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# Generalised Neutrosophic Soft Set and its Integration to Decision Making Problem

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**Abstract:** In 1999, Molodtsov [5] introduced the concept of soft set theory, which can be used as a generic mathematical tool for dealing with uncertainty. In this paper, we first generalise the concept of neutrosophic sof set defined by Maji [20]. We then study basic operations on the generalised neutrosophic soft set. We finally present an application of generalised neutrosophic soft set in decision making problem.

**Keywords:** Neutrosophic set, neutrosophic soft set, generalised neutrosophic soft set, decision making

## 1 Introduction

Most of the problems in engineering, medical science, economics, environments etc. have various uncertainties. Molodtsov [5] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. Since this theory has a parameterization tool, it is different from tradional theories concerning with uncertainties, such as the theory of fuzzy set, the theory of intuitionistic fuzzy set, the theory of rough set, and the theory of vague set. This feature make it indispensable for applications in many fields such as function smoothless, Riemann integration, measurement, game theory, decision making, ect.

Maji et al. [16] introduced some operations of soft sets, which makes a theoretical study of the soft set theory in more detail. Integration of soft sets to decision making problem was firstly proposed by Maji et al. [15]. After these two important studies, the soft set theory have been studied increasingly, see [2,3,11,12,13,14]. Also it has been applied to several algebra structures: groups [1,9], semirings [6], rings [24], BCK/BCI-algebras [27,28], d-algebras [29], ordered semigroups [30], and BL-algebras [10]. Recently, some authors have introduced some new mathematical tools by generalizing and extending Molodtsov's classical soft set theory; fuzzy soft sets [17], intuitionistic fuzzy soft sets [18,19], vague soft sets [25], interval-valued fuzzy soft sets [26], and interval-valued intuitionistic fuzzy soft sets [31]. Majumdar and Samanta [23] have defined the similarity measures for the soft sets. In [22], they have applied the generalised fuzzy soft sets to decision making problem by generalizing fuzzy soft sets.

Neutrosophic set, a part of neutrosophy introduced by Smarandache [7,8] as a new branch of philosophy, is a mathematical tool dealing with problems involving imprecise, indeterminacy and inconsistent knowledge. Contrary to intuitionistic fuzzy sets, a neutrosophic set consists of three basic membership functions independently of each other, which are truth, indeterminacy and falsity. By combining the soft set theory with neutrosophic set theory, Maji [20,21] has introduced the notion of neutrosophic soft set and showed an application of neutrosophic soft sets in object recognition problem.

In this paper we begin with some basic definitions of soft sets and neutrosophic(soft) sets. We define a new concept named generalised neutrosophic soft set by generalizing the neutrosophic soft sets defined by Maji in [20] and study its basic properties. To make more effective and realistic to neutrosophic soft sets, we attach to them a degree indicating the possibility of approximate value-set. We then give similarity measures for the neutrosophic soft sets and the generalised neutrosophic soft sets, respectively. We also present an application of generalised neutrosophic soft sets in decision making problem. In the final section, we make the general evaluation of this paper.

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## 2 Preliminaries

In the section we briefly recall the notions of soft set and neutrosophic(soft) set. For details see [2,5,7,8,13,14,23].

**Definition 1.**[5,14] *Let $U$ be an initial universe, $P(U)$ be the power set of $U$, $E$ be the set of all parameters and $A \subseteq E$. Then, a soft set $F_A$ over $U$ is a set defined by a set valued function $f_A$ representing a mapping $f_A : E \longrightarrow P(U)$ such that $f_A(x) = \varnothing$ if $x \notin A$.*

*Thus, a soft set $F_A$ over $U$ can be represented by the set of ordered pairs $F_A = \{\langle x, f_A(x)\rangle : x \in E,\ f_A(x) \in P(U)\}$.*

**Definition 2.**[7,8] *Let $U$ be a space of points (objects) and $x \in U$. A neutrosophic set $N$ in $U$ is characterized by a truth-membership function $\mathscr{T}_N$, an indeterminacy-membership function $\mathscr{I}_N$ and a falsity-membership function $\mathscr{F}_N$, where $\mathscr{T}_N(x)$, $\mathscr{I}_N(x)$ and $\mathscr{F}_N(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$. That is*

$$\mathscr{T}_N : U \longrightarrow ]0^-, 1^+[$$
$$\mathscr{I}_N : U \longrightarrow ]0^-, 1^+[$$
$$\mathscr{F}_N : U \longrightarrow ]0^-, 1^+[$$

*There is no restriction on the sum of $\mathscr{T}_N(x)$, $\mathscr{I}_N(x)$ and $\mathscr{F}_N(x)$, so $0^- \le sup\mathscr{T}_N(x) + sup\mathscr{I}_N(x) + sup\mathscr{F}_N(x) \le 3^+$. Here, for practical purposes and to keep the discussion relatively simpler we are assuming the range of $[0,1]$.*

**Definition 3.**[7] *A neutrosophic set $N$ is contained in the other neutrosophic set $M$, $N \subseteq M$, if and only if $\forall x \in U$, $\mathscr{T}_N(x) \le \mathscr{T}_M(x)$, $\mathscr{I}_N(x) \le \mathscr{I}_M(x)$ and $\mathscr{F}_N(x) \ge \mathscr{F}_M(x)$.*

**Definition 4.**[20] *Let $U$ be an initial universe set and $E$ be a set of parameters. Consider $A \subseteq E$. Let $\mathscr{N}(U)$ denotes the set of all neutrosophic sets of $U$. Then $F_A$ is termed to be the neutrosophic soft set over $U$, where $F$ is a mapping given by $F : A \longrightarrow \mathscr{N}(U)$.*

*Thus, we can represent the neutrosophic soft set $F_A$ over $U$ by*

*$F_A = \{\langle x, \mathscr{T}_{F_A(e)}(x), \mathscr{I}_{F_A(e)}(x), \mathscr{F}_{F_A(e)}(x)\rangle : e \in E,\ x \in U$ and $\mathscr{T}_{F_A}(x), \mathscr{I}_{F_A}(x), \mathscr{F}_{F_A}(x) \in P(U)\}$.*

## 3 Generalised neutrosophic soft sets

Throughout paper, $U$ is an initial universe, $E$ is a set of parameters and $\Lambda$ is an index set.

**Definition 5.***Let $U$ be an initial universe and $E$ be a set of parameters. Let $\mathscr{N}(U)$ be the set of all neutrosophic sets of $U$. A generalised neutrosophic soft set $F^\mu$ over $U$ is defined by the set of ordered pairs*

$$F^\mu = \{(F(e), \mu(e)) : e \in E, F(e) \in \mathscr{N}(U), \mu(e) \in [0,1]\},$$

*where $F$ is a mapping given by $F : E \longrightarrow \mathscr{N}(U)$ and $\mu$ is a fuzzy set such that $\mu : E \longrightarrow I = [0,1]$. Here, $F^\mu$ is a mapping defined by $F^\mu : E \longrightarrow \mathscr{N}(U) \times I$.*

*For any parameter $e \in E$, $F(e)$ is referred as the neutrosophic value set of parameter $e$, i.e,*

$$F(e) = \left\{\left\langle x, \alpha_{F(e)}(x), \gamma_{F(e)}(x), \beta_{F(e)}(x)\right\rangle\right\},$$

*where $\alpha, \gamma, \beta : U \longrightarrow [0,1]$ are the memberships functions of truth, indeterminacy and falsity respectively of the element $x (\in U)$. For any $x \in U$ and $e \in E$, $0 \le \alpha_{F(e)}(x) + \gamma_{F(e)}(x) + \beta_{F(e)}(x) \le 3$. In fact, $F^\mu$ is a parameterized family of neutrosophic sets on $U$, which has the degree of possibility of the approximate value set which is represented by $\mu(e)$ for each parameter $e$. So we can write it as follows:*

$$F^\mu(e) = \left\{\left(\frac{x_1}{F(e)(x_1)}, \frac{x_2}{F(e)(x_2)}, \ldots, \frac{x_n}{F(e)(x_n)}\right), \mu(e)\right\}$$

*Example 1.*Consider two generalised neutrosophic soft sets $F^\mu$ and $G^\delta$, where $U$ is a set of three cars under the consideration of a decision maker to purchase, which is indicated by $U = \{c_1, c_2, c_3\}$, and $E$ is a parameter set, where $E = \{e_1, e_2, e_3\} = \{$ performance, security, comfort$\}$. Then $F^\mu$ and $G^\delta$ describe two different attractiveness of the cars to the decision maker.

