Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.12785/amis/090446

Comparative Research on Optimal Purchasing Decision and Many Algorithms with Different Properties under Uncertain Demand

Chu Ye-ping[1,2], *Yang Ai-min*[3,*] *and Bi ya*[4,*]

[1] School of logistics and engineering management, Hubei University of Economics, Hubei Wuhan, 430205, China
[2] College of Science, Hebei United University, Tangshan, 063009, China
[3] College of Mechanical Engineering, Yanshan University, Qinhuangdao, 066004, China
[4] School of logistics and engineering management, Hubei University of Economics, Hubei Wuhan, 430205, China

Received: 16 Nov. 2014, Revised: 16 Feb. 2015, Accepted: 17 Feb. 2015
Published online: 1 Jul. 2015

Abstract: Accurate prediction to the uncertain demand is an important guarantee for preparing the optimal purchasing decision and reducing the inventory cost. Compared with the method of fitting the uncertain demand of commodity sensitive to time and season by general random probability distribution, the fuzzy mathematical method has less relative error and better goodness of fit. Therefore, the fuzzy mathematical method is used for forming the optimal purchasing decision model under uncertain demand. A numerical computation method based on the browsed exact analytic calculation method and two improved PSO algorithm is designed. A plurality of confirmatory experiments and comparative experiments analyze and compare the performance of algorithms with different properties and discus the disadvantages and adaptability thereof, providing new angle and thought for the theoretical research and providing reference basis for the method selection of enterprise.

Keywords: fuzzy mathematics; uncertain demand; purchasing decision model; analytic calculation method; numerical calculation method

1 Introduction

Demand is an important parameter directly determining the purchasing volume and inventory cost of enterprise, affecting the daily management and economic benefits of enterprise[1]. The market environment of the middle-sized and small enterprises is getting more and more complex, all factors affecting the market are coupled and limited more and more seriously, thereby enhancing the uncertainty of the product demand, greatly increasing the purchasing decision difficulty of enterprise and bringing challenge for controlling the inventory cost of enterprise. Therefore, the enterprise cares and urgently solves the problem of accurately predicting the product demand, preparing scientific purchasing strategy and greatly reducing the inventory cost under the uncertain condition.

Seen from the view of theoretical research, the demand can be divided into the deterministic demand and the uncertain demand. The deterministic demand indicates the demand under stable market environment or the ideal processing to the market environment, ignoring the demand under uncertain factor effect. This demand is hardly appeared in the actual situation and has only the value of theoretical research. The uncertain demand indicates the demand fluctuation and elasticity resulted from time, season and emergency, the fluctuation will cause the chaos between the supplier and the manufacturer [2][3]. There are two methods for processing the uncertain demand, 1, considering it as function changed along with the time or inventory, for instance, the linear function whose demand rate is time in limited period formed by Mandal and Maiti; the inventory model whose demand rate is trapezoid change formed by Cheng and Wang [4]; the inventory model whose demand rate is in power law relationship with current inventory level researched by Alfares[5]. 2, considering it as a independent and random probability distribution, it is used most frequently in the theory circle, for instance, the

* Corresponding author e-mail: aimin_heut@163.com, idabiya@126.com

two-level inventory optimization theory whose demand follows the Poisson distribution formed by TAN Yiyan and so on [6]; and the closed-loop supply chain model whose demand follows the uniform distribution function formed by YI Yuyin [7], having reward and punishment system. Besides, there are some special methods for processing uncertain demands, for instance, the Banker [8] and Nicholas [9] methods processing the uncertain demand are performed by adding a random disturbance item on the known demand function; people including LI Na [10] directly sets the demand as the random quantity provided by the random function in emulator program; people including ZHANG Fuli [11] divides the uncertain demand into high-grade demand and low-grade demand for researching the return strategy under different demand fluctuation situation.

The method of fitting the uncertain demand by general probability distribution is widely accepted, but is also has strict use premise as follows, i) the change of the demand is random and irregular; ii) there is large amount of solid historic statistical data. The obvious distortion will be formed if the premise is not followed. Along the appearance and gradual maturing of the fuzzy mathematical theory, the degree of fitting of the method for fitting the uncertain demand of commodities with obvious fluctuation rule is higher than the general probability distribution, this phenomenon is concerned and valued by wide scholars, however, the research on this field is just at the start phase, only few articles describe the research related to the method application, being short of effective evidence and quantitative comparative analysis.

