Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.12785/amis/090352

DOA Estimation Based on Compressive Sensing Method in Micro Underwater Location Platform

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Received: 22 Aug. 2014, Revised: 22 Nov. 2014, Accepted: 23 Nov. 2014
Published online: 1 May 2015

Abstract: Based on a space sparse representation of underwater target, and combining compressed sensing sparse reconstruction principle, we present a new high-accuracy DOA estimation method. The proposed method can be used in narrowband and wideband scenarios. We demonstrate the effectiveness of the proposed method on simulated data by plots of spatial spectra, computation time and root mean square error. We observe that our approach has number of advantages over other DOA estimation methods including decreasing the calculated quantity, and the use of feasibility and superiority in the micro underwater location platform, as well as not requiring high demand of array.

Keywords: sparse representation, compressed sensing, DOA estimation, underwater acoustic locating

1 Introduction

With the developing of ocean exploration and the strengthening of national defense construction, the study of underwater acoustic target location method has been an active research area. The underwater acoustic target DOA estimation is one of the important methods of the underwater acoustic location. Moreover, the research for underwater acoustic target location method which is based on micro underwater location platform (such as UUV) has been aroused great interest. This micro platform must have good mobility and accuracy in practical application. Considing the load capacity of the micro platform, the size of array aperture can not be too big, and the data acquisition and processing can not be too much. Therefore, the traditional high resolution DOA technology can not be completely applied to the micro underwater location platform, such as beam-forming [3], Capons method [5] and subspace-based methods such as MUSIC [4]. Capons method and MUSIC are able to resolve sources within a Rayleigh cell (i.e., achieve super-resolution), provided that the SNR is moderately high, the sources are not strongly correlated, and the number of snapshots is sufficient. There are some disadvantages when the above-mentioned algorithms are used in micro underwater location platform and the underwater acoustic target DOA estimation. Therefore, the research of underwater acoustic target location based on the micro underwater location platform is very necessary and has great application prospect.

In recent years, compressed sensing theory has been an active research area, playing a fundamental role in many applications involving visual electronics, medical imaging devices, radio receiver, radar imaging and so on. D. Malioutov [7] first proposed spatial sparsity for DOA estimation, which showed that the source localization problem can be cast as a sparse representations recovery problem in a redundant dictionary using the $\ell_1 - SVD$ [7] method. Recently, spatial sparsity was linked to the theoretical results of the compressed sensing (CS) [8] framework, utilizing a spatial CS approach for DOA estimation. A.C Gurbuz also achieved the DOA estimation based on the CS in time domain. W. Ying proposed space compressive sampling array for DOA estimation [8, 9], it can reduce the array dimension while still maintaining high accuracy [11, 12]. At present, the above-mentioned research of DOA estimation based on CS is not involved in the application of sonar, which also provides a new approach for micro underwater location platform (such as UUV). Therefore, based on the above research results and combined with the characteristics of acoustic location, we can realize a further study on the underwater acoustic target DOA estimation problem, which is based on micro underwater location platform.

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Then we propose a new high-accuracy DOA estimation method of underwater acoustic target, which can be apply to the micro underwater location platform. This method is compared with the existing high-accuracy DOA estimation method from different facets (such as the number of array element N, array element spacing d, the number of snapshots and the time consumed, etc) by simulation, and obtains some useful results. This method will play an active effect to the development of underwater acoustic target location. The paper is organized as follows. Firstly, the compressive sensing theory is simply introduced. Secondly, the underwater acoustic target DOA estimation method based on CS will be formulated. Then, the performance of the algorithms is experimentally analyzed. And some performance analysis and comparison will be drawn. Finally, we make some conclusions for this paper.

2 Compressed Sensing Theory

The main goal of the CS theory is to recover the signal x from the measurements y, in the presence of the white zero-mean Gaussian noise.