Suppose that $F^\mu$ and $G^\delta$ are given as follows, respectively;

$$\left\{\begin{array}{l} F^\mu(e_1) = \left(\frac{c_1}{(0.6,0.5,0.4)}, \frac{c_2}{(0.2,0.7,0.5)}, \frac{c_3}{(0.6,0.1,0.8)}\right), (0.1) \\ F^\mu(e_2) = \left(\frac{c_1}{(0.3,0.8,0.2)}, \frac{c_2}{(0.6,0.3,0.1)}, \frac{c_3}{(0.7,0.4,0.3)}\right), (0.4) \\ F^\mu(e_3) = \left(\frac{c_1}{(0.2,0.5,0.6)}, \frac{c_2}{(0.1,0.7,0.2)}, \frac{c_3}{(0.8,0.3,0.4)}\right), (0.6) \end{array}\right\}$$

$$\left\{\begin{array}{l} G^\delta(e_1) = \left(\frac{c_1}{(0.1,0.4,0.9)}, \frac{c_2}{(0.8,0.1,0.2)}, \frac{c_3}{(0.3,0.4,0.5)}\right), (0.2) \\ G^\delta(e_2) = \left(\frac{c_1}{(0.6,0.3,0.5)}, \frac{c_2}{(0.5,0.7,0.8)}, \frac{c_3}{(0.1,0.7,0.3)}\right), (0.7) \\ G^\delta(e_3) = \left(\frac{c_1}{(0.5,0.5,0.1)}, \frac{c_2}{(0.3,0.2,0.7)}, \frac{c_3}{(0.4,0.1,0.6)}\right), (0.1) \end{array}\right\}$$

For the purpose of storing a generalised neutrosophic soft set in a computer, we can present it in matrix form. For example, the matrix form of $F^\mu$ can be expressed as follows: for $m, n \in \Lambda$,

$$\begin{pmatrix} (0.6,0.5,0.4) & (0.2,0.7,0.5) & (0.6,0.1,0.8) & (0.1) \\ (0.3,0.8,0.2) & (0.6,0.3,0.1) & (0.7,0.4,0.3) & (0.4) \\ (0.2,0.5,0.6) & (0.1,0.7,0.2) & (0.8,0.3,0.4) & (0.6) \end{pmatrix},$$

where the $m$-th row vector shows $F(e_m)$ and and $n$-th column vector shows $c_n$ while the last column shows the values of $\mu$.

**Definition 6.***Let $F^\mu$ be a generalised neutrosophic soft set over $U$, where $F^\mu(e) = \{(F(e), \mu(e))\}$ and $F(e) = \left\{\left\langle x, \alpha_{F(e)}(x), \gamma_{F(e)}(x), \beta_{F(e)}(x)\right\rangle\right\}$ for all $e \in E$, $x \in U$. Then for $e_m \in E$ and $x_n \in U$,*

*(1)$F^\triangleright$ is said to be truth-membership part of $F^\mu$, where $F^\triangleright = \{(F^\triangleright_{mn}(e_m), \mu(e_m))\}$ and $F^\triangleright_{mn}(e_m) = \left\{\left\langle x_n, \alpha_{F(e_m)}(x_n)\right\rangle\right\}$;*

(2)$F^*$ *is said to be indeterminacy-membership part of* $F^\mu$,
*where* $F^* = \{(F^*_{mn}(e_m), \mu(e_m))\}$ *and* $F^*_{mn}(e_m) = \{\langle x_n, \gamma_{F(e_m)}(x_n)\rangle\}$;
(3)$F^\triangleleft$ *is said to be falsity-membership part of* $F^\mu$,
*where* $F^\triangleleft = \{(F^\triangleleft_{mn}(e_m), \mu(e_m))\}$ *and* $F^\triangleleft_{mn}(e_m) = \{\langle x_n, \beta_{F(e_m)}(x_n)\rangle\}$.

We say that every part of $F^\mu$ is a component itself and denote by $F^\mu = (F^\triangleright, F^*, F^\triangleleft)$. Then matrix forms of components of $F^\mu$ in Example 1 can be expressed as follows:

$$F^\triangleright = \begin{pmatrix} (0.6,0.2,0.6) & (0.1) \\ (0.3,0.6,0.7) & (0.4) \\ (0.2,0.1,0.8) & (0.6) \end{pmatrix}$$
$$F^* = \begin{pmatrix} (0.5,0.7,0.1) & (0.1) \\ (0.8,0.3,0.4) & (0.4) \\ (0.5,0.7,0.3) & (0.6) \end{pmatrix}$$
$$F^\triangleleft = \begin{pmatrix} (0.4,0.5,0.8) & (0.1) \\ (0.2,0.1,0.3) & (0.4) \\ (0.6,0.2,0.4) & (0.6) \end{pmatrix},$$

where

$$F^\triangleright_{mn}(e_m) = \{\langle x_n, \alpha_{F(e_m)}(x_n)\rangle\},$$
$$F^*_{mn}(e_m) = \{\langle x_n, \gamma_{F(e_m)}(x_n)\rangle\},$$
$$F^\triangleleft_{mn}(e_m) = \{\langle x_n, \beta_{F(e_m)}(x_n)\rangle\}$$

mean the truth, indeterminacy and falsity values of $n$-th element the according to $m$-th parameter, respectively.

*Remark.*Suppose that $F^\mu$ is a generalised neutrosophic soft set over $U$. Then we say that each component of $F^\mu$ can be seen as the generalised fuzzy soft set of Majumdar [22]. Also if it is taken $\mu(e) = 1$ for all $e \in E$, then our generalised neutrosophic soft set coincides with the neutrosophic soft set of Maji [20].

**Definition 7.***A generalised neutrosophic soft set* $F^\mu$ *over* $U$ *is said to be a generalised null neutrosophic soft set denoted by* $\emptyset^\mu$, *if* $\forall e \in E$, $\emptyset^\mu : E \longrightarrow \mathscr{N}(U) \times I$ *such that* $\emptyset^\mu(e) = \{(F(e), \mu(e))\}$, *where* $F(e) = \{\langle x, 0, 0, 0\rangle\}$ *and* $\mu(e) = 0$, $x \in U$.

**Definition 8.***A generalised neutrosophic soft set* $F^\mu$ *over* $U$ *is said to be a generalised absolute neutrosophic soft set denoted by* $U^\mu$, *if* $\forall e \in E$, $U^\mu : E \longrightarrow \mathscr{N}(U) \times I$ *such that* $U^\mu(e) = \{(F(e), \mu(e))\}$, *where* $F(e) = \{\langle x, 1, 1, 1\rangle\}$ *and* $\mu(e) = 1$, $x \in U$.

**Definition 9.***Let* $U$ *be an initial universe and* $E$ *be a set of parameters,* $F^\mu$ *and* $G^\delta$ *be two generalised neutrosophic soft sets, we say that* $F^\mu$ *is a generalised neutrosophic soft subset of* $G^\delta$ *if*

(1)$\mu$ *is a fuzzy subset of* $\delta$;
(2)$\forall e \in E$, $F(e)$ *is a neutrosophic subset of* $G(e)$, *i.e., for all* $e_m \in E$ *and* $m, n \in \Lambda$, $F^\triangleright_{mn}(e_m) \le G^\triangleright_{mn}(e_m)$, $F^*_{mn}(e_m) \le G^*_{mn}(e_m)$, *and* $F^\triangleleft_{mn}(e_m) \ge G^\triangleleft_{mn}(e_m)$.

*We denote this relationship by* $F^\mu \sqsubseteq G^\delta$. *Moreover, if* $G^\delta$ *is a generalised neutrosophic soft subset of* $F^\mu$, *then* $F^\mu$ *is called a generalised neutrosophic soft superset of* $G^\delta$. *This relationship is denoted by* $F^\mu \sqsupseteq G^\delta$.

