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Exponential Estimators for Population Mean Using the Transformed Auxiliary Variables

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Abstract: This paper deals with exponential ratio-cum-ratio and product-cum-product type estimators using transformed auxiliary variables under simple random sampling without replacement. The proposed estimators are useful for estimating the finite population mean. The development of estimators is based on the information of two transformed auxiliary variables. The generalized form of the proposed estimator has been developed and the special cases are discussed. The bias and mean square error expressions of the proposed estimators are derived up to the first order of approximation. An empirical study has been carried out to compare the efficiency of proposed estimators with some available estimators in literature. An improvement has been reflected in terms of mean square error (MSE).

Keywords: Transformed auxiliary variable, simple random sampling, exponential estimators, mean square error, bias.

1 Introduction

The role of auxiliary information is of prime importance in sampling theory. In survey sampling auxiliary variables are commonly used in order to obtain improved designs and to achieve higher precision in the estimates of some population parameters such as population total, population mean, population proportion, population ratio, etc. In case when the relation between auxiliary and study variable is positive then ratio estimation is often suggested. However, product method of estimation is usually considered, when the correlation coefficient between study and auxiliary variable is negative. In most of the survey situations the auxiliary information is always available. It may either be promptly available or may be collected without much difficulty by averting a part of survey resources. Laplace [7] made use of auxiliary information for the population census of France. The work of Neyman [8] may be referred to as the initial work where the use of auxiliary information has been established. The development is continue by using the exponential form of the ratio and product type estimators such as Bahl and Tuteja [2], Noor ul amin and Hanif [9] and Sanaullah et al. [12] etc .
Consider the finite population $S=\{x_1,x_2,......,x_N\}$ of size N. Let Y be the study variable with population mean $\overline{Y}$ while X and Z are auxiliary variables with population mean $\overline{X}$ and $\overline{Z}$, respectively. The sample means for variables Y, X, and Z are denoted by $\overline{y}$, $\overline{x}$, and $\overline{z}$, respectively. The sample are drawn by the simple random sampling without replacement (SRSWOR) of size n (n<N).

We also have following assumptions:

$$
\begin{aligned}
e_{\overline{y}} &= \frac{\overline{y}-\overline{Y}}{\overline{Y}}, \qquad e_{\overline{x}} = \frac{\overline{x}-\overline{X}}{\overline{X}}, etc,\\
E(e_{\overline{y}}) &= E(e_{\overline{x}}) = 0, E(e^2_{\overline{y}}) = \theta C_y^2,\\
E(e^2_{\overline{x}}) &= \theta C_x^2 \; E(e_{\overline{x}}e_{\overline{y}}) = \theta\rho_{xy}C_yC_x,\\
E(e_{\overline{z}}e_{\overline{y}}) &= \theta\rho_{yz}C_yC_z\\
\theta = \frac{1}{n}-\frac{1}{N}, \quad C_i &= \frac{Si}{\overline{i}} \quad ,K_{ij} = \rho_{ij}\frac{C_i}{C_j}\\
\text{where} \;\; i &= x,y,z, \;\; j = x,y,z \;\; \text{and} \;\; i\neq j
\end{aligned} \tag{1}
$$

These notations are similar for the other auxiliary variables.

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Cochran [4] and Robson [11] proposed classical ratio and product estimators for estimating the population mean as

$$t_1 = \bar{y}\frac{\bar{X}}{\bar{x}}, \quad (2)$$

and

$$t_2 = \bar{y}\frac{\bar{z}}{\bar{Z}}, \quad (3)$$

respectively, the mean square error equations of the estimators t_1 and t_2 are

$$MSE(t_1) \approx \bar{Y}^2\theta[C_y^2 + C_x^2(1-2K_{yx})], \quad (4)$$

and

$$MSE(t_2) \approx \bar{Y}^2\theta[C_y^2 + C_z^2(1+2K_{yz})], \quad (5)$$

respectively.
Srivenkatramana [13] suggested dual to ratio and product estimators t_3 and t_4 by applying a transformation $\bar{x}^* = \frac{N\bar{X}-n\bar{x}}{N-n}$ on auxiliary variables X and Z as

$$t_3 = \bar{y}\frac{\bar{x}^*}{\bar{X}}, \quad (6)$$

and

$$t_4 = \bar{y}\frac{\bar{Z}}{\bar{z}^*}, \quad (7)$$

where $\bar{x}^* = (1+g)\bar{X} - g\bar{x}$, $\bar{z}^* = (1+g)\bar{Z} - g\bar{z}$ and $g = \frac{n}{N-n}$

