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Model Updating and Damage Detection using Direct Mode Shape Expansion to Dealing with Spatial Incompleteness

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Abstract: In the study of finite element model updating or damage detection, most papers usually employ transformation matrix from the master coordinates to the full-order coordinates, by either model reduction or modal expansion schemes to deal with spatially incomplete situations. This article employs the direct mode shape expansion method, by applying a hybrid vector that is constructed by measured values at master degrees of freedom (dofs) and constant values at slave dofs, to expand the measured spatially incomplete mode shape based on a series of modification factors. One theoretical development is that model updating or damage detection using Cross Model Cross Mode (CMCM) method is firstly combined with the direct mode shape expansion scheme for dealing with spatial incompleteness. The other development is a new indicator, i.e., modal strain energy change indicator (MSECI) is presented based on traditional modal strain energy method. Numerical studies have been conducted for a three-dimensional four-story frame structure with multiple damaged elements, as the measured modes are synthesized from finite element models. The numerical results reveal that the direct mode shape expansion method outperforms Guyan expansion method on higher order mode shape expansion. Implementing the CMCM method together with the direct mode shape expansion scheme, proper damge detection results can be obtained.

Keywords: Model updating, Damage detection, Direct mode shape expansion, Spatial incompleteness, Hybrid vector

1 Introduction

Detection, location and quantification of damage in a structure via techniques that examine changes in measured structural vibration response have attracted much attention in recent years. The methods for damage detection are commonly classified into four levels. While a higher level method always includes issues covered in a lower level method, specific focus of each level is generally accepted as follows: Level 1-determining whether damage occurs in the structure, Level 2-identifying the geometric location of the damage, Level 3-quantifying the severity of the damage, and Level 4 -predicting the remaining service life of the structure.

The method widely used to detect damage in structures is using modal frequency changes for the lower natural frequency can be easily and precisely measured in practice. Vandiver (1976)[1] used the same principle to determine the occurrence of damage in offshore structures. Biswas et al.(1990)[2] demonstrated that a decrease in natural frequencies could be used to detect damage in a highway bridge. Messina et al. (1996)[3] used the natural frequency sensitivity analysis to determine damage locations and extents. These methods seem to fail to locate and quantify damages sometimes since modal frequencies are a global property of the structure, which are especially obvious for the symmetrical structure. As mode shapes can provide much information than natural frequency, many researchers have devoted their efforts to detect damages with mode shape information or both mode shape information and natural frequency information. Mannan and Richardson (1990)[4] located structural cracks by using the difference in the stiffness matrices of structures before and after damage. Pandey et al. (1991)[5] used the changes in the mode shape curvature to detect and locate damage. Pandey and Biswas (1994)[6] developed a method to locate damages using changes in the flexibility matrix of

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the structure. This approach is feasible since the structural flexibility matrix can be obtained accurately by using only a few of the lower frequency modes. Later they demonstrated the effectiveness of the flexibility change method using experimental data. The disadvantage of their method is that results of damage localization depend on the conditions of the structural boundary. However, in actual engineering the structural boundary condition is difficult to determine ideally. Shi et al. (1998)[7] proposed using the change of modal strain energy (MSE) in each element as damage indicator, and it was proved to be effective in locating the structural damage. However, because the damage elements are not known, the undamaged elemental stiffness matrix is used instead of the damaged one as an approximation during calculating modal strain energy change of the nth element for the jth mode. Li et al. (2006)[8] developed an effective damage localization method for three-dimensional frame structures, the modal strain energy decomposition (MSED) method. The MSED method defines two damage indicators, axial damage indicator and transverse damage indicator, for each member. Analyzing the joint information of the two damage indicators greatly improves the accuracy of localizing damage elements. But the MSED method cannot achieve satisfactory estimate for the corresponding damage severity. Hu et al. (2006)[9] developed a newly damage severity estimation method, termed as cross modal strain energy (CMSE) method. This damage severity estimation method can be applied sequentially after damage members have been correctly identified by any other damage localization method. The CMSE method is a non-iterative, exact solution method and uses both the mode shapes and modal frequencies for the damage severity estimation.