As one of the concerned matters of enterprise, the purchasing decision directly affects the circulating funds and inventory cost of the enterprise. The optimal purchasing design has rich research content, mainly including: the optimization for the purchasing decision of different purchasing individuals, for instance, the purchasing of the order-oriented enterprises; the optimization for the purchasing decision of different purchasing objects, for instance, the purchasing of emergency supplies; the optimization for the purchasing decision with different contents and levels, for instance, the consortium purchasing, procurement price, benefit sharing and procurement risk control; and the purchasing decision optimization under uncertain demand. Under the uncertain demand, the core problem of the purchasing decision is the balance of inventory and shortage risk. The purchasing decision is made according to many variable factors, for instance, the uncertain demand, the changed order cost, the sudden advance or delay, the limited service level and so on. As to these problems, the scholars continuously improve the purchasing optimal models [12] –[18]. The conclusion is as follows: the action of processing the uncertain demand by scientific method will bring important practical effect to the purchasing decision and application effect thereof.

In summary, the current article provides many effective and practical methods for the research on optimal purchasing decision under uncertain demand; the disadvantages thereof are as follows:

(1) As to the method fitting the uncertain demand, most researches adopt the general probability distribution function, but the data in the fitting by this method has obvious fluctuation rule and serious distortion.

(2) The known research shows that the fuzzy mathematical method fitting the uncertain demand of commodities sensitive to time and season has higher degree of fitting in comparison with the general probability distribution. However, in current stage, the related research mainly faces to the specific application, lacking theoretical quantitative analysis and proof.

(3) Currently, most researches on optimal purchasing decision model and algorithm face to specific problem; the comparative research is hardly performed for different solution methods. This is a brand new research angle of view and has positive effect on the theory and enterprise practice sides.

Therefore, this article tries to form the optimal purchasing decision model under uncertain demand based on fuzzy mathematical theory to the middle-sized and small enterprises in China, solve the optimal purchasing model by multiple algorithms with different properties, and perform the comparative research to these different algorithms. The research can help the middle-sized and small enterprises to prepare more accurate purchasing decision under uncertain demand, and realize the target of reducing the inventory cost of enterprise, decreasing the running risk of enterprise and improving the management ability and competition of enterprise. The research deepens the theory and method of the purchasing and inventory optimization management and has obvious economical effect and practical value.

2 Optimal purchasing decision model based on fuzzy mathematics

2.1 Question raise

There are a lot of medium-sized and small enterprises making toys for export enterprises in Guangzhou China. They organize the production and raw material procurement according to foreign order. The busy production seasons for these toy enterprise is from August to November and the slack seasons are from February to May. The main raw material for the toy production is the plastic particles. Therefore, the raw material purchasing plan for these toy enterprises will be changed along the order quantity, so it has obvious time and season fluctuation rule. After a one-year field research, we find that these enterprises are troubled by the inventory cost control problem; the basic reason is inaccurate prediction of purchasing volume for raw

Table 1: Plastic particles purchasing source data each month from year 2008 to 2012.

	1	2	3	4	5	6	7	8	9	10	11	12
2008	30	48	50	64	58	34	82	98	80	92	78	80
2009	19	40	35	67	54	37	84	81	80	79	49	70
2010	15	36	53	68	51	35	64	80	79	79	66	67
2011	16	37	51	67	51	34	98	80	82	76	65	67
2012	19	25	37	68	35	19	81	65	82	64	70	48

Table 2: Interval data.

Section	Frequency	Relative probability	Data points
[2, 18]	2	0.0909	10
(18, 34]	7	0.3182	26
(34, 50]	11	0.5000	42
(50, 66]	14	0.6364	58
(66, 82]	22	1.0000	74
(82, 98]	4	0.1818	90

material. The raw material is purchased before the order, so the purchasing volume should be estimated according to the historic data, which causes large derivation. Therefore, the optimal purchasing volume decision for the toy production enterprise and enterprises under similar condition in the premise of uncertain demand is an urgent to-be-solved practical and common issue, which is necessary for the theoretical research on the scientificalness and practical application sides.

2.2 Data sources

Here we take the month purchasing volume of plastic particles of a toy production enterprise from year 2008 to 2012 (see table 1) as the research data.

In order to perform the calculation and comparison analysis, the data in the table 1 is processed as follows: the data is formed as the interval data, calculating the frequency and relative probability of the purchasing volume of plastic particles each month, the middle interval value is used as the average purchasing value of said month (see table 2).

2.3 Discussion on Poisson flow and fuzzy mathematics

(1) Qualitative comparison

The Poisson distribution is a random probability distribution indicating the particle quantity in each unit time. The particles appear randomly, so the Poisson distribution has the stability, unfollow-up effect and universality. If the market data is complete and accurate and characterized by randomness and irregularity, the data is suitable to be fit by the Poisson distribution, but this requirement is hard to be strictly performed in the actual situation. i) The market data is fluctuated and clearly regular; ii) The market data is incomplete and deficient. In fact, many researches find serious distortion when fitting the uncertain demand with obvious time and season change rule by the general random probability distribution, so the final research result is derived from the actual situation. This phenomenon catches attention and interest of scholars at home and abroad, causing many new methods and measures, one of which is the fuzzy mathematical method. The fuzzy mathematics describes the intermediate transition process of the changed matter by the degree of membership, efficiently breaking the absolute theory in or out of the classic set theory proposed by the Descartes, driving people to find a method for describing the transition interval between the accuracy and the fuzziness, and providing a new mathematical method for processing the fuzzy problem [19] –[22]. The advantages for fitting the uncertain demand by this method are as follows: i) It is highly suitable for fitting the uncertain situation with fixed variation tendency; ii) It does not need complete and accurate historic data.

