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Modeling Interest Rate Dynamics for the Bank of Ghana Rates using the Hull-White Model

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Abstract: Interest rates play an important role in the financial environment, affecting business transactions directly or indirectly. Fluctuations in interest rates caused largely by demand and supply of credit, regulated by the apex banks, impacts business transactions in the Economy. Understanding the dynamics of interest rates is therefore very important to Financial institutions, individual and corporate investors. In this work, the dynamics of Bank of Ghana's, daily interest rates, (Jan 2020 - July 2021) is modeled using the Hull-White model. London interbank offer rates, for the same period are included in the analysis. By estimating the parameters of the model, and using a computation algorithm for the solution of the SDE of the model; It is found that the mean reverting model captured the BOG and LIBOR rates well, largely maintaining the trend of the data structure. It also pointed to the presence of jumps in the data sets.

Keywords: Hull-White, mean reverting, parameters, dynamics, BOG, LIBOR

1 Introduction

Basically, an interest rate is the amount of money a lender or creditor charges for access to money (usually expressed in annual percentages). Interest rate plays a big role in the financial environment. Interest rate to the borrower is the price paid for the use of money, such as a loan; whereas for the lender, interest rate is money earned or fee charged for taking the risk to loan money to borrower. The investor considers interest rate as the return on investment for a bank savings account or on a fixed income instrument like bonds and treasury bills. Interest rates are very important because a lot of business transactions depend on it directly or indirectly. They are determined largely by (i) central banks (such as Fed reserve bank) which issues short term interest rate (ii) demand for government treasury bills determines fixed interest rates on loans and credit (iii) banking sector also operates in competitive environment to influence rate of return on loans and deposit. Central banks have the biggest impact on interest rate; by monitoring the economy and setting rates at a level where money supply in the system is in balance, to keep the economy stable and keep inflation at bay. Recently the bank of England in a bid to check inflation, raised interest rate twice in two months. This is to dampen individual spending and reduce borrowing from banks. Interest is a cost for one entity, and income for another. Stock market investors also do better with declining rate, since the economy is improved with more company sales profit. Though it is a very important factor, interest rate fluctuates rapidly. Some uncertainty in the financial market, arise from fluctuations of interest rate. Against this backdrop, understanding the dynamics, becomes very important to both the lender and borrower (central banks, financial institutions, individual and corporate investors etc.).This fluctuation is a result of supply and demand of credit.

Modeling the fluctuations of interest rate i.e., obtaining a model that properly describes interest rate dynamics would aid better investment decisions. This is the purpose of this paper. Interest rate is not solely determined by central bank, such as the Bank of Ghana, (BOG), but also by interplay of economic variables, government policies, attitude of investors, inflation etc. as mentioned. Using historical data (trend) of interest rate over the years would be useful in forecasting future rates through a stochastic model, such as the Hull-White model. Some works have been done on interest rate using the Hull White. Charles Corrado et al [1] observed that the Hull-White model is flexible and allows a wide range of

