

Derivation of Thermal Relaxation Time between Two-Phase Flow under the Effect of Heating Sink

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Abstract: In the present paper, the thermal relaxation time of a mixture surrounding a growing vapour bubbles between two-phase temperatures under the effect of heating sink is investigated. The mathematical model obtained by Mohammadein [6] is extended and solved analytically by using similarity parameters method; which used by Mohammadein [5] between two finite boundaries. Under the initial and final boundaries of growing vapour bubbles, thermal relaxation time is derived in terms of void fraction and affected by initial superheating and thermal diffusivity. Thermal relaxation time under the effect of heating sink performs a lower values than the previous pressure and thermal relaxation times obtained by authors [4,5]. Thermal relaxation time performs a good agreement with Moby Dick experiment [2].

Keywords: Thermal relaxation time; Two-phase flow; Heating sink; Moby Dick experiment; Void fraction.

1 Introduction

The heat transfer between superheated liquid and the growing vapour bubble has been widely discussed by many authors [2,4,5,7,8,9,10,12,13]. The initial growth of the bubble nucleus depends strongly on the interfacial mechanical interactions like acceleration, pressure forces and surface tension forces. During this stage thermal phenomena are negligible. Therefore, this stage is called isothermal. As the nucleus radius increases, the bubble growth becomes mostly dependent on supply of the heat that is consumed to vaporize the liquid on the bubble surface. During this stage the rate of expansion of the bubble is much lower than during the isothermal stage. The bubble and the liquid surrounding it can be assumed to make up an isobaric system. This stage of the bubble growth is called isobaric. It is worth noticing that duration of isothermal stage is very short, compared to the isobaric stage.

Scriven [9] determined the field of temperature around the growing vapour bubbles in an infinite superheated liquid. One of the models including the phenomenon of non-equilibrium of two-phase flow under the effect of source and sink is the homogeneous thermal relaxation model. Within this framework, it is assumed

that vapour and liquid phases are moving with equal velocities, and the structure of the flow is homogeneous mixture of the two phases. Temperature distribution and thermal relaxation time between two-phase flow are obtained by many authors [2,4,5]. The heat equation of Scriven [9] under the effect of heating sink is extended to estimate temperature distribution surrounded a growing of vapour bubbles. The growth of bubble radius in terms of some physical parameters is obtained by Mohammadein and Elgammal [6] has the form

$$R(t) = \sqrt{R_0^2 + 2C(t - t_0)}. \quad (1)$$

On the basis of Appendix, the constant C can be written in the form

$$C = \left(\frac{a_l J_a}{\left(1 + \frac{bF}{3}\right) \left(1 - \varphi_0^{\frac{1}{3}}\right) \left[1 + \frac{(C_{pL} - C_{pV})\Delta\theta_0}{L}\right]} \right) \quad (2)$$

Where $b = -1$ for heating sink

In this paper, the thermal relaxation time θ_T is derived in terms of void fraction φ_0 for a mixture of superheated liquid and vapour under the effect of heating sink. The

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Mohammadein model [5] is extended to describe the current problem. Thermal relaxation time θ_T is compared with previous theoretical thermal and pressure relaxation times and with that obtained from Moby Dick experiment [2].

2 Relaxation time

Any thermodynamic system is described by means of certain thermodynamic parameters such as pressure, temperature, density and others characterizing the state of the system. Among many states of the system, the state of equilibrium is a distinguished one. In this state all thermodynamic parameters have well-defined values, being constant in space and time as long as the system is isolated from its surroundings. Any action of external thermodynamic forces on the system in equilibrium leads to a non-equilibrium state. The system regains its thermodynamic equilibrium spontaneously. However, as a result of interaction with the surroundings its physical parameters may obtain different values. The transition of the thermodynamic system from non-equilibrium to equilibrium is called *relaxation*. The time during which the transition takes place is called *relaxation time*. It is assumed that the relaxation time is equal to the time in which a thermodynamic parameter changes from its initial value to a value e times lower. This definition comes from the dynamic analysis of phenomena described by a quantity $Y(t)$; which is decreasing exponentially with time