One particular model-based approach to identify the damage location and to assess its severity is the finite element (FE) model updating method. The purpose of FE model updating is to calibrate the mass, damping and stiffness matrices of the FE model based on the test data so as to obtain better agreement between numerical model predictions and measured results. Clearly, comparing the updated matrices associated with a damaged structure to those of the baseline model provides an indication of the damage, for both location and severity. Friswell et al. (1998)[10] first conducted FE model updating to damped systems, who extended the traditional direct methods to estimating both the damping and stiffness matrices of a damaged cantilever beam while assuming that its mass matrix was known. Their algorithm has the drawback that it does not guarantee the connectivity of the original finite element model. Kuo et al. (2006)[11], extended the direct method to a more general problem that the analytical mass, damping, and stiffness matrices were all allowed to be updated. Li et al. (2008)[12] extended the CMCM method to the damped systems for damage detection using spatially incomplete complex modes. The method was demonstrated to be effective using a cantilever beam structure, which has been employed by Friswell et al. (1998). However, further studies indicate the method is sensitive to noise; thus, a more robust indicator is expected based on the CMCM method for damage localization.

In this paper, we will make improvements in two aspects: 1) employing the direct mode shape expansion method by Liu (2011)[13] to deal with spatial incompleteness of measured modes; and 2) a new damage localization indicator, i.e., modal strain energy change indicator (MSECI) is proposed based on the concept of modal strain energy. In numerical study, a three-dimensional four-story frame structure with multiple damaged elements will be chosen for the numerical studies, where the measured modal information will be synthesized from using a finite element model that is similar to the analytical model, but with different sets of system coefficients.

2 Preliminary

Throughout this paper, to distinguish symbols associated with the models before and after updating, a superscript "′" is used for the updated model in contrast to the original (or baseline) model, e.g., $\mathbf{M}'$ and $\mathbf{M}$ represent the mass matrix of the updated and baseline models, respectively.

The author (Liu, 2011)[13] presented a direct estimation method for expanding incomplete experimental mode shapes. The performance of the method was investigated using a 5 dofs mass-spring system and a steel cantilever-beam experiment. A hybrid vector for the jth mode, which includes the measured data at master coordinates and constant values at slave coordinates, is defined

$$\tilde{\mathbf{\Phi}}_j = \begin{Bmatrix} measured \\ measured \\ \vdots \\ constant \\ constant \\ \vdots \\ constant \end{Bmatrix} \tag{1}$$

in which, the constant can be assumed to be any integer, such as one or two.

The jth measured mode shape $\mathbf{\Phi}'_j$ is a modification of $\tilde{\mathbf{\Phi}}_j$ by

$$\mathbf{\Phi}_j{}' = \tilde{\mathbf{\Phi}}_j + \sum_{s=1}^{N} \delta_{s,j} \tilde{\mathbf{\Phi}}_{s,j} \tag{2}$$

where $\tilde{\mathbf{\Phi}}_{s,j}$ is a vector of which the value at the sth dof equals to the in Eq. (1), and other values are zero; N is the number of unmeasured dofs, and $\delta_{s,j}$ are a series of factors to modify the vector $\tilde{\mathbf{\Phi}}_{s,j}$, here $\delta_{s,j}$ are called mode-correction factors for the jth mode.