The simplest version of the sparse representation problem without noise is to find a sparse $x \in \mathbb{C}^N$, given $y \in \mathbb{C}^M$, which are related by $y = \Phi x$, with $M < N$. In practice, a noiseless measurement model is rarely appropriate; therefore, noise [10, 11] must be introduced. A sparse representation problem with additive Gaussian noise takes the following from:

$$y = \Phi x + n. \tag{1}$$

where Φ is an $M \times N$ sensing matrix giving us information about x. The assumption of sparsity of is crucial since the problem is ill-posed with it. The solution to the ill-posed problem is only possible when some properties of the signal x can be sparse or compressible in some sparsity basis providing the following representation.

$$x = \psi d. \tag{2}$$

where $\psi \in R^{N\times N}$is a sparsity basis, and d is a vector of the size $N \times 1$ with contains only $J \ll N$ nonzero elements. Then consider the signal x is $J-sparse$.

But the sensing matrix Φ obey the RIP [16] can use the CS theory recover the signal x form the measurements y at high accuracy. So it is important to choose a suitable sensing matrix. And the Gaussian and Bernoulli is often used.

For the problem of signal reconstruction, if the signal vector x is sparse or compressible, we can get the solutions for (1). An ideal measure of sparsity is the count of nonzero entries denoted by $\| x \|_0^0$,which we also call the $\ell_0 - norm$. Hence, mathematically, we need find the solution to min $\| x \|_0^0$ subject to $y = \Phi x$. Many approximations have been devised over the years, including greedy approximations (matching pursuit algorithm (MP), stepwise regression, and orthogonal matching pursuit algorithm (OMP)), as well as ℓ_1 and ℓ_p relaxations, where $\| x \|_0^0$ is replaced by $\| x \|_1$, and $\| x \|_p^p$, for $p < 1$. For the latter two, it has been shown recently that if x is sparse enough with respect to Φ, then these approximations in fact lead to exact solutions for precise definitions of these notions).These results are practically very significant, since the ℓ_1 relaxation min $\| x \|_1$ subject to belong to $y = \Phi x + n$ belong to a convex optimization problem, and the global optimum can be found for real-valued data by linear programming.

Therefore, equation (1) can be solved by the solution of convex optimization as following representation.

$$min \| x \|_{\ell_1} \, s.t \| Y - \Phi X \|_{\ell_2} \ll \varepsilon. \tag{3}$$

where ε is a parameter specifying how much noise we wish to allow. The $\ell_2 - term$ forces the residual $Y - \Phi X$ to be small, whereas the $\ell_2 - term$ enforces sparsity of the representation.

3 Underwater Acoustic Target DOA Estimation Method Base On CS

3.1 Space sparse representation of underwater acoustic target signal

Now, we consider a uniform linear array (ULA) of N sensors by inter-sensor spacing of d. Then J narrowband sources from the far-field, which have a known center frequency of f, impinge on the uniform linear array. The sources arrive on the array from the unknown $\theta_n (n = 1, 2, 3, 4, J)$, which we wish to estimate. The linear array response to the impinging plane waves can be expressed as:

$$a(\theta_n) = [1, e^{-j\pi \cos(\theta_n)}, ..., e^{-j\pi(N-1)\cos(\theta_n)}]. \tag{4}$$

The transposed matrix $A_J(\theta)$ can be expressed as:

$$A_J(\theta) = [a(\theta_1), a(\theta_2), ..., a(\theta_J)]. \tag{5}$$

Uniformly we discrete the underwater acoustic target signal space into $K >> J$ possible angles of arrival and construct a redundant matrix of K atoms corresponding to the array responses of the respective angles of arrival. Then the transposed matrix $A(\theta)$ can be expressed as:

$$A(\theta) = [a(\theta_1), a(\theta_2), ..., a(\theta_K)]. \tag{6}$$

where $\theta = [\theta_1, ..., \theta_K]$ is the vector of unknown source locations. According to the theory of compressed sensing, the array received signal can be expressed as:

$$x(t) = A(\theta)S(t) + n(t). \tag{7}$$

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where t is the discrete time index, and $S(t)$ is sparse vector of the size $K \times 1$, with $K \gg J$, and it only has J nonzero elements which correspond to the signal angles of arrival. And $n(t)$ is the noise vector of the size $N \times 1$. Then consider $x(t)$ is $J-sparse$, and $A(\theta)$ is a sparsity basis for array received signal $x(t)$.

