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Analysis of EEG Signals using Nonlinear Dynamics and Chaos: A review

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Abstract: Nonlinear dynamics and chaos theory have been used in neurophysiology with the aim to understand the complex brain activity from electroencephalographic (EEG) signals. Although linear methods have been the most used in EEG analysis, nonlinear approaches have been increased their presence because they reveal aspects that cannot be measured from linear approaches. However, published works in this scientific field is still very low. This work describes the fundamentals of EEG signals and its basic concepts related with nonlinear dynamics and chaotic measures of complexity and stability. After that, a short review of the most common EEG-based applications is given in medical and non-medical contexts.

Keywords: Nonlinear analysis, Chaos, Dynamical Systems, EEG, Complexity, Stability.

This paper is dedicated to the memory of Professor José Sousa Ramos.

1 Introduction

Many complex real-world phenomena are characterized by nonlinear dynamics and the chaos theory [1]. This important mathematical subject has aroused great interest in a lot of scientific fields as physics, chemistry, economics, electronics, biomedical engineering, just to name a few. From the first studies of Pointcaré in 1890 [2], distinguished mathematicians have significantly contributed in the field of chaotic dynamics, such as Birkhoff, Kolmogorov, Cartwright, Littlewood, Smale, Lorenz, Mandelbrot, among others. At this point, we would like to highlight the recent notable contributions of Prof. Jose Sousa-Ramos in this research field [3,4,5,6] and its multidisciplinary applications, ranging from electronic circuits [7,8], economics [9] and biological systems [10].

Since biomedical data can be properly acquired through sensors and peripheral devices, the analysis of biosignals [11], which reflects typically complex dynamics, has been widely studied in the area of nonlinear analysis. During the last years, nonlinear dynamic methods have been successfully used in biomedical applications based on electrocardiogram (ECG), electromyogram (EMG), electrooculogram (EOG), magnetoencephalogram (MEG) and electroencephalogram (EEG) data. In particular, this work is focused in nonlinear brain dynamics. As it is widely accepted [12,13], a brain is considered a chaotic dynamical system and, then, their generated EEG signals are generally chaotic. Besides that, an EEG signal is chaotic in another sense, because its amplitude changes randomly with respect to time. In this review paper, we give the mathematical background of the most widely-used nonlinear dynamic methods for EEG data and, also, an state-of-the-art of some of the most relevant and recent EEG-based applications with nonlinear methods.

The rest of this paper is organized as follows. Section 2 describes the fundamentals of EEG signals and, then, in section 3, the basic concepts related with nonlinear dynamics and chaos are introduced for time series analysis. Section 4 describes the notions of the most popular nonlinear methods to measure the level of chaos in EEG data: measures of complexity (correlation and fractal dimension) and stability (Lyapunov exponents and entropy). In section 5, we give a short review about applications of nonlinear analysis and characterization of

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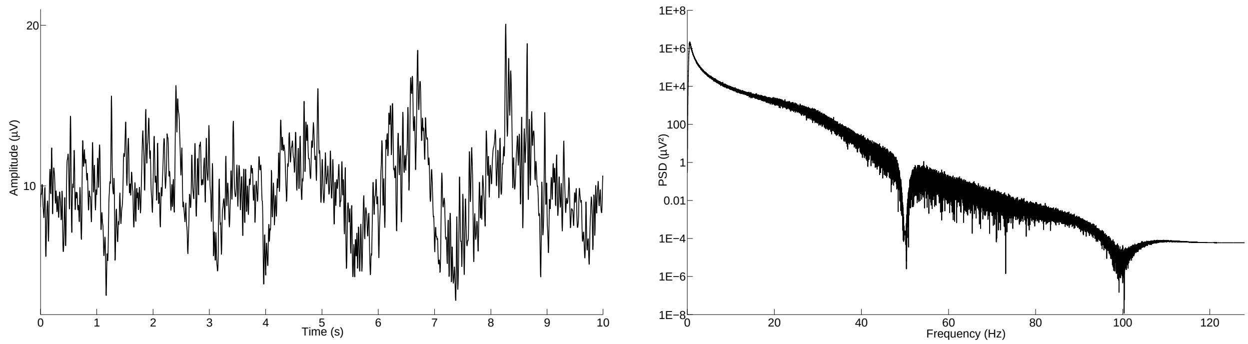

(a) (b)

Fig. 1: EEG signal in the time (a) and frequency (b) domain.

EEG signals. Finally, the paper ends with the main concluding remarks.

