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Decision-making in Diagnosing Heart Failure Problems Using Dual Hesitant Fuzzy Sets

Cover Page Footnote

In this paper, we have introduced dual-hesitant fuzzy sets, a generalization of fuzzy sets that permits us to represent the situation in which different membership and non-membership functions are considered possible. We have established a relationship on DHFS and have shown that application can use in many real life applications. For example, medicine, political or social case.

Decision-making in Diagnosing Heart Failure Problems Using Dual Hesitant Fuzzy Sets

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Abstract- Almost all real life problems are characterized by uncertainty, for modelling uncertainty several types of sets, such as fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, type 2 fuzzy sets, type n fuzzy sets, and hesitant fuzzy sets, have been introduced and investigated widely and used in modelling real life data. In this paper, we propose to construct the concept of dual hesitant fuzzy sets (DHFSs), which encompass fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, and fuzzy and multi-sets as special cases. Then we investigate the basic operations and properties of DHFSs. We also discuss the relationships among the sets mentioned above, and then propose an extension principle of DHFSs. Additionally, we give an example to illustrate the application of DHFSs in Diagnosing Heart Failure Problems which is explained in details and puts new view of some operations of DHFS.

Keywords- Fuzzy sets; Intuitionistic Fuzzy sets; Dual Hesitant Fuzzy Sets; Diagnosing Heart Failure; New View of Some Operations of DHFS.

I. INTRODUCTION

To deal with real-world uncertain and ambiguous problems, the strategies commonly used in classical mathematics are not always useful. In 1965, Zadeh [1] proposed the concept of fuzzy set (FS) as an extension of the classical notion of sets. The fuzzy set theory allows for a gradual determination of the membership of elements in a set, which is represented using a membership function having a value in the real unit interval [0,1]. In many cases, however, because the membership function is a single-valued function, it cannot be used to represent both support and objection evidence. The intuitionistic fuzzy set (IFS), which is a generalization of Zadeh's fuzzy set, was introduced by Atanassov [2]. IFS has both a membership and a non-membership function, allowing it to better express the fuzzy character of data than Zadeh's fuzzy set, which only has a membership function. In

some real-life scenarios. IFS are useful in real life problems connected to the linguistic variable which is made only as a function of membership like rough. As a result of IFS flexibility in dealing with uncertainty, we gave an IFS approach in selecting specialization [3]. Since there is possible to be hesitation part which represents unknown object [4] and studied recently [5, 6]. Hesitant fuzzy sets [7] were defined another extension. That is, we introduce. The rationale is that, when defining an element's membership, it is not because we have a margin of error that makes determining the membership degree difficult (as in A-IFS), or a distribution of possibilities among the available values, but since we have a range of potential values. This is the case if we consider as possible values for the membership of into the set only two values and . This situation can arise in a multi criteria decision-making problem. For example, when only some values of membership are possible for a given element because, e.g., some experts have only assigned such small and finite set. In this context, instead of considering just an aggregation operator [8], it is useful to deal with all the possible values. This situation, as we will discuss later, can be modeled using multi sets. Never-the less, such model is not completely adequate because the operations for multi sets do not apply correctly to our sets according to their interpretation. Because of this, we introduce in this paper the new definition and we name such sets as the dual hesitant fuzzy sets. Then, we introduce appropriate operators. The structure of the paper is as follows: In Section 1, we review IFS. Then, in Section 2. We define dual hesitant fuzzy sets, some basic operations. Finally, in Section 4. We present a new application of DHFS as in Diagnosing Heart Failure. We provide a fresh method for diagnostics in medicine by Eulalia Szmids et al. [9] using dual hesitant fuzzy sets by Bin Zhu et al. [15] as using in DHFS. Finding the lowest distance between symptoms is how Jawad Ali et al. [16] arrive at a solution. The issues with decision-making, especially when it comes to diagnosing medical conditions. Every time an unknown object is being evaluated, there is a good likelihood that a non-null hesitation portion will exist. To be more exact, dual hesitant fuzzy sets allow us to describe things

like a patient's fluctuating body temperature and unclear other symptoms. This article will introduce dual hesitant fuzzy sets as a method for reasoning with incomplete information and facts. Furthermore, the efficacy of the proposed technique is demonstrated through the analysis of heart failure diagnosis data, showcasing high accuracy compared to previous methods, thus highlighting its clinical significance. The paper finishes with some conclusions and points out some future work..