Then a series of equations are constructed

$$\sum_{n=1}^{N_e} \alpha_n \mathbf{K}^{\diamondsuit}_{n,ij} + \sum_{s=1}^{N} \delta_{s,j} \mathbf{K}^{\diamondsuit}_{s,ij} - \sum_{s=1}^{N} \delta_{s,j} \mathbf{M}^{\diamondsuit}_{s,ij} + \sum_{s=1}^{N}\sum_{n=1}^{N_e} \alpha_n \delta_{s,j} \tilde{\mathbf{K}}^{s}_{n,ij} = \mathbf{f} \quad (3)$$

Assume δ_j is known, then δ_j is reproduced iteratively and the proposed method is again implemented; and the process is repeated until convergence are obtained. The jth estimated mode shape from Eq. 3 is denoted as $\mathbf{\Phi}_{j,es}{}^{*}$, and readers can find detailed information in reference[14].

Cross model cross mode method. First, the stiffness, damping and mass matrices of the baseline model, denoted as $\mathbf{K}$, $\mathbf{C}$ and $\mathbf{M}$, respectively, have been initially modeled. In the proposed cross-model cross-mode approach, the stiffness matrix $\mathbf{K}'$ of the updated model is a modification of $\mathbf{K}$ via

$$\mathbf{K}' = \mathbf{K} + \sum_{n=1}^{N_K} \alpha_n \mathbf{K}_n \quad (4)$$

where individual $\mathbf{K}_n$ is a pre-selected stiffness sub-matrix of the baseline model; α_n are unknown stiffness correction factors to be determined; and N_K is the number of stiffness correction terms for the stiffness matrix. Likewise, one writes the corresponding expression for the mass matrix $\mathbf{M}'$ and viscous damping matrix $\mathbf{C}'$, respectively, as

$$\mathbf{M}' = \mathbf{M} + \sum_{n=1}^{N_M} \beta_n \mathbf{M}_n \quad (5)$$

and

$$\mathbf{C}' = \mathbf{C} + \sum_{n=1}^{N_C} \gamma_n \mathbf{M}_n \quad (6)$$

where the individual $\mathbf{C}_n$ and $\mathbf{M}_n$ are pre-selected sub-matrices of the baseline model; β_n and γ_n are correction coefficients to be determined; N_M and N_C are the numbers of mass and damping correction coefficients, respectively.

When N_i and N_j modes are taken from the analytical model and the measured model, respectively, total $N_m = N_i \times N_j$ complex equations can be formed

$$K_{ij}{}^{\dagger} + \sum_{n=1}^{N_K} \alpha_n K^{\dagger}_{n,ij} + \lambda_j{}' \left(C_{ij}{}^{\dagger} + \sum_{n=1}^{N_C} \gamma_n C^{\dagger}_{n,ij} \right) + \lambda_j{}'^2 \left(M_{ij}{}^{\dagger} + \sum_{n=1}^{N_M} \beta_n M^{\dagger}_{n,ij} \right) = 0 \quad (7)$$

Those equations are named cross-model cross-mode (CMCM) equations in view of the fact that they are formed by crossing over two models, for two arbitrary modes. From Eq. 7 α_n is solved and used to detect damages' location and severity. Note that in the following presentation, the damping is not taken into account. Readers can find detailed information in reference[12].

3 CMCM Method for Spatial Incompleteness

In dealing with spatially incomplete situations, one usually applies either model reduction or modal expansion schemes. The transformation matrix from the master coordinates to the full-order coordinates for the baseline model is denoted as $\mathbf{T}$, which can follow either $\mathbf{T}_G$ for Guyan transformation[14] or $\mathbf{T}_S$ for SEREP[15].