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stochastic volatility specifications. They used S&P 500 index (SPX) options in their work and found that S&P 500 index corresponds to a mean-reverting stochastic volatility process from Hull-White model. Antoon Pelsser [2] worked on the Hull White model and found that in addition to its analytical tractability, the probability that interest rates become negative is much smaller than in the Ho-Lee model. Analytically, he showed that the Hull-White model can be fitted to initial term structure of interest rate and prices of exotic options can be calculated from it. The model in use in this paper is credited to Hull and White [3] who worked on pricing of interest rate derivatives. For a model of the short-term interest rate, which is a mean reverting process they extended the model to reflect time dependence and added a time-dependent drift, allowing both the reversion rate and volatility factor to be functions of time. This model can be fitted effectively to the term structure of interest rates, spot and forward rate volatilities. Xiao Lu [4] worked on Treasury bond data from January 3rd, 1994 to Dec 31st 2003. They extended the Hull White model, presented a new way of describing properties of interest rates, and provided a more accurate estimation of future interest rate. Guisepe O and Michele B.[5] in their work, did some forecasting using LIBOR time series, (from December 31st 2009, to January 31st 2020) and test data of EUR overnight and a random time series. They used the Hull-White as extension of the CIR model. This is done in order to overcome foundations of possible negative interest rate values. In the result, this extension preserves the positivity of interest rate. The Hull-White model has also been used in actuaries by Zeddouk and Devolder [6] to model longevity .risk. Longevity was considered by them, as having stochastic behavior. Sinem Kozpinar [7] looked at some mean reverting models including Hull-White, in terms of their actuarial applications. Force of mortality, longevity risk via mortality intensity, and pricing of vulnerable options were studied using the Hull-White model and other models. Their work revealed that the Hull-White model over performed in modeling mortality intensity, compared to Ornstein Uhlenbeck and Vasicek models. Studying the dynamics of interest rate using Hull-White model is useful not just for understanding the nature of fluctuations, but for possible forecasting. Orlando et al in a further work to their work [5] in 2019 compared the Hull-White model with CIR model in terms of forecasting.

In this work we apply the Hull White model to interest rate data from the BOG and LIBOR, and determine if it performs well as a good model to explain the fluctuations in the interest rates. Also to examine how close the behavior of BOG interest rates are, to the dynamics of the standard London interbank offer rates, (LIBOR) used in the study of interest rates. The data used was daily data from 2nd January 2020 to 2nd July 2021.As stated apart from BOG, interplay of economic variables, government policies, attitude of investors, inflation etc, and play a role in interest rates. As a result, it is expected, that performance of the data set may reflect behavior of interest rate in Ghana, compared to other Economies. The model would also be used to attempt forecasting future interest rate via out-of-sample data. The work could ascertain that interest rate fluctuation pattern, and trend is country specific. The rest of the paper is arranged as follows: the second section discusses the Mathematical Framework and presents the Model, the third section gives the model application to BOG data, the model fitting results and analysis are also presented here. The conclusion is given in the fourth section.

2 The Hull-White Model

2.1 Introduction

Stochastic models such as the Hull-White model, are preferred to deterministic models in studying important processes that fluctuate over time in the market. They capture randomness and uncertainty in the process, by using random variables (time dependent). The Hull-White model belongs to a general family of mean reverting models. It is a single factor interest rate model. Interest rate models are generally for estimating the spot interest, using it for bond pricing. The Hull-White model is an extension of the Vasicek and CIR models. It is appealing, and of interest for the study because it is very tractible and has a closed form solution; According to Kozpinar [7] it outperforms other affine models in most applications . The general Hull-White SDE is given as;

$$dX = [\theta(t) + a(t)(b - X)dt] + \sigma x^{\beta} dW \tag{1}$$

$\theta(t)$ is the time dependent drift, $a(t)$ is the reversion rate (estimated from historical data). σ is volatility and the variable of interest, X (interest rate) is also a function of time.

Equation (1) is shifted for $\beta = 0$, as;

$$dX = [\theta(t) + a(t)(b - X)dt + \sigma dW] \tag{2}$$

$\theta(t)$ is calculated from initial yield curve describing the current term structure of interest-rate. $a(t)$, $\theta(t)$ and $\sigma(t)$ are deterministic functions of time and W(t) is the usual Brownian motion.

In the classic work by Hull and white [3] all parameters were time dependent. Subsequent work by Hull J. [8] showed that this time dependency in the parameters, $a(t)$, $\theta(t)$ and $\sigma(t)$ can lead to non-stationary volatility term structure. This is not desirable when pricing some instruments whose value depends on term structure of future volatility. Non stationarity implies that the present volatility structure may not be preserved in the future [9]. As a result of this, the model is also used with constant volatility. This is the version to be used in this work. A short coming of the Hull-White model is the possibility of negative interest rate which it shares with the Vasicek and O-U Ornstein Uhlenbeck models. This arises from the fact that it considers interest rate as normally distributed. However, the probability of the negative rates occurring as a model output is low.

