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A New Hybrid Root-Finding Algorithm to Solve Transcendental Equations Using Arcsine Function

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Abstract: The objective of this paper is to propose a new hybrid root finding algorithms for solving non-linear equations (NLEs) or transcendental equations (TEs). The proposed algorithm is based on the trigonometrical algorithm using arcsine function to find a root. Several numerical examples are presented to illustrate the proposed algorithms, and comparisons are presented with other existing methods to show efficiency and accuracy. Implementation of the proposed algorithms is presented in a mathematical software tool Maple.

Keywords: Root-finding algorithm, arcsine function, Transcendental equations, Maple.

1 Introduction

The applications of NLEs of the type $f(x) = 0$ arise in numerous branches of pure sciences as well as applied sciences, such as computer science, chemical engineering, physics, etc. Getting the root of transcendental equations is of great importance. In the current scenario, many academicians have focused to solve NLEs numerically/ mathematically as well as logically/ analytically. In the research work of scientists, there are quite more new iterative methods/ algorithms and many hybrid methods existing. These hybrid algorithms are created with help of different techniques, see, for example, [1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 24, 25, 26, 27]. In general, the roots of NLEs or TEs cannot be stated in a particular form or cannot be calculated systematically. The root-finding methods/algorithms offer us to calculate estimations/approximations to the roots; these estimations are stated either as small separate intervals or as floating-point numbers. The concept of creating a hybrid method combining two classic approaches is not new, but it has a long history.

2 Methodologies

In this paper, we develop a new mixed/hybrid root finding algorithm for solving TEs. This algorithm is created with the help of trigonometrical algorithm using arcsine function. Using numerical examples, we show that the proposed algorithms converge faster than the other related methods. The main idea of the proposed hybrid algorithm is based on the regula-falsi method and trigonometrical algorithm. A number of examples are presented to explain the proposed algorithm. The assessments are made to relate the calculated results using the proposed algorithm with other current algorithms/methods to show the efficiency and accuracy. Execution of the proposed algorithms is presented in Maple.

3 Main Results

In this section, we present the proposed hybrid algorithm. The regula-false method gives guarantee the existing of root, and the trigonometrical method gives faster convergent. The iterative formula used in trigonometrical method is as follows, more details about this method can be found in [14]. For $n = 0, 1, 2, \dots$

$$x_{n+1} = x_n \left(1 + \arcsin \left(\frac{-f(x_n)}{x_n f'(x_n)} \right) \right), \quad (1)$$

In regula-falsi method [20, 21], we take two initial guesses say a and b such that the product of $f(a)$ and $f(b)$ should be less than zero. The approximate root is calculated by discovering the point of intersection of the straight line combining

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the coordinates $(a, f(a))$ and $(b, f(b))$ with the x -axis. Hence the estimated root can be calculated by the formula,

$$x_r = a - \frac{f(a)(b-a)}{f(b)-f(a)} \quad (2)$$

Now, we have to choose the appropriate interval to compute the second iteration. We have the following possible cases:

- If the product of $f(a)$ and $f(x_r)$ is less than zero, then the root exists in $[a, x_r]$, and set $b = x_r$ to find the second iteration using formula (2).
- If the product of $f(a)$ and $f(x_r)$ is greater than zero, then the root exists in $[x_r, b]$, and set $a = x_r$ to find the second approximate root using (2).
- If $f(x_r)$ is zero, then the required root is x_r and the process is stopped.

3.1 Algorithm

Input: The given function, interval, where the root lies in, $[a, b]$, number of iterations (n).

Output: The approximate root (x), function value ($f(x)$).

```

i = 0
while i == n do
  i = i + 1;
   $x_{rf} = a - \frac{f(a)(b-a)}{f(b)-f(a)}$ 
   $x_{n+1} = x_n \left( 1 + \arcsin \left( \frac{-f(x_n)}{x_n f'(x_n)} \right) \right)$ 
  if  $|f(x_i)| < |f(x)|$  and  $x_i \in (a, b)$ 
    if  $f(x_i) * f(a) < 0$  then  $b = x_i$  else  $a = x_i$ 
  else if  $f(x_{rf}) * f(a) < 0$  then  $b = x_{rf}$ 
  else  $a = x_{rf}$ 
end (while)

