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Real Coded Genetic Algorithm for Design of IIR Digital Filter with Conflicting Objectives

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Abstract: A solution methodology for the design of digital infinite impulse response (IIR) filter considering multiple conflicting objectives has been proposed. The nucleus of the method is that multiple objectives can be attuned suitably, to optimize the performance in pass bands, stop bands and transition bands of IIR digital filters. The proposed method uses multiobjective optimization approach for designing stable IIR filter using real-coded genetic algorithm (RCGA). Digital IIR filters are designed by minimizing magnitude response and phase response simultaneously, using weighted sum approach. The value of weights are searched using RCGA along with filter coefficients thus assigning different weight vector to each individual population thereby finding multiple pareto-optimal solutions in one simulation run. The order of the filter is controlled by a control gene whose value is also optimized along with the filter coefficients to obtain optimum order of designed IIR filter. The computational experiments show that the proposed approach gives better digital IIR filters than the existing genetic algorithm based methods.

Keywords: infinite-impulse response (IIR) filters, genetic algorithms (GAs), multiple criteria, optimization.

1 Introduction

Infinite-impulse-response (IIR) digital filters are effective in wide range of applications where high selectivity and efficient processing of discrete signals are desirable. IIR filters with approximate linear phase response are difficult to design as compared to finite impulse response (FIR) filter, but IIR filter requires lower order than FIR filter to get the desired amplitude response. Digital IIR filter design primarily follows two approaches: transformation approach and optimization approach. Several well-known filter design techniques, such as Butterworth, Chebyshev, and the Elliptic function, have been developed using transformation techniques [1, 2]. Filter designed with transformation techniques are not efficient in terms of phase response, filter structure and coefficient quantization error. To implement optimization technique with some criteria, various optimization methods have been applied where p-error, mean-square-error and ripple magnitudes (tolerances) of both pass-band and stop-band are used to measure performance for the design of digital IIR filters [3–6]. Conventional gradient-based design may easily stick in the local minima due to non-linear and multimodal nature of error surface of IIR filters. Therefore, researchers have developed design methods [7–18], based on modern heuristics optimization algorithms.

The main hindrance in the design of recursive digital filters is to establish the lowest order for the purpose of obtaining specified magnitude response and linear phase response. In most of the research work IIR filter designing problem is taken as a single objective problem in terms of magnitude response or phase response without considering filter structure, which means that the order of IIR filter must be determined in advance. In Hierarchical genetic algorithm (HGA) [7] filter order and magnitude response error are considered, but phase response error is neglected. The IIR filter with linear phase response is important in most of practical applications because non-linear phase response alter the frequency components of the signal, which causes distortion. The cooperative co-evolutionary genetic algorithm (CCGA) [8] considers the magnitude response error, phase response error, and lowest order simultaneously and uses NSGA-II [10] to maintain the diversity in the three objectives. A new local search operator enhanced multi-objective evolutionary

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algorithm (LS-MOEA) [9] simultaneously optimizes the structure and coefficients of digital IIR filter to obtain relatively better linear phase response and lower order, besides the good magnitude response.

In many multidimensional and multimodal engineering design problems, the GA has been used as a robust and proficient search technique. GA requires less iterative computational equations as compared to traditional search algorithms like calculus based searches, dynamic programming, random searches and gradient methods.

Out of various existing coding schemes used for coding of search space solutions, real coding technique seems particularly natural when tackling optimization problems. Handling of continues search space is very easy with real-coded genetic algorithm (RCGA) and solution representation is very close to natural formulation of real-world problems. Because of these reasons, most of real-world multi-objective optimization problems are solved using RCGA [19, 20]. Whenever a parameter is binary coded, there is a poor precision as it does not represent parameter values that produce the best solution. The RCGA improves the final local tuning capabilities of a binary-coded genetic algorithm, which is a must for high precision optimization problems.

The intent of this paper is to apply a real-coded genetic algorithm with arithmetic-average-bound-blend (AABBX) crossover [23, 24] and wavelet mutation operator [22] for the design of digital IIR filter. The values of the filter coefficients are optimized with RCGA approach to simultaneously achieve minimum magnitude response error and phase response error alongwith optimal order of the filter. Multiobjective constrained optimization problem is converted into scalar objective constrained optimization problem employing weighting method. The weighting technique is used to generate non-inferior solutions, which allow explicit trade-off between conflicting objective levels. The weighting patterns are either presumed on the basis of decision makers intuition or simulated with suitable step size variation. Further, the weightage pattern can also be searched in the non-inferior domain. In the paper, the weightage pattern is searched using RCGA search technique along with the decision vector. On violation of inequality constraints, decision variables are updated randomly till inequality constraints are satisfied.