2.2 Mathematical Background

Let X be the short rate interest rate. Suppose it is driven by Hull-White model then

$$dX = [(\theta(t) - a(t)x)dt + \sigma dW] \tag{3}$$

Equation (3) is the Hull-White model, SDE. Following the evolution of the model explained in the introduction above, we will use the constant coefficient version in this work.

To obtain the closed form solution for (3) we employ Itô's lemma. Given a process

X(t), described by

$$dX = adt + \sigma dW_t$$

For $f = f(X,t)$ where $f \in C^{1,2}$, $f : [0,\infty] \times R \to R$,

$$df = \{f_t + af_x + \frac{1}{2}\sigma^2 f_{xx}\}dt + \sigma f_x dw$$

For equation (3). Let $f(x,t) = e^{at}X$, then $f_x = e^{at}, f_{xx} = 0, f_t = ae^{at}X$

Using Itô's lemma

$$\begin{aligned} df &= de^{at}X \qquad (4)\\ &= [(\theta(t) - a(t)X)e^{at} + 0 + a(t)e^{at}X]dt + \sigma e^{at}dW \qquad (5)\\ &= [(\theta(t)e^{at} - a(t))Xe^{at} + a(t)e^{at}X]dt + \sigma e^{at}dW\\ de^{at}X &= \theta(t)e^{at}dt + \sigma e^{at}dW \qquad (6)\end{aligned}$$

Integrating both sides of (5) from t to T yields

$$\begin{aligned} e^{aT}X_T - e^{at}X_t &= \int_t^T \theta(u)e^{au}du + \int_t^T \sigma(u)e^{au}dW(u)\\ e^{aT}X_T &= e^{at}X_t + \int_t^T \theta(u)e^{au}du + \int_t^T \sigma(u)e^{au}dW(u)\\ X_T &= \frac{1}{e^{aT}}[e^{at}X_t + \int_t^T \theta(u)e^{au}du + \int_t^T \sigma(u)e^{au}dW(u)]\\ X_T &= e^{-aT}[e^{at}X_t + \int_t^T \theta(u)e^{au}du + \int_t^T \sigma(u)e^{au}dW(u)]\\ X_T &= e^{-a(T-t)}X_t + \int_t^T \theta(u)e^{-a(T-u)}du + \int_t^T \sigma(u)e^{-a(T-u)}dW(u) \qquad (7)\end{aligned}$$

For $t \in [0,t]$

$$X_t = e^{-at}X_0 + \int_0^t \theta(u)e^{-a(t-u)}du + \int_0^t \sigma(u)e^{-a(t-u)}dW(u) \tag{8}$$

Since the constant coefficient version is being used in this work, from (7) we have

$$\begin{aligned}
X_t &= e^{-at}X_0 + \theta e^{-at}\int_0^t e^{au}du + \sigma\int_0^t e^{-a(t-u)}dW(u) \\
&= e^{-at}X_0 + \theta e^{-at}|_0^t \frac{1}{a}e^{au}du + \sigma\int_0^t e^{-a(t-u)}dW(u) \\
X_t &= e^{-at}X_0 + \theta e^{-at}(\frac{1}{a}e^{at} - \frac{1}{a}) + \sigma\int_0^t e^{-a(t-u)}dW(u) \\
&= e^{-at}X_0 + \frac{\theta}{a} - \frac{\theta}{a}e^{-at} + \sigma\int_0^t e^{-a(t-u)}dW(u) \\
&= e^{-at}X_0 + \frac{\theta}{a}(1-e^{-at}) + \sigma\int_0^t e^{-a(t-u)}dW(u) \qquad (9) \\
&= e^{-at}X_0 + \frac{\theta}{a}(1-e^{-at}) + \sigma I
\end{aligned}$$