```

3.2 Flow-diagram

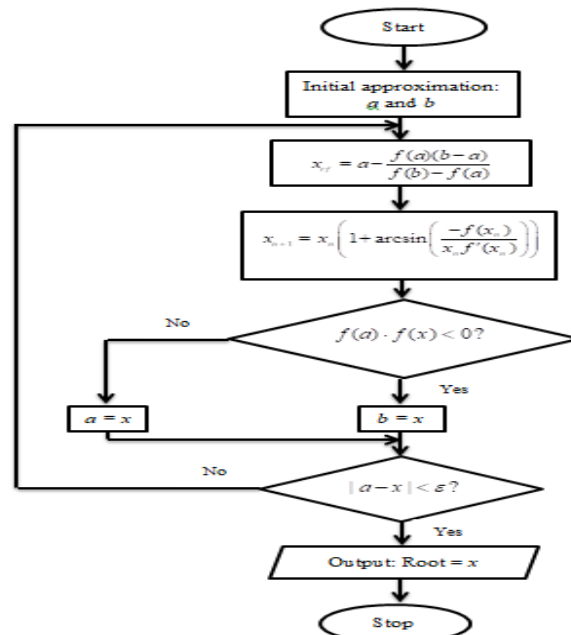


Fig. 1: Flow-chart of the algorithm

3.3 Maple Executions

```

RFARCSIN := proc (a, b, Eq, eps, n)
  local a1, b1, f, c, i, c1;
  i := 0;
  a1 := evalf(a);
  b1 := evalf(b);
  f := unapply(lhs(Eq), x);
  if f(a1) = 0 then
    return a1
  else if f(b1) = 0 then
    return b1
  else if 0 < f(a1)*f(b1) then
    error "Should be f(a)*f(b)<0"
  end if; end if; end if;
  do
    c1 := (a1*f(b1)-b1*f(a1))/(f(b1)-f(a1));
    c := c1*(1+arcsin(-f(c1)/(c1*(D(f))(c1))));
    i := i+1;
    if f(c) = 0 or c-a1 < eps or i = n then
      return c
    else if f(a1)*f(c) < 0 then
      b1 := c
    else a1 := c
    end if; end if;
    printf("Iteration %g : x = %g \n", i, c)
  end do
end proc

```

4 Discussions with Numerical Examples

Example 1: Consider an equation $e^x - 3x - 2 = 0$ with initial approximations $a = 2$ and $b = 3$, to illustrate the proposed algorithm, as follows:

Iteration 1: $x_f = 2.063006766$, $x_1 = 2.128613403$ and $f(x_1) = 0.017366659$.

Iteration 2: $x_f = 2.125058497$, $x_2 = 2.125391285$ and $f(x_2) = 4.63727E-07$.

Iteration 3: $x_f = 2.12539119$, $x_3 = 2.125391199$ and $f(x_3) = 0$.

We obtain the required root in three iterations and the required root is 2.125391199.

Example 2: In this example we present the comparisons of the calculations with existing root finding methods. Consider an equation $\sin(x) - x^2 = 0$ with initial guesses $a = 0.5$ and $b = 1$. The exact root of the given equation is 0.8767262154. Using various numerical existing algorithms, we compute the root of this equation given in Table 1 up to correct ten decimal places. In the table, BM, NRF, RFM, Steffensen, Halley, EXP, TRIG and PA indicate bisection method, Newton-Raphson method, regula-falsi method, Steffensen's method, Helley's method, exponential method [13],

trigonometric method [14] and proposed algorithm respectively.

Table 1: Number iterations required to compute a root with various methods

BM	RFM	NRM	Halley	Steffensen	EXP	TRIG	PA
30	12	5	4	4	4	5	3

Example 3: Recall the equation in Example 1, $e^x - 3x - 2 = 0$ with $a = 2$ and $b = 3$ for sample computations using maple, as follows.

```
> RFMARCSIN (2, 3, exp(x)-3*x-2 = 0, 10^(-10), 20);
```

Iteration 1 : x = 2.12861

Iteration 2 : x = 2.12539

Iteration 3 : x = 2.12539

Iteration 4 : x = 2.12539

2.125391198

5 Conclusions

In this paper, we planned a new hybrid root finding algorithms for solving the given NLEs or TEs. The proposed algorithm is mainly based on the arcsine function trigonometrical algorithm. To illustrate the proposed algorithm, we presented several numerical examples and also evaluations with existing methods are presented to show the efficiency and accuracy of the proposed method. Maple implementation is presented with sample computations.

6 Recommendations

This research recommends to compute a real root of a given TEs with fast converging rate. The idea of main result can be used to create new hybrid root-finding algorithms in the relevant research.

Conflicts of Interest Statement

The authors certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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