When mode shape values for the damaged structure are available only at master coordinates, applying the modal expansion $\mathbf{\Phi}'_j = \mathbf{T}' \left(\mathbf{\Phi}'_j\right)_m$, one can write

$$K^{\dagger}_{ij} = \mathbf{\Phi}^t_i \mathbf{K} \mathbf{T}' \left(\mathbf{\Phi}'_j\right)_m \quad (8)$$

where $\left(\mathbf{\Phi}'_j\right)_m$ is the jth mode shape of the damaged structure measured only at the master coordinates, and $\mathbf{T}'$ is the counterpart of $\mathbf{T}$ for the damaged structure. Likewise, following the concept of the modal expansion, one writes

$$K^{\dagger}_{n,ij} = \left(\mathbf{\Phi}_i\right)^t \mathbf{K}_n \mathbf{T}' \left(\mathbf{\Phi}'_j\right)_m \quad (9)$$

$$M^{\dagger}_{ij} = \left(\mathbf{\Phi}_i\right)^t \mathbf{M} \mathbf{T}' \left(\mathbf{\Phi}'_j\right)_m \quad (10)$$

$$M_{n,ij}{}^{\dagger} = \left(\mathbf{\Phi}_i\right)^t \mathbf{M}_n \mathbf{T}' \left(\mathbf{\Phi}'_j\right)_m \quad (11)$$

$$C_{ij}{}^{\dagger} = \left(\mathbf{\Phi}_i\right)^t \mathbf{C} \mathbf{T}' \left(\mathbf{\Phi}'_j\right)_m \quad (12)$$

and

$$C_{n,ij}{}^{\dagger} = \left(\mathbf{\Phi}_i\right)^t \mathbf{C}_n \mathbf{T}' \left(\mathbf{\Phi}'_j\right)_m \quad (13)$$

In this paper, we will employ the direct mode shape expansion method to deal with spatial incompleteness, thus $\mathbf{T}' \left(\mathbf{\Phi}'_j\right)_m$ in Eq. 8 is replaced by the estimated one of $\mathbf{\Phi}'_j$ in Eq. 2, denoted by $\mathbf{\Phi}'_{j,es}$, then Eq. 8 becomes

$$K^{\dagger}_{ij} = \mathbf{\Phi}^t_i \mathbf{K} \mathbf{\Phi}'_{j,es} \quad (14)$$

4 Modal Strain Energy Change Indicator

The elemental modal strain energy (MSE) is defined as the product of the elemental stiffness matrix and the second power of the mode shape component (Shi et al, 1998). For the nth element, the ith mode for the baseline model, and the jth mode for the damaged model, MSE is given as

$$MSE_{ni} = \left(\mathbf{\Phi}_i\right)^t \mathbf{K} \mathbf{\Phi}_i \quad (15)$$

$$MSE'_{nj} = \left(\mathbf{\Phi}'_j\right)^t \mathbf{K}'_n \mathbf{\Phi}'_j \quad (16)$$

From Eq. 4 , Eq. 16 can be rewritten as

$$MSE'_{nj} = \left(\mathbf{\Phi}'_j\right)^t \left(\mathbf{K}_n + \alpha_n \mathbf{K}_n\right) \mathbf{\Phi}'_j \tag{17}$$

where α_n is correction factor corresponding to the nth element.

In the above equation, the jth mode shape from the damaged model, $\mathbf{\Phi}_j{}'$, which includes the information of the mass, stiffness and damping after the occurrence of damage, is slightly different from its traditional counterpart, which only includes the stiffness information after the occurrence of damage.

If the MSE for several modes are considered together, the MSE_n and $MSE_n{}'$ of the nth element is defined as the average of the summation of MSE_{ni} and $MSE_{nj}{}'$ respectively.

$$MSE_n = \frac{1}{N_i}\sum_{i=1}^{N_i} MSE_{ni} = \frac{1}{N_i}\sum_{i=1}^{N_i} \left(\mathbf{\Phi}_i\right)^t \mathbf{K}_n \mathbf{\Phi}_i \tag{18}$$

$$MSE'_n = \frac{1}{N_j}\sum_{j=1}^{N_j} MSE'_{nj} = \frac{1}{N_j}\sum_{j=1}^{N_j} \left(\mathbf{\Phi}'_j\right)^t \mathbf{K}'_n \mathbf{\Phi}'_j \tag{19}$$