The paper is structured as follows. Section 2 describes the IIR filter design problem statement. The real-coded genetic algorithm for designing the optimal digital IIR filters is described in Section 3. In Section 4, the performance of the proposed method has been evaluated and achieved results are compared with the design obtained in [7–9] for the low-pass (LP), high-pass (HP), band-pass (BP), and band-stop (BS) filters. Finally, the conclusions and discussions are outlined in Section 5.

2 IIR Filter Design Problem

The transfer function of IIR is represented by cascading first and second order sections to avoid the coefficient quantization problem which causes instability. In cascade realization coefficient range is limited. The structure of cascading type digital IIR filter [13] is:

$$H(z) = x_1 \times \left(\prod_{k=1}^{m} \frac{1 + x_{2k}z^{-1}}{1 + x_{2k+1}z^{-1}} \times \prod_{i=1}^{n} \frac{1 + x_{4i+2m-2}z^{-1} + x_{4i+2m-1}z^{-2}}{1 + x_{4i+2m}z^{-1} + x_{4i+2m-1}z^{-2}} \right) \quad (1)$$

$X = [x_1, x_2, \ldots\ldots, x_{2m+4n+1}]^T_{V\times 1}$ is a vector decision variable of dimension $V \times 1$ with $V = 2m + 4n + 1$. x_1 represents the gain, $[x_1, x_2, \ldots\ldots, x_{2m+4n+1}]$ denotes the filter coefficients of first and second order sections.

2.1 Magnitude response error

To design IIR filter, main aim is to minimize the magnitude response of defined frequency band in which either frequency is allowed to pass or restricted. Magnitude response error for passband frequency is stated as [7–9]:

$$\Delta H_P(\omega) = \begin{cases} \left|H\left(e^{j\omega}\right)\right| - 1, & \left|H\left(e^{j\omega}\right)\right| > 1 \\ 1 - \delta_P - \left|H\left(e^{j\omega}\right)\right|, & \left|H\left(e^{j\omega}\right)\right| < 1 - \delta_P \\ 0, & \left|H\left(e^{j\omega}\right)\right| \geq 1 - \delta_P \end{cases} \quad (2)$$

where ω represents passband frequency and $\Delta H_p(\omega)$ is the magnitude response error in passband.

Similarly magnitude response error is stated below for stopband frequency.

$$\Delta H_s(\omega) = \begin{cases} \left|H\left(e^{j\omega}\right)\right| - \delta_s, & \left|H\left(e^{j\omega}\right)\right| > \delta_s \\ 0, & \left|H\left(e^{j\omega}\right)\right| \leq \delta_s \end{cases} \quad (3)$$

where ω represents stop band frequency and $\Delta H_s(\omega)$ is the magnitude response error in stop band.

The first objective is to minimize the magnitude response error in passband and stopband. Mathematically the objective 1 is defined as:

$$Minimize O_1 = \frac{1}{A_n}\sum_{j=1}^{A_n} \Delta H_P(\omega_j) + \frac{1}{B_n}\sum_{k=1}^{B_n} \Delta H_s(\omega_k) \quad (4)$$

where A_n and B_n are the sampling frequency points in passband and stopband, respectively. The best fitness function value is achieved when magnitude response of the designed IIR filter lies within the prescribed range in passband and stopband.

2.2 Phase response error

The linear phase response is optimized for both passband and transition band [9], because sometimes non-linearity in phase response of transition band may cause distortion. The phase response is calculated at different frequency sampling points $(\alpha_1, \alpha_2, \ldots\ldots, \alpha_l)$. The first order difference in phase response can be calculated as:

$$O_2 = \Delta phase = \{\Delta\alpha_1, \Delta\alpha_2, \ldots\ldots\ldots, \Delta\alpha_{l-1}\} \quad (5)$$

where $\Delta\alpha_l = \Delta\alpha_{l+1} - \Delta\alpha_l$; l is the total number of sampling points in passband and transition band. The phase response is linear if all the elements of $\Delta phase$ have the same value. The second objective function in terms of linear phase response error is represented as variance of phase differences.

