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# New Operation Defined over Dual-Hesitant Fuzzy Set and Its Application in diagnostics in medicine

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# New Operation Defined over Dual-Hesitant Fuzzy Set and Its Application in diagnostics in medicine

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***Abstract-*** **In recent decades, several types of sets, such as fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, type 2 fuzzy sets, type *n* fuzzy sets, and hesitant fuzzy sets, have been introduced and investigated widely. In this paper, we propose dual hesitant fuzzy sets (DHFSs), which encompass fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, and fuzzy multi-sets as special cases. Then we investigate the basic operations and properties of DHFSs. We also discuss the relationships among the sets mentioned above, and then propose an extension principle of DHFSs. Additionally, we give an example to illustrate the application of DHFSs in diagnostics in medicine which is explained in details and puts new view of some operations of DHFS.**

***Keywords-*** **Intuitionistic Fuzzy sets, Dual Hesitant Fuzzy Sets, diagnostics in medicine, and New Operation Defined over DHFS.**

## I. INTRODUCTION

It is not always helpful to apply the methods usually employed in classical mathematics to handle unclear and uncertain real-world problems. The notion of fuzzy set (FS) was introduced by Zadeh [1] in 1965 as a development of the traditional idea of sets. The membership of elements in a set can be gradually ascertained according to fuzzy set theory. This is represented by a membership function with a value in the real unit interval [0,1]. However, as the membership function is a single-valued function, it is frequently unable to capture evidence for both the objection and the support. Atanassov created the intuitionistic fuzzy set (IFS) [2], which is a generalization of Zadeh's fuzzy set. Unlike Zadeh's fuzzy set, which only has a membership function, IFS has both a membership and a non-membership function, which enables it to better convey the fuzzy nature of data. Given that reluctance can be interpreted as an unknown object [3, 4], it has been researched recently [5, 6]. An additional extension was defined for hesitant fuzzy sets [7]. In other words, we present. The reasoning behind this is that, when defining an element's membership, it is helpful to consider all possible values rather than just an aggregation operator [8], because we have a range of potential values rather than a distribution of possibilities among the available values or a margin of error that makes determining the membership degree difficult (as in A-IFS). We provide a new definition for dual hesitant fuzzy sets in this study since, according to their understanding, procedures for multi sets do not apply appropriately to our sets. Next, we present the relevant operators. The paper is organized as follows: FS and IFS are reviewed in Section 1. Next, we create dual hesitant fuzzy sets and a few fundamental operations in Section 2. Lastly, we introduce a novel use of DHFS for medical diagnostics in Section 4. We offer a novel approach to medical diagnostics by Eulalia Szmidt et al. [9] utilizing Bin Zhu et al.'s dual hesitant fuzzy sets [15] as a basis for DHFS. Jawad Ali et al. [16] arrive at a solution by determining the lowest distance between symptoms. the problems with decision-making, particularly in the context of medical diagnosis. There is a good chance that there will be a non-null hesitation section each time an unknown item is evaluated. More precisely, dual hesitant fuzzy sets enable us to characterize variables such as the patient's erratic body temperature and ambiguous additional symptoms. In order to make the determination of illness, dual hesitant fuzzy sets will be introduced in this article. The report concludes with several recommendations for further research.

## I. DEFINITIONS

Definition [1]: consider that $X$ denotes a non-empty set which is a fuzzy set consisting of $X$ having $A=\{(x,\mu_A(x)):x\in X\}$, where $\mu_A(x):X\rightarrow$ *[0, 1]* denotes the function of the membership of the fuzzy set *A*.