3.2 Underwater acoustic target DOA model based on CS

According to the theory of compressed sensing and the above (7), the underwater acoustic target DOA estimation problem can be formulated as:

$$y(t) = \Phi x(t) = \Phi A(\theta) S(t) + \Phi n(t). \tag{8}$$

where Φ is Gaussian random measurement matrix of the size of $M \times N$. And $y(t)$ is the array output of the size of $M \times 1$. Compared with the traditional array output, this array output contains much less data. As a result, it greatly reduces the hardware complexity and the amount of data processing, which is benefit to use in the micro underwater location platform.

In order to ensure the accuracy of the estimation, it must need more snapshot samples. Then equation (8) can be reformulated into the matrix form:

$$Y = \Phi X = \Phi AS + N. \tag{9}$$

where $Y = [y(1), y(2), ..., y(L)]$, $S = [s(1), s(2), ..., s(L)]$ and $N = [n(1), n(2), ..., n(L)]$, and the L represents the number of snapshots. The sparse vectors S can be solved from (9) using the joint CS recovery algorithms such as regularized M-FOCUSS, SOMP, etc.

3.3 Underwater acoustic target DOA estimation method based on CS

In this paper, we use the convex relaxation methods of mixed norm to realize the DOA estimation of underwater acoustic target. According to the above (3) and (9), the solution of it can be expressed as:

$$min \parallel S \parallel_{\ell_1} s.t \parallel Y - \Phi AS \parallel_{\ell_2} \ll \varepsilon. \tag{10}$$

where ε is a constant and relate to the noise, and the result is better when $\varepsilon = 0.6$.We calculate the average of each row of $\hat{S}$ to get a column vector, when the S is fixed by (10), and the column vector can be expressed as following equation:

$$\hat{S} = [\sum_{j=1}^{L} S_{1j}/L \ \sum_{j=1}^{L} S_{2j}/L \ ... \ \sum_{j=1}^{L} S_{Kj}/L]. \tag{11}$$

where the $S_{ij}(i = 1,2,3,...,K; j = 1,2,3,...,L)$ correspond to the element of matrix S of $i-th$ row and $j-th$ column, and $\hat{S}$ was the sparse column vector which only contain a small number of non-zero elements. We illustrate the steps for underwater acoustic target DOA estimation method based on CS in Fig.1.

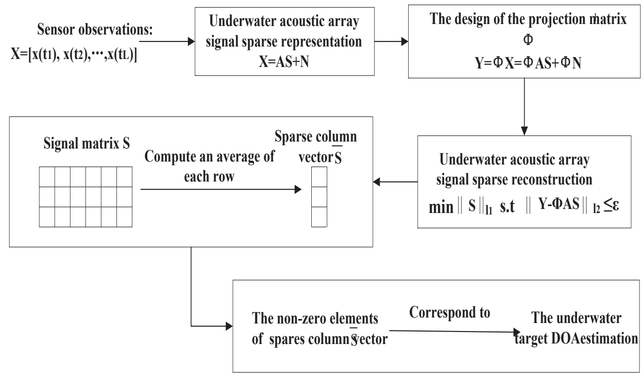

Fig. 1: Block diagram of steps for underwater acoustic target DOA estimation method based con CS.

Combined with the discussion of the section 3.1, the underwater acoustic target signal space is discredited into $K >> J$ possible angles of arrival. The location of each angle corresponds to the location of each element in the sparse vector $\hat{S}$.And the method can be shown as the Fig.2.

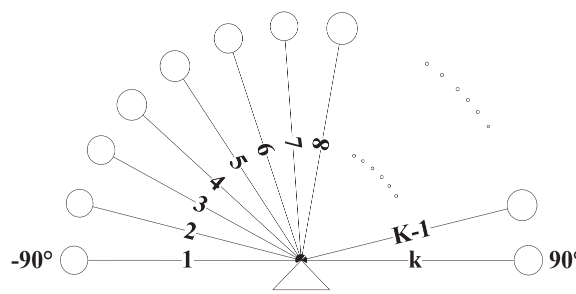

Fig. 2: The space sparsity model of underwater acoustic target.