$$Minimize O_2 = var\{\alpha\Delta_l\} \quad (6)$$

$where \alpha_l \in passband \cup transitionband$

2.3 Multiobjective IIR filter design problem formulation

The IIR filter design task is to find optimum structure having optimal order, minimum magnitude and minimum phase response error. Mathematically, multiobjective optimization problem for the design of IIR filter is stated below:

$$Minimize\{O_1(X), O_2(X)\} \quad (7)$$

Subject to: Stability constraints [21]

$$1 + x_{2k+1} \geq 0\,(k = 1, 2, \ldots\ldots, m) \quad (7a)$$

$$1 - x_{2k+1} \geq 0\,(k = 1, 2, \ldots\ldots, m) \quad (7b)$$

$$1 - x_{4i+2m+1} \geq 0\,(i = 1, 2, \ldots\ldots, n) \quad (7c)$$

$$1 + x_{4i+2m} + x_{4i+2m+1} \geq 0\,(i = 1, 2, \ldots\ldots, n) \quad (7d)$$

$$1 - x_{4i+2m+1} \geq 0\,(i = 1, 2, \ldots\ldots, n) \quad (7e)$$

where $O_1(X)$ given by equation (4) is magnitude response error, and $O_2(X)$ given by equation (6) is variance of phase difference. X is a vector decision variable of dimension $V \times 1$ $with$ $V = 2m + 4n + 1$. The aim is to find the value of filter coefficients being decision variables, X which optimizes all the objective functions, simultaneously.
The multiobjective constrained optimization problem for the design of digital IIR filter is converted into a scalar constrained optimization problem by using a weighted sum of the objectives of $O_1(X)$ and $O_2(X)$ to generate non-inferior solutions.

$$F = \sum_{j=1}^{M} w_j O_j(X) \quad (8a)$$

Subject to:

$$(i) \sum_{j=1}^{M} w_j = 1, w_j \geq 0 \text{ and } w_j = \frac{\alpha_j}{\sum_{j=1}^{M}}\,(j = 1, 2, \ldots., M) \quad (8b)$$

(ii) Satisfaction of stability constraint given by equation (7a) to equation (7e).
where $O_j(X)$ is the j^{th} objective function, and α is non-negative real number weight between 0 to 100, assigned to j^{th} objective and M is number of objectives.
The population-based approach of RCGA is exploited to search the value of weights, assigned to different objectives corresponding to each individual particle. This approach yields multiple pareto-optimal solutions in one simulation run. The decision variable becomes $X = [x_1, x_2, \ldots\ldots, x_{2m+4n+1}, x_{2m+4n+2}, x_{2m+4n+3}]^T_{V\times 1}$ where $V = 2m + 4n = 3$. $x_{2m+4n+2}$ corresponds to w_1 and $x_{2m+4n+3}$ corresponds to w_2.

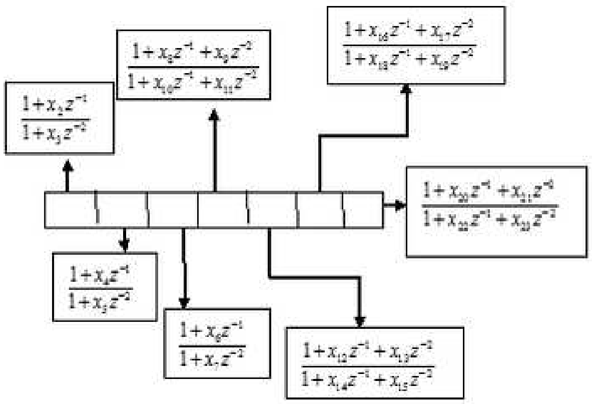

Fig. 1: Activation / Deactivation of filter coefficients with control gene.

2.4 Order

The order of the IIR filter is determined as follows:

$$Order = \sum_{j=1}^{m} p_j + 2\sum_{k=1}^{n} q_k \quad (9)$$

where p_j and q_k are j^{th} and k^{th} control genes respectively for corresponding first order and second order blocks, m and n are the number of first and second order blocks respectively. The maximum order of the filter is $m + 2n$.
The order of digital IIR filter is represented by control gene (Figure 1). The coding method followed has been inherited from [7–9]. The control genes determine activation/deactivation of corresponding blocks of filter coefficients by setting 1/0, respectively. The value of binary bits used to generate control genes is evaluated based on the integer value of real variable, $x_{2m+4n+4}$ of decision vector X. The integer value of real variable $x_{2m+4n+4}$ is optimized along with the filter coefficients to

obtain optimum Order of designed IIR filter. $X = [x_1, x_2, \ldots\ldots, x_{2m+4n+1}, x_{2m+4n+2}, x_{2m+4n+3}]^T_{V\times 1}$ is the final decision variable where $V = 2m+4n+4$

2.5 Constraint handling

The design of causal recursive filters requires the inclusion of stability constraints. Therefore, the stability constraints are obtained by using the Jury method [21] on the coefficients of the digital IIR filter in equation (1).