(2) Quantitative comparison

By using the table 1 and the table 2 as the sample data, the relative error and goodness of fit of the Poisson distribution and fuzzy triangular distribution are analyzed and compared from the statistics side.

i) Check for relative error

The relative percentage error of Poisson distribution and fuzzy triangular distribution is calculated by the sample quantity, the specific result is shown as the table 3.

ii) Check for goodness of fit

The goodness of fit for the Poisson distribution and fuzzy triangular distribution is calculated by the sample quantity, the specific result is shown as the table 4.

Wherein, $p_i$indicates the appearance possibility of the sample in the interval; N_i indicates the quantity of the

Table 3: Relative error comparison of two distributions

Section	Frequency	The real data The overall Probability	Relative error	The Poisson distribution The overall Probability	Relative error	The fuzzy triangular distribution The overall probability	Relative error
$[2,18]$	2	0.03	0	0.0000	100.0	0.03	0.12
$(18,34]$	7	0.02	0	0.0006	99.44	0.11	9.50
$(34,50]$	11	0.18	0	0.1890	3.06	0.20	8.20
$(50,66]$	14	0.23	0	0.6996	199.8	0.29	24.78
$(66,82]$	22	0.37	0	0.1100	69.99	0.38	4.84
$(82,98]$	4	0.07	0	0.0008	98.81	0.08	19.98

Table 4: Goodness of fit for the Poisson distribution and the fuzzy triangular distribution

		Poisson distribution			fuzzy distribution		
Section	Frequency	p_i	np_i	$(N_i - np_i)^2/np_i$	p_i	np_i	$(N_i - np_i)^2/np_i$
$[2,18]$	2	0	0	0	0.0309	2.4558	0.0846
$(18,34]$	7	0.0006	0.0389	2063.2	0.1132	6.7912	0.0064
$(34,50]$	11	0.1890	11.3370	0	0.1960	11.6421	0.0354
$(50,66]$	14	0.6996	41.9750	18.6	0.2749	16.4930	0.3768
$(66,82]$	22	0.1100	6.6015	35.9	0.3769	22.2541	0.0029
$(82,98]$	4	0.0008	0.0476	328.4	0.0809	5.2147	0.2829

sample in the i interval; indicates the total sample quantity ($n = 60$); the goodness of fit for the Poisson distribution is χ_1^2, the goodness of fit for the fuzzy triangular distribution is χ_2^2. Providing the freedom degree of the χ_1^2 distribution is $r - s - 1 = 5 - 1 - 1 = 3$, $\alpha = 0.05$, and $\chi^2_{0.05,3} = 7.815$. The calculation result is $\chi_1^2 = \sum_{i=1}^{5} \frac{(N_i - np_i)^2}{np_i} = 2446.2$.

The $\chi_1^2 = 2446.2 >> 7.815 = \chi^2_{0.05,3}$, so the distortion will be great if the sample is fit by the Poisson distribution. Providing the freedom degree of the χ_2^2 distribution is $r - s - 1 = 6 - 1 - 1 = 4$, $\alpha = 0.05$, and $\chi^2_{0.05,4} = 9.488$, the calculation result is $\chi_2^2 = \sum_{i=1}^{6} \frac{(N_i - np_i)^2}{np_i} = 0.7891$.

$\chi_2^2 = 0.7891 << 9.488 = \chi^2_{0.05,4}$, so it is well suitable for partially fitting the sample by fuzzy triangle.

In summary, no matter by the qualitative or quantitative analysis and comparison, compared with the Poisson distribution, the fuzzy mathematical method has obvious fitting advantage to the data with clear time or season change rule.

3 Formation of optimal purchasing decision model based on fuzzy mathematics

(1) Model formation:

$$f_{qi} = max[p_q(d_i - w_{i-1}) + p_d Q + p_0, 0] \quad (1)$$

$$f_{ii} = max[p_i(w_{i-1} - d_i), 0] \quad (2)$$

$$U_{x_i} = \begin{cases} \dfrac{d_i - d_1}{d_m - d_1}, d_1 \le d_i \le d_m \\ \dfrac{d_2 - d_i}{d_2 - d_m}, d_1 \le d_m \le d_2 \\ 0, \quad d_i < d_1, d_i > d_2 \end{cases} \quad (3)$$

$$F_t = \sum_{i=1}^{12} (f_{qi} + f_{ii}) = min\, f_i(d_i) \quad (4)$$

$$mindeF_t(d_i) = \frac{\sum F(d_i u(d_i))}{\sum u(d_i)} \quad (5)$$

$$Q = minQ = (\frac{defF_i(d_i)}{12} - p_0)/p_d \quad (6)$$

(2) Sign and model specification:
d_i indicates the demand in the i phase;
p_i indicates the unit inventory cost in the i phase;
p_d indicates the unit commodity price in the i phase;
p_q indicates the unit shortage cost in the i phase;
p_o indicates the unit order cost;
f_i indicates the demand point cost in the i phase;
F_i indicates the annual total cost of demand point;
w_i indicates residual inventory in the i phase;