*Example 2.*Consider two generalised neutrosophic soft sets $F^\mu$ and $G^\delta$. Suppose that $U = \{c_1, c_2, c_3\}$ is the set of cars and $E = \{e_1, e_2, e_3\}$ is the set of parameters where $e_1$ =performance, $e_2$ =security, $e_3$ =comfort. Suppose that $F^\mu$ and $G^\delta$ are given as follows, respectively;

$$\begin{Bmatrix} F^\mu(e_1) = \left(\frac{c_1}{(0.4,0.7,0.4)}, \frac{c_2}{(0.2,0.5,0.5)}, \frac{c_3}{(0.6,0.4,0.4)}\right), (0.2) \\ F^\mu(e_2) = \left(\frac{c_1}{(0.7,0.2,0.4)}, \frac{c_2}{(0.6,0.4,0.9)}, \frac{c_3}{(0.3,0.5,0.7)}\right), (0.5) \\ F^\mu(e_3) = \left(\frac{c_1}{(0.2,0.5,0.7)}, \frac{c_2}{(0.4,0.2,0.4)}, \frac{c_3}{(0.5,0.2,0.8)}\right), (0.6) \end{Bmatrix}$$

and

$$\begin{Bmatrix} G^\delta(e_1) = \left(\frac{c_1}{(0.7,0.8,0.1)}, \frac{c_2}{(0.8,0.7,0.5)}, \frac{c_3}{(0.6,0.5,0.2)}\right), (0.4) \\ G^\delta(e_2) = \left(\frac{c_1}{(0.7,0.8,0.3)}, \frac{c_2}{(0.6,0.5,0.7)}, \frac{c_3}{(0.7,0.7,0.7)}\right), (0.7) \\ G^\delta(e_3) = \left(\frac{c_1}{(0.4,0.5,0.6)}, \frac{c_2}{(0.6,0.7,0.3)}, \frac{c_3}{(0.8,0.3,0.4)}\right), (0.8) \end{Bmatrix}$$

Then $F^\mu$ is a generalised neutrosophic soft subset of $G^\delta$, that is, $F^\mu \sqsubseteq G^\delta$.

**Definition 10.***Let* $F^\mu$ *and* $G^\delta$ *be two generalised neutrosophic soft sets over same universe* $U$, $F^\mu$ *and* $G^\delta$ *are called generalised neutrosophic soft set equal, denoted by* $F^\mu = G^\delta$ *if* $F^\mu \sqsubseteq G^\delta$ *and* $F^\mu \sqsupseteq G^\delta$. *In other words, if* $\mu(e_m) = \delta(e_m)$ *and* $F(e_m) = G(e_m)$ *for all* $e_m \in E$, *i.e.,* $F^\triangleright_{mn}(e_m) = G^\triangleright_{mn}(e_m)$, $F^*_{mn}(e_m) = G^*_{mn}(e_m)$ *and* $F^\triangleleft_{mn}(e_m) = G^\triangleleft_{mn}(e_m)$, *then* $F^\mu = G^\delta$.

**Proposition 1.***Let* $F^\mu$ *and* $G^\delta$ *be two generalised neutrosophic soft sets over* $U$. *Then*

(i)$F^\mu \sqsubseteq F^\mu$;
(ii)$F^\mu \sqsubseteq U^\mu$ *if* $F^\triangleleft_{mn}(e_m) = 1$ *for all* $e_m \in E$ *and* $m, n \in \Lambda$ *and*
$\emptyset^\mu \sqsubseteq F^\mu$ *if* $F^\triangleleft_{mn}(e_m) = 0$ *for all* $e_m \in E$ *and* $m, n \in \Lambda$ ;
(iii)$F^\mu \sqsubseteq G^\delta$ *and* $G^\delta \sqsubseteq H^\lambda \Longrightarrow F^\mu \sqsubseteq H^\lambda$.

*Proof.*It is clear from Definition 9.

**Definition 11.***[4] A binary operation* $\otimes : [0,1] \times [0,1] \longrightarrow [0,1]$ *is continuous* $t-$ *norm if* $\otimes$ *satisfies the following conditions.*

*(i)* $\otimes$ *is commutative and associative, (ii)* $\otimes$ *is continuous, (iii)* $a \otimes 1 = a$, $\forall a \in [0,1]$ *and (iv)* $a \otimes b \le c \otimes d$ *whenever* $a \le c, b \le d$ *and* $a, b, c, d \in [0,1]$.

**Definition 12.***[4] A binary operation* $\oplus : [0,1] \times [0,1] \longrightarrow [0,1]$ *is continuous* $t-$*conorm if* $\oplus$ *satisfies the following conditions:*

*(i)* $\oplus$ *is commutative and associative, (ii)* $\oplus$ *is continuous, (iii)* $a \oplus 0 = a$, $\forall a \in [0,1]$ *and (iv)* $a \oplus b \le c \oplus d$ *whenever* $a \le c, b \le d$ *and* $a, b, c, d \in [0,1]$.

**Definition 13.** *The union of two generalised neutrosophic soft sets $F^{\mu}$ and $G^{\delta}$ over $U$, denoted by $H^{\lambda} = F^{\mu} \sqcup G^{\delta}$, is a generalised neutrosophic soft set $H^{\lambda}$ defined by $H^{\lambda} = (H^{\triangleright}, H^{*}, H^{\triangleleft})$, where $\lambda(e_m) = \mu(e_m) \oplus \delta(e_m)$ and*

$$\begin{aligned} H^{\triangleright}_{mn}(e_m) &= F^{\triangleright}_{mn}(e_m) \oplus G^{\triangleright}_{mn}(e_m) \\ H^{*}_{mn}(e_m) &= F^{*}_{mn}(e_m) \oplus G^{*}_{mn}(e_m) \\ H^{\triangleleft}_{mn}(e_m) &= F^{\triangleleft}_{mn}(e_m) \otimes G^{\triangleleft}_{mn}(e_m) \end{aligned}$$

*for all $e_m \in E$ and $m, n \in \Lambda$.*

**Definition 14.** *The intersection of two generalised neutrosophic soft sets $F^{\mu}$ and $G^{\delta}$ over $U$, denoted by $K^{\varepsilon} = F^{\mu} \sqcap G^{\delta}$, is a generalised neutrosophic soft set $K^{\varepsilon}$ defined by $K^{\varepsilon} = (K^{\triangleright}, K^{*}, K^{\triangleleft})$, where $\varepsilon(e_m) = \mu(e_m) \otimes \delta(e_m)$ and*

$$\begin{aligned} H^{\triangleright}_{mn}(e_m) &= F^{\triangleright}_{mn}(e_m) \otimes G^{\triangleright}_{mn}(e_m) \\ H^{*}_{mn}(e_m) &= F^{*}_{mn}(e_m) \otimes G^{*}_{mn}(e_m) \\ H^{\triangleleft}_{mn}(e_m) &= F^{\triangleleft}_{mn}(e_m) \oplus G^{\triangleleft}_{mn}(e_m) \end{aligned}$$

*for all $e_m \in E$ and $m, n \in \Lambda$.*

*Example 3.* Let us consider the generalised neutrosophic soft sets $F^{\mu}$ and $G^{\delta}$ defined in Example 1. Suppose that the $t-conorm$ is defined by $\oplus(a,b) = \max\{a,b\}$ and the $t-norm$ by $\otimes(a,b) = \min\{a,b\}$ for $a,b \in [0,1]$. Then $H^{\lambda} = F^{\mu} \sqcup G^{\delta}$ is defined as follows:

$$\left\{ \begin{array}{l} H(e_1) = \left( \frac{c_1}{(0.6,0.5,0.4)}, \frac{c_2}{(0.8,0.7,0.2)}, \frac{c_3}{(0.6,0.4,0.5)} \right), (0.2) \\ H(e_2) = \left( \frac{c_1}{(0.6,0.8,0.2)}, \frac{c_2}{(0.6,0.7,0.1)}, \frac{c_3}{(0.7,0.7,0.3)} \right), (0.7) \\ H(e_3) = \left( \frac{c_1}{(0.5,0.5,0.1)}, \frac{c_2}{(0.3,0.7,0.2)}, \frac{c_3}{(0.8,0.3,0.4)} \right), (0.6) \end{array} \right\}$$