The stability constraints given by equation (7a) to equation (7e) have been forced to satisfy by updating the coefficients with random variation as given below. The variation is given as small so that the characteristic of population should not be changed.

$$x_{2k+1} = \begin{cases} x_{2k+1}(1-r); & (1+x_{2k+1}) < 0 \\ & or \\ & (1-x_{2k+1}) < 0 \\ x_{2k+1}, & Otherwise \end{cases} \quad (10a)$$

$$x_{4i+2m+1} = \begin{cases} x_{4i+2m+1}(1-r)^2; & (1+x_{4i+2m+1}) < 0 \\ & or \\ & (1-x_{4i+2m+1}) \geq 0 \\ x_{4i+2m+1}, & Otherwise \end{cases} \quad (10b)$$

$$x_{4i+2m} = \begin{cases} x_{4i+2m}(1-r)^2; & (1+x_{4i+2m}+x_{4i+2m+1}) < 0 \\ & or \\ & (1-x_{4i+2m}+x_{4i+2m+1}) < 0 \\ x_{4i+2m} & Otherwise \end{cases} \quad (10c)$$

where r is any uniform random number which is varied between [0, 1]. Square term has been used in random variation to give small increment.

3 Solution Methodology

RCGA search for many points in the search space at once, and continually narrow the focus of the search to the areas of the observed best performance. The basic elements of RCGA are reproduction, selection, crossover and mutation. In reproduction operation, the individuals possessing higher fitness values are selected from the existing population. In the crossover operation, two individuals are selected at random from the mating pool and a crossover is performed using mathematical relations. Mutation is an important part of genetic search, it helps to prevent the population from stagnating at any local optima. Mutation is intended to prevent the search falling into local optimum of the search space.

In this paper, a RCGA with genetic operators including arithmetic-average-bound-blend (AABBX) crossover [23, 24] and wavelet mutation is applied for optimizing the filter coefficients to simultaneously minimize magnitude response error, phase response error and order of the filter by employing stability constraints. The arithmetic-average-bound-blend crossover operator combines the arithmetic, average, bound and blend crossover operators. The arithmetic crossover operation produces some children with their parents features; average crossover manipulates the genes of the selected parents and the minimum and maximum possible values of the genes and bound crossover is capable of moving the offspring near the domain boundary. The offspring such obtained spreads over the domain so that a higher chance of reaching the global optimum can be obtained.

The wavelet mutation operation based on wavelet theory is a powerful tool for fine tuning of the genes to search the solution space locally. This property of wavelet mutation operation enhances the searching performance and provides a faster convergence than conventional RCGA. Short pseudocode for RCGA is given below:
1. Generate initial population strings randomly.
2. Calculate fitness values of population members.
3. Search for solution among the population? If yes then GOTO Step 8.
4. Using stochastic remainder roulette wheel selection choose highly fit member of population as parents and generate off-springs according to their fitness.
5. Breed new strings by mating current off-springs. Apply AABBX crossover and wavelet mutation operator to introduce variations and generate offsprings.
6. Substitute existing offsprings with new offsprings by applying competition and selection.
7. GOTO Step 3 and repeat.
8. Stop.

3.1 Initialization

Random search is applied to record the starting point. Global search is applied to explore the starting point and then the starting point is perturbed in local search space to record the best starting point. The search process is started by initializing the variable using equation (11).

$$x^0_{ij} = x^{min}_1 + rand()\left(x^{max}_i - x^{min}_i\right) \quad (11)$$

$(i = 1, 2, \ldots..V; j = 1, 2, \ldots.NV)$
where NV is size of population, t represents an iteration of RCGA, and represents the maximum and minimum limits of i^{th} decision variable of vector X.

3.2 Fitness function

Expected fitness function, f is derived from the objective function and is used in successive genetic operations. The expected fitness function used to solve design of IIR filter is given below:

$$f^t_j = max\left\{\frac{1}{1+F^t_j(X)}\right\}(j = 1, 2, \ldots., NV) \quad (12)$$

where $F_j^t(x)$ is obtained from equation (8a).

3.3 Reproduction

The initial and most important genetic algorithm operator is reproduction. In reproduction good members from population are selected to form a mating pool. The reproduction operator is also known as selection operator. Many reproduction operators exist and they all essentially pick the strings of above average from the current population and insert their multiple copies in the mating pool in a probabilistic manner. The commonly used reproduction operator is the proportionate reproduction operator where a string is selected for the mating pool with a probability proportional to its fitness. The basic roulette wheel selection method is stochastic sampling with replacement (SSR). The segment size and selection probability remain same throughout the selection phase and individuals are selected accordingly. Stochastic sampling with partial replacement (SSPR) extends upon SSR by resizing an individual's segment if it is selected. After the selection of each individual, the size of its segment is reduced by one. If the segment size becomes negative, then it is set to zero. Remainder sampling methods involve two distinct phases. In the integral phase, the individuals are selected deterministically according to the integer part of their expected trials. The remaining individuals are then selected probabilistically from fractional part of the individuals expected values. In this paper the stochastic remainder roulette wheel selection has been applied [23].