$$Y(t) = Y(0) e^{-\left(\frac{t}{\theta}\right)}. \quad (3)$$

The relaxation time θ appears here as a time constant, setting the rate of decay of the quantity $Y(t)$. The state of the thermodynamic system is usually described by a function of chosen thermodynamic parameters. The relaxation time corresponding to different parameters may be different. The largest value is considered to be the relaxation time of the entire thermodynamic system. In two-phase flow the highest value belongs to the relaxation time related to evaporation or condensation. The relaxation time due to the growth of the vapour bubble of initial radius R_0 has been determined from time variations of the average temperature T_l of the superheated liquid surrounding the growing vapour bubbles. To represent the bubble surroundings a sphere of outer radius $R_m > R_0$ has been chosen. The average temperature in this domain is a function of time defined as follows

$$\bar{T}(t_i) = \frac{1}{V_l} \int_{R(t_i)}^{R_m} (4\pi r^2) T(r, t_i) dr, \quad (4)$$

where

$$V_l = \frac{4}{3}\pi (R_m^3 - R^3(t_i));$$

is the instantaneous volume of the liquid. For a fixed value of R_m , the function $T(t)$ may be calculated between instants

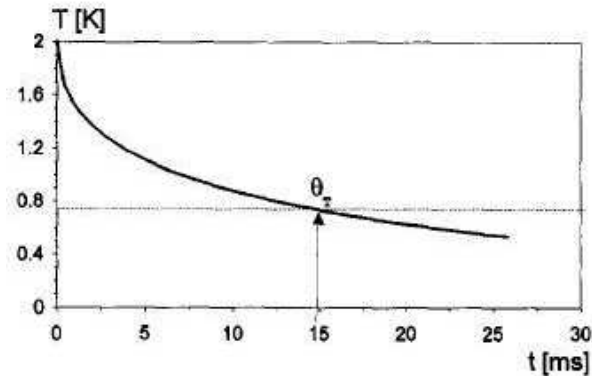


Fig. 1: Determination of the relaxation time from time evolution of the average liquid superheating in the surroundings of growing vapour bubbles.

$t_0 = 0$ and t_m , the final instant time of t_i determined from the relation

$R(t_m) = R_m$. The total drop of the average temperature of the liquid is equal to its initial superheating ΔT_0 . According to the definition (1), the relaxation time θ_T related to the temperature of the liquid is equal to the time during which the average temperature decreases to the following value

$$\bar{T}(\theta_T) = T_s + \Delta T_0 e^{-1} \approx T_s + 0.368 \Delta T_0. \quad (5)$$

Let the average superheating of the liquid $\Delta T = T_l - T_s$ be introduced. In Fig. 1, the evaluation of the relaxation time based on time evolution of the average superheating ΔT is presented. It has been assumed that the initial value of void fraction $\phi_0 = 0.01$, initial superheating of the liquid $\Delta T_0 = 2.0 \text{ K}$ and the initial bubble radius $R_0 = 0.1 \text{ mm}$.

The relaxation time determined from growth of a single vapour bubble can describe the heat and mass exchange in the entire bubble flow if the flow is homogeneous, that is when the distance between all neighboring bubbles in a control volume is the same and all bubbles have the same radius.

The derivation of pressure relaxation time between two-phase bubbly flow in a pure mixture of vapour and superheated liquid is obtained by Mohammadein and Omran [4] in the form

$$\theta_p = \frac{2\sigma^2 e^2}{C_0^2 \Delta P_0^2} + \frac{\rho C_0^2 e}{8\Delta P_0} + \frac{2\sigma e}{C_0 \Delta P_0} \sqrt{\frac{\sigma^2 e^2}{C_0^2 \Delta P_0^2} + \frac{\rho C_0^2 e}{8\Delta P_0}}, \quad (6)$$

where

$$C_0 = \sqrt{\frac{12}{\pi}} J_a (a_l)^{\frac{1}{2}} \left(1 - \frac{\phi_0^{\frac{1-n}{3}}}{B} \right), \quad (7)$$

$$B = 1 + \frac{3\rho_l R_0 \dot{R}_0^2}{4\sigma}. \quad (8)$$

The above equation (6) represents the formula of pressure relaxation time in terms of void fraction and some physical parameters.