Then the modal strain energy change (MSEC) of the nth element could be obtained from Eq. 18 and Eq. 19

$$\begin{aligned} MSEC_n =& \frac{1}{N_j}\sum_{j=1}^{N_j} \left(\mathbf{\Phi}'_j\right)^t \mathbf{K}'_n \mathbf{\Phi}'_j \\ & - \frac{1}{N_i}\sum_{i=1}^{N_i} \left(\mathbf{\Phi}_i\right)^t \mathbf{K}_n \mathbf{\Phi}_i \end{aligned} \tag{20}$$

Substituting $\mathbf{K}'_n = \mathbf{K}_n + \alpha_n \mathbf{K}_n$ into the above equation, one obtains

$$\begin{aligned} MSEC_n =& \frac{1}{N_j}\sum_{j=1}^{N_j} \left(\mathbf{\Phi}'_j\right)^t \mathbf{K}_n \mathbf{\Phi}'_j \\ & + \frac{1}{N_j}\sum_{j=1}^{N_j} \alpha_n \left(\mathbf{\Phi}'_j\right)^t \mathbf{K}_n \mathbf{\Phi}'_j \\ & - \frac{1}{N_i}\sum_{i=1}^{N_i} \left(\mathbf{\Phi}_i\right)^t \mathbf{K}_n \mathbf{\Phi}_i \end{aligned} \tag{21}$$

Neglecting the difference between the first and third terms, Eq. 21 becomes

$$MSEC_n \approx \frac{1}{N_j}\sum_{j=1}^{N_j} \alpha_n \left(\mathbf{\Phi}'_j\right)^t \mathbf{K}_n \mathbf{\Phi}'_j \tag{22}$$

Following the normalization procedure by Stubbs et al. (1995)[16], a new damage indicator of the nth element based on modal strain energy change (MSECI) is defined as

$$MSECI_n = \frac{MSEC_n - \overline{MSEC_n}}{\sigma_{MSEC_n}} \tag{23}$$

where $\overline{MSEC_n}$ and σ_{MSEC_n} represent the sample mean and standard deviation of $MSEC_n$, respectively.

When mode shape values for the damaged structure are available only at master coordinates, applying the direct mode shape expansion, Eq. 15 and Eq. 16 become

$$MSE_{ni} = \left(\mathbf{\Phi}_i\right)^t \mathbf{K} \mathbf{\Phi}_i \tag{24}$$

$$MSE'_{nj} = \left[\mathbf{\Phi}^*_{j,es}\right]^t \left(\mathbf{K}_n + \alpha_n \mathbf{K}_n\right) \mathbf{\Phi}^*_{j,es} \tag{25}$$

Employing Eq. 18, the MSECI of the nth element can be obtained, which can be utilized for damage detection.

5 Numerical Study

The structure adopted in the numerical studies is a three-dimensional four-story frame structure shown in Fig. 1, which is synthesized from a finite element model where each structural member is modeled as a three-dimensional uniform beam element, and is distinguished by assigning an element number. The essential geometrical and material properties of the frame structure are given below. The length of all horizontal members oriented in the x direction is 1m, all horizontal members oriented in the y direction 3m, and all vertical members 1 m. For all members, the Young's module $E = 210GPa$, the mass density per unit length $\bar{m} = 22.035Kg/m$, the cross-section are $A = 2.825 \times 10^{-3}m^2$, and the moment of inertia $I = 2.89 \times 10^{-6}m^4$. Performing an eigen analysis, one obtains that the structure has the first three modal frequencies 8.8626, 11.95, and 15.434 Hz, respectively.