3.4 Crossover operators

The arithmetic-average-bound-blend crossover has been used for the selection of chromosomes and is based on the stochastic remainder Roulette-wheel mechanism [23]. The AABBX operator is the combination of arithmetic crossover, average crossover, bound and blend crossovers. Suppose two vectors are selected chromosomes in the t^{th} iteration of the RCGA execution. Each chromosome has V genes, which are real numbers. The AABBX operator creates ten children from the parents x_{iv}^t and x_{iu}^t as follows:
(i) Arithmetic crossover

$$x_{i1}^{t+1} = w_\alpha x_{iv}^t + (1-w_\alpha)x_{iu}^t \,(i=1,2....,V) \tag{13}$$

$$x_{i2}^{t+1} = (1-w_\alpha)x_{iv}^t + w_\alpha x_{iu}^t \,(i=1,2....,V) \tag{14}$$

$$x_{i3}^{t+1} = Min\left[x_{iv}^t, x^t iu\right]\,(i=1,2....,V) \tag{15}$$

$$x_{i4}^{t+1} = Max\left[x_{iv}^t, x^t iu\right]\,(i=1,2....,V) \tag{16}$$

(ii) Average crossover

$$x_{i5}^{t+1} = \frac{1}{2}\left(x_{iv}^t, x_{iu}^t\right)(i=1,2....,V) \tag{17}$$

$$x_{i6}^{t+1} = \tfrac{1}{2}\left[w_b\left(x_{iv}^t + x_{iy}^t\right) + (1-w_b)\left(x_i^{min} + x_i^{max}\right)\right]\,(i=1,2....,V) \tag{18}$$

(iii) Bound crossover

$$x_{i7}^{t+1} = w_c Min\left[x_{iv}^t, x_{iu}^t\right] + (1-w_c)x_i^{min}\,(i=1,2....,V) \tag{19}$$

$$x_{i8}^{t+1} = w_c Min\left[x_i^t v, x_{iu}^t\right] + (1-w_c)x_i^{max}\,(i=1,2....,V) \tag{20}$$

(iv) Blend crossover

$$x_{i9}^{t+1} = w_d x_{iv}^t + (1-w_d)x_{iu}^t\,(i=1,2....,V) \tag{21}$$

$$x_{i10}^{t+1} = (1-w_d)x_{iv}^t + w_d x_{iu}^t\,(i=1,2....,V) \tag{22}$$

w_a, w_b and w_c are constant weights. The values are adjusted such that $0 < w_a, w_b, w_c < 1$. w_d is also constant weight such that $1 \le w_d < 2$. Two children having the highest fitness values are selected as the offspring chromosomes for the crossover operation. These two offspring chromosomes are added to the previous population including the parents. The enlarged population formed after the execution of the crossover operator is considered for the mutation.

3.5 Mutation operator

Mutation is a genetic operator used to maintain genetic diversity from one generation of population of chromosomes to next. Mutation alters one or more gene values in a chromosome from its initial state. This can result in entirely new gene values being added to the gene pool. With the new gene values, the genetic algorithm may be able to arrive at better solution than was previously available. Each gene of the chromosome is given an opportunity to mutate, governed by the probability of the mutation p_m. For each gene of the chromosome, a random number in the range of [0, 1] is generated. If the random number is less than p_m, that gene is selected for the mutation, otherwise it is not selected. In this algorithm, p_m is set at 0.2. The new gene, x_{ij}^t after mutation will be as follows:

$$x_{ij}^t = \begin{cases} x_{ij}^t + \Delta(\phi, t, t_{max})\left(x_i^{max} - x_{ij}^t\right) & if\Delta \ge P_m \\ x_{ij}^t + \Delta(\phi, t, t_{max})\left(x_{ij}^t - x_{ij}^{min}\right) & if\Delta < P_m \end{cases} \tag{23}$$

where $(i=1,2,....V; j=1,2,....,NV)$ t_{max} is the maximum number of iterations of the RCGA and t is the current iteration number.

Morlet wavelet as the mother wavelet can be rewritten as

$$\Delta(\phi, t, t_{max}) = \psi_d(\phi) = \frac{1}{\sqrt{d}}e^{\frac{-\phi^2}{2}}\cos 5\phi \tag{24}$$

where ϕ is randomly generated in the range of [-4, 4] because this wavelet has [-4, 4] as its effective support. The dilation parameter d is set to vary with the value of (t/t_{max}), giving the adaptive search capability to the proposed real coded genetic algorithm.

$$d = exp\left(lng\left(1-\left(1-\frac{t}{t_{max}}\right)^{\xi}\right)\right) \quad (25)$$

where ξ is the shape parameter of the monotonic increasing function of d. g is the upper limit of the dilation parameter. The dilation parameter d is a function of t and t_{max}, and so δ is really a function of ϕ, t and t_{max}.