where $I = \int_0^t e^{-a(t-u)}dW(u)$

Integrating by parts, I is obtained as

$$I = W(t) - e^{-at}W_0 - ae^{-at}\int_0^t e^{au}W(u)du$$

$W_0 = W(0)$
Since $W_0 = 0$ for the Brownian motion

$$\therefore I = W(t) - ae^{-at}\int_0^t e^{au}W(u)du$$

From (8)

$$\begin{aligned}
X_t &= e^{-at}X_0 + \frac{\theta}{a}(1-e^{-at}) + \sigma[W(t) - ae^{-at}\int_0^t e^{au}W(u)du] \\
X_t &= e^{-at}X_0 + \frac{\theta}{a}(1-e^{-at}) + \sigma W(t) - \sigma ae^{-at}\int_0^t e^{au}W(u)du] \qquad (10)
\end{aligned}$$

Equation (9) is equivalent to

$$X_t = e^{-at}X_0 + \frac{\theta}{a}(1-e^{-at}) + \sigma ae^{-at}\int_0^t e^{au}W(u)du] \qquad (11)$$

Hence

$$X_t \sim N\left(e^{-at}X_0 + \frac{\theta}{a}(1-e^{-at}), \frac{\sigma^2}{2a}(1-e^{-2at})\right) \qquad (12)$$

X_t follows a normal distribution with mean $e^{-at}X_0 + \frac{\theta}{a}\left(-e^{-at}\right)$ and variance $\frac{\sigma^2}{2a}\left(-e^{-2at}\right)$

3 Methodology

Parameters, a, theta, sigma of the Hull White model were estimated for the two data sets, using MLE and obtained via R-code. These parameters were then imported into an R-program developed for the closed form solution of the Hull white SDE. From the interest rate process, i.e. closed form solution of the model SDE, given in equation (10), X_t values were computed using an R-program. The program runs as an iterative process. The estimated parameters θ (drift), σ (volatility) and reversion rate, a , were used for simulation of the mean reverting process. It uses interest rate for 2^{nd} January 2020 as initial rate X_0 for both data sets. The program generates W_t, randomly from normal distribution, $N(0,1)$. Time was defined in days. This simulation using the R-program generates the theoretical X_t interest rate values and the plots for the data set used. Generally, for the various data sets used in the work, In -sample data were obtained. i.e. the theoretical X_t values were generated for the various data sets by simulating the process of the closed form solution. This was done using the estimated model parameters in an iterative process using an R-codes.

3.1 Calibration of Data

To estimate the parameter θ, a and σ of the model, the likelihood function $L(\theta, a, \sigma, X_t)$ from (10) were derived and used. Hence $X_t \sim N(\mu_t, \rho_t^2)$ where mean, $\mu_t = e^{-at}X_0 + \frac{\theta}{a}(1-e^{-at})$ and variance $\sigma_t = \frac{\theta^2}{2a}(1-e^{-2at})$

The likelihood function $L(\theta, a, \sigma, X_t) = \prod_{t=1}^{n} f(X_t)$ where $f(X_t) = \frac{1}{\sqrt{2\pi\sigma_t}} e^{\frac{1}{2}\sigma_t(X_t-\mu_t)^2}$

The log-likelihood function

$$
\begin{aligned}
l(\theta, a, \sigma, X_t) &= \log L(\theta, a, \sigma, X_t) \\
&= \sum_{t=2}^{n} \log\left[(2\pi\sigma_t)^{-\frac{1}{2}} e^{\frac{1}{2}\sigma_t(X_t-\mu_t)^2}\right] \\
&= -\frac{1}{2}\sum_{t=1}^{n}\left[\log(2\phi\sigma_t) + \left(\frac{X_t-\mu_t}{\sigma_t}\right)^2\right]
\end{aligned}
$$

Where $\mu_t = e^{-at}X_0 + \frac{\theta}{a}(1-e^{-at})$
and $\sigma_t = \sqrt{\frac{\theta^2}{2a}(1-e^{-2at})}$

Table 1: Parameter Estimates of the model for BOG and LIBOR daily Interest rate data (Jan 2020 ? Jul 2021)