In this study, the test structure is considered to be a damaged one. The damage is modeled as a reduction in the stiffness of element 2, 16 and 21 by 30%, 20% and 35% respectively, from the analytical model. These three elements represent different type of structure members and are highlighted in Fig.1(b). Specifically, other elements of the tested model are produced with a series of quantities β_n generated based on a Gaussian distribution with the mean equal to 0 and standard deviation equal to 0.05. Firstly, the feasibility of MSECI for damage localization using spatially complete and noise-free complex modes is investigated. Assume all dofs of the structure can be measured, and 96 modes from the analytical model and 1 modes from the true model are utilized; thus 96 real-valued CMCM equations can be formed, which are sufficient to solve 96 unknowns. Resulting estimates of damage locations using α and MSECI are shown at the top and the bottom panels of Fig. 2 respectively, plotted against the element number n. From Fig. 2, one concludes that the proposed damage indicator MSECI can be employed for damage localization in structures, and we also can see that the proposed indicator MSECI is more effective to damages occurred in legs. If only a subset of the dofs are measured

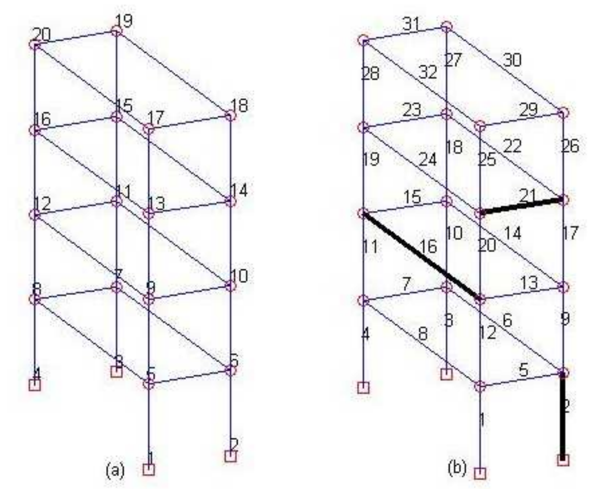

Fig. 1: A three-dimensional four-story frame structure: (a) node numbers; (b) element numbers

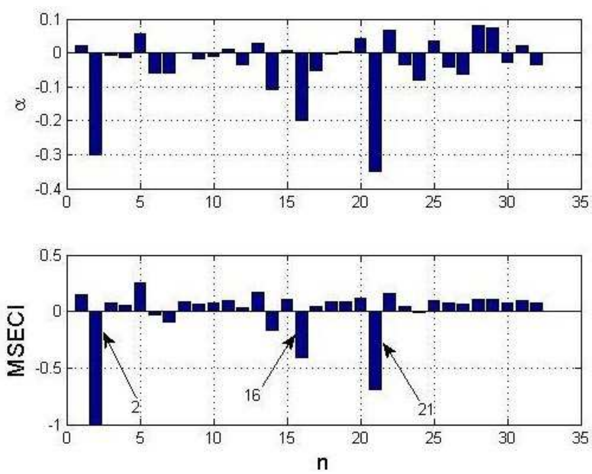

Fig. 2: Damage localization employing spatially complete, noise-free modes

(a spatially incomplete situation), then the analytical model must be reduced or the measured mode shapes must be expanded. In the following presentation, only the translational dofs in x, y and z directions at nodes 9 to 20 are measured for the first two modes. When the traditional Guyan reduction scheme is used to reduce the analytical model, implementing the proposed indicator based on the same previous consideration yields the result shown in Fig. 3. From Fig.3 one concludes that errors caused by Guyan reduction have a great influence on damage detection, even on the proposed indicator. In the following presentation, we will study whether mode shape expansion could be improved employing the direct mode shape expansion method, compared to Guyan expansion