3.6 Competition and selection

Each individual in the combined population has to compete with some other individuals to have a chance to be copied to the next iteration. The score for each trial vector after stochastic competition is given by

$$w_i^t = \sum_{n=1}^{NV} w_n^t; (1,2,......NV) \quad (26)$$

where

$$w_n^t = \left\{1,\ if u_1 < \frac{f_r^t}{f_i^t+f_r^t}\ 0,\ otherwise \right. \quad (27)$$

NV is the population size.
f_r^t is the fitness value of the randomly selected competitor in the combined population.
f_i^t is the fitness value of x_i^t.
u_1 and u_2 are randomly selected from a uniform distribution set u (0, 1)
$m = int\,(2\times NV \times u_2 + 1)$

After competing, the trial 2NV solutions, including the parents and the offspring, are ranked in descending order of the score obtained in equation (26). The first NV trial solutions survive and are copied along with their objective functions into survivor set as the individuals of the next iteration..

4 Design Examples and Comparisons

Digital IIR filter have been realized by taking 200 equally spaced points within the frequency domain $[0,\pi]$. The design parameters followed for the design of IIR filter are presented in Table 1. The control parameter values such as population size, crossover, and mutation rate employed for the RCGA algorithm are given in Table 2. The magnitude and phase response diagrams of LP and HP filters are presented in figure 2 and figure 3 presents magnitude and phase response diagrams of BP and BS filters. The pole zero diagrams for LP, HP, BP and BS filters are presented in figure 4 for all types of digital IIR filters. It can be observed from figure 4 that the designed filters follow the stability constraints imposed in the design procedure as all the poles lie inside the unit circle. The stability of filter is not influenced by the zeros lying outside the unit circle. The designed IIR filter models obtained by the RCGA approach for LP, HP, BP and BS are given by equation (28), equation (29), equation (30) and equation (31) respectively.

$$H_{LP}(z) = 0.211090 \times \frac{(z+0.230950)\left(z^2-0.950127z+0.949332\right)}{(z+0.403023)\left(z^2-1.207782z+0.643844\right)} \quad (28)$$

$$H_{HP}(z) = 0.196099 \times \frac{(z-0.519345)\left(z^2+0.932477z+0.929194\right)}{(z+0.332503)\left(z^2+1.171756z+0.616812\right)} \quad (29)$$

$$H_{BS}(z) = 0.475804 \times \frac{\left(z^2+0.300735+0.873221\right)\left(z^2-0.306113z+0.862739\right)}{\left(z^2+0.737646z+0.468787\right)\left(z^2-0.743670z+0.478598\right)} \quad (30)$$

$$H_{BS} = 0.475804 \times \frac{\left(z^2+0.300735z+0.873221\right)\left(z^2-0.306113z+0.862739\right)}{\left(z^2+0.737646z+0.468787\right)\left(z^2-0.743670z+0.478598\right)} \quad (31)$$

Table 1: Table 1. Prescribed Design Conditions on LP HP, BP and BS Filters.

Filter *type*	Pass-band $(\delta_p = 0.1088)$	Stop-band $(\delta_p = 0.17783)$	Order
LP	$0 \le \omega \le 0.2\pi$	$0.3\pi \le \omega \le \pi$	11
HP	$0.8\pi \le \omega \le \pi$	$0 \le \omega \le 0.7\pi$	11
BP	$0.4\pi \le \omega \le 0.6\pi$	$0 \le \omega \le 0.25\pi$ $0.75 \le \omega \le \pi$	
BS	$0 \le \omega \le 0.25\pi$ $0.75 \le \omega \le \pi$	$0.4\pi \le \omega \le 0.6\pi$	11

Table 2: Value of Control Parameters.

