 \int_0^{\infty} \alpha^{n+2a+1} \beta^{-\alpha(n+a+h/\varphi+1)-1} \prod_{i=1}^{n} x_i{}^{\alpha-1} exp(-\alpha g) exp\left(-\beta^{-\alpha} Z_1\right) d\alpha. \tag{12}$$

As the reliability function is a function of the parameters α and β and hence it is a parameter itself. To obtain the posterior pdf of R we use the joint posterior of α and β and transform (α,β) to (α,R) and then integrate with respect to α, the marginal posterior pdf of R can be derived as

$$g_3^*(R|data) = C_1 \int_0^\infty \alpha^{n+2a} t^{-\alpha(n+a+h/\varphi+1)} \prod_{i=1}^n x_i^{\alpha-1} exp(-\alpha g) R^{Z_1 t^{-\alpha}-1} (-log(R))^{n+a+h/\varphi} d\alpha, \tag{13}$$

where the normalizing constant C_1 can be derived as

$$C_1^{-1} = \Gamma(n+a+h/\varphi+1) \int_0^\infty \alpha^{n+2a} exp(-\alpha g) \prod_{i=1}^n x_i^{\alpha-1} Z_1^{-\left(n+a+\frac{h}{\varphi}+1\right)} d\alpha,$$

and $Z_1 = b\psi(\alpha) + \sum_{i=1}^n x_i^{\alpha}$.

3.2 Informative Prior

[4,5] used the prior information on the reliability level at any arbitrary prior fixed time τ, say $R_\tau = exp\left(-\left(\frac{\tau}{\beta}\right)^{\alpha}\right)$, thus using the Log-Inverse Gamma distribution (LIG) to represent this prior knowledge and changing the variables from R_τ to a conditional prior density on β given α , which has non-informative prior. Thus, a unified form for the joint prior on α and β can be derived as:

$$h_2(\alpha,\beta) \propto \beta^{-a\alpha-1} \alpha^{1-\delta} \tau^{a\alpha} exp\left(-b\left(\frac{\tau}{\beta}\right)^{\alpha}\right), \tag{14}$$

for $a = b = 0$ and $\delta = 2$ we get the non-informative prior for α and β.
If the prior information on R_τ is given in terms of the prior mean μ_R and the standard deviation σ_R then the LIG parameters can be obtained by solving the equations:

$$1 + \frac{2}{b} = \left[1 + \frac{1}{b}\right]^K \quad a = \frac{ln(\mu_R)}{ln(b/(1+b))}, \tag{15}$$

where

$$K = \frac{ln(\mu_R^2 + \sigma_R^2)}{ln(\mu_R)}.$$

Using (7) and (14) in (8) the joint posterior density on α and β can be written as

$$h(\alpha,\beta|data) = C\alpha^{n-\delta+1} \beta^{-\alpha(n+a)-1} \tau^{a\alpha} \prod_{i=1}^n x_i^{\alpha-1} exp(-\beta^{-\alpha}(b\tau^{\alpha} + \sum_{i=1}^n x_i^{\alpha})).$$

The marginal posterior pdf of α and β can be given respectively as

$$h_1^*(\alpha|data) = C\Gamma(n+a)\alpha^{n-\delta} \tau^{a\alpha} \textstyle\prod_{i=1}^n x_i^{\alpha-1} Z^{-(n+a)}, \tag{16}$$

$$h_2^*(\beta|data) = C \int_0^\infty \alpha^{n-\delta+1} \beta^{-\alpha(n+a)-1} \tau^{a\alpha} \prod_{i=1}^n x_i^{\alpha-1} exp\left(-\beta^{-\alpha} Z\right) d\alpha. \tag{17}$$

Similarly, the marginal pdf of R can be derived as

$$h_3^*(R|data) = C \int_0^\infty \alpha^{n-\delta} t^{-\alpha(n+a)} \tau^{a\alpha} \prod_{i=1}^n x_i^{\alpha-1} R^{Zt^{-\alpha}-1} (-log(R))^{n+a-1} d\alpha,$$

where

$$C^{-1} = \Gamma(n+a) \int_0^\infty \alpha^{n-\delta} \prod_{i=1}^n x_i^{\alpha-1} \tau^{a\alpha} Z^{-(n+a)} d\alpha,$$

and $Z = b\tau^{\alpha} + \sum_{i=1}^n x_i^{\alpha}$.