Parameters	BOG Daily Rates(**2020 -2021**)	LIBOR Daily Rates (**2020 -2021**)
Reversion Rate a	0.0102	0.0440
Drift θ	0.1362	0.0166
Volatility σ	0.1132	0.2171

4 Result and Discussion

4.1 In-sample data (BOG daily interest rate 2020 -2021)

Data used was daily data from 2^{nd} January 2020 to 2^{nd}July 2021. The interest rate series excludes weekend when there is no banking activities. Data was obtained from BOG website and LIBOR websites. Plot on Fig 1 shows the generated BOG daily data (2020-2021) fitted to the empirical data using MLE. The trend of the data structure is averagely maintained by the theoretical (model) data values, showing that the likelihood estimates are good enough. From the plot, presence of peaks and periods of quick close fluctuations are observed in the empirical data, which is not captured by the model. This is because the model is a diffusion model which is mean reverting.

Table 2 and Table 3 below show the BOG in- sample data values, $X(t)$ generated for the model, along with the empirical interest rate values for comparison.

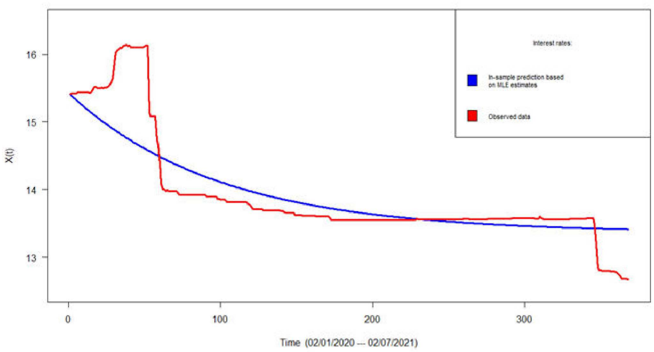

Fig. 1: Plot showing actual BOG interest rate and interest rate values generated using the Model

[1] **Table 2 below presents the empirical and theoretical X_t values for first 35 banking days (BOG daily interest rate, (2020 -2021)**

Effective Date	Emperical Values	**Theoretical Values**
02-Jan-20	15.41	15.38927
03-Jan-20	15.42	15.36874
06-Jan-20	15.42	15.34842
08-Jan-20	15.42	15.32831
09-Jan-20	15.42	15.30841
10-Jan-20	15.44	15.2887
13-Jan-20	15.44	15.2692
14-Jan-20	15.44	15.24989
15-Jan-20	15.44	15.23078
16-Jan-20	15.44	15.21186
17-Jan-20	15.44	15.19313
20-Jan-20	15.44	15.1746
21-Jan-20	15.44	15.15625
22-Jan-20	15.43	15.13809
23-Jan-20	15.44	15.12011
24-Jan-20	15.48	15.10232
27-Jan-20	15.52	15.0847
28-Jan-20	15.52	15.06727
29-Jan-20	15.5	15.05001
30-Jan-20	15.5	15.03293
31-Jan-20	15.5	15.01603
03-Feb-20	15.51	14.99929
04-Feb-20	15.5	14.98273
05-Feb-20	15.51	14.9663
06-Feb-20	15.51	14.95011
07-Feb-20	15.52	14.93405
10-Feb-20	15.55	14.91815
11-Feb-20	15.58	14.90241
12-Feb-20	15.64	14.88683
13-Feb-20	15.8	14.87141
14-Feb-20	16.02	14.85615
17-Feb-20	16.06	14.84104
18-Feb-20	16.07	14.82609
19-Feb-20	16.1	14.81129
20-Feb-20	16.09	14.79664

Table 3 below presents the empirical and theoretical X_t values for last 35 banking days (BOG daily interest rate 2020 -2021)