Population Size	50
Represenation	Real number representation
Crossover	Arithmetic-average-bound-blend crossover
Crossover Rate	0.9
Mutation	Wavelet Mutation
Mutation Rate	0.01

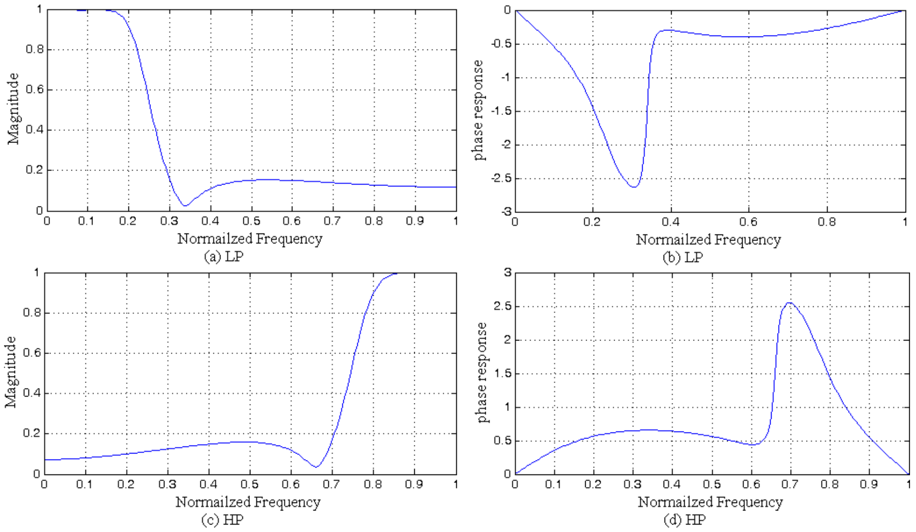

Fig. 2: Magnitude and phase response of LP and HP filter.

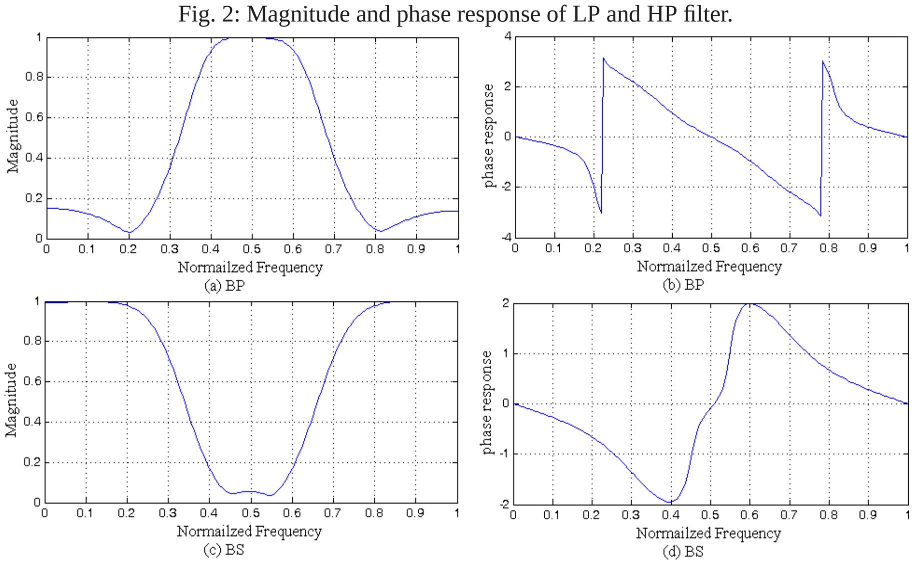

Fig. 3: Magnitude and phase response of BP and BS filter.

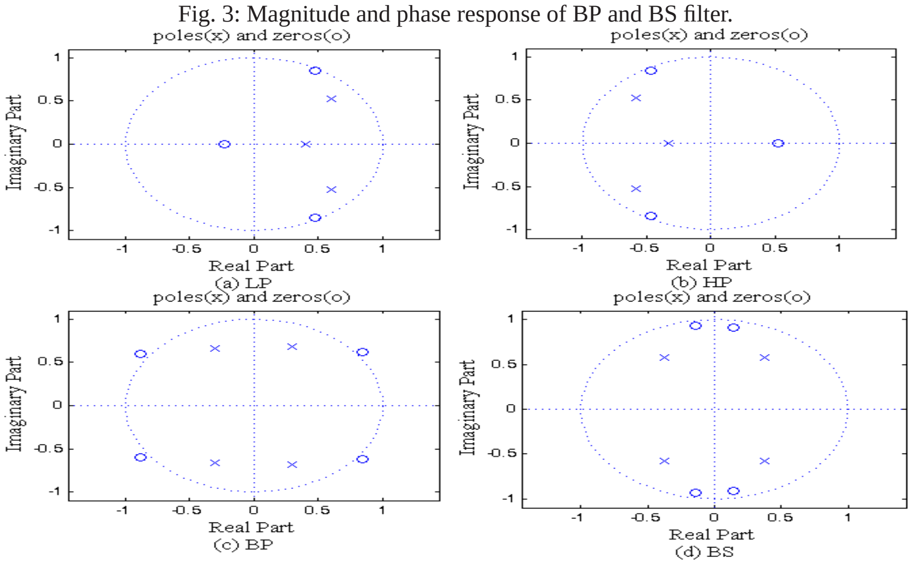

Fig. 4: Pole-zero plots for LP, HP, BP and BS filters.