Definition [2]: consider that *X* denotes a non-empty set, which is an IFS where *A* in *X* refers to an object with the form $A=\{(x,\mu_A(x),\nu_A(x)):x\in X\}$, where the functions $\mu_A(x),\nu_A(x):\ X\longrightarrow[0,1]$ *are* defined, respectively; the degree of membership and the degree of non-membership of

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the element $x \in X$ to the set *A* that is a subset of *X*, and for every element $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition [2]: IF A, B be IFS in X, then:

1. [union] $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x))\rangle : x \in X\}$
2. [intersection] $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle : x \in X\}$

Definition [15]: consider that *X* denotes a non-empty set, which is an DHFS where *A* in *X* refers to an object with the form $A = \{(x, h(x), g(x)) : x \in X\}$, where the functions $h(x), g(x) : X \to [0,1]$ *are* defined, respectively; the degree of membership and the degree of non-membership of the element $x \in X$ to the set *A* that is a subset of *X*, and for every element $x \in X$, $0 \le \mu^2{}_A(x) + \nu^2{}_A(x) \le 1$.

Definition [15]: for two DHFS: $d_1, d_2$, the following operations are defined:

(1) $d_1 \cup d_2 = \{h \in (h_1 \cup h_2) \setminus h \ge \max(h^-{}_1, h^-{}_2), g \in (g_1 \cup g_2) \setminus g \le \min(g^+{}_1, g^+{}_2)\};$

(2) $d_1 \cap d_2 = \{h \in (h_1 \cap h_2) \setminus h \le \min(h^+{}_1, h^+{}_2), g \in (g_1 \cup g_2) \setminus g \ge \max(g^-{}_1, g^-{}_2)\}.$

## A. New Operation Defined over DHFS

In the section we use fuzzy intersection and union to define intuitionistic fuzzy set instead of standard intersection and union.

Where the standard form defined as general:

$$A \cap B = \{\langle x, h \le \min(\mu_A(x), \mu_B(x)), h \ge \max(\nu_A(x), \nu_B(x))\rangle : x \in X\}$$

$$A \cup B = \{\langle x, h \ge \max(\mu_A(x), \mu_B(x)), h \le \min(\nu_A(x), \nu_B(x))\rangle : x \in X\} \quad (1)$$

For each element x of the universal set, this function takes as its argument the pair consisting of the element's membership and non-membership grades in set A and in set B, and yield the membership and non-membership grade of the element in the set constituting the intersection and union of A and B,

Thus

$$\min(\mu_A(x), \mu_B(x)) = i[(\mu_A(x), \mu_B(x)]$$

$$\max(\mu_A(x), \mu_B(x)) = u[(\mu_A(x), \mu_B(x)] \quad (2)$$

Similarly

$$\min(\upsilon_A(x), \upsilon_B(x)) = i[(\upsilon_A(x), \upsilon_B(x)]$$

$$\max(\upsilon_A(x), \upsilon_B(x)) = u[(\upsilon_A(x), \upsilon_B(x)] \quad (3)$$

*For i must satisfy the following axioms for all a, b , d* ∈ [0,1].

$$i : [0,1]\ X\ [0,1] \longrightarrow [0,1] \quad (4)$$

*Axiom i1.* *i (a,1) = a (boundary condition).*

*Axiom i2.* *b < d implies i (a, b) < i (a, d) (monotonicity).*

*Axiom i3.* *i (a, b) = i(b, a) (commutatively).*

*Axiom i4.* *i (a ,i (b, d)) = i (i (a, b), d) (associativity).*

*Axiom i5.* *i is a continuous function (continuity).*

*Axiom i6.* *i (a, a) = a (subidempotency).*

*Axiom i7.* $a_1 < a_2$ *and* $b_1 < b_2$ *implies* $i(a_1, b_1) < i(a_2, b_2)$ *(strict monotonicity.)*

*So, we can discuss:*

*Standard intersection:* $i(a, b) = h \le \min(a, b)$ (5)

*Algebraic product:* $i(a, b) = h \le ab$ (6)

*Bounded difference:* $i(a, b) = h \le \max(0, a+b-1)$ (7)

*Drastic intersection:*
$$i(a,b) = \begin{cases} h \le a & , b = 1 \\ h \le b & , a = 1 \\ 0 & , O.W. \end{cases} \quad (8)$$

Seen in Fig. 1, Fig. 2, Fig. 3 and Fig. 4 that drawn using MatLab.