3 Analysis

The non-equilibrium of two-phase temperatures introduces some relaxation phenomena to reach the equilibrium state. Thermal relaxation time is obtained as constant time as in the following relaxation equation

$$\frac{d\bar{T}(t)}{dt} = -\frac{\bar{T}(t) - T_s}{\theta_T}, \quad (9)$$

with the initial and boundary conditions

$$\text{At } t = t_0, T(r, t_0) = T_0, \quad (10)$$

$$T(R_m, t_i) = T_0. \quad (11)$$

The temperature distribution $T(r, t)$ between two-phase mixture under the effect of heating sink [6] is the basis for estimation of thermal relaxation time θ_T , then by integrating equation (9) w. r. to t , we have

$$\bar{T}(t) = T_s + (T_0 - T_s) \exp\left(-\frac{t_0 - t}{\theta_T}\right), \quad (12)$$

and at $t = \theta_T$, then

$$\bar{T}(\theta_T) = T_s + (T_0 - T_s) \exp\left(-\frac{t_0}{\theta_T} - 1\right). \quad (13)$$

On the basis of physical rule

$$\bar{T}(t_i) = \frac{3}{R_m^3 - R^3(t_i)} \int_{R(t_i)}^{R_m} r^2 T(r, t_i) dr. \quad (14)$$

Equation (14) can be rewritten in the form

$$\bar{T}(t_i) = \frac{3}{R_m^3 - R^3(t_i)} \times \left[R_m^3 T_0 - R^3(t_i) T\{R(t_i), t_i\} \int_{R(t_i)}^{R_m} r^3 \frac{dT(r, t_i)}{dr} dr \right]. \quad (15)$$

At $t = \theta_T$

$$\bar{T}(\theta_T) = \frac{1}{R_m^3 - R^3(\theta_T)} \times \left[R_m^3 T_0 - R^3(\theta_T) T\{R(\theta_T), \theta_T\} \int_{R(\theta_T)}^{R_m} r^3 \frac{dT(r, \theta_T)}{dr} dr \right], \quad (16)$$

where

$$\frac{dT(r, \theta_T)}{dr} = \frac{\partial T(r, \theta_T)}{\partial r}. \quad (17)$$

Substituting from Eq. (16) into Eq.(13), then

$$T_s + (T_0 - T_s) \exp\left(-\frac{t_0}{\theta_T} - 1\right) = \frac{1}{R_m^3 - R^3(\theta_T)} \times \left[R_m^3 T_0 - R^3(\theta_T) T\{R(\theta_T), \theta_T\} \int_{R(\theta_T)}^{R_m} r^3 \frac{\partial T(r, \theta_T)}{\partial r} dr \right], \quad (18)$$

where

$$\int_{R(\theta_T)}^{R_m} r^3 \frac{\partial T(r, \theta_T)}{\partial r} dr = -\frac{CM}{a_l} R^{(1+b)}(t_i) \times \int_{R(\theta_T)}^{R_m} \frac{1}{r^{(b-1)}} \exp\left\{-\frac{C}{a_l} \left(\frac{r^2}{2R^2(t_i)} + \frac{\varepsilon R(t_i)}{r} - \left(\varepsilon + \frac{1}{2}\right)\right)\right\} dr, \quad (19)$$

Substituting from integral (19) into Eq.(18), and by using Eq.(1) then

$$\sqrt{2C(\theta_T - t_0) + R_0^2} = R_0 \phi_0^{-\frac{1}{3}} \left\{ 1 + \frac{M}{\Delta \theta_0 [1 - \exp(-\frac{t_0}{\theta_T} - 1)]} \right\}^{-\frac{1}{3}}. \quad (20)$$

For $t_0 \ll \theta_T$

$$\theta_T = \frac{R_0^2}{2C} \left(\left[\phi_0 \left\{ 1 + \frac{M}{\Delta \theta_0 [1 - \exp(-1)]} \right\} \right]^{-\frac{2}{3}} - 1 \right), \quad (21)$$

where

$$M = \frac{\Delta \theta_0}{J_a} \left[1 + \frac{(C_{PL} - C_{PV}) \Delta \theta_0}{L} \right], \quad (22)$$