Mean square error of t_3 and t_4 up to the first order of approximation is

$$MSE(t_3) \approx \bar{Y}^2\theta[C_y^2 + gC_x^2(g-2K_{yx})], \quad (8)$$

and

$$MSE(t_4) \approx \bar{Y}^2\theta[C_y^2 + gC_x^2(g+2K_{yz})], \quad (9)$$

respectively . Bahl and Tuteja [2] proposed the following exponential ratio and product type estimators . The exponential estimators are preferable on classical ratio and product estimators, when the linear relationship between study variable and auxiliary variable is not very strong.

$$t_5 = \bar{y}exp[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}], \quad (10)$$

and

$$t_6 = \bar{y}exp[\frac{\bar{z}-\bar{Z}}{\bar{Z}+\bar{z}}], \quad (11)$$

Mean square error of t_5 and t_6 up to the first order of approximation is

$$MSE(t_5) \approx \bar{Y}^2\theta[C_y^2 + \frac{C_x^2}{4} - K_{yx}C_x^2], \quad (12)$$

and

$$MSE(t_6) \approx \bar{Y}^2\theta[C_y^2 + \frac{C_z^2}{4} + K_{yz}C_z^2], \quad (13)$$

respectively, the exponential dual to ratio type estimator proposed by Sharma and
Tailor [14] as

$$t_7 = \bar{y}exp[\frac{\bar{x}^*-\bar{X}}{\bar{x}^*+\bar{X}}], \quad (14)$$

$$MSE(t_7) \approx \bar{Y}^2\theta[C_y^2 + gC_x^2(\frac{g}{4} - K_{yx})], \quad (15)$$

the exponential dual to product type estimator is

$$t_8 = \bar{y}exp[\frac{\bar{Z}-\bar{z}^*}{\bar{Z}+\bar{z}^*}], \quad (16)$$

$$MSE(t_8) \approx \bar{Y}^2\theta[C_y^2 + gC_z^2(\frac{g}{4} + K_{yz})]. \quad (17)$$

2 Proposed Estimators

Let $\bar{x}^* = \frac{N\bar{X}-n\bar{x}}{N-n}$ and $\bar{z}^* = \frac{N\bar{Z}-n\bar{z}}{N-n}$ then $\bar{x}^* = (1+g)\bar{X} - g\bar{x}$ and $\bar{z}^* = (1+g)\bar{Z} - g\bar{z}$, where $g = \frac{n}{N-n}$. Using the transformation $\bar{x}^*$ and $\bar{z}^*$ we propose exponential ratio-cum-ratio and exponential product-cum-product estimators in generalized form, given by

$$t_{9G} = \bar{y}exp[a(\frac{\bar{x}^*-\bar{X}}{\bar{x}^*+\bar{X}}) + b(\frac{\bar{z}^*-\bar{Z}}{\bar{z}^*+\bar{Z}})], \quad (18)$$

and

$$t_{10G} = \bar{y}exp[c(\frac{\bar{X}-\bar{x}^*}{\bar{X}+\bar{x}^*}) + d(\frac{\bar{Z}-\bar{z}^*}{\bar{Z}+\bar{z}^*})], \quad (19)$$

respectively, where a, b, c and d are positive real numbers. In order to obtain the bias and mean square error of the proposed estimators, using the notations (1), the proposed exponential ratio-cum-ratio type estimator may be written as

$$t_{9G} = \bar{Y}(1+\bar{e}_y)exp[-\frac{ag\bar{e}_x}{2}(1-\frac{g\bar{e}_x}{2})^{-1} - \frac{bg\bar{e}_z}{2}(1-\frac{g\bar{e}_z}{2})^{-1}], \quad (20)$$

Opening the exponential function in (20) and ignoring the terms with power three or greater, we get

$$t_{9G} \approx \bar{Y}[\bar{e}_y - \frac{ag\bar{e}_x}{2} + \frac{a^2g^2\bar{e}_x^2}{8} - \frac{gb\bar{e}_z}{2} + \frac{b^2g^2\bar{e}_z^2}{8} + \frac{g^2ab\bar{e}_x\bar{e}_z}{4} - \frac{ga\bar{e}_x\bar{e}_y}{2} - \frac{gb\bar{e}_z\bar{e}_y}{2}], \quad (21)$$