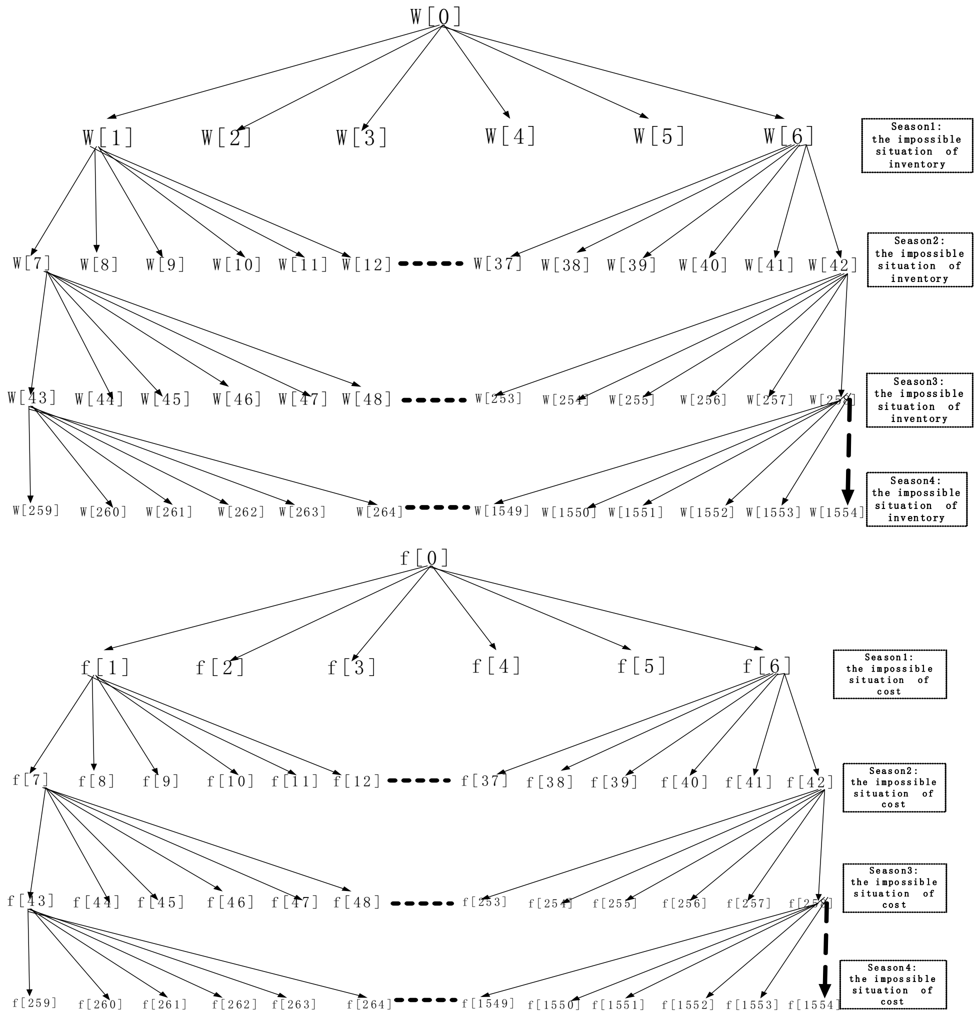

Fig. 1: the line of searching the $f_{[i]}$ and $w_{[i]}$

Q indicates the month demand quantity;

$Q_0 = W_0$, Q is the optimal purchasing volume.

providing each month is a demand phase, the formulas (1) and (2) indicates that the shortage cost will be generated if the demand is greater than the inventory in the demand phase, otherwise the inventory cost will be generated.

the formula (3) indicates that the uncertain demand of the demand point is represented by the triangular fuzzy number $d_i = (d_1, d_m, d_2)$, the grade of membership is the appearance possibility of the demand quantity and expressed as U_{x_i}.

the formula (4) indicates the minimum value of all total purchasing cost of the demand point in 12 months, f_i is the fuzzy expression form, so the total cost F_i is fuzzy.

the formula (5) performs the fuzzing treatment to the formula (4) by the centroid method. the formula (6) indicates the optimal purchasing volume each month in one year.