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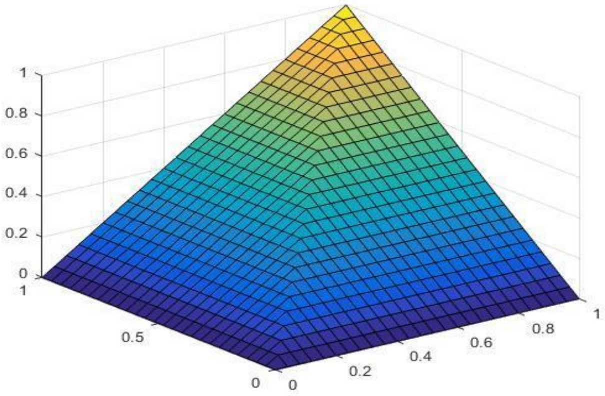


**Figure 1. Standard intersection**

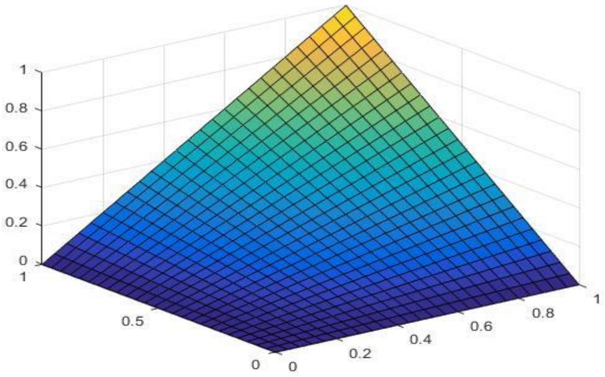


**Figure 2. Algebraic product**

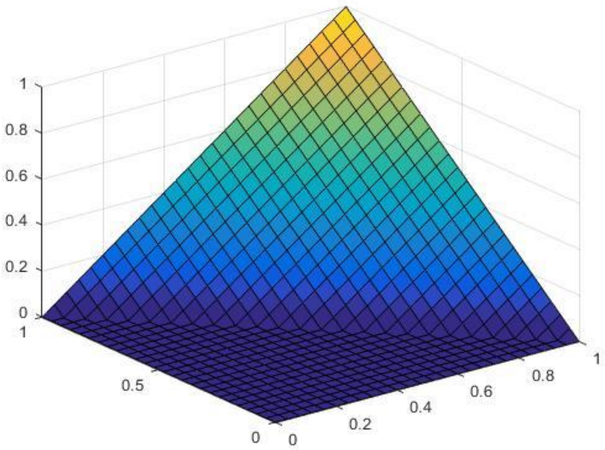


**Figure 3. Bounded difference**

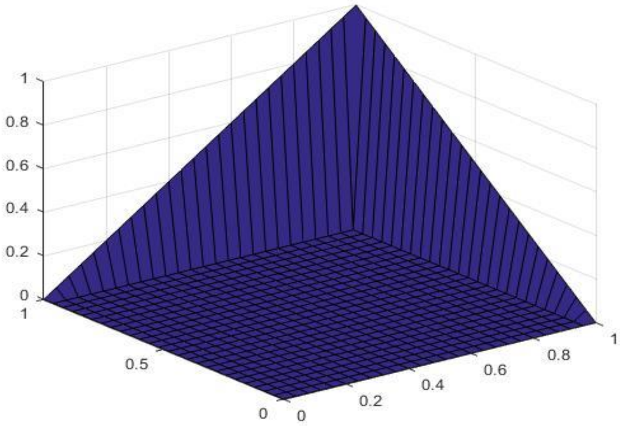


**Figure 4. Drastic intersection**

*For u must satisfy the following axioms for all a, b, d ∈ [0,1]*

$$u : [0,1] \, X \, [0,1] \longrightarrow [0,1] \tag{9}$$

*Axiom u1.* *u (a,0) = a (boundary condition).*

*Axiom u2.* *b < d implies u (a, b) < u (a, d) (monotonicity).*

*Axiom u3.* *u (a, b) = u(b, a) (commutatively).*

*Axiom u4.* *u (a, u (b, d)) = u(u(a, b),d) (associativity).*

*Axiom u5.* *u is a continuous function (continuity).*

*Axiom u6.* *u (a, a) > a (subidempotency).*

*Axiom u7.* $a_1 < a_2$ *and* $b_1 < b_2$ *implies* $u(a_1, b_1) < u(a_2, b_2)$ *(strict monotonicity).*

*So, we can discuss:*

*Standard union:* $u(a, b) = h \geq max(a, b)$ (10)

*Algebraic sum:* $u(a, b) = h \geq a+b-ab$ (11)

*Bounded sum:* $u(a, b) = h \geq min(1,a+b)$ (12)

*Drastic union:*
$$u(a,b) = \begin{cases} h \geq a & ,b = 0 \\ h \geq b & ,a = 0 \\ 1 & ,OW. \end{cases} \tag{13}$$

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Seen in Fig. 5, Fig. 6, Fig. 7 and Fig. 8 and drawn with MatLab.