Then, the DOA estimation of the underwater acoustic target can be achieved by locating the nonzero elements of $\hat{S}$. For example, when $K = 181$, the target space is evenly divided into 181 spatial angles from $-90°$ to $90°$, each of the elements and the corresponding to the signal vector containing 181 elements, i.e. $-90°$ corresponds to the first element in the vector matrix, $-89°$ corresponds to a vector of second elements, and so on. For example, the $20-th$ element in the matrix $\hat{S}$ is nonzero, so you can determine a target's DOA angle of $-70°$. Specific process as shown in Fig.3:

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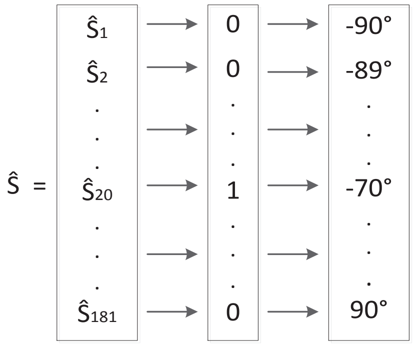

Fig. 3: The corresponding relation between signal vector and space angle.

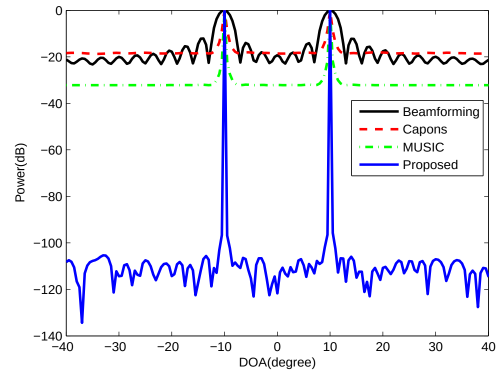

Fig. 4: Spatial spectra for beam-forming, Capons method, MUSIC, and the proposed method for two underwater acoustic targets. DOAs: -10° and 10°, $SNR = 10dB$.

4 Experimental Results

In this section, we present several experimental results for our convex relaxation underwater acoustic target localization scheme. First, we compare the spectra of our convex relaxation method to those of MUSIC [4], Capons method [5] and beam-forming [3]. Next, we analyze the performance of the proposed method by computer simulation. The performance of our method is compared with the Capons method, beam-forming, and MUSIC.

4.1 Spectra for convex relaxation of mixed-norm algorithm

We consider a uniform linear array of $N = 40$ hydrophones separated by half a wavelength of actual narrowband underwater acoustic target signals. Two narrowband underwater acoustic target signals in the far-field impinge on this array from distinct *DOAs*. The signal direction angles of these two targets are set to -10° and 10° respectively. And the frequencies of these two targets are set to $800kHz$ and $1000kHz$ respectively. The total number of snapshots is $L = 1024$, and squeezed number is $M = 8$. The additive noise is Gaussian with the $SNR = 10dB$ and the number of potential locations is $K = 721$,which is used convex relaxation of mixed-norm algorithm to recover. In Fig.4, we compare the spectrum obtained using our proposed method with those of beam-forming, Capons method and MUSIC. Beam-forming, Capons method, MUSIC and our technique are able to resolve the two sources. However, our proposed method gives the sharper peaks and showcases the best performance, then the estimation performance of Capons method and MUSIC is relatively poor. Therefore, estimation of underwater target of CS based on DOA method is effective and feasible. Then we will continue to analyze the comparison and analysis of the proposed method and several common methods.

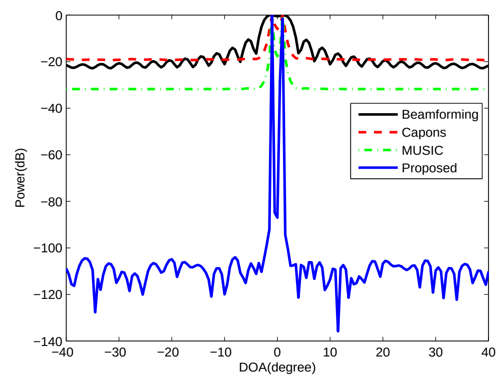

Fig. 5: Comparison of the ability to distinguish for beam-forming, Capons method, MUSIC, and the proposed method (convex relaxation of mixed-norm algorithm) *DOAs*: -1° and 1°, $SNR = 10dB$.

4.2 Analysis the performance of these algorithms

Next we analyze the performance of these algorithms in different *DOAs*, N, d and L.