4 Searching algorithm based on analytic calculation

The above-mentioned model is solved by the searching algorithm based on analytic calculation. The line of thought is as follows: fully considering possible inventory or shortage each month, describing the situation by fuzzy way, calculating the total cost under each situation, and selecting the situation with lowest cost. The algorithm has two main parameters, one of which is the demand quantity $w_{[i]}$ each month, and the other is the total demand point cost $f_{[i]}$ each month. The figure 1 brings the line of searching the two variables.

Taking the cost of the first month as example, other months have similar situation.

(1) when the inventory is $w_{[0]}$, there are 6 fuzzy situations of the possible inventory balance$w_{[1]}$, $w_{[2]}$, $w_{[3]}$, $w_{[4]}$, $w_{[5]}$, $w_{[6]}$, the possibility for the 6 situations are $U_{[0]}, U_{[1]}, U_{[2]}, U_{[3]}, U_{[4]}, U_{[5]}$.

2) when the inventory is $w_{[0]}$, there are 6 total cost situations of the possible inventory balance: $F_{[1]}$, $F_{[2]}$, $F_{[3]}$, $F_{[4]}$, $F_{[5]}$, $F_{[6]}$,the possibility for the 6 situations are $U_{[0]}, U_{[1]}, U_{[2]}, U_{[3]}, U_{[4]}, U_{[5]}$.

(3) therefore, the total demand point cost in the first month is

$$F_{[1]} = \frac{f(1)*U_0 + f(2)*U_1 + \cdots + f(6)*U_5}{U_{[0]} + U_{[1]} + U_{[2]} + U_{[3]} + U_{[4]} + U_{[5]}} \quad (7)$$

5 Two improved particle swarm optimization algorithm based on numeric calculation

5.1 Two improved strategies for basic PSO algorithm

The basic particle swarm optimization algorithm is described as the follows: the D dimension feasible region space Ω of the object function is equipped with the swarm containing M particles, $U = \{x_1, x_2, \cdots, x_m\}$, wherein U indicates the swarm set, x_i indicates individual particle. Each particle indicates one potential feasible solution for the optimization problem. The particle is expressed by two vectors which are the position and the viscosity, $x_i = \{x_i, v_i\}$. Providing the position of the i particle is expressed as $x_i = (x_{i1}, x_{i2}, \cdots, x_{iD})^T$, indicating a potential feasible solution of the object function; $v_i = (v_{i1}, v_{i2}, \cdots, v_{iD})^T$ indicates the search viscosity of the particle. The adaptability value i of the particle is used for checking the effect of the solution, generally, the object function or the reciprocal thereof is used as the adaptability function.

The updating formula of the basic particle swarm optimization algorithm is:

$$v_i^{t+1} = w * v_i^t + c_1 * rand_1() * (pbest_i^t - x_i^t) + c_2 * rand_2() * (gbest^t - x_i^t) \quad (8)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (9)$$

Specification of formula signs:

t indicates the iteration time number;

v_i^{t+1} and x_i^{t+1} indicate the viscosity and position of the particle i in the iteration process;

c_1 and c_2 indicate the learning factors of the particle;

w indicates the inertia particle of the particle;

$pbest_i^t$ indicates the historic optimal position of the particle i itself;

$gbest^t$ indicates the historic optimal position of all particles.

The formula shows that the basic particle swarm algorithm contains few parameters; the algorithm is a simple excellent global searching algorithm. All particles at the post algorithm phase will be converged to the particle at better position, missing the diversity of the particle, so the algorithm will be trapped in the disadvantages of local optimization and slow convergency. Therefore, the basic particle swarm algorithm can be modified from the following two angles so as to improve the integral performance of the algorithm. i) Increasing the diversity of the particle for solving the problem that the algorithm is easily trapped in the local optimization; ii) Increasing the searching dynamism of the particle so as to solve the problem that the algorithm has slow convergency at the post phase.

(1) Clustering strategy

Creatures will stay in a certain geographic area with the same specie autonomously in the growth and evolution process, this phenomenon is called as clustering. The clustering in the scientific issue is a process of dividing a group into many clusters by similarity. The individuals in the cluster develop by learning from other excellent individuals in the cluster. The known research article shows that the performance and structure of the individuals in the cluster greatly affect the development of the cluster. We introduce the clustering method in the basic particle swarm algorithm so as to guarantee the diversity of the cluster and prevent the algorithm from the local optimization. The specific improvement strategy is dynamically clustering all particles according to the distance and increasing the dependence of the particle to the cluster.

(2) Dynamic adaptive strategy

The basic reason for the slow convergency viscosity at the post phase of the basic particle swarm algorithm is the fixed inertia weight limit to the updating viscosity of the particle, affecting the searching efficiency of the particle to the optimal value. Yunji Wang presented one improved algorithm and discussed the relationship between the convergence speed and stability of the system[23]. Therefore, the particles at different positions are dynamically applied with different inertia weights and optimization search viscosity so as to efficiently improve integral optimization search ability and viscosity of the particle. The specific improvement strategies are as follows: determining the “advantages and disadvantages”of the particles in the optimization search process according to the adaptability thereof. If the adaptability value of the particle is high, the particle is close to the optimal value, then the inertial weight and the optimization search viscosity thereof should be reduced, and the searching ability close to the optimal value should be enhanced; if the adaptability value of the particle is little, the particle is far away from the optimal value, then the inertia weight and the optimization viscosity thereof should be increased so as to quickly get away from the area, and the ability thereof expanding to other areas should be enhanced.