Table (1): The parameter estimates (*), the interval length, the Lower level (LL), and the Upper level (UL) of the 90% and 95% intervals for the parameters α, β and the reliability R based on the kernel and Bayes priors for ball bearing data.

Level γ			0.90			0.95		
Approaches	P	*	LL	UL	Length	LL	UL	Length
KERNEL	α	2.2653	1.7118	2.9518	1.2400	1.6179	3.0814	1.4635
	β	78.01	60.25	94.72	34.47	57.604	97.192	39.588
	R	0.6827	0.5218	0.8441	0.3223	0.4966	0.8747	0.3780
NON-INFOR	α	2.0339	1.5097	2.4057	0.8959	1.4153	2.4262	1.0108
	β	82.55	68.14	98.52	30.38	65.49	102.25	36.76
	R	0.6896	0.5649	0.8521	0.2871	0.5352	0.8636	0.3284
CALABRIA PRIOR	α	2.1481	1.6190	2.5321	0.9131	1.5226	2.5508	1.02826
	β	84.28	73.45	93.05	19.60	71.20	93.62	22.42
	R	0.7103	0.5999	0.8616	0.2617	0.5732	0.8702	0.2969
NIGM PRIOR	α	2.0373	1.5583	2.3852	0.8269	1.4736	2.4008	0.9272
	β	88.68	65.65	92.45	26.80	62.96	93.78	30.82
	R	0.6549	0.5295	0.8169	0.2873	0.5003	0.8301	0.3298

4 Numerical Example

Consider the results of tests on the endurance of deep-groove ball bearings. The data, given by [7], consists of a complete sample of size n = 23. The results of the test, in millions of revolutions before failure, are:
17.88, 28.92, 33.00, 41.52,41.12, 45.60, 48.48, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64,68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40. These data have been used by many authors such as [8] who used the Weibull distribution as the best fitting for these kinds of data, the MLE for the parameters α and β based on this data are given respectively as 2.1666 and 89.1165.

The hyper-parameter for Calabria prior are derived from (15) based on the prior information on R_τ in terms of the prior mean $\mu_R = 0.90$ and the standard deviation $\sigma_R = 0$. which are equals a = 0.801 and $b = 7.11$ at $\delta = 2$ and $\tau = 2$. The hyper-parameter for Nigm prior have been taken arbitrary for giving good results as $a = -1, b = g = 2, h = 0.2$. Based on the different priors the point estimates and the 90% and 95% probability intervals for the parameters are derived for the purpose of comparison with the kernel estimates. The results in table (4) have been indicated the kernel approach is good enough,despite the sample size is small, though the bootstrap does not work well for small n regardless of how many bootstrap samples are used see [9]. Thus, the kernel estimation technique is more reliable than the Bayesian method despite the length of intervals is little larger due to the flatness of the distribution parameters as clear from Figures (1, 2, 3). Moreover, the hyper-parameter for the prior distributions are very confusing for choosing the good ones where some choice make the shapes are very variables as clear in graph (2) for the scale parameter, on the other hand this problem not exist in the kernel inference.

5 Simulations Study and Comparisons

The statistical performances of the point and intervals estimates have been studied, via Monte Carlo simulation in terms of:

i) The covering percentage (CP), which is defined as the fraction of times the 90% intervals covering the true value of the parameter in repeated sampling.

ii) Also measuring the efficiency for the point estimates based on the root mean square errors (RMSE) and for the interval estimates based on their average lengths.

Via Monte Carlo simulations 1000 samples of sizes n= 20(20)100 have been generated for values of the shape and scale parameters equal one.The hyper-parameter for Calabria prior are derived from (15) based on the prior information on R_τ in terms of the prior mean μ_R= 0.90 and the standard deviation σ_R =0.10, which are equal $a = 0.801$ and $b = 7.11$ at $\delta = 2$ and $\tau = 2$. The hyper-parameter for Nigm prior have been taken arbitrary for giving good results as shown in table (3).
From the results based on Bayes priors in tables (2, 3, 4), we can summarize the following main points:

Table (2): The estimates (*), the Root Mean Square Errors (RMSE), the average interval lengths (TL), and the Coverage Percentage (CP) of the 90% intervals for the kernel and Bayes methods based on the non-informative and Calabria priors for the parameters α and β.