II. MASTER DEFINITIONS

Definition [1]: consider that X denotes a non-empty set

which is a fuzzy set consisting of X having $A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x) : X \rightarrow [0, 1]$ denotes the function of the membership of the fuzzy set A . Definition [2]: consider that X denotes a non-empty set, which is an IFS where A in X refers to an object with the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where the functions $\mu_A(x), \nu_A(x) : X \rightarrow [0, 1]$ are defined, respectively; the degree of membership and the degree of non-membership of the element $x \in X$ to the set A that is a subset of X , and for every element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition [15]: consider that X denotes a non-empty set, which is an DHFS where A in X refers to an object with the form $A = \{(x, h(x), g(x)) : x \in X\}$, where the functions $h(x), g(x) : X \rightarrow [0, 1]$ are defined, respectively; the degree of membership and the degree of non-membership of the element $x \in X$ to the set A that is a subset of X , and for every element $x \in X$, $0 \leq \mu_A^2(x) + \nu_A^2(x) \leq 1$.

Definition [2]: IF A, B be IFS in X , then:

1. [inclusion]

$$A \subseteq B \leftrightarrow \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x)$$

$$\forall x \in X$$

2. [equality]

$$A = B \leftrightarrow \mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x)$$

$$\forall x \in X$$

3. [complement] $A^c = \{(x, \nu_A(x), \mu_A(x)) : x \in X\}$

4. [union]

$$A \cup B = \{(x, \max(\mu_A(x), \mu_B(x)),$$

$$\min(\nu_A(x), \nu_B(x)) : x \in X\}$$

5. [intersection]

$$A \cap B = \{(x, \min(\mu_A(x), \mu_B(x)),$$

$$\max(\nu_A(x), \nu_B(x)) : x \in X\}$$

6. [addition]

$$A \oplus B = \{(x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x),$$

$$\nu_A(x)\nu_B(x)) : x \in X\}$$

7. [multiplication]

$$A \otimes B = \{(x, \mu_A(x)\mu_B(x),$$

$$\nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)) : x \in X\}$$

8. [difference]

$$A - B = \{(x, \min(\mu_A(x), \nu_B(x)),$$

$$\max(\nu_A(x), \mu_B(x)) : x \in X\}$$

9. [symmetric difference]

$$A \Delta B = \{(x, \max[\min(\mu_A, \nu_B), \min(\nu_A, \mu_B)],$$

$$\min[\max(\nu_A, \mu_B), \max(\mu_A, \nu_B)] : x \in X\}$$

10. [Cartesian product]

$$A \times B = \{(x, \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)) : x \in X\}$$

Definition [7]: for three HFS: h, h_1, h_2 , the following operations are defined:

$$(1) \text{ Lower bound: } h(x) = \min h(x);$$

$$(2) \text{ Upper bound: } h(x) = \max h(x);$$

$$(3) h_1 \cup h_2 = \{h \in h_1 \cup h_2 \mid h \geq \max(h_1, h_2)\};$$

$$(4) h_1 \cap h_2 = \{h \in h_1 \cap h_2 \mid h \leq \min(h_1, h_2)\}.$$

Definition [15]: for two DHFS: d_1, d_2 , the following operations are defined:

$$d_1 \cup d_2 = \{h \in (h_1 \cup h_2) \mid h \geq \max(h_1^-, h_2^-),$$

$$(1) g \in (g_1 \cup g_2) \mid g \leq \min(g_1^+, g_2^+)\};$$

$$d_1 \cap d_2 = \{h \in (h_1 \cap h_2) \mid h \leq \min(h_1^+, h_2^+),$$

$$(2) g \in (g_1 \cap g_2) \mid g \geq \max(g_1^-, g_2^-)\}.$$

III. NEW VIEW OF SOME OPERATIONS IN DHFS

In intuitionistic fuzzy set, we use t-norm to talk about the minimum between memberships in intersection operation and use t-conorm to talk about the maximum between memberships in union operation. So, we can use t-norm and t-conorm in dual-hesitant fuzzy set where it is useful to the intersection or the union operations that we need the minimum and maximum in the same operation in DHFS, but with the hesitant numbers to the membership

and the non-membership $D = (x, \mu^2(x), \nu^2(x))$; let

$\mu^2(x) = \nu^2(x) = d(x)$; for intersection operation:

$d_1 \cap d_2 = \{h \in (h_1 \cap h_2) \setminus h \leq \min(h_1^+, h_2^+), g \in (g_1 \cup g_2) \setminus$

$g \geq \max(g_1^-, g_2^-)\}$ and for union operation: $d_1 \cup d_2 =$

$\{h \in (h_1 \cup h_2) \setminus h \geq \max(h_1^-, h_2^-), g \in (g_1 \cup g_2) \setminus$

$g \leq \min(g_1^+, g_2^+)\}$, where the intersection operation, the membership degree can use t-norm and the non-membership degree can use t-conorm and where the union operation, the membership degree can use t-conorm and the non-membership degree can use t-norm. Consider that i is a t-norm and u is a t-conorm. Considering that $x \in [0, 1]$, we determine:

$$x_i^{(n)} = i(x, \dots, x) = i(x_i^{(n-1)}, x); n \geq 2$$

n -times

$$x_u^{(n)} = u(x, \dots, x) = u(x_u^{(n-1)}, x); n \geq 2$$

n -times

Definition: Consider that i and u are a t-norm and a t-conorm from $[0, 1]^n$ to $[0, 1]$. Accordingly:

i. A t-norm i (and a t-conorm u , respectively) is considered to be continuous if it continues as a function on the unit interval.

ii. A t-norm i and a t-conorm u , respectively, of DHFS

$D = (x, \mu^2(x), \nu^2(x))$ if we talk about intersection operation: $\mu^2(x)$ tends to minimum, at case that there are

two number for $\mu^2(x)$ which is considered Archimedean

if $\lim_{n \rightarrow \infty} (\mu_i^2(x))^n = 0$, and $\nu^2(x)$ tends to maximum,

at case that there are two number for $\nu^2(x)$ which is considered Archimedean if $\lim_{n \rightarrow \infty} (\nu_u^2(x))^n = 1$, for respect, for any $x \in (0, 1)$.

Proposition: Consider that θ is a set of all DHFS, i and u represent a t-norm and a t-conorm. Then:

i. In intersection operation. If i and u are Archimedean, then $i(\mu^2(x), \mu^2(x)) < \mu^2(x)$ and

$u(\nu^2(x), \nu^2(x)) > \nu^2(x)$ for all

$D = (x, \mu^2(x), \nu^2(x))$.

ii. In union operation, if i and u are Archimedean, then

$u(\mu^2(x), \mu^2(x)) > \mu^2(x)$ and

$i(\nu^2(x), \nu^2(x)) < \nu^2(x)$ for all

$D = (x, \mu^2(x), \nu^2(x))$

Proof:

i. Consider the contrary, that's to say that

$\mu^2(x)_i^{(2)} = i(\mu^2(x), \mu^2(x)) \geq \mu^2(x)$ and

$\nu^2(x)_u^{(2)} = u(\nu^2(x), \nu^2(x)) \leq \nu^2(x)$ for some

$x \in (0, 1)$. Then, we have

$\mu^2(x)_i^{(3)} = i(i(\mu^2(x), \mu^2(x)), \mu^2(x)) \geq$ and

$i(\mu^2(x), \mu^2(x)) \geq \mu^2(x)$

$\nu^2(x)_u^{(3)} = u(u(\nu^2(x), \nu^2(x)), \nu^2(x)) \leq$

$u(\nu^2(x), \nu^2(x)) \leq \nu^2(x)$

from

DHFS properties and by inference we will also have

$\mu^2(x)_i^{(n)} = i(\mu^2(x), \dots, \mu^2(x)) \geq \mu^2(x)$. Then

$\lim_{n \rightarrow \infty} (\mu^2(x))_i^{(n)} \neq 0$ and

$\nu^2(x)_u^{(n)} = u(\nu^2(x), \dots, \nu^2(x)) \leq \nu^2(x)$. Then

$\lim_{n \rightarrow \infty} (\nu^2(x))_u^{(n)} \neq 1$, which is a contradiction.