Table 3. Comparison of design results for LP, HP, BP and BS filters.

	Lowest filter order	Pasg-band magnitude performance	Stop-rand magnitude performance	Phase response Error
LP Filter				
HGA	3	$0.8862 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1800$	1.6485×10^{-4}
CCGA	3	$0.9034 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1669$	1.4749×10^{-4}
LS-MOEA	3	$0.9083 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1586$	1.0959×10^{-4}
RCGA	3	$0.9141 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1556$	1.1788×10^{-4}
HP Filter				
HGA	3	$0.9221 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1819$	1.1212×10^{-4}
CCGA	3	$0.9044 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1749$	9.7746×10^{-4}
LS-MOEA	3	$0.9004 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1746$	9.6150×10^{-5}
RCGA	3	$0.9004 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1742$	9.5757×10^{-5}
BP Filter				
HGA	6	$0.8956 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1772$	1.1222×10^{-4}
CCGA	4	$0.8920 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1654$	8.1751×10^{-5}
LS-MOEA	4	$0.9285 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1734$	6.0371×10^{-5}
RCGA	4	$0.9333 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1641$	5.9070×10^{-5}
BS Filter				
HGA	4	$0.8920 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1726$	2.7074×10^{-4}
CCGA	4	$0.8966 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1733$	1.6119×10^{-4}
LS-MOEA	4	$0.8967 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1725$	1.5084×10^{-4}
RCGA	4	$0.8975 \leq \lvert H(e^{j\omega}) \rvert \leq 1.0$	$\lvert H(e^{j\omega}) \rvert \leq 0.1708$	1.5144×10^{-4}

Table 4. Variation of fitness function for LP, HP, BP and BS filters.

	Minimum	Maximum	Average	Standard Deviation
LP	0.9998624	0.9999988	0.999950641	5.10948E-05
HP	0.9998645	0.9999990	0.999966836	3.84241E-05
BP	0.9998758	0.9999993	0.999954749	3.02676E-05
BS	0.9998702	0.9999981	0.999915193	4.31160E-05

The values have been recorded giving 100 random run.

Table 5. Effect of parameter variation on fitness function values.

Parameter	Perturbation	LP	HP	BP	BS
w_a	0.2 ± 10 %	0.012910015	0.013485016	0.003122007	0.002964006
w_b	0.2 ± 10 %	0.003774004	0.002336003	0.002114005	0.003506007
w_c	0.2 ± 10 %	0.003446004	0.004117005	0.003474008	0.002870006
w_d	1.2 ± 10 %	0.001545002	0.002486003	0.002580006	0.003021006
p_m	0.22 ± 10 %	0.002938003	0.001647002	0.003467008	0.003313007
ξ	0.5 ± 10 %	0.003562004	0.001901002	0.002931007	0.002948006

The results obtained for LP, HP, BP and BS filters with RCGA are summarized and are compared with HGA [7], CCGA [8], and LS-MOEA [9], in Table 3. By studying Table 3, it is concluded:

• In comparision to EAs (CCGA and HGA) the RCGA offers best performance in terms of phase response quality for LP, HP, BP and BS filters. In term of lowest order: RCGA and CCGA are equivalent and surpass HGA. The magnitude response obtained with RCGA is better in almost all types of digital IIR filters.

• When compared with optimization algorithm LS-MOEA, RCGA provides better magnitude response for all types of digital IIR filters. In terms of lowest order RCGA is comparable with LS-MOEA. The phase response linearity obtained with RCGA is comparable with LS-MOEA.

• In addition to above the RCGA was tested for the robustness by performing 100 independent runs with random variation. The minimum, maximum, average and standard deviation values obtained have been summarized in Table 4, which clearly depicts that RCGA is a robust algorithm being a small value of standard deviation. Last but not least the results obtained by perturbing different parameters of RCGA shown in Table 5 justify that RCGA is insensitive to parameters.

In view of above three points it can be concluded that RCGA is a robust and effective algorithm for the design of digital IIR filters of better responses and lower order.

5 Conclusion

This paper proposes a RCGA approach for optimization of digital IIR filters considering multiple conflicting objectives. On the basis of results obtained for the design of digital IIR filter, it can be concluded that RCGA is a robust algorithm and possesses the capacity for the local tuning of the solutions. RCGA can design a digital filter of any type, while the lowest order of the filter is achieved; it can design the IIR filter with better magnitude and phase performance. Concluding, Simulation studies show that the proposed method is accurate, robust and an efficient optimizer for digital IIR filter design.

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