Parameters		α				β			
Approaches	N	*	RMSE	TL	CP	*	RMSE	TL	CP
KERNEL	20	1.1351	0.2672	0.7846	0.87	1.0336	0.2470	0.8335	0.87
	40	1.0744	0.1482	0.4824	0.92	1.0346	0.1789	0.5923	0.94
	60	1.0284	0.0965	0.3134	0.90	1.0139	0.1415	0.4746	0.92
	80	1.0283	0.0965	0.3164	0.90	1.0069	0.1295	0.4148	0.89
	100	1.0372	0.0911	0.2824	0.89	1.0141	0.1019	0.3714	0.93
NON-INFOR	20	1.0072	0.1793	0.6728	0.96	1.0449	0.2377	0.8590	0.93
	40	1.0157	0.1315	0.6285	0.98	1.0371	0.1593	0.5777	0.95
	60	1.0118	0.1062	0.6400	0.97	1.0246	0.1383	0.4640	0.92
	80	1.0023	0.0844	0.6419	0.98	1.0173	0.1165	0.4012	0.91
	100	0.9959	0.0705	0.6399	0.99	1.0053	0.1081	0.3559	0.92
INFOR. PRIOR	20	1.0376	0.1697	0.6727	0.98	1.0811	0.2322	0..8274	0.95
	40	1.0316	0.1276	0.6405	0.96	1.0537	0.1566	0.5633	0.94
	60	1.0232	0.1048	0.6495	0.96	1.0358	0.1362	0.4557	0.90
	80	1.0112	0.0822	0.6484	0.98	1.0261	0.1142	0.3957	0.91
	100	1.0034	0.0695	0.6449	0.99	1.0127	0.1076	0.3519	0.91

1. The CP for the intervals based on Nigm prior are larger than the other priors for large samples but not for small samples.

2. The RMSE and the average lengths based on Nigm prior are smaller than the other priors.

3. Based on the Kernel results, we found the CP is almost greater than the nominal levels for all sample sizes.The average lengths are almost close to the ones, which are based on Nigm prior and get better as the sample sizes increase.

4. Finally, unsurprisingly, the RMSE based on the Kernel results are too small than the corresponding ones via Bayes inferences which ensure the results of data ball bearing are in favor the kernel estimates.

As a conclusion, the results have been indicated the robustness of the proposed method than the Bayesian approach even if we used conjugate priors, which encourage the statisticians for using this method.

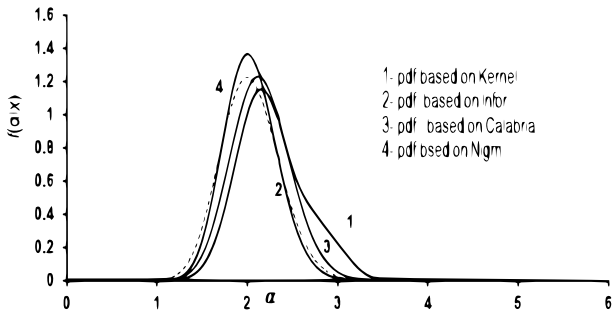

Fig. 1: The pdf of the α parameter based on the kernel function and the different priors.

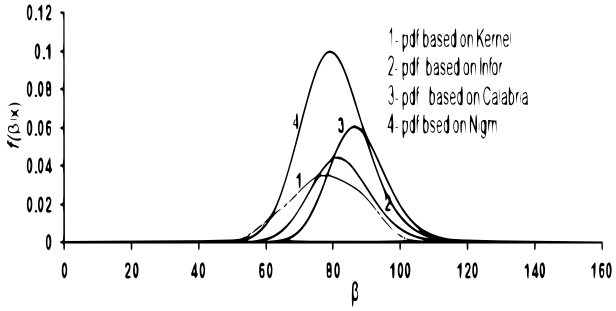

Fig. 2: The pdf of the β parameter based on kernel function and the different priors.

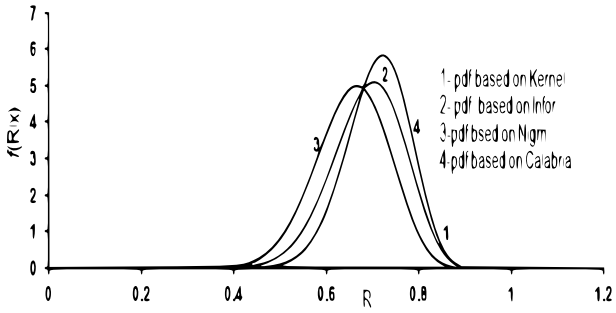

Fig. 3: The pdf of the reliability R based on the kernel and other priors.