By substituting from Eq. (22) in Eq.(21), then

$$\theta_T = \frac{R_0^2}{2C} \left(\left[\phi_0 \left\{ 1 + \frac{1 + \frac{(C_{PL} - C_{PV}) \Delta \theta_0}{L}}{J_a [1 - \exp(-1)]} \right\} \right]^{-\frac{2}{3}} - 1 \right). \quad (23)$$

The above equation (23) represents thermal relaxation time θ_T in terms of void fraction ϕ_0 and other physical parameters. The thermal relaxation time obtained by Mohammadein [5] is considered as a special case of the present relaxation time when $b=0$.

4 Discussion of results

The system (9-11) is solved analytically in terms of average temperature. The temperature distribution surrounded a growing vapour bubbles is obtained (see ref. [6]). Thermal relaxation time θ_T in terms of void fraction and Jacob number is obtained by relation (23). The values of physical parameters considered in this study is taken from the ref. [10] for a superheated steam water as in the following Table :

The thermal relaxation time in terms of void fraction for two different values of thermal diffusivity under effect of heating sink is shown in Fig.2. It is observed that, thermal relaxation time θ_T is decreasing with the increasing of thermal diffusivity values. The thermal

Table 1: title

	Value		Value
a_l	$16.9 \times 10^{-8} \text{ m}^2 \cdot \text{s}^{-1}$	L	$419100 (\text{W} / \text{m}^2) / (\text{K}^0 / \text{m})$
ρ_l	958.3 Kg m^{-3}	λ	$0.679 \text{ W} / (\text{m K}^0)$
ρ_v	0.597 Kg m^{-3}	R_0	0.0001 m
C_{pl}	$4220 \text{ J} / (\text{kg K}^0)$	R_m	0.001 m
C_{pv}	$2030 \text{ J} / (\text{kg K}^0)$		

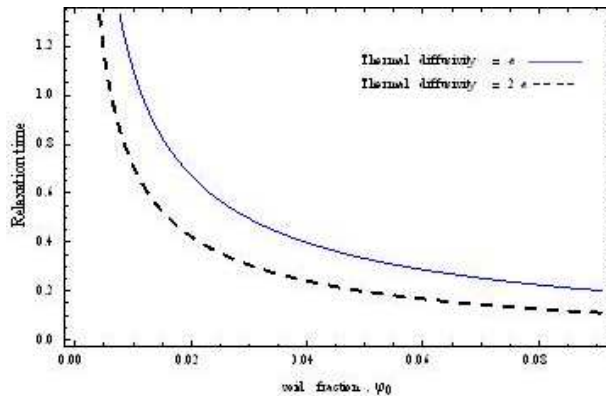


Fig. 2: Thermal relaxation time in terms of void fraction for two different values of thermal diffusivity under the effect of heating sink.

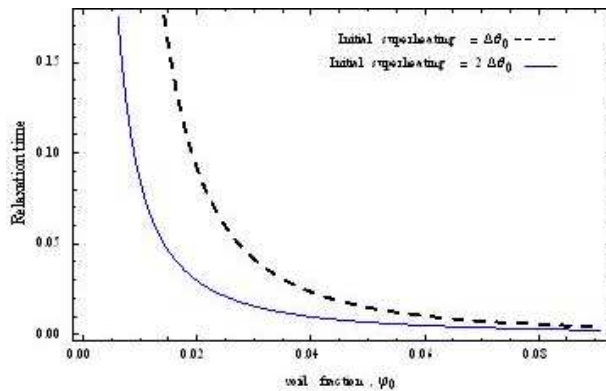


Fig. 3: Thermal relaxation time in terms of void fraction for two different values of initial superheating under the effect of heating sink.

relaxation time in terms of void fraction for two different values of superheating is shown in Fig.3. It is observed that, thermal relaxation time θ_T is decreasing with the increasing of superheating values.