*Example 4.* Let us consider the generalised neutrosophic soft sets $F^{\mu}$ and $G^{\delta}$ defined in Example 1. Suppose that the $t-conorm$ is defined by $\oplus(a,b) = \max\{a,b\}$ and the $t-norm$ by $\otimes(a,b) = \min\{a,b\}$. Then $K^{\varepsilon} = F^{\mu} \sqcap G^{\delta}$ is defined as follows:

$$\left\{ \begin{array}{l} K(e_1) = \left( \frac{c_1}{(0.1,0.4,0.9)}, \frac{c_2}{(0.2,0.1,0.5)}, \frac{c_3}{(0.3,0.1,0.8)} \right), (0.1) \\ K(e_2) = \left( \frac{c_1}{(0.3,0.3,0.5)}, \frac{c_2}{(0.5,0.3,0.8)}, \frac{c_3}{(0.1,0.4,0.3)} \right), (0.4) \\ K(e_3) = \left( \frac{c_1}{(0.2,0.5,0.6)}, \frac{c_2}{(0.1,0.2,0.7)}, \frac{c_3}{(0.4,0.1,0.6)} \right), (0.1) \end{array} \right\}$$

**Proposition 2.** *Let $F^{\mu}$, $G^{\delta}$ and $H^{\lambda}$ be three generalised neutrosophic soft sets over $U$. Then*

(1)$F^{\mu} \sqcup G^{\delta} = G^{\delta} \sqcup F^{\mu}$,
(2)$F^{\mu} \sqcap G^{\delta} = G^{\delta} \sqcap F^{\mu}$,
(3)$\left(F^{\mu} \sqcup G^{\delta}\right) \sqcup H^{\lambda} = F^{\mu} \sqcup \left(G^{\delta} \sqcup H^{\lambda}\right)$,
(4)$\left(F^{\mu} \sqcap G^{\delta}\right) \sqcap H^{\lambda} = F^{\mu} \sqcap \left(G^{\delta} \sqcap H^{\lambda}\right)$.

*Proof.* The proofs can be easily obtained from relative definitions.

**Proposition 3.** *Let $F^{\mu}$, $G^{\delta}$ and $H^{\lambda}$ be three generalised neutrosophic soft sets over $U$. If we consider the $t-conorm$ defined by $\oplus(a,b) = \max\{a,b\}$ and the $t-norm$ by $\otimes(a,b) = \min\{a,b\}$ for $a,b \in [0,1]$, then the following holds:*

(1)$H^{\lambda} \sqcap \left(F^{\mu} \sqcup G^{\delta}\right) = \left(H^{\lambda} \sqcap F^{\mu}\right) \sqcup \left(H^{\lambda} \sqcap G^{\delta}\right)$,
(2)$H^{\lambda} \sqcup \left(F^{\mu} \sqcap G^{\delta}\right) = \left(H^{\lambda} \sqcup F^{\mu}\right) \sqcap \left(H^{\lambda} \sqcup G^{\delta}\right)$.

*Remark.* The relations in above proposition does not hold in general.

**Definition 15.** *The complement of a generalised neutrosophic soft set $F^{\mu}$ over $U$, denoted by $F^{\mu(c)}$ is defined by $F^{\mu(c)} = \left(F^{\triangleright(c)}, F^{*(c)}, F^{\triangleleft(c)}\right)$, where $\mu^{(c)}(e_m) = 1 - \mu(e_m)$ and*

$$\begin{aligned} F^{\triangleright(c)}_{mn}(e_m) &= F^{\triangleleft}_{mn}(e_m) \\ F^{*(c)}_{mn}(e_m) &= 1 - F^{*}_{mn}(e_m) \\ F^{\triangleleft(c)}_{mn}(e_m) &= F^{\triangleright}_{mn}(e_m) \end{aligned}$$

*for each $e_m \in E$ and $m, n \in \Lambda$.*

*Example 5.* Consider Example 1. Complement of the generalised neutrosophic soft set $F^{\mu}$ denoted by $F^{\mu(c)}$ is given as follows:

$$\left\{ \begin{array}{l} F^{\mu(c)}(e_1) = \left( \frac{c_1}{(0.4,0.5,0.6)}, \frac{c_2}{(0.5,0.3,0.2)}, \frac{c_3}{(0.8,0.9,0.6)} \right), (0.9) \\ F^{\mu(c)}(e_2) = \left( \frac{c_1}{(0.2,0.2,0.3)}, \frac{c_2}{(0.1,0.7,0.6)}, \frac{c_3}{(0.3,0.6,0.7)} \right), (0.6) \\ F^{\mu(c)}(e_3) = \left( \frac{c_1}{(0.6,0.5,0.2)}, \frac{c_2}{(0.2,0.3,0.1)}, \frac{c_3}{(0.4,0.7,0.8)} \right), (0.4) \end{array} \right\}$$

**Proposition 4.** *Let $F^{\mu}$ and $G^{\delta}$ be two generalised neutrosophic soft sets over $U$. Then*

(1)$F^{\mu}$ *is a generalised neutrosophic soft subset of* $F^{\mu} \sqcup F^{\mu(c)}$,
(2)$F^{\mu} \sqcap F^{\mu(c)}$ *is a generalised neutrosophic soft subset of* $F^{\mu}$.

*Proof.* It is clear from definition.

**Definition 16.** *"AND" operation on two generalised neutrosophic soft sets $F^{\mu}$ and $G^{\delta}$ over $U$, denoted by $H^{\lambda} = F^{\mu} \wedge G^{\delta}$, is the mapping $H^{\lambda} : C \longrightarrow \mathcal{N}(U) \times I$ defined by $H^{\lambda} = (H^{\triangleright}, H^{*}, H^{\triangleleft})$, where $\lambda(e_m) = \min\{\mu(e_k), \delta(e_h)\}$ and*

$$\begin{aligned} H^{\triangleright}_{mn}(e_m) &= \min\left\{F^{\triangleright}_{kn}(e_k), G^{\triangleright}_{hn}(e_h)\right\} \\ H^{*}_{mn}(e_m) &= \tfrac{1}{2}\left\{F^{*}_{kn}(e_k) + G^{*}_{hn}(e_h)\right\} \\ H^{\triangleleft}_{mn}(e_m) &= \max\left\{F^{\triangleleft}_{kn}(e_k), G^{\triangleleft}_{hn}(e_h)\right\} \end{aligned}$$

*for all $e_m = (e_k, e_h) \in C \subseteq E \times E$ and $m, n, h, k \in \Lambda$.*

**Definition 17.** *"OR" operation on two generalised neutrosophic soft sets $F^{\mu}$ and $G^{\delta}$ over $U$, denoted by $K^{\varepsilon} = F^{\mu} \vee G^{\delta}$, is the mapping $K^{\varepsilon} : C \longrightarrow \mathcal{N}(U) \times I$ defined by $K^{\varepsilon} = (K^{\triangleright}, K^{*}, K^{\triangleleft})$, where $\varepsilon(e_m) = \max\{\mu(e_k), \delta(e_h)\}$ and*

$$\begin{aligned} K^{\triangleright}_{mn}(e_m) &= \max\left\{F^{\triangleright}_{kn}(e_k), G^{\triangleright}_{hn}(e_h)\right\} \\ K^{*}_{mn}(e_m) &= \tfrac{1}{2}\left\{F^{*}_{kn}(e_k) + G^{*}_{hn}(e_h)\right\} \\ K^{\triangleleft}_{mn}(e_m) &= \min\left\{F^{\triangleleft}_{kn}(e_k), G^{\triangleleft}_{hn}(e_h)\right\} \end{aligned}$$

*for all $e_m = (e_k, e_h) \in C \subseteq E \times E$ and $m, n, h, k \in \Lambda$.*

**Definition 18.***Let $F^{\mu}$ and $G^{\delta}$ be two generalised neutrosophic soft sets over $U$ and $C \subseteq E \times E = E^2$. A function $R : C \longrightarrow \mathcal{N}(U) \times I$, defined by $R = F^{\mu} \wedge G^{\delta}$ and $R(e_m, e_h) = F^{\mu}(e_m) \wedge G^{\delta}(e_h)$ is said to be a neutrosophic soft relation from $F^{\mu}$ to $G^{\delta}$ for all $(e_m, e_h) \in C$.*

**Definition 19.***Let $\mathfrak{F} = \left\{F_t^{\mu_t} : t \in \Lambda\right\}$ be any collection of generalised neutrosophic soft sets over $U$. Suppose that $C \subseteq E^t$ for $t \in \Lambda$. Then a generalised neutrosophic soft relation $R$ on $\mathfrak{F}$ is the mapping $R : C \longrightarrow \mathcal{N}(U) \times I$ and is defined by $R(e_{m_1}, e_{m_2}, \ldots, e_{m_t}) = \wedge_{i=1}^{t} F_{t_i}^{\mu_{t_i}}(e_{m_i})$, where $(e_{m_1}, e_{m_2}, \ldots, e_{m_t}) \in C$.*