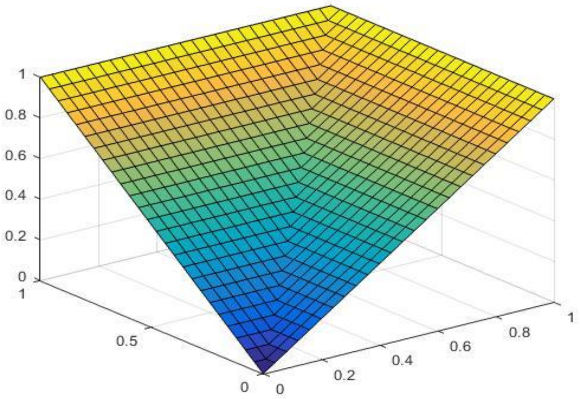


**Figure 5. Standard *union***

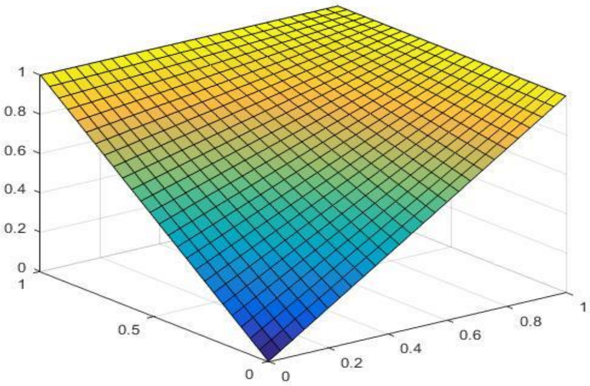


**Figure 6. Algebraic *sum***

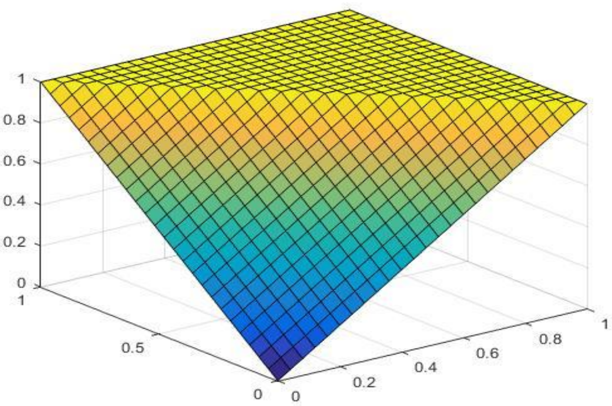


**Figure 7. Bounded sum**

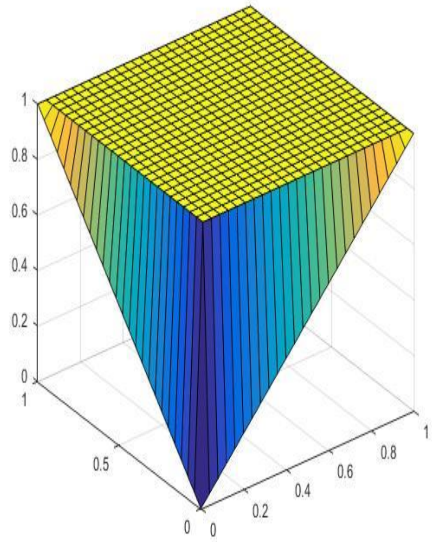


**Figure 8. Drastic union**.

### B. *APPLICATION OF DHFS IN DIAGNOSTICS IN MEDICINE*

Given the presence of a database, an example of a diagnostics in medical that is, a description of a set of symptoms $S$ and a set of diagnoses $D$ will be given. We'll talk about the patient's condition when they are aware of the outcomes of their testing. A Dual- hesitant fuzzy set is used to describe the difficulty. The dual-hesitant fuzzy distances as introduced in the suggested diagnosis technique by Jawad Ali et al. [16]. The following three steps are involved: The following three steps are involved:

1) Identification of the symptoms as indicated by Table 1.

Journal of Engineering Research (JER)
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**Table 1. Identification of symptoms**

| $Q$ | $Temperature$ | $Headache$ | $Stomach\ pain$ | $Cough$ | $Chest\ pain$ |
|---|---|---|---|---|---|
| $Al$ | $\{(0.8,0.7),(0.2,0.3)\}$ | $\{(0.6,0.5),(0.2,0.3)\}$ | $\{(0.2,0.1),(0.8,0.9)\}$ | $\{(0.7,0.6),(0.3,0.4)\}$ | $\{(0.1,0.0),(0.1,0.2)\}$ |