5.2 Algorithm processes of two improved particle swarm algorithms

5.2.1 PSO algorithm based on K-mean cluster (KM-PSO)

(1) Initialization.

If there are 20 particle swarms; the inertial weight $w = 0.65$, the learning factor $c_1 = c_2 = 1$, the maximum viscosity v_{max} is indicated in specific problems; the $rand()$ is valued in the section of [0-1].

(2) Clustering operation.

1) selecting $k(k = 6)$ particles from the swarm as the cluster center (randomly selecting at the first iteration); 2) calculating the Euclidean distance from all particles to the cluster center, and sorting the particle to the clusters according to the minimum distance rule; 3) re-clustering all particles by using the average particle positions in the cluster as the cluster center; 4) if there is empty cluster, using the average value of other non-empty cluster centers as the center point of the empty cluster, directly entering the next cluster calculation; repeating steps 2), 3) and 4) for 5 times, stopping and finishing one clustering operation; 5) performing one clustering operation after each iteration update.

(3) Updating formula of KM-PSO algorithm.

$$v_i^{t+1} = w * v_i^t + c_1 * rand_1() * (pbest_i^t - x_i^t) + c_2 * rand_2() * (gbest^t - x_i^t) \quad (10)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (11)$$

$$v_i^{t+1} = \begin{cases} v_{max}, if \ \ v_i^{t+1} > v_{max} \\ v_{min}, if \ \ v_i^{t+1} < v_{min} \end{cases} \quad (12)$$

Wherein, *Nelbest* indicates the historic optimal position of the particles clustering; formula (12) shows the speed limit of the particles.

(4) The iteration stopping condition.

1) the iteration time number T is 500; or 2) the optimal values of continuous 10 calculations are the same.

5.2.2 PSO algorithm based on K-mean cluster and dynamic adaptive algorithm (KMDA-PSO)

(1) Initialization.

Providing there are 20 particle swarms; w indicates the adaptive dynamic inertia weight; w_i^{t+1} indicates the inertia weight of the particle i at the $t+1$ generation, $w_0 = 0.65$; the learning factor $c_1 = c_2 = 1$; indicates the average adaptability value of the i cluster; $Average_g$ indicates the average adaptability value of the cluster; providing the adaptive dynamic v_{max} is the maximum allowed viscosity of the particle i at the $t+1$ generation; the adaptive dynamic v_{max} is indicated in specific problems.

(2) Clustering operation: the same as the (2) in 4.2.1.

(3) Updating formula of KMDA-PSO algorithm.

$$v_i^{t+1} = w * v_i^t + c_1 * rand_1() * (pbest_i^t - x_i^t) + c_2 * rand_2() * (Nelbest_i^t - x_i^t) \quad (13)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (14)$$

$$w_i^{t+1} = \begin{cases} w_i^t * (1 - average_i)/Average_g, if \ \ average_i \geq Average_g \\ w_i^t * (1 - average_i)/Average_g, if \ \ average_i < Average_g \end{cases} \quad (15)$$

$$w_i^{t+1} = \begin{cases} w_{max}, \ if \ \ w_i^{t+1} > w_{max} \\ w_{min}, \ if \ \ w_i^{t+1} < w_{min} \end{cases} \quad (16)$$

$$v_i^{t+1} = \begin{cases} v_i^t * (1 - average_i)/Average_g, if \ \ average_i \geq Average_g \\ v_i^t * (1 - average_i)/Average_g, if \ \ average_i < Average_g \end{cases} \quad (17)$$

$$v_i^{t+1} = \begin{cases} v_{max}, \ if \ \ v_i^{t+1} > v_{max} \\ v_{min}, \ if \ \ v_i^{t+1} < v_{min} \end{cases} \quad (18)$$

wherein, the formula (15) and (17) show the dynamic adaptive process of the inertia weight and the speed of the particle. If $average_i \geq Average_g$, the particle is close to the optimal value in the global situation and in a good phase, so the inertia weight and the speed of this particle should be adaptively reduced in the next iteration so as to prevent missing the optimal solution when studying the inertia weight; if $average_i < Average_g$, the particle is far away from the optimal value in the global situation and in a bad state, so the inertia weight and the speed of the particle should be adaptively increased in the next iteration in order to increase the optimization search viscosity. The formula (16) and (18) show the limit to the inertia weight and the speed, preventing the particle from flying away from the limit area.

(4) Iteration stops condition.