ii. Consider the contrary, that's to say that

$\mu^2(x)_u^{(2)} = u(\mu^2(x), \mu^2(x)) \leq \mu^2(x)$ and

$\nu^2(x)_i^{(2)} = i(\nu^2(x), \nu^2(x)) \geq \nu^2(x)$ for some

$x \in (0, 1)$. Then, we have

$\mu^2(x)_u^{(3)} = u(u(\mu^2(x), \mu^2(x)), \mu^2(x)) \leq$ and

$u(\mu^2(x), \mu^2(x)) \leq \mu^2(x)$

$\nu^2(x)_i^{(3)} = i(i(\nu^2(x), \nu^2(x)), \nu^2(x)) \geq$ from

$u(\nu^2(x), \nu^2(x)) \geq \nu^2(x)$

the DHFS properties and by inference we will also have:

$\mu^2(x)_u^{(n)} = u(\mu^2(x), \dots, \mu^2(x)) \leq \mu^2(x)$. Then
 $\lim_{n \rightarrow \infty} (\mu^2(x))_u^{(n)} \neq 1$ and
 $\nu^2(x)_i^{(n)} = i(\nu^2(x), \dots, \nu^2(x)) \geq \nu^2(x)$. Then,
 $\lim_{n \rightarrow \infty} (\nu^2(x))_i^{(n)} \neq 0$, which is a contradiction.

Proposition: In case of a continuous function where $i, u : [0, 1]^2 \rightarrow [0, 1]$, there equivalent statements for any DHFS as follows:

- i denotes a continuous Archimedean t-norm and u is a continuous Archimedean t-conorm.
- There are a continuous additive generator for any DHFS $D = (x, \mu^2(x), \nu^2(x))$ if we talk about intersection operation $\mu^2(x)$ tends to minimum, , at case that there are two number for $\mu^2(x), \nu^2(x)$ tends to maximum, , at case that there are two number for $\nu^2(x)$ that's to say that there exists a continuous strictly decreasing $i(\mu^2(x)) : [0, 1] \rightarrow [0, \infty[$, $i(1) = 0$ that is uniquely defined up to a multiplicative constant such that for all $x, y \in [0, 1]$, we have:
 $i(\mu^2(x), \mu^2(y)) = i^{-1}(\min\{i(\mu^2(x)) + i(\mu^2(y)), i(0)\})$
and strictly increasing
 $u(\nu^2(x)) : [0, 1] \rightarrow [0, \infty[$, $u(1) = 0$
that is uniquely defined up to a multiplicative constant such that for all $x, y \in [0, 1]$, we will have:
 $u(\nu^2(x), \nu^2(y)) = u^{-1}(\max\{u(\nu^2(x)) + u(\nu^2(y)), u(1)\})$.

Proof:

Let $i(x, y) = i^{-1}(\min\{i(x) + i(y), i(0)\})$. We should prove that they are a continuous Archimedean t-norm.

$$\begin{aligned} H_1 : i(\mu^2(x), 1) &= i^{-1}(\min\{i(\mu^2(x)) + i(\mu^2(1)), i(0)\}) \\ &= i^{-1}(\min\{i(\mu^2(x)) + 0, i(0)\}) \\ &= i^{-1}(\min\{i(\mu^2(x)), i(0)\}) = i^{-1}(i(\mu^2(x))) = \mu^2(x) \end{aligned}$$

(Boundary).

H_2 : To prove that i is increasing we consider $\mu^2(x) < \mu^2(w)$ and $\mu^2(y) < \mu^2(z)$. Then $i(\mu^2(x)) > i(\mu^2(w))$ and $i(\mu^2(y)) > i(\mu^2(z))$ and

$i(\mu^2(x)) + i(\mu^2(y)) > i(\mu^2(w)) + i(\mu^2(z))$. Since i^{-1} is decreasing we obtain:

$$\begin{aligned} i(\mu^2(x), \mu^2(y)) &= i^{-1}(\min\{i(\mu^2(x)) + i(\mu^2(y)), i(0)\}) \\ &\leq i^{-1}(\min\{i(\mu^2(w)) + i(\mu^2(z)), i(0)\}) \leq i(\mu^2(w), \mu^2(z)) \end{aligned}$$

(Monotonicity).

ty).

$H_3 : i(\mu^2(x), \mu^2(y)) = i(\mu^2(y), \mu^2(x))$ is obvious.

(Commutative).