The comparison of thermal relaxation time θ_T in terms of void fraction under the effect of heating sink with Mohammadein [5] is shown in Fig.4. It is observed that,

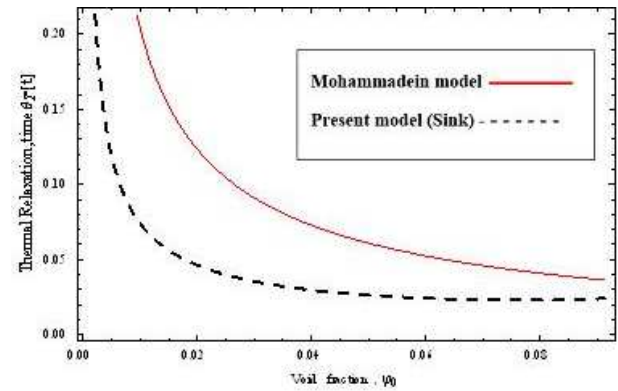


Fig. 4: The comparison of thermal relaxation time in terms of void fraction under effect of heating sink with Mohammadein model[5].

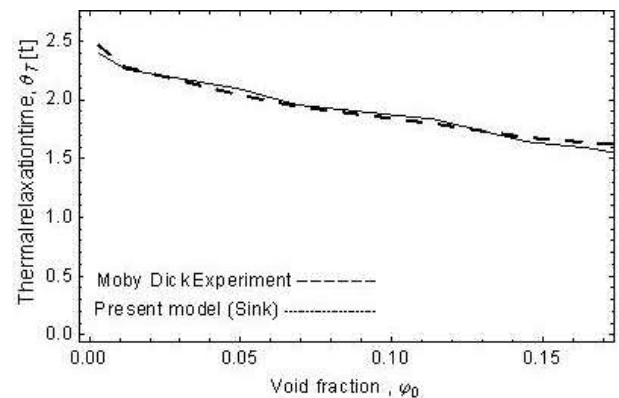


Fig. 5: The comparison of thermal relaxation time in terms of void fraction under the effect of heating sink with Moby Dick experiment at $\Delta \theta_0 = 2\text{K}$.

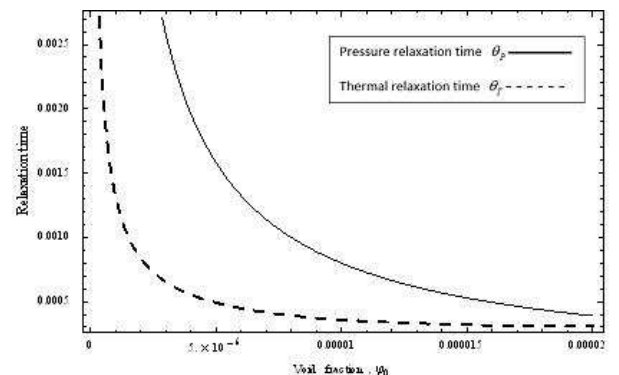


Fig. 6: The comparison of thermal relaxation time in terms of void fraction under effect of heating sink with Pressure relaxation time (Mohammadein and Omran model [4]) at $\Delta \theta_0 = 2\text{K}$.

thermal relation time θ_T performs lower values than Mohammadein [5].

The thermal relaxation time in terms of void fraction under effect of heating sink is compared with Moby Dick experiment as shown in Fig.5. It is observed that, thermal relaxation time θ_T coincide with experiment for some values of physical parameters. The thermal relaxation time is compared with pressure relaxation time θ_p (Mohammadein and Omran model [4]) as shown in Fig. 6. It observed that, thermal relaxation time θ_T in terms of void fraction performs lower values than pressure relaxation time θ_p .

5 Conclusion

The temperature distribution surrounded a growing vapour bubbles is obtained by Mohammadein et al [6] to calculate average temperature (16). The system (9-11 and 12) is solved analytically to find thermal relaxation time θ_T (23). The thermal relaxation time under the effect of heating sink is obtained in terms of initial void fraction ϕ_0 thermal diffusivity, and superheating. The discussion of results and figures concluded the following remarks:

1. Thermal relaxation time in terms of void fraction ϕ_0 is proportional inversely with void fraction, superheating and thermal diffusivity values.
2. Thermal relaxation time performs lower values than thermal relaxation time obtained by Mohammadein model [5] and pressure relaxation time θ_p
3. The relaxation time θ_T under the effect of heating sink coincides with thermal relaxation time obtained by Moby Dick experiment [2] for some values of physical parameters.
4. In the current study, thermal and pressure relaxation times and experiment have the same order of magnitudes.
5. Thermal relaxation time is very sensitive to the effect of heating sink for different values of physical parameters.
6. Thermal relaxation time obtained by Mohammadein [5] is considered as a special case of the thermal relaxation time in this study when b equal to zero.