Now, we present an application of the generalised neutrosophic soft set relation in a decision making problem. Suppose that $U = \{c_1, c_2, c_3\}$ is a set of cars and $E = \{e_1, e_2, e_3\} = \{\text{performance, security, comfort}\}$ is a set of parameters which is attractivess of cars. Suppose Mr. X wants to buy one the most suitable car according to himself depending on three of the parameters only. Here, the selection is dependent on the choice parameters of buyer. Suppose that there are two observations $F^{\mu}$ and $G^{\delta}$ on cars by two committees based on the choice parameters of Mr. X. The committees construct the following generalised neutrosophic soft sets $F_{\mu}$ and $G^{\delta}$, respectively;

$$\begin{pmatrix} (0.7,0.5,0.3) & (0.5,0.4,0.5) & (0.9,0.9,0.6) & (0.2) \\ (0.3,0.2,0.5) & (0.6,0.7,0.3) & (0.5,0.8,0.4) & (0.5) \\ (0.6,0.6,0.5) & (0.4,0.2,0.5) & (0.2,0.7,0.3) & (0.7) \end{pmatrix}$$

$$\begin{pmatrix} (0.7,0.4,0.7) & (0.3,0.6,0.3) & (0.5,0.4,0.8) & (0.4) \\ (0.3,0.5,0.2) & (0.8,0.6,0.7) & (0.4,0.3,0.3) & (0.1) \\ (0.9,0.7,0.7) & (0.2,0.5,0.1) & (0.8,0.1,0.4) & (0.6) \end{pmatrix}$$

Let us consider generalised neutrosophic soft set relation $R$ which is the mapping $R : C \longrightarrow \mathcal{N}(U) \times I$, given as follows:

| $R$ | $c_1$ | $c_2$ | $c_3$ | $\mu$ |
|---|---|---|---|---|
| $e_{11}$ | (0.7, 0.45, 0.7) | (0.3, 0.5, 0.5) | (0.5, 0.65, 0.8) | (0.2) |
| $e_{12}$ | (0.3, 0.5, 0.3) | (0.5, 0.5, 0.7) | (0.4, 0.6, 0.6) | (0.1) |
| $e_{13}$ | (0.7, 0.6, 0.7) | (0.2, 0.45, 0.5) | (0.8, 0.5, 0.6) | (0.2) |
| $e_{21}$ | (0.3, 0.3, 0.7) | (0.3, 0.65, 0.3) | (0.5, 0.6, 0.8) | (0.4) |
| $e_{22}$ | (0.3, 0.35, 0.5) | (0.6, 0.65, 0.7) | (0.4, 0.55, 0.4) | (0.1) |
| $e_{23}$ | (0.3, 0.45, 0.7) | (0.2, 0.6, 0.3) | (0.5, 0.45, 0.4) | (0.5) |
| $e_{31}$ | (0.6, 0.5, 0.7) | (0.3, 0.4, 0.5) | (0.2, 0.55, 0.8) | (0.4) |
| $e_{32}$ | (0.3, 0.55, 0.5) | (0.4, 0.4, 0.7) | (0.2, 0.5, 0.3) | (0.1) |
| $e_{33}$ | (0.6, 0.65, 0.7) | (0.2, 0.35, 0.5) | (0.2, 0.4, 0.4) | (0.6) |

Matrix form of relation R

| $R$ | $c_1$ | $c_2$ | $c_3$ | $\mu$ |
|---|---|---|---|---|
| $e_{11}$ | $\underline{0.7}$ | 0,3 | 0.5 | 0.2 |
| $e_{12}$ | 0.3 | $\underline{0.5}$ | 0.4 | 0.1 |
| $e_{13}$ | 0,7 | 0.2 | $\underline{0.8}$ | 0.2 |
| $e_{21}$ | 0.3 | 0,3 | $\underline{0.5}$ | 0.4 |
| $e_{22}$ | 0.3 | $\underline{0.6}$ | 0.4 | 0.1 |
| $e_{23}$ | 0.3 | 0.2 | $\underline{0.5}$ | 0.5 |
| $e_{31}$ | $\underline{0.6}$ | 0,3 | 0.2 | 0.4 |
| $e_{32}$ | 0.3 | $\underline{0.4}$ | 0.2 | 0.1 |
| $e_{33}$ | $\underline{0.6}$ | 0.2 | 0.2 | 0.6 |

Matrix $F^{\triangleright}$ of $R$

| $R$ | $c_1$ | $c_2$ | $c_3$ | $\mu$ |
|---|---|---|---|---|
| $e_{11}$ | 0,45 | 0.5 | $\underline{0.65}$ | 0.2 |
| $e_{12}$ | 0.5 | 0.5 | $\underline{0.6}$ | 0.1 |
| $e_{13}$ | $\underline{0.6}$ | 0.45 | 0.5 | 0.2 |
| $e_{21}$ | 0.3 | $\underline{0.65}$ | 0.6 | 0.6 |
| $e_{22}$ | 0.35 | $\underline{0.65}$ | 0.55 | 0.1 |
| $e_{23}$ | 0.45 | $\underline{0.6}$ | 0.45 | 0.5 |
| $e_{31}$ | 0,5 | 0.4 | $\underline{0.55}$ | 0.4 |
| $e_{32}$ | $\underline{0.55}$ | 0.4 | 0.5 | 0.1 |
| $e_{33}$ | $\underline{0.65}$ | 0.35 | 0.4 | 0.6 |

Matrix $F^{*}$ of $R$

| $R$ | $c_1$ | $c_2$ | $c_3$ | $\mu$ |
|---|---|---|---|---|
| $e_{11}$ | 0.7 | 0.5 | $\underline{0.8}$ | 0.2 |
| $e_{12}$ | 0.3 | $\underline{0.7}$ | 0.6 | 0.1 |
| $e_{13}$ | $\underline{0.7}$ | 0.5 | 0.6 | 0.2 |
| $e_{21}$ | 0.7 | 0.3 | $\underline{0.8}$ | 0.4 |
| $e_{22}$ | 0.5 | $\underline{0.7}$ | 0.4 | 0.1 |
| $e_{23}$ | $\underline{0.7}$ | 0.3 | 0.4 | 0.5 |
| $e_{31}$ | 0.7 | 0.5 | $\underline{0.8}$ | 0.4 |
| $e_{32}$ | 0.5 | $\underline{0.7}$ | 0.3 | 0.1 |
| $e_{33}$ | $\underline{0.7}$ | 0.5 | 0.4 | 0.6 |

Matrix $F^{\triangleleft}$ of $R$

We present the matrices of three basic components of $R$, which are truth-membership, indeterminacy membership and falsity-membership part. To choose the best car, we firstly mark the highest numerical grade (underline) in each row of each matrix. But here, since the last column is the grade of such belongingness of a car for each pair of parameters, it is not taken into account while marking. Then we calculate the score of each component of $R$ by taking the sum of products of these numerical grades with the corresponding values of $\mu$. Next, we calculate the final score by subtracting the score of falsity-membership part of $R$ from the sum of scores of truth-membership part and of indeterminacy membership part of $R$. The car with the highest score is the desired car by Mr. X.

Now, we calculate the score of each component of $R$, respectively.

$$\begin{array}{l} \text{Score } (c_1) = 0.74 \\ \text{Score } (c_2) = 0.15 \\ \text{Score } (c_3) = 0.61 \end{array}, \quad \begin{array}{l} \text{Score } (c_1) = 0.56 \\ \text{Score } (c_2) = 0.62 \\ \text{Score } (c_3) = 0.41 \end{array},$$

$$\begin{array}{l} \text{Score } (c_1) = 0.91 \\ \text{Score } (c_2) = 0.21 \\ \text{Score } (c_3) = 0.80 \end{array}$$

Thus we conclude the problem by calculating final score, i.e.,

$$\begin{array}{l} \text{Score } (c_1) = (0.74) + (0.56) - (0.91) = 0.39 \\ \text{Score } (c_2) = (0.15) + (0.62) - (0.21) = 0.56 \\ \text{Score } (c_3) = (0.61) + (0.41) - (0.80) = 0.22 \end{array}$$

Then the optimal selection for Mr. X is the $c_2$.