1) the iteration time number T is 500; or 2) the optimal values of continuous 10 calculations are the same.

(5) The KMDA-PSO algorithm flow chart (see the Fig 2)

The following 2 experiments are designed to comprehensively evaluate the algorithm of the article. The

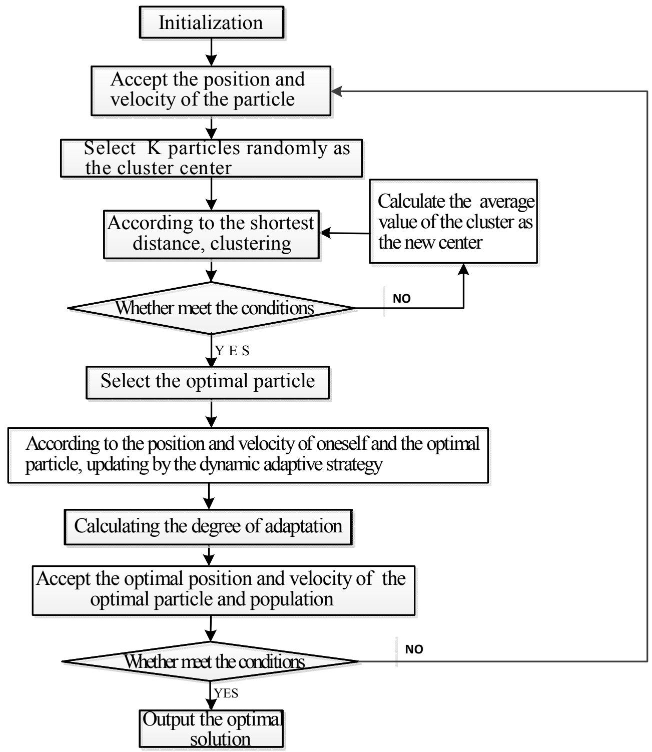

Fig. 2: The KMDA-PSO algorithm flow chart

initial value of the experiment parameter is: $w_0 = 0.65$; $c_1 = c_2 = 1$; the $rand()$ is valued in the section of [0-1]; the population size is 20; each algorithm is operated for 20 times separately and iterated for 500 times.

(1) Benchmark test

This experiment aims to analyzing and comparing all general specification of the basic PSO algorithm and the other two improved algorithms. We select 4 standard functions to test these three algorithms (see the table 5 and 6).

4 standard functions are used for testing two improved particle swarm algorithms. Each algorithm is performed to each function for 20 times in the experiment, and there are 500 iterations in each performance, the swarm scale is 20.

(3) Comparative experiment of algorithm convergency performance

Through testing the Rastrigin function, this experiment aims to compare the convergence viscosity and convergence precision of the basic PSO algorithm and the another two improved algorithms under the initial condition (see the fig 3).

The figures 3 shows that under the same initial value, the KMDA-PSO algorithm has the best convergence viscosity and convergence precision, and the KM-PSO algorithm has the better convergence viscosity and convergence precision than the basic PSO algorithm. After adjusting comprehensive improvement strategy, the two improved algorithms overcome the disadvantage of the basic PSO algorithm at great extent, improving all convergence performance of the algorithm.

6 Example Calculation

Specification: in the searching algorithm based on analytic calculation, the algorithm should calculate all possible inventory and shortage each month by the formula (5); the calculation should be performed for 6^{12} times by 6 possibility statistical intervals in the table 2, which exceeds the calculation ability of the current computer and causes the data overflow. In order to realize the system, the following modification is made: the 2 to 6 possibility statistical intervals in the table 2 are not changed, the time interval of optimal purchasing volume and inventory cost solution is changed from month to season. The calculation time after the modification is 6^4 , meeting the calculation ability requirement of the computer. The formula (6) in the past is changed as the formula (19) now.

$$\min Q_{Season} = \left(\frac{defF_{tseason}(d_i)}{4} - p_0\right)/p_d \quad (19)$$

This example calculation is performed by the PSO algorithm and the KMDA-PSO algorithm to the optimal