A Calculation of constant C in Eq.(2)

The solution of heat diffusion as given by author [6] under the effect of heating sink has the following form

$$T(r, t_i) - T_o = -\frac{CM}{a} R^{(1+b)}(t_i) \times \int_r^{R_m} \frac{1}{r^{(2+b)}} \exp \left\{ -\frac{C}{a} \left(\frac{r^2}{2R^2(t_i)} + \frac{\varepsilon R(t_i)}{r} - \left(\varepsilon + \frac{1}{2} \right) \right) \right\} dr, \quad (i)$$

where at $r = R_m$, then $T(R_m, t_i) = T_o$,

and at $r = R(t_i)$, let $z = 1 - \frac{R(t_i)}{r}$, Eq. (i) becomes

$$T(R(t_i), t_i) - T_o = -\frac{CM}{a} R^{(1+b)}(t_i) \times \int_{R(t_i)}^{R_m} \frac{1}{r^{(2+b)}} \exp \left\{ -\frac{C}{a} \left(\frac{r^2}{2R^2(t_i)} + \frac{\varepsilon R(t_i)}{r} - \left(\varepsilon + \frac{1}{2} \right) \right) \right\} dr, \quad (ii)$$

or

$$T(R(t_i), t_i) - T_o = -\frac{CM_1}{a} \times \int_0^{1 - \frac{R(t_i)}{R_m}} (1-z)^b \exp \left\{ -\frac{3C}{2a} \left(z + \frac{F}{3} \right)^2 \right\} dz, \quad (iii)$$

Where

$$M_1 = M \exp \left\{ \frac{CF^2}{6a} \right\}. \quad (iv)$$

$$\text{Let } h = \sqrt{\frac{3C}{2a}} \left(z + \frac{F}{3} \right), \quad (v)$$

$$\text{then } (1-z)^b = 1 + \frac{bF}{3} - b\sqrt{\frac{2a}{3C}}h. \quad (vi)$$

Substituting from relations (v) and (vi) into equation (iii) then

$$T(R(t_i), t_i) - T_o = -\frac{CM_1}{a} \sqrt{\frac{2a}{3C}} \times \int_{h_1}^{h_2} \left(1 + \frac{bF}{3} - b\sqrt{\frac{3C}{2a}}h \right) e^{(-h^2)} dh. \quad (vii)$$

The following mathematical rules are satisfied

$$\int_{h_1}^{h_2} e^{(-h^2)} dh = \int_0^{h_2} e^{(-h^2)} dh - \int_0^{h_1} e^{(-h^2)} dh, \quad (viii)$$

$$= \frac{\sqrt{\pi}}{2} [\text{erf}(h_2) - \text{erf}(h_1)]$$

and

$$\int_{h_1}^{h_2} h e^{(-h^2)} dh = \frac{1}{2} (e^{-h_1^2} - e^{-h_2^2}). \quad (ix)$$

On the basis of mathematical rules (viii) and (ix), the integral is evaluated and Eq.(vii) becomes

$$T(R(t_i), t_i) - T_o = -\frac{CM_1}{a} \sqrt{\frac{2a}{3C}} \times \left\{ \left(1 + \frac{bF}{3} \right) \frac{\sqrt{\pi}}{2} \left[\text{erf} \left(\sqrt{\frac{3C}{2a}} \left(1 - \frac{R(t_i)}{R_m} \right) + F \sqrt{\frac{C}{6a}} \right) \right] - \text{erf} \left(F \sqrt{\frac{C}{6a}} \right) \right\} + \frac{bCM_1}{2a} \sqrt{\frac{2a}{3C}} \left(e^{-\left(F \sqrt{\frac{C}{6a}} \right)^2} - e^{-\left(\sqrt{\frac{3C}{2a}} \left(1 - \frac{R(t_i)}{R_m} \right) + F \sqrt{\frac{C}{6a}} \right)^2} \right) \quad (x)$$