## 4 Similarity measures of neutrosophic soft sets based on distance

In this section we introduce a similarity measure based on distance for neutrosophic soft sets, which can be used in the field of pattern recognition, feature extraction, region extraction, image processing, coding theory etc. We first define a distance function between two neutrosophic soft sets. We then give the similarity measure by means of the distance function.

**Definition 20.***Let $U$ be the universal set, $U=\{x_1,x_2,...,x_k\}$ and $E$ be the set of parameters, $E=\{e_1,e_2,...,e_t\}$.. Suppose that $F^\mu$ and $G^\delta$ are two neutrosophic soft sets over $U$, i.e., $\mu(e_m)=\delta(e_m)=1$ for each $e_m\in E$, $F^\mu=(F^\triangleright,F^*,F^\triangleleft)$ and $G^\delta=(G^\triangleright,G^*,G^\triangleleft)$. Then we define the distance function as follows:*

$$d\left(F^\mu,G^\delta\right)=\max_m\left(d_m\left(F^\mu,G^\delta\right)\right).$$

*Here, $d_m\left(F^\mu,G^\delta\right)$ denotes the distance between $F^\mu$ and $G^\delta$ the according to $m-$th parameter and is defined by*

$$d_m\left(F^\mu,G^\delta\right)=\left(\frac{1}{k}\sum_{i=1}^{k}\sum_{j=1}^{3}\left(F^\mu_{mn_{ij}}-G^\delta_{mn_{ij}}\right)^2\right)^{\frac{1}{2}}$$

*where $F^\mu_{mn_{ij}}(e_m)=\left(F^\triangleright_{mn_{ij}}(e_m),F^*_{mn_{ij}}(e_m),F^\triangleleft_{mn_{ij}}(e_m)\right)$ and $G^\delta_{mn_{ij}}(e_m)=(G^\triangleright_{mn_{ij}}(e_m),G^*_{mn_{ij}}(e_m),G^\triangleleft_{mn_{ij}}(e_m))$ for each $e_m\in E$ and $m,n,i,j\in\Lambda$.*

*Then similarity measure between $F^\mu$ and $G^\delta$ based on distance, denoted by $D\left(F^\mu,G^\delta\right)$, is defined by*

$$D\left(F^\mu,G^\delta\right)=\min_m D_m\left(F^\mu,G^\delta\right),$$

*where* $D_m\left(F^\mu,G^\delta\right)=\dfrac{1}{1+d_m\left(F^\mu,G^\delta\right)}$.

*Example 6.*Let us consider the neutrosophic soft sets $F^\mu$ and $G^\delta$ as follows, respectively:

$$\begin{pmatrix}(0.5,0.6,0.5)&(0.4,0.7,0.2)&(0.9,0.8,0.1)\\(0.4,0.4,0.5)&(0.7,0.1,0.8)&(0.7,0.8,0.5)\\(0.1,0.9,0.4)&(0.2,0.8,0.3)&(0.2,0.6,0.4)\end{pmatrix}$$
$$\begin{pmatrix}(0.3,0.9,0.6)&(0.9,0.5,0.6)&(0.2,0.1,0.4)\\(0.7,0.5,0.2)&(0.6,0.3,0.6)&(0.4,0.3,0.7)\\(0.9,0.1,0.6)&(0.8,0.1,0.9)&(0.9,0.5,0.8)\end{pmatrix}$$

Here, $d_1\left(F^\mu,G^\delta\right)\cong0.74$, $d_2\left(F^\mu,G^\delta\right)\cong0.46$ and $d_3\left(F^\mu,G^\delta\right)\cong1.03$. Then $D_1\left(F^\mu,G^\delta\right)\cong0.57$, $D_2\left(F^\mu,G^\delta\right)\cong0.68$ and $D_3\left(F^\mu,G^\delta\right)\cong0.49$ and so $D\left(F^\mu,G^\delta\right)\cong0.49$.

## 5 Similarity measures between generalised neutrosophic soft sets

Let $U$ be the universal set, $U=\{x_1,x_2,...,x_k\}$ and $E$ be the set of parameters, $E=\{e_1,e_2,...,e_t\}$. Suppose that $F^\mu$ and $G^\delta$ are two generalised neutrosophic soft sets over $U$, $F^\mu=(F^\triangleright,F^*,F^\triangleleft)$ and $G^\delta=(G^\triangleright,G^*,G^\triangleleft)$, where $F^\mu_{mn}(e_m)=(F^\triangleright_{mn}(e_m),F^*_{mn}(e_m),F^\triangleleft_{mn}(e_m))$ and $G^\delta_{mn}(e_m)=(G^\triangleright_{mn}(e_m),G^*_{mn}(e_m),G^\triangleleft_{mn}(e_m))$ for each $e_m\in E$ and $m,n,i\in\Lambda$.

For the calculation of similarity between $F^\mu$ and $G^\delta$, denoted by $S\left(F^\mu,G^\delta\right)$, we do three different calculation, which are $s(\mu,\delta)$, $s\left(F^\mu,G^\delta\right)$ and finally $S\left(F^\mu,G^\delta\right)=s\left(F^\mu,G^\delta\right)\times s(\mu,\delta)$. Let us define

$$s_{truth}\left(F^\mu,G^\delta\right)=\max_m s^\triangleright_m\left(F^\mu,G^\delta\right)$$
$$s_{indeter}\left(F^\mu,G^\delta\right)=\max_m s^*_m\left(F^\mu,G^\delta\right)$$
$$s_{falsity}\left(F^\mu,G^\delta\right)=\max_m s^\triangleleft_m\left(F^\mu,G^\delta\right),$$

where

$$s^\triangleright_m\left(F^\mu,G^\delta\right)=1-\frac{\sum_{i=1}^{k}\left|F^\triangleright_{mn_i}-G^\triangleright_{mn_i}\right|}{\sum_{i=1}^{k}\left|F^\triangleright_{mn_i}+G^\triangleright_{mn_i}\right|},$$
$$s^*_m\left(F^\mu,G^\delta\right)=1-\frac{\sum_{i=1}^{k}\left|F^*_{mn_i}-G^*_{mn_i}\right|}{\sum_{i=1}^{k}\left|F^*_{mn_i}+G^*_{mn_i}\right|},$$
$$s^\triangleleft_m\left(F^\mu,G^\delta\right)=1-\frac{\sum_{i=1}^{k}\left|F^\triangleleft_{mn_i}-G^\triangleleft_{mn_i}\right|}{\sum_{i=1}^{k}\left|F^\triangleleft_{mn_i}+G^\triangleleft_{mn_i}\right|}$$

for each $m,n,i\in\Lambda$.

Then we define the similarity between $F^\mu$ and $G^\delta$ as follows

$$S\left(F^\mu,G^\delta\right)=s\left(F^\mu,G^\delta\right)\times s(\mu,\delta),$$

where

$$s\left(F^\mu,G^\delta\right)=\frac{1}{3}\left(s_{truth}\left(F^\mu,G^\delta\right)+s_{indeter}\left(F^\mu,G^\delta\right)+s_{falsity}\left(F^\mu,G^\delta\right)\right)$$

and where

$$s(\mu,\delta)=1-\frac{\sum|\mu_m-\delta_m|}{\sum|\mu_m+\delta_m|},\ \mu_m=\mu(e_m)\text{ and }\delta_m=\delta(e_m)$$

for each $e_m\in E$.