Effective Date	Emperical Values	Theoretical Values
14-May-21	13.57	13.43321
17-May-21	13.57	13.43251
18-May-21	13.57	13.43182
19-May-21	13.57	13.43114
20-May-21	13.57	13.43046
21-May-21	13.57	13.42979
24-May-21	13.57	13.42913
25-May-21	13.57	13.42847
26-May-21	13.58	13.42782
27-May-21	13.58	13.42718
28-May-21	13.58	13.42654
31-May-21	13.58	13.42529

[1] Bank of Ghana website: https://www.bog.gov.gh/economic-data/interest-rates/

Effective Date	Emperical Values	Theoretical Values
04-Jun-21	12.81	13.42285
07-Jun-21	12.81	13.42226
08-Jun-21	12.8	13.42167
09-Jun-21	12.8	13.42109
10-Jun-21	12.8	13.42052
11-Jun-21	12.8	13.41995
14-Jun-21	12.8	13.41938
15-Jun-21	12.8	13.41883
16-Jun-21	12.79	13.41827
17-Jun-21	12.79	13.41773
18-Jun-21	12.79	13.41719
21-Jun-21	12.78	13.41665
22-Jun-21	12.76	13.41612
23-Jun-21	12.74	13.4156
24-Jun-21	12.71	13.41508
25-Jun-21	12.68	13.41456
29-Jun-21	12.68	13.41406
30-Jun-21	12.68	13.41355
01-Jul-21	12.68	13.41305
02-Jul-21	12.67	

4.2 In-sample data (LIBOR daily interest rate 2020 -2021)

Plot on Fig 2 shows LIBOR daily data (2020-2021) fitted using MLE. Also in this case the trend of the data structure is averagely maintained showing that the likelihood estimates are good enough. From the plot, presence of peaks and periods of quick close fluctuations are observed in the empirical data, also periods of high spikes (jumps) which are not captured by the model. This is because the model is a diffusion model which is mean reverting.

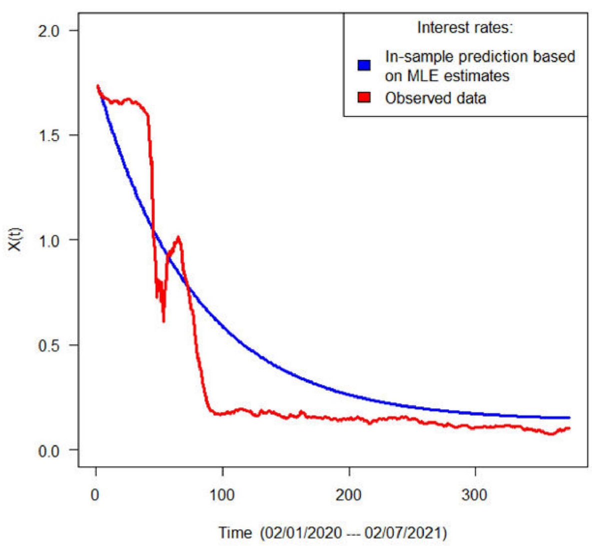

Fig. 2: Plot showing actual LIBOR interest rate and generated interest rate values using the Model

Table 4 and Table 5 below show the LIBOR in- sample data values $X(t)$ generated for the model, along with the empirical interest rate values for comparison.

Table 4 below presents the empirical and theoretical X_t values for first 35 banking days: LIBOR daily interest rate, (2020 -2021)