4.2.1 Resolution analysis

In Fig.5, we compare the ability to distinguish the two sources using our proposed method with those of beam-forming, Capons method and MUSIC. In the plot, the SNR is $10dB$, and the sources are closely spaced (2° separation). We can find that the beam-forming algorithm cannot distinguish two targets. The performance of Capons method is also a sharp variation, the two peaks cannot be obviously distinguished. Although MUSIC

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algorithm can estimate two target direction angles, but the performance changes obviously . However, our proposed method can still accurately distinguish two targets, with a relatively high resolution and the sharper peaks.

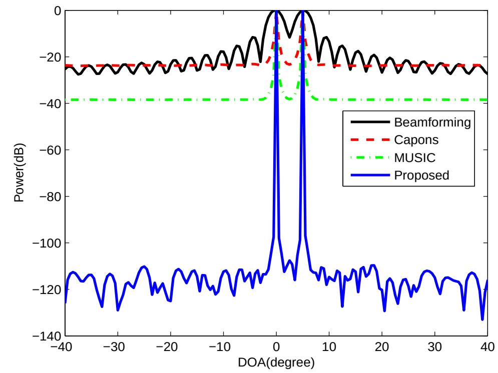
4.2.2 Analyze the performance of these algorithms in different *SNR*, *N*, *d* and *L*

Next we analyze the performance of these algorithms in different *SNR*, *N*, *d* and *L*. Fig.6 shows that the performance of different algorithms with different *SNR*. We compare the spectrum obtained using our proposed method with those of beam-forming, Capons method, and MUSIC. In the top plot, the *SNR* is $15dB$, and the sources are closely spaced (5° separation). Our technique and MUSIC are able to resolve the two sources, whereas beam-forming and Capons method merge the two peaks. In the bottom plot, we decrease the *SNR* to $0dB$, and our proposed method is still able to resolve the two sources. Therefore, the proposed method has the relatively superior robustness at high levels of noise.

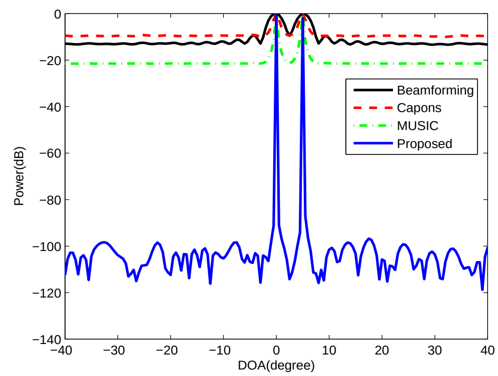
Fig.7 shows that the performance of different algorithms in different *N* and *d* respectively. Fig.7(a), (b), (c) and (d) represent four cases respectively. Compared with (a) and (b), we can discover that with the decrease of *N*, beam-forming and Capons method are not able to estimate the *DOAs*, but the proposed method shows better performance. Compared with (a) and (c), we can also conclude that beam-forming algorithm has bad effect due to the limited array aperture, when *d* is reduced to 0.2. MUSIC and Capons Method performance also become very poor. In the plot of (d), when we decrease both *N* and *d*, we can easily find that beam-forming, Capons method and MUSIC can hardly distinguish the two angles. However, our technology is still having high resolution, with two sharp spectral peaks. This is very helpful to the micro underwater location platform as it can reduce the size of the array which is used in location platform. Continue to analyze the performance of these algorithms in different *L*. As we can see from Fig.8, when the snapshot number is reduced from 1024 to 20, Capons method is almost failed to distinguish the two targets. As we know, MUSIC algorithm needs the sufficient number of snapshots to realize the estimation of angles. So the performance of this method is also very poor. However, beam-forming can estimate two target direction angles, but tow spectrum peaks are not very sharp. Obviously, the proposed method has the best performance when the number of snapshots is very few.

4.2.3 Analysis of the algorithm complexity

According to the above analysis, our proposed method only needs few snapshots, so we can draw a conclusion that the proposed method will take less time to estimate the DOA. Then we calculate the running time of these

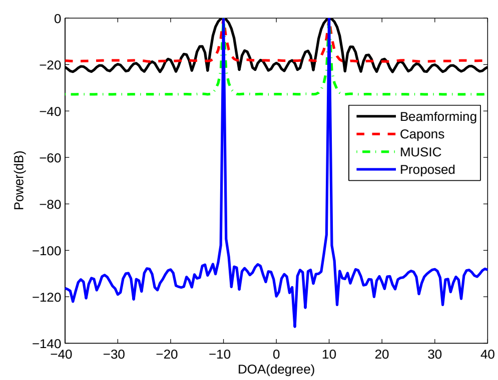


(a)



(b)

Fig. 6: (a) and (b). Spatial spectra for beam-forming, Capons method, MUSIC, and the proposed method.DOAs: 0° and 5°. Top: $SNR = 15dB$. Bottom:$SNR = 0dB$.