Taking expectation on both sides of (21) and after simplifications, the bias of t_{9G} is obtained as

$$Bias(t_{9G}) \approx \frac{\overline{Y}\theta}{8}[g^2(C_z^2(2abK_{xz}+b^2)+a^2C_x^2) -4g(aK_{yx}C_x^2+bK_{yz}C_z^2)], \quad (22)$$

For the derivation of mean square error of t_{9G}, we again use (20), ignoring the terms with power two or higher, we get

$$t_{9G} = \overline{Y}(1+\overline{e}_y)exp[-\frac{ag\overline{e}_x}{2} - \frac{bg\overline{e}_z}{2}], \quad (23)$$

Expanding exponential terms, ignoring higher order terms, we get

$$t_{9G} - \overline{Y} \approx \overline{Y}[\overline{e}_y - \frac{ga\overline{e}_x}{2} - \frac{gb\overline{e}_z}{2}], \quad (24)$$

Taking square and expectations on the both sides of (24), we get

$$MSE(t_{9G}) \approx \overline{Y}^2\theta[C_y^2 + \frac{g^2}{4}(a^2C_x^2 +bC_z^2(b+2aK_{xz}) - g(aK_{yx}C_x^2+bK_{yz}C_z^2))]. \quad (25)$$

Partially differentiating (25) w.r.t. a and b, equating to zero, the optimal values of a and b are obtained as

$a = \frac{2C_y(\rho_{yx}-\rho_{yz}\rho_{zx})}{gC_x(1-\rho_{xz}^2)}$ and $b = \frac{2C_y(\rho_{yz}-\rho_{yx}\rho_{zx})}{gC_z(1-\rho_{xz}^2)}$

respectively, after substituting the values of a and b in mean square error of t_{9G} the minimum MSE of t_{9G} is denoted by $(t_{9G})_{min}$, and is given by

$$MSE(t_{9G})_{min} = \overline{Y}^2\theta C_y^2[1- \frac{(\rho_{yx}^2+\rho_{yz}^2-2\rho_{yx}\rho_{xz}\rho_{yz})}{(1-\rho_{xz}^2)}]. \quad (26)$$

Likewise, the mean square error and bias of t_{10G} may be obtained as

$$MSE(t_{10G}) \approx \overline{Y}^2\theta[C_y^2 + \frac{g^2}{4}(c^2C_x^2+ dC_z^2(d+2cK_{xz}) + g(cK_{yx}C_x^2+dK_{yz}C_z^2))]. \quad (27)$$

$$Bias(t_{10G}) \approx \frac{\overline{Y}\theta}{8}[g^2(C_z^2(2cdK_{xz}+d^2) +c^2C_x^2) - 4g(cK_{yx}C_x^2+dK_{yz}C_z^2)], \quad (28)$$

Partially differentiating (27) w.r.t. c and d, equating to zero, the optimal values of c and d are obtained as

$c = \frac{-2C_y(\rho_{yx}-\rho_{yz}\rho_{zx})}{gC_x(1-\rho_{xz}^2)}$ and $d = \frac{-2C_y(\rho_{yz}-\rho_{yx}\rho_{xz})}{gC_z(1-\rho_{xz}^2)}$

respectively, after substituting the values of c and d in mean square error of t_{10G}, the minimum MSE of t_{10G} is denoted by $(t_{10G})_{min}$, and is given by

$$MSE(t_{10G})_{min} = \overline{Y}^2\theta C_y^2[1+ \frac{3(\rho_{yx}^2+\rho_{yz}^2-2\rho_{yx}\rho_{xz}\rho_{yz})}{(1-\rho_{xz}^2)}]. \quad (29)$$

2.1 Special Cases of Proposed Estimators

The several estimators may be obtained for the different values of a,b,c and d.

1. For $a=1$ and $b=1$, the following exponential ratio-cum-ratio type estimator may be obtained from proposed estimator (t_{9G}).

$$t_9 = \overline{y}exp[\frac{\overline{x}^* - \overline{X}}{\overline{x}^* + \overline{X}} + \frac{\overline{z}^* - \overline{Z}}{\overline{z}^* + \overline{Z}}], \quad (30)$$

2. For $c=1$ and $d=1$, the following exponential product-cum-product type estimator may be obtained from proposed estimator (t_{10G}).

$$t_{10} = \overline{y}exp[\frac{\overline{X} - \overline{x}^*}{\overline{X} + \overline{x}^*} + \frac{\overline{Z} - \overline{z}^*}{\overline{Z} + \overline{z}^*}], \quad (31)$$

Note that various estimators may be obtained in the family of ratio-cum-ratio type estimator and product-cum-product type estimator for the different values of a,b and c,d respectively.