H_4 : We have:

$$\begin{aligned} i(i(\mu^2(x), \mu^2(y)), \mu^2(z)) &= i^{-1}(i^{-1}(\min\{i(\mu^2(x)) + i(\mu^2(y)), i(0)\}), \mu^2(z)) \\ &= i^{-1}(\min\{i(i^{-1}(\min\{i(\mu^2(x)) + i(\mu^2(y)), i(0)\})), i(\mu^2(z)), i(0)\}) \\ &\because i(\mu^2(x)) + i(\mu^2(y)) < i(0). \text{ Since } i^{-1} \text{ is strictly decreasing. We obtain:} \\ i(i(\mu^2(x), \mu^2(y)), \mu^2(z)) &= i^{-1}(\min\{i(i^{-1}(i(\mu^2(x)) + i(\mu^2(y)))) + i(\mu^2(z)), i(0)\}) \\ &= i^{-1}(\min\{(i(\mu^2(x)) + i(\mu^2(y)) + i(\mu^2(z))), i(0)\}), i(0) = 1 \\ &= i^{-1}(i(\mu^2(x)) + i(\mu^2(y)) + i(\mu^2(z))) \end{aligned}$$

and

$$\begin{aligned} i(\mu^2(x), i(\mu^2(y), \mu^2(z))) &= i^{-1}(\mu^2(x), i^{-1}(\min\{i(\mu^2(y)) + i(\mu^2(z)), i(0)\})) \\ &= i^{-1}(\min\{i(\mu^2(x)) + i(i^{-1}(\min\{i(\mu^2(y)) + i(\mu^2(z)), i(0)\})), i(0)\}) \\ &\because i(\mu^2(y)) + i(\mu^2(z)) < i(0). \text{ Since } i^{-1} \text{ is strictly decreasing. We obtain:} \end{aligned}$$

$$\begin{aligned} i(\mu^2(x), i(\mu^2(y), \mu^2(z))) &= i^{-1}(\min\{i(\mu^2(x)) + i(i^{-1}(i(\mu^2(y)) + i(\mu^2(z))))), i(0)\}) \\ &= i^{-1}(\min\{i(\mu^2(x)) + i(\mu^2(y)) + i(\mu^2(z)), i(0)\}), i(0) = 1 \\ &= i^{-1}(i(\mu^2(x)) + i(\mu^2(y)) + i(\mu^2(z))) \end{aligned}$$

$$\therefore i(i(\mu^2(x), \mu^2(y)), \mu^2(z)) = i(\mu^2(x), i(\mu^2(y), \mu^2(z)))$$

(Associative).

tive).

Similarity to t-conorm.

IV. APPLICATION OF HFS IN DIAGNOSING HEART FAILURE PROBLEM

Definition Jawad Ali et al. [16]:

Heart failure occurs when the heart muscle doesn't pump blood as well as it should. When this happens, blood often backs up and fluid can build up in the lungs, causing shortness of breath. Certain heart conditions gradually leave the heart too weak or stiff to fill and pump blood properly. These conditions include narrowed arteries in the heart and high blood pressure. Proper treatment may improve the symptoms of heart failure and may help some people live longer. Lifestyle changes can improve quality of life. Try to lose weight, exercise, use less salt and manage stress. But heart failure can be life-threatening. People with heart failure may have severe symptoms. Some may need a heart transplant or a device to help the heart pump blood. Heart failure is sometimes called congestive heart failure.

Given the presence of a database, an example of a in Diagnosing Heart Failure Problems that is, a description of a set of symptoms S and a set of diagnoses D will be given. We'll talk about the patient's condition when they are aware of the outcomes of their testing. A dual-hesitant fuzzy set is used to describe the difficulty. The dual-hesitant fuzzy distances as introduced in the suggested diagnosis technique by Jawad Ali et al. [16].

Allow the set of diagnosis to be $D = \{ \text{Heart Failure Problems} \}$. The considered set of symptoms is $S = \{ \text{Chest pain}, \text{weakness}, \text{shortness of breath}, \text{Cough} \}$. For *Cough*, for instance, coughing up white or pink ($d = \{(0.9, 0.8), (0.1, 0.2)\}$) or another symptom *shortness of breath*, rapid or irregular heartbeat with shortness of breath $d = \{(0.7, 0.6), (0.3, 0.4)\}$. The data is same in reality, but we want to emphasize that our approach requires the values of both heist parameters, so we also explicitly include the hesitation margin.

The following three steps are involved:

- 1) Identification of the symptoms as indicated by Table 1.