| $Bob$ | $\{(0.1,0.0),(0.7,0.8)\}$ | $\{(0.5,0.4),(0.2,0.3)\}$ | $\{(0.6,0.5),(0.0,0.1)\}$ | $\{(0.1,0.0),(0.7,0.8)\}$ | $\{(0.1,0.0),(0.1,0.2)\}$ |
| $Joe$ | $\{(0.8,0.7),(0.2,0.3)\}$ | $\{(0.8,0.7),(0.2,0.3)\}$ | $\{(0.1,0.0),(0.9,1.0)\}$ | $\{(0.2,0.1),(0.2,0.3)\}$ | $\{(0.7,0.6),(0.1,0.2)\}$ |
| $Ted$ | $\{(0.6,0.5),(0.1,0.2)\}$ | $\{(0.5,0.4),(0.2,0.3)\}$ | $\{(0.3,0.2),(0.7,0.8)\}$ | $\{(0.7,0.6),(0.2,0.3)\}$ | $\{(0.3,0.2),(0.2,0.3)\}$ |

2) The medical knowledge is simulated via dual-hesitant fuzzy relations, as Table 2 illustrates.

Allow the set of diagnosis to be $D$ = {$Viral\ fever$, $Malaria$, $Typhoid$, $Chest\ problem$ }.The considered set of symptoms is $S$ ={$Temperature$, $Headache$, $Stomach\ pain$, $Cough$, $Chest\ pain$ }.

Table 2 presents the data. Two hesitantion numbers are used to describe each symptom: membership $h$. For $Malaria$, for instance, a high temperature ($h = \{(0.8, 0.7), (0.2, 0.3)\}$, On the other hand, the $Chest\ problem$ : low temperature ($h = \{(0.2, 0.1), (0.8, 0.9)\}$). The data is same in reality, but we want to emphasize that our approach requires the values of both heist parameters, so we also explicitly include the hesitation margin.