3 Efficiency Comparison

This section provides theoretical comparison between some existing estimators of population mean and the proposed estimator with respect to mean square error. The comparison of exponential ratio-cum-ratio type estimator is made with $\overline{y}, t_1, t_3, t_5$ *and* t_7 and exponential product-cum-product estimator is made with $\overline{y}, t_2, t_4, t_6$ *and* t_8. In simple random sampling without replacement the variance of sample mean $\overline{y}$ is $V(\overline{y}) = \theta S_y^2$. Reddy [10] has proved that in repetitive surveys K_{xz} is stable so, we obtained the conditions (32) - (41) in terms of K_{xz} . Grover et al. [5] has noticed that K_{xz} has range $(-\infty, 0)$ when both variables have positive values and negative correlation coefficient (ρ_{xz}) . Similarly,K_{xz} has range $(0, \infty)$ for positive correlation coefficient (ρ_{xz}). Keeping this in mind we can define the preference region of proposed estimators

$MSE(t_9) < V(\overline{y})$if

$$K_{xz} \ \varepsilon \ (0, (\frac{2K_{yz}}{g} - \frac{1}{2}) + \frac{C_x^2}{C_z^2}(\frac{2K_{yz}}{g} - \frac{1}{2})) \quad (32)$$

$MSE(t_9) < MSE(t_1)$ if

$$K_{xz}\ \varepsilon\ (0, (\frac{2K_{yz}}{g} - \frac{1}{2}) + \frac{C_x^2}{C_z^2} (\frac{2K_{yz}}{g} - \frac{1}{2}) + \frac{2C_x^2}{C_z^2 g^2}(1-2k_{yx})) \quad (33)$$

$MSE(t_9) < MSE(t_3)$ if

$$K_{xz}\ \varepsilon\ (0, (-\frac{1}{2} + \frac{2K_{yz}}{g}) - \frac{C_x^2}{C_z^2} (\frac{1}{2} - \frac{1}{2g^2}) + \frac{2C_x^2 K_{yx}}{gC_z^2}(1-\frac{1}{g})) \quad (34)$$

$MSE(t_9) < MSE(t_5)$ if

$$K_{xz}\ \varepsilon\ (0, (-\frac{1}{2} + \frac{2K_{yz}}{g}) - \frac{C_x^2}{2C_z^2} - \frac{K_{yx}C_x^2}{C_z^2}(1-\frac{2}{g})) \quad (35)$$

$MSE(t_9) < MSE(t_7)$ if

$$K_{xz}\ \varepsilon\ (0, (-\frac{1}{2} + \frac{2K_{yz}}{g})) \quad (36)$$

$MSE(t_{10}) < V(\bar{y})$ if

$$K_{xz}\ \varepsilon\ ((-\frac{1}{2} + \frac{2K_{yz}}{g}) - \frac{C_x^2}{C_z^2}(\frac{1}{2} + \frac{2K_{yx}}{g}), 0)) \quad (37)$$

$MSE(t_{10}) < MSE(t_2)$ if

$$K_{xz}\ \varepsilon\ ((-\frac{1}{2} + \frac{2K_{yz}}{g}) - \frac{C_x^2}{C_z^2} (\frac{1}{2} + \frac{2k_{yx}}{g}) - \frac{2}{g^2}(2K_{yx}-1), 0)) \quad (38)$$

$MSE(t_{10}) < MSE(t_4)$ if

$$K_{xz}\ \varepsilon\ (-\frac{1}{2}(1-\frac{1}{g^2}) - \frac{2K_{yz}}{g}(1-\frac{1}{g}) - \frac{C_x^2}{C_z^2}(\frac{1}{2} + \frac{2K_{yx}}{g}), 0) \quad (39)$$

$MSE(t_{10}) < MSE(t_6)$ if

$$K_{xz}\ \varepsilon\ ((-K_{xz} - \frac{C_x^2}{C_z^2} + \frac{3}{2} + \frac{4K_{yz}}{g}), 0) \quad (40)$$

$MSE(t_{10}) < MSE(t_8)$ if

$$K_{xz}\ \varepsilon\ (-\frac{C_x^2}{C_z^2}(\frac{1}{2} - \frac{2K_{yx}}{g}), 0) \quad (41)$$

When the condition (32) to (41) is fulfilled, the suggested ratio-cum-ratio and product-cum- product exponential estimator will be more competent in comparison with available estimators.