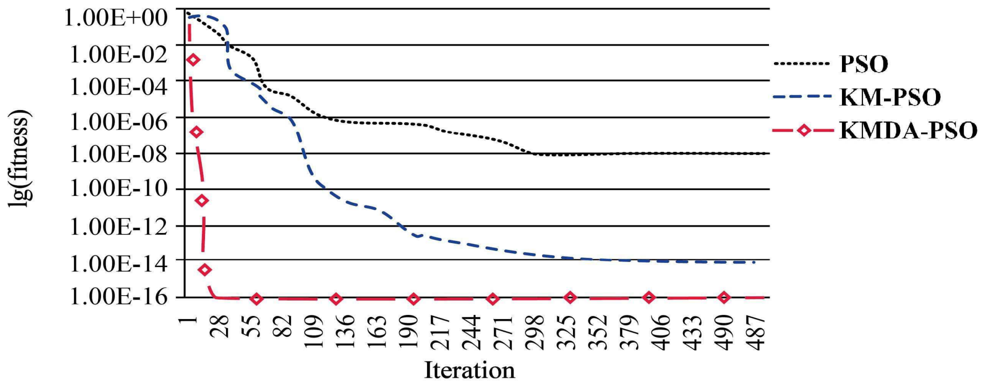

Fig. 3: the result of the convergence viscosity and convergence precision

Table 5: The test function

Function name	The function	The Parameters of test function
Sphere	$f(x) = \sum_{i=1}^{n} x_i^2$	The search area (-100,100) The Dimension20 The search precision0.01
Rosebrock	$f(x) = \sum_{i=1}^{n} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	The search area (-30,30) The Dimension20 The search precision100
Griewank	$f(x) = \frac{1}{4000} \sum_{i=1}^{n} (x_i - 100)^2 - \prod_{i=1}^{n} cos \frac{x_i - 100}{\sqrt{(i)}} + 1$	The search area (-600,600) The Dimension20 The search precision0.01
Rastrigin	$f(x) = \sum_{i=1}^{n} x_i^2 - 10cos(2\pi * x_i) + 10$	The search area (-5.12,5.12) The Dimension20 The search precision0.01

purchasing decision model under the uncertain demand. There are the 16 inputsthe demands in 1-12 monthsunitTON16, 48, 50, 64, 58, 34, 85, 98, 80, 92, 78, 80; the unit shortage cost 2000 Yuan; the unit commodity price 3000 Yuan; the unit inventory cost 1000Yuan; the unit order cost 1000 Yuan. The result is the table 7.

7 Conclusion

(1) The statistic performances of the method for fitting the uncertain demand with obvious time and season change feature by the fuzzy mathematical method are better than that of the general random probability distribution.

(2) The basic PSO algorithm is simple and easy, but it also easily traps in the local optimization and slow convergency viscosity at the post algorithm phase, which can be improved by the following methods: (1) introducing the cluster operation, guaranteeing the particle diversity and efficiently preventing the algorithm from trapping in the local optimization; (2) performing the dynamic adaptive adjustment to the inertia weight and viscosity of the particle, and efficiently improving the convergency viscosity of the algorithm.

(3) The searching algorithm based on analytic calculation and the improved particle swarm algorithm based on numerical calculation can solve the optimal purchasing decision model based on fuzzy mathematics and they are two algorithms with opposite properties having different application difficulty and precision. The analytic calculation method is direct and simple on the algorithm process, the optimization search result is sole and reliable, but certain precision should be removed is the problem is large scaled. The improved particle swarm algorithm is the currently researched hot algorithm having obvious advantage for processing large-scale non-linear problem, but it cannot guarantee the result optimization

Table 6: The Benchmark result

Function	Algorithm	Worst	Best	The mean best	SD
Sphere	PSO	8.54E-13	1.59E-15	1.38E-13	1.77E-13
	KM-PSO	1.79E-15	7.53E-18	3.62E-16	3.95E-16
	KMDA-GP	4.41E-29	3.19E-31	7.16E-30	1.07E-29
Rosebrock	PSO	3.37 E-02	2.42 E-01	2.93E-01	2.71E-01
	KM-PSO	2.89 E-05	3.59 E-06	5.49 E-06	1. 16E-05
	KMDA-GP	2.86 E-12	2.89 E-13	3.12 E-13	1.51E-13
Griewank	PSO	7.85 E-06	2.06E-07	5.36E-07	6.87E-07
	KM-PSO	0	0	0	0
	KMDA-GP	0	0	0	0
Rastrigin	PSO	3.19E-08	1.17E-08	2.75E-08	2.35E-08
	KM-PSO	2.72E-14	5.59E-15	7.46 E-15	4.62E-15
	KMDA-GP	0	0	0	0

Table 7: The optimal purchasing decision and inventory cost

System output	KMDA-PSO	The analytic algorithm
The optimal Purchasing decision(TON)	60.66	65.34
The optimal Inventory cost(Yuan)	732000	763533

and is lack of strong mathematical evidence on the general framework, optimization search mechanism and convergency problem of the algorithm.

Acknowledgment

This work was financially supported by Hubei Education Department Scientific Research Project (Project Number: Q20132206); Hubei Logistic Development Research Center Sponsored Project

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Yeping Chu PhD. Associate professor, School of logistics and engineering management, HUBEI University of Economics, China. The main research direction: Agricultural Logistics, Supply Chain Optimization

Ai-min Yang Chinese. Associate professor, College of Science, Hebei United University, Tangshan, China. His current research interests include control, dynamics and local fractional derivative.

Bi Ya Chinese. PhD. Associate professor, School of logistics and engineering management, HUBEI University of Economics, China. The main research direction: Optimization algorithm, System modeling, simulation and optimization.