(a)

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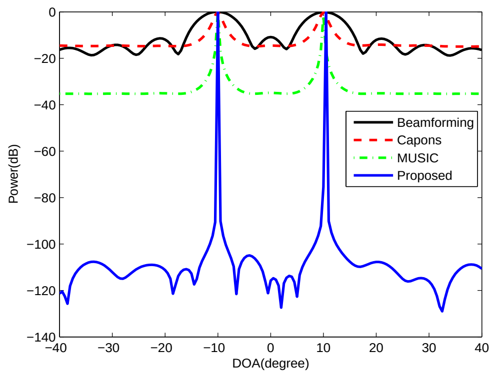

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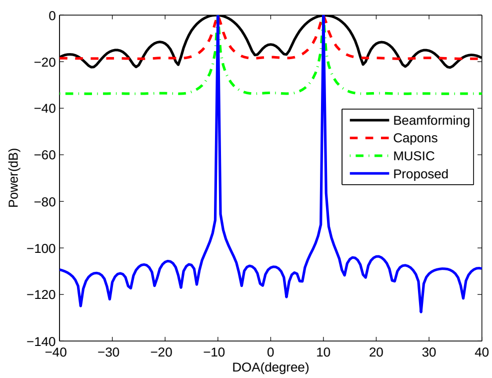
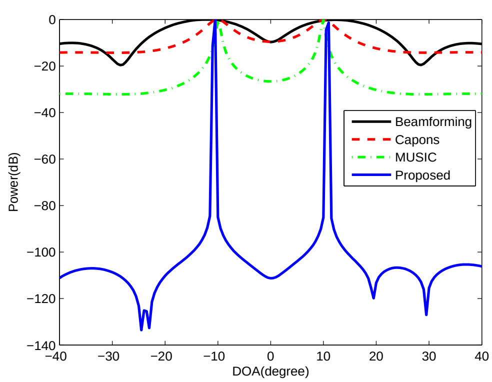
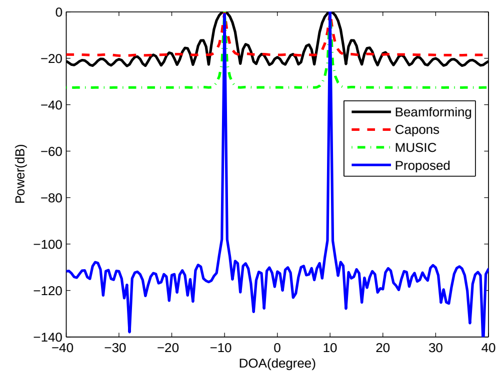


(c)

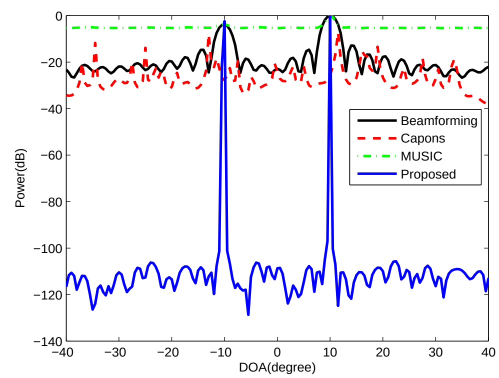


(d)

Fig. 7: The performance of these algorithms with *DOAs* of -10° and 10° ($SNR = 10dB$), $L = 1024$.(a)$N = 40$,$d = 0.5$.(b)$N = 16$,$d = 0.5$.(c)$N = 40$,$d = 0.2$.(d)$N = 16$,$d = 0.2$.