4 Empirical Study

This section includes empirical study for suggested estimators. This study has been performed using three real populations (see appendix for parameters). The description of the populations is given by

Population I [Ahmed [1]]
y : No of literate persons
x : No of cultivators
z : Total population

Population II [Population census report of Multan district in 1998, Pakistan]
y : Population matric and above
x : Primary but below matric
z : Population of both sexes

Population III [Gujarati [6]]
y : Average miles per gallon
x : Top speed, miles per
z : Cubic feet of Cab space

The optimum values for the constants, MSE and relative efficiencies are given in Table 4.1- 4.3. The comparison of each estimator has been done with mean per unit estimator.

Table 4.1: Optimum Values of the Constants

Population	a	b	c	d
1	1.23	1.32	*	*
2	0.028	3.07	*	*
3			4.87	1.53

*Data is not applicable

For the evaluation, the classical ratio and product estimator, dual to classical ratio-type estimator proposed by Srivenkatramana [13], dual to classical product-type estimator by Bandyopadhyay [3], exponential ratio-type estimators and product-type estimators proposed by Bahl-Tuteja [2], exponential dual to ratio type estimator by Sharma and Tailor [14] and Bahl-Tuteja [2] using transformed auxiliary variables have been considered.

Table 4.2: Mean Square error (MSE) for the Estimators

Estimators	1	2	3
$\bar{y}$	216.96	3383.92	1.82
t_1	44.39	2310.56	*
t_3	65.71	2178.14	*
t_5	39.62	2074.34	*
t_7	93.41	2435.25	*
t_9	32.77	1325.29	*
t_{9G}	23.92	600.018	*
t_2	*	*	1.85
t_4	*	*	1.57
t_6	*	*	1.61
t_8	*	*	1.59
t_{10}	*	*	1.26
t_{10G}	*	*	0.66

The percentage relative efficiencies have been calculated by applying the following formula:

$$PRE = \frac{VAR(\bar{y})}{MSE(t_*)} \times 100 \qquad (42)$$

Table 4.3: Percentage Relative Efficiencies for the Estimators

Estimators	1	2	3
$\bar{y}$	100	100	100
t_1	488.77	146.45	*
t_3	330.18	155.36	*
t_5	547.58	163.14	*
t_7	232.26	138.96	*
t_9	662.11	255.34	*
t_{9G}	907.16	563.97	*
t_2	*	*	98.27
t_4	*	*	115.68
t_6	*	*	112.61
t_8	*	*	114.18
t_{10}	*	*	144.30
t_{10G}	*	*	274.51

5 Conclusion

From Table 4.3, we can see that the suggested exponential ratio-cum-ratio type estimator is competent as compare to classical ratio-type estimator, dual to classical ratio-type estimator proposed by Srivenkatramana [13], exponential ratio estimator introduced by Bahl and Tuteja [2] and exponential dual to ratio-type estimator suggested by Sharma and Tailor [14]. Similarly we can see that the suggested exponential product-cum-product type estimator is more efficient than classical product-type estimator, dual to classical product-type estimator proposed by Bandyopadhyay [3], exponential product estimator given by Bahl and Tuteja [2] and adapted estimator of Bahl and Tuteja [2] using transformed auxiliary variables. According to the results of population 1-2 we may conclude that suggested generalized exponential ratio-cum-ratio type estimator is efficient on comparison with all considered estimators. Further for population 3, proposed generalized form of exponential product-cum-product type estimator has shown improved performance as compare to all considered product type estimators.

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Appendix:

Table A: Parameters of Populations

Population

Parameter	**1**	**2**	**3**
N	376	424	81
n	159	169	33
$\overline{Y}$	316.65	646.215	33.8345
$\overline{X}$	141.13	4533.981	112.4568
$\overline{Z}$	1075.31	325.0325	98.7654
C_y	0.7721	1.509	0.2972
C_x	0.845	1.342	0.1256
C_z	0.7746	1.335	0.2258
ρ_{yx}	0.9106	0.623	-0.6908
ρ_{yz}	0.9094	0.907	-0.3683
ρ_{xz}	0.8614	0.682	-0.0427

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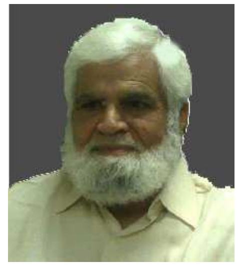

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