Effective Date	Emperical Values	**Theoretical Values**
02/01/2020	1.73438	1.7223934
03/01/2020	1.71425	1.7063503
06/01/2020	1.69213	1.687005
07/01/2020	1.699	1.6670781
08/01/2020	1.67713	1.6471995
09/01/2020	1.68363	1.6275425
10/01/2020	1.67663	1.6081572
13/01/2020	1.67625	1.5890554
14/01/2020	1.66963	1.5702355
15/01/2020	1.669	1.5516915
16/01/2020	1.65775	1.5334162
17/01/2020	1.65438	1.5154023
20/01/2020	1.65338	1.497643
21/01/2020	1.6595	1.4801318
22/01/2020	1.65938	1.4628629
23/01/2020	1.66088	1.4458308
24/01/2020	1.6595	1.4290304
27/01/2020	1.64925	1.4124572
28/01/2020	1.65	1.3961068
29/01/2020	1.64525	1.3799749
30/01/2020	1.655	1.3640578
31/01/2020	1.66188	1.3483516
03/02/2020	1.66775	1.332853
04/02/2020	1.66625	1.3175585
05/02/2020	1.66963	1.3024649
06/02/2020	1.67088	1.287569
07/02/2020	1.66525	1.2728678
10/02/2020	1.65788	1.2583584
11/02/2020	1.65275	1.244038
12/02/2020	1.65013	1.2299038
13/02/2020	1.6585	1.2159531
14/02/2020	1.65825	1.2021832
17/02/2020	1.64675	1.1885917
18/02/2020	1.647	1.1751759
19/02/2020	1.63938	1.1619336

Table 5 below presents the empirical and theoretical X_t values for Last 35 banking days LIBOR daily interest rate, (2020 -2021)

Effective Date	Emperical Values	**Theoretical Values**
11/05/2021	0.09375	0.1584126
12/05/2021	0.09813	0.1581468
13/05/2021	0.10088	0.1578843
14/05/2021	0.0975	0.1576251
17/05/2021	0.0975	0.1573693
18/05/2021	0.09925	0.1571167
19/05/2021	0.0965	0.1568673
20/05/2021	0.0925	0.1566211
21/05/2021	0.09163	0.156378
24/05/2021	0.091	0.156138
25/05/2021	0.09	0.155901
26/05/2021	0.0925	0.1556671
27/05/2021	0.09213	0.1554361

Effective Date	Emperical Values	**Theoretical Values**
28/05/2021	0.08588	0.1552081
01/06/2021	0.08875	0.154983
02/06/2021	0.0855	0.1547607
03/06/2021	0.08	0.1545413
04/06/2021	0.08125	0.1543247
07/06/2021	0.08125	0.1541108
08/06/2021	0.077	0.1538996
09/06/2021	0.07463	0.1536911
10/06/2021	0.07263	0.1534853
11/06/2021	0.07288	0.1532821
14/06/2021	0.07463	0.1530814
15/06/2021	0.08175	0.1528833
16/06/2021	0.0825	0.1526878
17/06/2021	0.09338	0.1524947
18/06/2021	0.091	0.1523041
21/06/2021	0.09588	0.1521159
22/06/2021	0.09075	0.1519301
23/06/2021	0.0915	0.1517466
24/06/2021	0.095	0.1515655
25/06/2021	0.09613	0.1513867
28/06/2021	0.10425	0.1512101
29/06/2021	0.10025	0.1510358

4.3 Discusion on results from the Hull-White Model: Interest rate data fit and trend for BOG and LIBOR

From the plots on Fig 1 and Fig 2, both the BOG and LIBOR data seem to exhibit jumps over the period, though with varying intensity. Both data sets (for about the first 100 days) started with high values, then jump down to longer periods of rate stability, implying little or no fluctuations. The data sets fit the model well, and follow similar trend. The nature of the data sets account for slight disparity in jump type and intensity. This explains that, besides the jumps, their dynamics are captured by the mean reverting Hull- White model.

5 Conclusion

The Hull White model has performed well in capturing the dynamics of interest rates for BOG and LIBOR. It performed better for BOG data set. This suggests that the BOG interest rates revert to a long run mean over time. The presence of jumps is glaring for the two data sets, especially for the LIBOR data set. Further work will be done by using a Hull white model with jumps incorporated. It is expected, that with that jump model the dynamics of interest rates will be captured better. Out-of?Sample data will also be obtained using the proposed Hull White jump model. It is expected that predictions of the interest rate can be made with accurate results.

R-codes for parameter estimation, and R-Program for Simulation of the closed form solution of the model SDE, are available on request in addition to and complete X_t values

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