Table (3): The Bayes estimates (*), the Root Mean Square Errors (RMSE), the average interval lengths (TL), and the Coverage Percentage (CP) of the 90% intervals based on the different priors for the parameters α, β based on Nigm prior for different values of b, g, and h.

Parameters				α				β			
n	b	g	h	*	RMSE	TL	CP	*	RMSE	TL	CP
40			2	0.9869	0.102	0.3663	0.89	0.9551	0.149	0.5985	0.94
	1	8	3	1.0290	0.107	0.3774	0.94	0.9267	0.157	0.5786	0.91
			2	0.9667	0.103	0.3592	0.85	0.9701	0.146	0.6222	0.95
	2	9	3	1.0099	0.100	0.3707	0.92	0.9406	0.152	0.6020	0.94
60			2	0.9937	0.089	0.3587	0.97	0.9691	0.132	0.4747	0.91
	1	8	3	1.0322	0.096	0.3682	0.96	0.9504	0.135	0.4651	0.89
			2	0.9850	0.089	0.3537	0.95	0.9790	0.129	0.4868	0.92
	2	9	3	1.0184	0.090	0.3634	0.96	0.9599	0.132	0.4768	0.90
80			2	0.9979	0.074	0.3517	0.99	0.9753	0.112	0.4076	0.92
	1	8	3	1.0248	0.079	0.3597	0.97	0.9613	0.115	0..4016	0.92
			2	0.9870	0.074	0.3481	0.98	0.9828	0.111	0..4152	0.92
	2	9	3	1.0144	0.075	0.3561	0.98	0.9685	0.112	0..4089	0.92
100			2	0.9954	0.064	0.3464	0.99	0.9719	0.108	0.3603	0.91
	1	8	3	1.0186	0.067	0.3532	0.99	0.9607	0.107	0.3560	0.89
			2	0.9866	0.065	0.3435	0.99	0.9778	0.107	0.3656	0.92
	2	9	3	1.0100	0.064	0.3503	1.0	0.9648	0.105	0.3613	0.93

6 Conclusion

Kernel estimation technique constitute a strong basis for statistical inference and it is uniquely determined on the basis of the information content of the classical statistical model alone, which is not possible for the Bayesian analysis that does not provide an objective frame work for choosing a prior density, which represents an additional assumption, external to the classical statistical model.

Thus, from the results of this paper kernel inference strengthens traditional inference statements and allows the

Table (4): The parameter estimates (*), the Root Mean Square Errors (RMSE), the average interval lengths (TL), and the Coverage Percentage (CP) of the 90% intervals based on the kernel and Bayesian approaches for the reliability R at t=0.5.

Approaches	N	20	40	60	80	100
KERNEL	*	0.6194	0.6187	0.6159	0.6082	0.6141
	RMSE	0.0929	0.0698	0.0577	0.0502	0.0412
	TL	0.3363	0.2276	0.1866	0.1607	0.1405
	CP	0.92	0.92	0.89	0.90	0.91
NONINF. PRIOR	*	0.5978	0.6107	0.6089	0.6069	0.6028
	RMSE	0.0852	0.0606	0.0537	0.0441	0.0419
	TL	0.3119	0.2319	0.2029	0.1900	0.1833
	CP	0.92	0.94	0.92	0.94	0.95
NIGM PRIOR	*	0.5761	0.6004	0.5901	0.6026	0.6036
	RMSE	0.0817	0.0592	0.0505	0.0451	0.0423
	TL	0.3699	0.2303	0.1997	0.1889	0.1834
	CP	0.96	0.94	0.95	0.93	0.93
CALABRIA PRIOR	*	0.6149	0.6189	0.6147	0.6113	0.6066
	RMSE	0.0728	0.0563	0.0510	0.0424	0.0402
	TL	0.2865	0.2237	0.2002	0.1892	0.1834
	CP	0.94	0.94	0.93	0.93	0.95

construction of alternative stronger types of inferences than the Bayesian inference.

Conflict of Interest The authors declare that there is no conflict of interest regarding the publication of this paper.

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