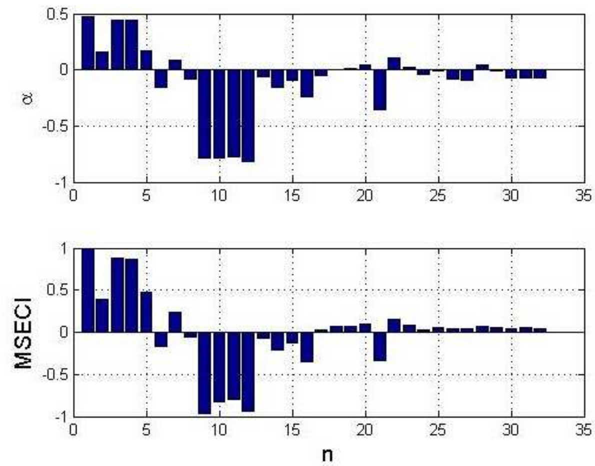

Fig. 3: Damage localization employing traditional Guyan reduction scheme

scheme. Therefore the first mode shape values at slave coordinates from the Guyan expansion and the direct mode shape expansion are investigated, respectively. Shown at the top panel of Fig.4 are translational mode shape values in x, y and z directions at nodes 5 to 8, plotted against the degrees of freedom. Likewise, rotational values in x, y and z directions at nodes 5 to 20 are plotted in Fig. 4 (b) and Fig.4(c), respectively. From Fig.4 one can find that traditional Guyan expansion and direct mode shape method have similar performance on the first mode shape values corresponding to slave coordinate. While for the second mode shape, the direct mode shape expansion method outperforms the traditional Guyan expansion, as shown in Fig.5. Then one may predict that implementing the CMCM method with the direct modes shape expansion scheme could perform better in damage detection. Shown in Fig.6 are the results of damage detection using the direct mode shape expansion method based on the same considerations yields Fig.3, and Fig.6 demonstrated our consideration.

Note that the robustness of the proposed method has not been discussed here, one reason is that our aim is to expand mode shapes using the measured values which maybe include a certain measurement noise to replace ones at master dofs, while not using the uncontaminated ones because they are not known in practice; the other reason is we can adopt noise elimination techniques to separate noise from the measurement response, which will be further studied in our future experiment work.

6 Conclusion

Cross model cross mode (CMCM) method performs better in model updating and damage detection when a number of spatially complete modes are available, which

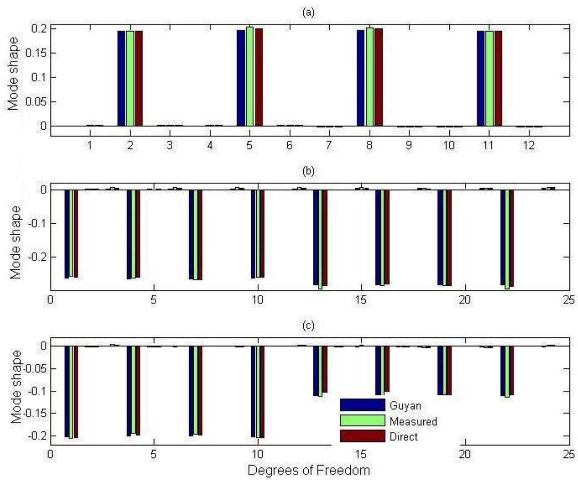

Fig. 4: The first mode shape values comparison using Guyan expansion and direct mode shape expansion, respectively: (a) translational values at nodes 5 to 8; (b) rotational values at nodes 5 to 12; (c) rotational values at nodes 13 to 20

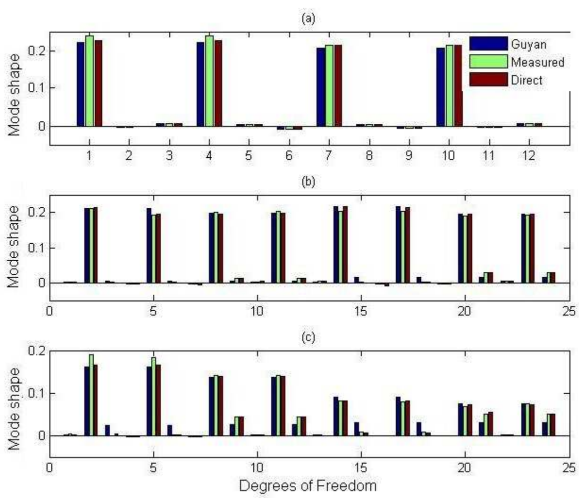

Fig. 5: The sencond mode shape values comparison using Guyan expansion and direct mode shape expansion, respectively: (a) translational values at nodes 5 to 8; (b) rotational values at nodes 5 to 12; (c) rotational values at nodes 13 to 20