When $t_i = t_o$, $R(t_o) = R_o$ then $T(R_o, t_o) = T_o$, then Eq. (x) becomes

$$\Delta \theta_o = \frac{CM_1}{a} \sqrt{\frac{2a}{3C}} \times \left\{ \left(1 + \frac{bF}{3} \right) \frac{\sqrt{\pi}}{2} \left[\text{erf} \left(\sqrt{\frac{3C}{2a}} \left(1 - \frac{R_o}{R_m} \right) + F \sqrt{\frac{C}{6a}} \right) \right] - \text{erf} \left(F \sqrt{\frac{C}{6a}} \right) \right\} + \frac{bCM_1}{2a} \sqrt{\frac{2a}{3C}} \left(e^{-\left(F \sqrt{\frac{C}{6a}} \right)^2} - e^{-\left(\sqrt{\frac{3C}{2a}} \left(1 - \frac{R_o}{R_m} \right) + F \sqrt{\frac{C}{6a}} \right)^2} \right) \quad (xi)$$

The following mathematical rule is satisfied

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \sum_{n=1}^{\infty} \frac{(-1)^n z^{2n+1}}{n!(2n+1)} = \frac{2}{\sqrt{\pi}} \left(z - \frac{z^3}{3} + \frac{z^5}{10} - \frac{z^7}{42} + \frac{z^9}{216} - \dots \right), \quad (\text{xii})$$

where

$$\operatorname{erf} \left(\sqrt{\frac{3C}{2a}} \left(1 - \frac{R_0}{R_m} \right) + F \sqrt{\frac{C}{6a}} \right) - \operatorname{erf} \left(F \sqrt{\frac{C}{6a}} \right) = \frac{2}{\sqrt{\pi}} \sqrt{\frac{3C}{2a}} \left(1 - \frac{R_0}{R_m} \right) \quad (\text{xiii})$$

When $F \rightarrow 1$, $F^2 \rightarrow 0$, and $a_l \ll 1$, Eq.(xi) becomes

$$\frac{J_a}{\left[1 + \frac{(C_{PL} - C_{PV}) \Delta \theta_0}{L} \right]} = \frac{C}{a} \left(1 + \frac{bF}{3} \right) \left(1 - \frac{R_0}{R_m} \right).$$

or

$$C = \left(\frac{a_l J_a}{\left(1 + \frac{bF}{3} \right) \left(1 - \varphi_0^{\frac{1}{3}} \right) \left[1 + \frac{(C_{PL} - C_{PV}) \Delta \theta_0}{L} \right]} \right). \quad (\text{xiv})$$

B List of Symbols:

Nomenclature:

a	Thermal diffusivity of the liquid ($m^2.s^{-1}$)
C	Constant defined by equation (16)
J_a	$= \frac{\rho_l C}{\rho_v L} \Delta \theta_0$, dimensionless Jakob number for pure liquids
k	Thermal conductivity of the liquid ($J.(s.m.K)^{-1}$)
L	Latent heat of vapourization ($J.kg^{-1}$)
M	constant defined by equation (15)
r	The distance from the origin of the bubble (m)
R_0	Initial bubble wall radius (m)
R	Instantaneous bubble wall radius (m)
t	Time elapsed (s)
T	Absolute temperature of liquid [K]
T_0	Initial absolute temperature of liquid [K]
ε	$= 1 - \frac{\rho_v}{\rho_l}$, constant, taking effect of radial convection on bubble growth
$\Delta \theta_0$	Initial uniform liquid superheating above saturation temperature or liquid superheating at great distance from bubble or superheating of bulk liquid [deg C]
θ_T	Thermal relaxation time [s]
θ_p	Pressure relaxation time [s]
ρ_l	Liquid density [kg/m^3]
ρ_v	Saturated vapour density [kg/m^3]

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