Table 1. Identification of symptoms

Q	<i>Chest pain</i>	<i>weakness</i>	<i>shortness of breath</i>	<i>Cough</i>
<i>Al</i>	$\{(0.8, 0.7), (0.2, 0.3)\}$	$\{(0.6, 0.5), (0.3, 0.4)\}$	$\{(0.2, 0.1), (0.7, 0.8)\}$	$\{(0.5, 0.4), (0.4, 0.5)\}$
<i>Bob</i>	$\{(0.1, 0.0), (0.9, 1.0)\}$	$\{(0.4, 0.3), (0.5, 0.6)\}$	$\{(0.6, 0.5), (0.3, 0.4)\}$	$\{(0.2, 0.1), (0.7, 0.8)\}$
<i>Joe</i>	$\{(0.8, 0.7), (0.2, 0.3)\}$	$\{(0.8, 0.7), (0.2, 0.3)\}$	$\{(0.1, 0.0), (0.8, 0.9)\}$	$\{(0.2, 0.1), (0.7, 0.8)\}$
<i>Ted</i>	$\{(0.6, 0.5), (0.4, 0.5)\}$	$\{(0.5, 0.4), (0.4, 0.5)\}$	$\{(0.3, 0.2), (0.7, 0.8)\}$	$\{(0.7, 0.6), (0.3, 0.4)\}$

- 2) The medical knowledge is simulated via dual-hesitant fuzzy relations, as Table 2 illustrates.

Table 2 presents the data. Two hesitation numbers are used to describe each symptom: membership and non-membership for h .

The set of patients under consideration is $P = \{ \text{Al}, \text{Bob}, \text{Joe}, \text{Ted} \}$. As before, Table 1 lists the patients' distinctive symptoms. All two heist parameters are required where explaining each symptom but the facts are the same as in Table 2.

Table 2. The simulated medical knowledge

R	<i>Chest pain</i>	<i>weakness</i>	<i>shortness of breath</i>	<i>Cough</i>
<i>Heart Failure Problems</i>	$\{(0.8, 0.7), (0.2, 0.3)\}$	$\{(0.7, 0.6), (0.3, 0.4)\}$	$\{(0.9, 0.8), (0.0, 0.1)\}$	$\{(0.6, 0.5), (0.3, 0.4)\}$

The set of patients under consideration is $P = \{Al, Bob, Joe, Ted\}$. As before, Table 3-1 lists the patients' distinctive symptoms. All two heist parameters are required where explaining each symptom but the facts are the same as in Table 2.

It is our responsibility to accurately diagnose every patient $p_i, i = 1, \dots, 4$. In order to complete the work, we suggest that for each patient, p_i , we compute the distance between his symptoms in Table 3-1 from a set of symptoms, $j = 1, \dots, 4$ distinctive for each diagnostic, κ in Table 3. The lowest distance found indicates an appropriate diagnosis.

In Jawad Ali et al. [16], we demonstrated that considering both of the hesit parameters—the membership and non-membership functions—are the sole appropriate method for computing the most popular distances for dual-hesitant fuzzy collections. More precisely, this is defined in the generalized normal wiggly dual hesitant weighted Hausdorff distances for all the symptoms of the i – th patient from the κ – th diagnosis.

Table 3 lists each patient's distances from the considered set of potential diagnoses. The smallest distance indicates the diagnosis heart failure problem where the number big than 0.5 refers that this person has this problem: Al and Bob have heart failure problems.

- 3) Determination of diagnosis as shown as in at Table 3:

Table 3. Diagnosis knowledge

	<i>Heart Failure Problem</i>
<i>Al</i>	0.6145
<i>Bob</i>	0.5325
<i>Joe</i>	0.4770
<i>Ted</i>	0.4770

Thus, we conclude that fuzzy sets with dual-hesitation are used in databases; we can express a hesitation with respect to items under examination. All symptom values are taken into account in the diagnosis process, which is based on calculating the distances between a case and all illnesses under consideration. Thus, our method allows to introduce weights for all symptoms (certain symptoms can be more essential for certain conditions). And finally, this application can using in discovering many diagnosis as corona [13] or lung cancer [14].

CONCLUSION AND FUTURE WORK

The suggested concept, dual-hesitant fuzzy sets, is a generalization of some types of uncertainty theories, that can be applied in information system analysis. We have established a relationship on DHFS and have shown that application can use in many real life applications. We aim infuture to apply this concept in probabilistic and stochastic information systems.

Conflicts of Interest: The authors declare no conflict of interest.

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