(a)



(b)

Fig. 8: The performance of these algorithms with *DOAs* of -10° and 10° ($SNR = 10dB$). (a)$L = 1024$. (b)$L = 20$.

algorithms by experiments. Through the analysis of data in the table, it is easy to draw a conclusion that proposed method takes the fewest time.

Table 1: The time of these algorithms to achieve the DOA estimation.

Algorithm	proposed	Beam-forming	Capons	MUSIC
Time/s	0.1076652	0.1144021	0.1122652	0.1250766

In summary,when changing these parameters ($DOAs$,N,d,L), the proposed method is more robust than beam-forming, Capons method and MUSIC algorithm. The proposed method has strong resolution. And the calculation of proposed method is least. These characteristics make the proposed method more suitable for micro underwater location platform of the underwater acoustic target DOA estimation.

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4.3 Wideband Underwater Acoustic Target Location

The technology and method of narrowband signal processing array is relatively mature, but with the array signal environment is increasingly complex and the signal has more diversified forms, so that the density and range of signal distribution in space and frequency domain has been increased greatly. However, disadvantage of narrowband array signal processing technology is also increasingly prominent. Therefore, the research on method of wideband signal processing array has great practical significance. Therefore, in this section, we will continue to study the underwater target wideband DOA problem based on CS.

The main difficulty arises when wideband signals are conside impossibility to represent the delays by simple phase shifts. A way we used to deal with this issue is to separate the signal spectrum into several narrowband regions, each of which yields to narrowband processing. To work in frequency domain, the time-samples are grouped into several snapshots, and transformed into the frequency domain:

$$X(f_j) = A(f,\theta)S(f_j) + N(f_j). \quad (12)$$

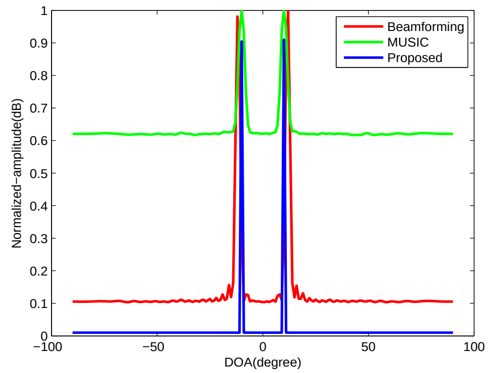
where j represents for the points of Fourier Transform. In this section, we also select Gauss projection matrix as our sparse sampling matrix of wideband signal. Then the equation (8) can be reformulated into the frequency form:

$$Y(f_j) = \Phi X(f_j) = \Phi A(f,\theta)S(f_j) + \Phi N(f_j). \quad (13)$$

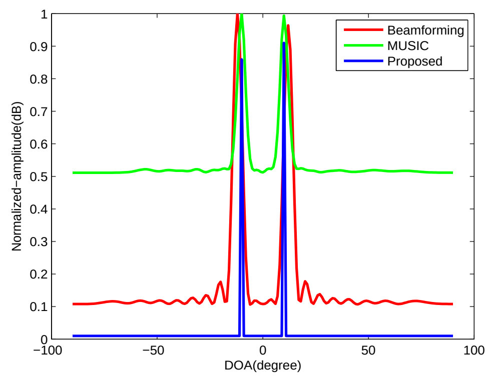
where Φ is an $M \times N$ sensing matrix giving us information about X, $X(f_j)$ is an $M \times 1$ array received matrix. $A(f_j,\theta)$ is $M \times K$ array manifold. $S(f_j)$ is $K \times 1$ signal vector. $N(f_j)$ is $M \times 1$ noise vector. According to the above analysis, the solution of it can be expressed as:

$$min \parallel S(f_j) \parallel_{\ell_0} s.t\ Y(f_j) = \Phi A(f_j,\theta) + \Phi N(f_j). \quad (14)$$

To solve the $\ell_0 - norm$ problem, the main method is the basis pursuit algorithm (BP [27]) and the matching pursuit algorithm (such as MP, OMP [28] etc.) at present. Now we analyze the performance of these algorithms in different N, $DOAs$. In the plot(a) of Fig.9, we present an example of using the same $40-elements$ uniform linear array as one used throughout this paper, but the signals are now wideband. We consider two chips with $DOAs$ -10° and 10° with frequency span from $100Hz$ to $140Hz$, and $L = 100$ time samples for beam-forming, MUSIC and our proposed method. Plot(a) shows that our proposed method has an excellent performance over the conventional methods. Then we reduce the number of arrays from 40 to 20, the others are kept the same as (a). Compare with (a) and (b), we can find that the performance of Capons method and MUSIC begin to degenerate. However, the performance of our method is still very good. In the plot(c), we reduce the array dimension, but also change the $DOAs$. Now two wideband sources are closely spaced (4° separation). Using beam-forming method, the two peaks are merged, but using our convex relaxation of mixed-norm algorithm, they are well resolved, and the side lobes are suppressed almost to zero.