not only provides damage locations, but also damage severities. While employing several lower order spatially incomplete modes, results become unstable. Therefore, our recent development is used, i.e., the direct mode shape expansion method, to deal with spatial incompleteness of measured modes, expected to improve damage detection implementing the CMCM method. Numerical results from a three-dimensional four-story

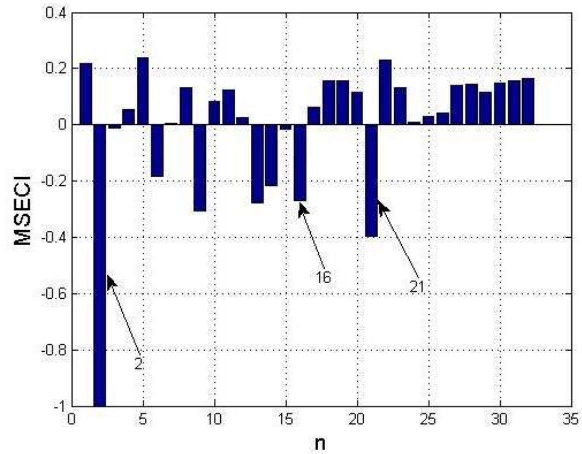

Fig. 6: Damage detection employing direct mode shape expansion

frame structure show that the direct mode shape expansion method outperforms Guyan expansion method on higher order mode shape expansion. Implementing the CMCM method and combined with the direct mode shape expansion to deal with spatial incompleteness of the measured modes, better damage detection could be obtained.

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References

[1] J.K. Vandiver, Proceedings of the Seventh Annual Offshore Technology Conference **2**, 243-252 (1976).
[2] M. Biswas, A.K. Pandey, M.M. Samman, The International Journal of Analytical and Experimental Modal Analysis **5**, 33-42(1990).
[3] A. Messina, J. E. Williams, T. Contursi, Journal of Sound and Vibration **216**, 791-808 (1996).
[4] M. A. Mannan, M. H. Richardson, Proceedings of the Eighth International Modal Analysis Conference **1**, 652-657(1990).
[5] A.K. Pandey, M. Biswas, M.M. Samman, Journal of Sound and Vibration **145**, 321-332(1991).
[6] A.K. Pandey, M. Biswas, Journal of Sound and Vibration **169**, 3-17 (1994).
[7] Z.Y. Shi, S.S. Law, L.M. Zhang, Journal of Sound and Vibration **218**, 714-733 (1998).

[8] H.J. Li, H.Z. Yang, S.-L J. Hu, Journal of Engineering Mechanics **132**, 941-951 (2006).
[9] S.-L. J. Hu, S.Q. Wang, and H.J Li, Journal of Engineering Mechanics **132**, 429-437 (2006).
[10] M.I. Friswell, J.E Mottershead, Kluwer Academic Publishers, the Netherlands, 1995.
[11] Y. Kuo, W. Lin, S. Xu, AIAA Journal **44**, 1310-1316 (2006).
[12] H.J. Li, F.S. Liu, S-L J. Hu, Science in China Series E: Technological Sciences **51**, 2254-2268 (2008).
[13] F.S. Liu, Journal of Sound and Vibration **330**, 4633-4645 (2011).
[14] R.J. Guyan, AIAA Journal **3**, 380 (1965).
[15] J.C. O'Callahan, P. Avitabile, R. Riemer, Seventh International Modal Analysis Conference, 29-37 (1989).

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