*Example 7.*Let $U=\{c_1,c_2,c_3\}$ and $E=\{e_1,e_2,e_3\}$. Let us consider two generalised neutrosophic soft sets $F^\mu$ and $G^\delta$ as follows, respectively;

$$\begin{pmatrix}(0.4,0.1,0.1)&(0.1,0.4,0.9)&(0.2,0.1,0.8)&(0.5)\\(0.3,0.4,0.3)&(0.4,0.3,0.2)&(0.3,0.3,0.9)&(0.9)\\(0.4,0.2,0.5)&(0.1,0.2,0.1)&(0.5,0.2,0.2)&(0.2)\end{pmatrix}$$
$$\begin{pmatrix}(0.1,0.4,0.3)&(0.8,0.1,0.6)&(0.1,0.7,0.1)&(0.7)\\(0.2,0.3,0.2)&(0.4,0.3,0.2)&(0.5,0.3,0.1)&(0.8)\\(0.7,0.1,0.2)&(0.8,0.2,0.8)&(0.3,0.2,0.6)&(0.3)\end{pmatrix}$$

Then

$s_1^{\triangleright}\left(F^{\mu},G^{\delta}\right) \cong 0,35$, $s_2^{\triangleright}\left(F^{\mu},G^{\delta}\right) \cong 0.85$, and $s_3^{\triangleright}\left(F^{\mu},G^{\delta}\right) \cong 0.57 \Longrightarrow s_{truth}\left(F^{\mu},G^{\delta}\right) \cong 0.85$,

$s_1^{*}\left(F^{\mu},G^{\delta}\right) \cong 0.33, s_2^{*}\left(F^{\mu},G^{\delta}\right) \cong 0.94$ and $s_3^{*}\left(F^{\mu},G^{\delta}\right) \cong 0.90 \Longrightarrow s_{indeter}\left(F^{\mu},G^{\delta}\right) \cong 0.94$,

$s_1^{\triangleleft}\left(F^{\mu},G^{\delta}\right) \cong 0.57$, $s_2^{\triangleleft}\left(F^{\mu},G^{\delta}\right) \cong 0.52$ and $s_3^{\triangleleft}\left(F^{\mu},G^{\delta}\right) \cong 0.41 \Longrightarrow s_{falsity}\left(F^{\mu},G^{\delta}\right) \cong 0.57$.

So

$$s\left(F^{\mu},G^{\delta}\right) = \frac{0.85+0.94+0.57}{3} \cong 0.78.$$

Moreover,
$s(\mu,\delta) = 1-\frac{\sum|\mu_m-\delta_m|}{\sum|\mu_m+\delta_m|} = 1-\frac{0.2+0.1+0,1}{1.2+1.7+0.5} = 0.88.$

Thus by definition of similarity between two generalised neutrosophic soft sets, it is follows that

$$S\left(F^{\mu},G^{\delta}\right) = s\left(F^{\mu},G^{\delta}\right).s(\mu,\delta) = (0.78)\times(0.88) \cong 0.69.$$

**Definition 21.***Let $F^{\mu}$ and $G^{\delta}$ be two generalised neutrosophic soft sets over $U$. We say that $F^{\mu}$ and $G^{\delta}$ are significantly similar if $S\left(F^{\mu},G^{\delta}\right) > \frac{1}{2}$.*

**Proposition 5.***The above defined similarity measure between generalised neutrosophic soft sets $F^{\mu}$ and $G^{\delta}$ satisfies the following properties:*

(1)$0 \le S\left(F^{\mu},G^{\delta}\right) \le 1$;
(2)$F^{\mu} = G^{\delta} \Longrightarrow S\left(F^{\mu},G^{\delta}\right) = 1$;
(3)$S\left(F^{\mu},G^{\delta}\right) = S\left(G^{\delta},F^{\mu}\right)$;
(4)*If $F^{\mu} \sqsubseteq G^{\delta} \sqsubseteq H^{\delta}$, then $S\left(F^{\mu},H^{\lambda}\right) \le S\left(F^{\mu},G^{\delta}\right)$ and $S\left(F^{\mu},H^{\lambda}\right) \le S\left(G^{\delta},H^{\lambda}\right)$.*

*Proof.*The results (1),(2) and (3) holds trivially from definition. We only prove (4).

Suppose that $F^{\mu} \sqsubseteq G^{\delta} \sqsubseteq H^{\delta}$. Then

$$\begin{array}{l} F^{\triangleright}_{mn_i}(e_m) \le G^{\triangleright}_{mn_i}(e_m) \le H^{\triangleright}_{mn_i}(e_m), \\ F^{*}_{mn_i}(e_m) \le G^{*}_{mn_i}(e_m) \le H^{*}_{mn_i}(e_m), \\ F^{\triangleleft}_{mn_i}(e_m) \ge G^{\triangleleft}_{mn_i}(e_m) \ge H^{\triangleleft}_{mn_i}(e_m), \end{array}$$

and $\mu(e_m) \le \delta(e_m) \le \lambda(e_m)$ for each $e_m \in E$ and $m,n,i \in \Lambda$. Since

$$\frac{\sum_{i=1}^{k}\left|F^{\triangleright}_{mn_i} - G^{\triangleright}_{mn_i}\right|}{\sum_{i=1}^{k}\left|F^{\triangleright}_{mn_i} + G^{\triangleright}_{mn_i}\right|} \le \frac{\sum_{i=1}^{k}\left|F^{\triangleright}_{mn_i} - H^{\triangleright}_{mn_i}\right|}{\sum_{i=1}^{k}\left|F^{\triangleright}_{mn_i} + H^{\triangleright}_{mn_i}\right|}$$

and

$$1-\frac{\sum_{i=1}^{k}\left|F^{\triangleright}_{mn_i} - H^{\triangleright}_{mn_i}\right|}{\sum_{i=1}^{k}\left|F^{\triangleright}_{mn_i} + H^{\triangleright}_{mn_i}\right|} \le 1-\frac{\sum_{i=1}^{k}\left|F^{\triangleright}_{mn_i} - G^{\triangleright}_{mn_i}\right|}{\sum_{i=1}^{k}\left|F^{\triangleright}_{mn_i} + G^{\triangleright}_{mn_i}\right|}$$

for each $e_m \in E$ and $m,n,i \in \Lambda$, we have $s_m^{\triangleright}\left(F^{\mu},H^{\lambda}\right) \le s_m^{\triangleright}\left(F^{\mu},G^{\delta}\right)$ for all $m \in \Lambda$. Similarly, it can be proved that $s_m^{*}\left(F^{\mu},H^{\lambda}\right) \le s_m^{*}\left(F^{\mu},G^{\delta}\right)$ and $s_m^{\triangleleft}\left(F^{\mu},H^{\lambda}\right) \le s_m^{\triangleleft}\left(F^{\mu},G^{\delta}\right)$ for all $m \in \Lambda$. By max-operation, this implies that $s_{truth}\left(F^{\mu},H^{\lambda}\right) \le s_{truth}\left(F^{\mu},G^{\delta}\right)$, $s_{indeter}\left(F^{\mu},H^{\lambda}\right) \le s_{indeter}\left(F^{\mu},G^{\delta}\right)$ and $s_{falsity}\left(F^{\mu},H^{\lambda}\right) \le s_{falsity}\left(F^{\mu},G^{\delta}\right)$. Hence $s\left(F^{\mu},H^{\lambda}\right) \le s\left(F^{\mu},G^{\delta}\right)$.

On the other hand, since $\mu(e_m) \le \delta(e_m) \le \lambda(e_m)$ for each $e_m \in E$, we have that

$$\frac{\sum|\mu_m-\delta_m|}{\sum|\mu_m+\delta_m|} \le \frac{\sum|\mu_m-\lambda_m|}{\sum|\mu_m+\lambda_m|},$$

$$1-\frac{\sum|\mu_m-\lambda_m|}{\sum|\mu_m+\lambda_m|} \le 1-\frac{\sum|\mu_m-\delta_m|}{\sum|\mu_m+\delta_m|}$$

and so

$$s(\mu,\lambda) \le s(\mu,\delta).$$

By $s\left(F^{\mu},H^{\lambda}\right) \le s\left(F^{\mu},G^{\delta}\right)$ and $s(\mu,\lambda) \le s(\mu,\delta)$, it follows that $S\left(F^{\mu},H^{\lambda}\right) \le S\left(F^{\mu},G^{\delta}\right)$.

Similarly, we have that $S\left(F^{\mu},H^{\lambda}\right) \le S\left(G^{\delta},H^{\lambda}\right)$.

## 6 Decision-making method based on the similarity measure

In this section, we present a handling method for the decision-making problem by means of the similarity measure between generalised neutrosophic soft sets. Note that the similarity measure depends on both the neutrosophic set value, i.e., $F(e)$ and the reliability of the value, i.e., $\mu(e)$ for any generalised neutrosophic soft set $F^{\mu}$.

Let us consider the universal set $U = \{y,n\}$,contain only two elements "yes and no". Suppose that $P = \{p_1,p_2,p_3,p_4,p_5\}$ are five projects offered to State Planning Organization in Turkey. Let $E = \{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8\}$ be the set of parameters (criteria for every project), where $e_1$=health, $e_2$=education, $e_3$=economy, $e_4$=environment, $e_5$=culture, $e_6$=tourism, $e_7$=industry and $e_8$=European Union.