The set of patients under consideration is $P = \{Al, Bob, Joe, Ted\}$. As before, Table 1 lists the patients' distinctive symptoms. All two heist parameters are required ($h = \{h_1, h_2\}$) explaining each symptom but the facts are the same as in Table 2.

It is our responsibility to accurately diagnose every patient $p_i, i = 1, ..., 4$. In order to complete the work, we suggest that for each patient, $p_i$, we compute the distance between his symptoms in Table 1 from a set of symptoms, $j = 1, ..., 4$ distinctive for each diagnostic, $k = 1, ..., 4$ in Table 3. The lowest distance found indicates an appropriate diagnosis.

In Jawad Ali et al. [16], We demonstrated that considering both of the heist parameters—the membership function—is the sole appropriate method for computing the most popular distances for hesitant fuzzy collections. More precisely, this is defined in Jawad Ali et al. [16] for all the symptoms of the $i-th$ patient from the $k-th$ diagnosis.

Table 3 lists each patient's distances from the considered set of potential diagnoses. The smallest distance indicates an accurate diagnosis: Al has $Malaria$, Bob has a $Viral\ fever$, Joe may have a $Chest\ problem$ or $Malaria$, and Ted may have $Typhoid$ or $Malaria$.

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Table 2. The simulated medical knowledge

| *R* | *Viral fever* | *Malaria* | *Typhoid* | *Chest problem* |
|---|---|---|---|---|
| *Temperature* | {(0.4, 0.3), (0.6, 0.7)} | {(0.7, 0.6), (0.0, 0.1)} | {(0.3, 0.2), (0.7, 0.8)} | {(0.1, 0.0), (0.6, 0.7)} |
| *Headache* | {(0.3, 0.2), (0.6, 0.7)} | {(0.2, 0.1), (0.4, 0.5)} | {(0.6, 0.5), (0.4, 0.5)} | {(0.2, 0.1), (0.5, 0.6)} |
| *Stomach pain* | {(0.1, 0.0), (0.2, 0.3)} | {(0.1, 0.0), (0.6, 0.7)} | {(0.2, 0.1), (0.0, 0.1)} | {(0.8, 0.7), (0.2, 0.3)} |
| *Cough* | {(0.4, 0.3), (0.6, 0.7)} | {(0.7, 0.6), (0.0, 0.1)} | {(0.2, 0.1), (0.2, 0.3)} | {(0.2, 0.1), (0.7, 0.8)} |
| *Chest pain* | {(0.1, 0.0), (0.1, 0.2)} | {(0.1, 0.0), (0.1, 0.2)} | {(0.1, 0.0), (0.7, 0.8)} | {(0.2, 0.1), (0.3, 0.4)} |

3) Determination of diagnosis as shown as in at Table 3.

Table 3. Diagnosis knowledge

| | *Viral fever* | *Malaria* | *Typhoid* | *Chest problem* |
|---|---|---|---|---|
| *Al* | 0.4 | 0.8 | 0.6 | 0.7 |
| *Bob* | 0.8 | 0.7 | 0.6 | 0.6 |
| *Joe* | 0.5 | 0.8 | 0.5 | 0.8 |
| *Ted* | 0.6 | 0.8 | 0.8 | 0.5 |

Thus, we conclude that fuzzy sets with hesitation are used in databases; we can express a hesitation with respect to items under examination. All symptom values are taken into account in the diagnosis process, which is based on calculating the distances between a case and all illnesses under consideration. Thus, our method allows to introduce weights for all symptoms (certain symptoms can be more essential for certain conditions). And finally, this application can using in discovering many diagnosis as corona [13] or lung cancer [14].

## VI. CONCLUSION AND FUTURE WORK

In this paper, we have introduced hesitant fuzzy sets, a generalization of fuzzy sets that permits us to represent the situation in which different membership functions are considered possible. We have established a relationship on HFS and have shown that application can use in many real life applications. For example, medicine, political or social case.

**Conflicts of Interest:** The authors declare no conflict of interest.

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