(a)



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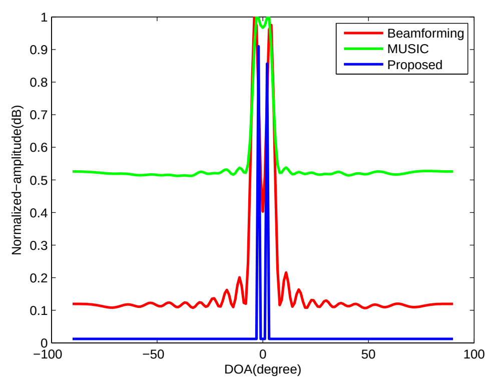

(c)

Fig. 9: The performance of these algorithms with $L = 100$ $SNR = 10dB$. (a)$N = 40$, $DOAs$: -10° and -10°; (b)$N = 20$, $DOAs$:-10° and -10°; (c) $N = 20$, $DOAs$: -2° and 2°.

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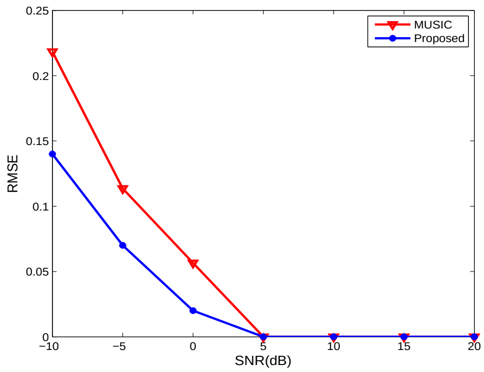

Fig. 10: The *RMSE* comparison of the three estimation methods.

We next compare the root mean square error of the DOA estimation produced by our approach and MUSIC algorithm. In Fig.10, we assume that a wideband underwater acoustic target signal in the far-field impinge on the array from a certain incident angle, and in the *SNR* of $10dB$ to $20dB$. The number of experiments is set to 100. We can get the root mean square error of the two methods. From the figure, we can obviously find that the root mean square error of MUSIC is higher than our proposed method, when *SNR* is less than $5dB$. We can draw a conclusion that, with the increase of the *SNR*, *RMSE* value is smaller and almost close to the zero. Our proposed method has little error even in a low *SNR*. So our technology has higher precision in DOA estimation.

5 Conclusions

In this paper, we explored a formulation of the sensor array underwater acoustic target localization problem in a sparse signal representation framework. According to the distribution characteristics of the underwater target space, we started with a scheme for targets localization with an underwater target signal sparse representation model in spatial domain. The scheme can be applied to narrowband and wideband scenarios. We described how to estimate each signal direction angle, through the corresponding relationship between signal vectors and space angles. Then we used the convex relaxation method of mixed norm to realize underwater acoustic target DOA estimation method based on compressed sensing. Finally, we examined various aspects of our approach, such as the number of snapshots. Several advantages over existing underwater acoustic target localization were identified, including increased resolution, decreased the calculated quantity, reduced the demand of the array. And there are a lot of research and application value to underwater acoustic array especially the underwater acoustic array of micro underwater platforms (such as UUV) to locate the targets.

Acknowledgement

The study was sponsored by Project of the Priority Academic Program Development of Jiangsu Higher Education Institutions, the National Natural Science Foundation of China (Grant No.11204109), the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (Grant No.12KJB510003, No.13KJB510007) and Qing Lan Project.

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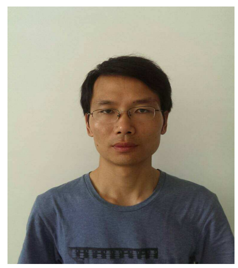

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