It has been developed a great number of criteria for acceptability testing both a single project and more than one independently of each other. Here, the our aim is to select the optimal project according to given parameters. For to evaluate the projects in terms of the parameters, it is established by the government a supervisory board. Model generalised neutrosophic soft set of this supervisory board is given in Table 1.

**Table 1:** Model generalised neutrosophic soft set

| $M^{\sigma}$ | health | education | economy | environment |
|---|---|---|---|---|
| $y$ | $(1,1,1)$ | $(1,1,1)$ | $(1,1,1)$ | $(0,0,0)$ |
| $n$ | $(0,0,0)$ | $(0,0,0)$ | $(0,0,0)$ | $(1,1,1)$ |
| $\sigma$ | 1 | 1 | 1 | 1 |

| $M^{\sigma}$ | culture | tourism | industry | European Union |
|---|---|---|---|---|
| $y$ | $(0,0,0)$ | $(1,1,1)$ | $(0,0,0)$ | $(1,1,1)$ |
| $n$ | $(1,1,1)$ | $(0,0,0)$ | $(1,1,1)$ | $(0,0,0)$ |
| $\sigma$ | 1 | 1 | 1 | 1 |

For each project, the supervisory board report the results given follow:

**Table 2:** Generalised neutrosophic soft set for $p_1$

| $F^{\mu}$ | health | education | economy | environment |
|---|---|---|---|---|
| $y$ | $(0.8,0.1,0.2)$ | $(0.6,0.2,0.8)$ | $(0.1,0.4,0.2)$ | $(0.7,0.4,0.8)$ |
| $n$ | $(0,4,0.3,0.5)$ | $(0.1,0.2,0.2)$ | $(0.6,0.5,0.7)$ | $(0.5,0.1,0.4)$ |
| $\mu$ | 0.4 | 0.6 | 0.2 | 0.8 |

| $F^{\mu}$ | culture | tourism | industry | European Union |
|---|---|---|---|---|
| $y$ | $(0.5,0.3,0.5)$ | $(0.6,0.4,0.4)$ | $(0.2,0.6,0.9)$ | $(0,0.7,0.3)$ |
| $n$ | $(0.2,0.9,0.1)$ | $(0.7,0.1,0.5)$ | $(0,0.4,0.8)$ | $(0.8,0.1,0.1)$ |
| $\mu$ | 0.4 | 0.5 | 0.2 | 0.1 |
| | | | | |

**Table 3:** Generalised neutrosophic soft set for $p_2$

| $G^{\delta}$ | health | education | economy | environment |
|---|---|---|---|---|
| $y$ | $(0.7,0.3,0.9)$ | $(0.8,0.1,0.7)$ | $(0.4,0.4,0.4)$ | $(0.5,0.4,0.4)$ |
| $n$ | $(0.5,0.1,0.5)$ | $(0.2,0.7,0.1)$ | $(0.4,0.5,0.2)$ | $(0.6,0.5,0.7)$ |
| $\delta$ | 0.4 | 0.3 | 0.7 | 0.8 |

| $G^{\delta}$ | culture | tourism | industry | European Union |
|---|---|---|---|---|
| $y$ | $(0.1,0.1,0.1)$ | $(0.1,0.7,0.3)$ | $(0.7,0.2,0.5)$ | $(0.4,0.4,0.1)$ |
| $n$ | $(0.9,0.7,0.9)$ | $(0.5,0.5,0.5)$ | $(0.9,0.4,0.1)$ | $(0.1,0.3,0.3)$ |
| $\delta$ | 0.3 | 0.3 | 0.1 | 0.5 |

**Table 4:** Generalised neutrosophic soft set for $p_3$

| $H^{\lambda}$ | health | education | economy | environment |
|---|---|---|---|---|
| $y$ | $(0.1,0.8,0.5)$ | $(0.8,0.5,0.2)$ | $0.4,0.3,0.1$ | $(0.3,0,0.5)$ |
| $n$ | $(0.3,0.5,0.5)$ | $(0.6,0.1,0.3)$ | $(0.2,0.2,0.3)$ | $(0.8,0.8,0.8)$ |
| $\lambda$ | 0.8 | 0.6 | 0.7 | 0.7 |

| $H^{\lambda}$ | culture | tourism | industry | European Union |
|---|---|---|---|---|
| $y$ | $(0.2,0.9,0.4)$ | $(0.6,0.8,0.1)$ | $(0.4,0.9,0.4)$ | $(0.3,0.3,0.7)$ |
| $n$ | $(0.3,0.5,0.2)$ | $(0.1,0.1,0.4)$ | $(0,0.1,0.3)$ | $(0.9,0.8,0.1)$ |
| $\lambda$ | 0.3 | 0.6 | 0.8 | 0.8 |

**Table 5:** Generalised neutrosophic soft set for $p_4$

| $K^{\varepsilon}$ | health | education | economy | environment |
|---|---|---|---|---|
| $y$ | $(0.4,0.6,0.1)$ | $(0.8,0.1,0.2)$ | $(0.3,0.5,0.8)$ | $(0.4,0.6,0.7)$ |
| $n$ | $(0.1,0.1,0.8)$ | $(0.4,0.1,0.4)$ | $(0.6,0.7,0.2)$ | $(0.9,0.3,0.3)$ |
| $\varepsilon$ | 0.1 | 0.4 | 0.9 | 0.3 |

| $K^{\varepsilon}$ | culture | tourism | industry | European Union |
|---|---|---|---|---|
| $y$ | $(0.3,0.2,0.1)$ | $(0.5,0.6,0.1)$ | $(0.9,0.2,0.4)$ | $(0.4,0.1,0.3)$ |
| $n$ | $(0.3,0.1,0.5)$ | $(0.1,0.1,0.7)$ | $(0.4,0.3,0.2)$ | $(0.2,0.1,0.5)$ |
| $\varepsilon$ | 0.2 | 0.7 | 0.3 | 0.7 |

**Table 6:** Generalised neutrosophic soft set for $p_5$

| $T^{\omega}$ | health | education | economy | environment |
|---|---|---|---|---|
| $y$ | $(0.5,0.1,0.3)$ | $(0.5,0.7,0.2)$ | $(0.4,0.7,0.9)$ | $(0.8,0.2,0.1)$ |
| $n$ | $(0.3,0.7,0.3)$ | $(0.7,0.4,0.4)$ | $(0.2,0.8,0.8)$ | $(0.3,0.9,0.7)$ |
| $\omega$ | 0.4 | 0.6 | 0.6 | 0.2 |

| $T^{\omega}$ | culture | tourism | industry | European Union |
|---|---|---|---|---|
| $y$ | $(0.4,0.5,0.5)$ | $(0.2,0.4,0.9)$ | $(0.1,0.3,0.1)$ | $(0.4,0.7,0.2)$ |
| $n$ | $(0.4,0.7,0.4)$ | $(0.9,0.6,0.4)$ | $(0.8,0.8,0.9)$ | $(0.3,0.5,0.5)$ |
| $\omega$ | 0.7 | 0.5 | 0.5 | 0.1 |

Now, we compute the similarty between the model generalised neutrosophic soft set and the generalised neutrosophic soft set of each project as follows:

$$S(M^{\sigma},F^{\mu}) \cong 0.44 < \frac{1}{2}, S\left(M^{\sigma},G^{\delta}\right) \cong 0.51 > \frac{1}{2},$$

$$S\left(M^{\sigma},H^{\lambda}\right) \cong 0.64 > \frac{1}{2},$$

$$S(M^{\sigma},K^{\varepsilon}) \cong 0.47 < \frac{1}{2} \text{ and } S(M^{\sigma},T^{\omega}) \cong 0.53 > \frac{1}{2}.$$

Thus we conclude that the project $p_3$ is should be selected by the supervisory board.

## 7 Conclusion

In this paper we have introduced the concept of generalised neutrosophic soft set and studied some of the related properties. By generalizing the similarity measure given in [22], we also presented a new method to find out the similarity measure of two generalised neutrosophic soft sets and discussed an application of this to decision making problem. In future one could study algebraic structures such as group, ring and field of the neutrosophic soft set and also its generalization.

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