2 EEG signals: notions

The EEG signals measure the electrical activity of the brain [14], which is recorded at many locations simultaneously by one electrode at each position on the human scalp (the term *channel* is usually used to refer to a recording position). Note that EEG signals are electrical potentials with respect to a reference electrode (usually placed at the earlobe) and the number of required electrodes depends on the application (from 2 to 128 positions). As the recorded signals are in the order of ± 100 μV, acquired EEG values are amplified (e.g. to ± 5 V) before the signals are sampled. Sampling frequencies above 256 Hz are enough to typical EEG signals, which have frequency components of 0 Hz to approximately 100 Hz. The standard EEG frequency bands are the delta (0.1 to 3.5 Hz), theta (4 to 7.5 Hz), alpha (8 to 13 Hz), and beta (14 to 30 Hz) bands [14,13]. EEG signals with frequencies greater than 30 Hz are known as gamma waves and they have been found in the cerebellar structures of animals. In general, an EEG signal has complex behavior with nonlinear dynamic properties and it can be represented after digitization as a sequence of time samples [13]. Figure 1(a) shows a time series of 10 seconds duration recorded via an EEG channel; and its corresponding Power Spectral Density (PSD) is shown in Figure 1(b). As we can observe in Figure 1(b), the most energy of the EEG signal is located below 30 Hz. The same figure also shows the effects of a notch filter at 50 Hz, which is typically used for avoiding artifact caused by power line interference.

2.1 Linear and nonlinear analysis of EEGs

As it is explained in Section 5, there are a broad range of cutting-edge EEG-based applications and, depending on the specific application, different relevant descriptors (also known as *features*) have to be extracted from EEG signals, which can be generally divided in two main feature categories [13]: *linear* and *nonlinear*.

Linear analysis of EEG signals includes frequency analysis (e.g. Fourier and Wavelet Tranforms) and parametric modeling (e.g. autoregressive models). In general, linear methods can be successfully applied in the study of several problems [15,16,17,18,19,20,21]. However, despite good results have been obtained with linear techniques, they only provide a limited amount of information about the electrical activity of the brain because they ignore the underlying nonlinear EEG dynamics. As it is widely accepted, the underlying subsystems of the nervous system that generates the EEG signals are considered nonlinear or with nonlinear counterparts [22]. Even in healthy subjects, the EEG signals show the chaotic behavior of the nervous system. Therefore, due to this nonlinear nature of EEGs, additional information provided by techniques from nonlinear dynamics has been progressively incorporated in order to reveal aspects that cannot be measured from linear methods [23]. Nonlinear dynamic measures of complexity (e.g., the correlation dimension) and stability (e.g., the Lyapunov exponent and Kolmogorov entropy) quantify critical aspects of the brain dynamics. Before describing the most widely-used nonlinear methods in Section 3, the basic concepts related with chaos are now introduced.

3 Basic concepts of nonlinear dynamics and chaos theory

Given a dynamical system [24], its *state* is given by a set of values of all variables that describe the system at a particular time; while, its *dynamics* is a set of ordinary differential equations (for continuous-time dynamical system) or a mapping function (for discrete-time dynamical system) that describes how the state changes over time. According the nature of the dynamics, we can distinguish between *linear* and *nonlinear* dynamical systems [1,24]. A dynamical system is linear if all the corresponding equations of its dynamics are linear; otherwise, it is nonlinear.

Considering that a system is defined by m variables, its state in a particular moment in time can be represented by a point in an m-dimensional space [1]. This space is usually known as *state space* or *phase space*. The sequence of consecutive states over the time defines a curve in the phase space which is called *trajectory*. In some cases, after observing the evolution of a dynamical system for a sufficiently long time, its trajectory tends to converge to a bounded subspace of the phase space. This kind of dynamical system is known as dissipative: systems with a volume contraction in the phase space. This bounded subspace is referred to as an *attractor*, because it attracts trajectories from all initial conditions. According to the resulting geometrical object, attractors can be grouped in [1,25]:

–*Steady State (Fixed Point)*. The attractor evolves towards a point (steady state), whatever the initial conditions. A classical example is a damped pendulum.

–*Limit Cycle*. The attractor is a closed one-dimensional curve, which represents a periodic motion. An example is the heartbeat while resting.

–*Limit Torus*. The attractor is a toroidal surface (in an integer dimension). It represents a quasiperiodic motion with an integer number of incommensurable frequencies.

–*Strange or Chaotic*. The system exhibits complex behaviors (chaos) and its attractor is a complex object. In this case, points that are initially close in the phase space, may become exponentially separated after time. The dynamics corresponding to a strange attractor is deterministic chaos: same initial conditions converge to same final state; but the final state is very different for small changes to initial conditions.

To characterize attractors, and then the corresponding dynamics of the system, different measures can be used. In one hand, the dimension of the attractor measures the spatial distribution of the corresponding geometrical object, i.e., its ‘complexity’. A point attractor has dimension zero, a limit cycle is one-dimensional, a torus has an integer dimension corresponding to the number superimposed periodic oscillations and, lastly, a strange attractor has a non integer dimension, i.e., a *fractal dimension*. The dimension do not give information on the evolution of trajectories over time and, then, it is an static measure. There are several techniques for estimating the dimension of the attractor [26], being the correlation dimension (D_2) the most popular approach. On the other hand, there are dynamic measures, such as Lyapunov exponents [27] and entropy measures, that give information about the ‘stability’ of the attractor, i.e., quantify the chaos of the attractor. Among the different available methods used to study dynamical systems in the state space, the next section describes the most popular and widely-used nonlinear approaches for EEG signal processing.

4 Nonlinear dynamic analysis

In a EEG-based study, there is a set of observations in the form of an EEG record, i.e., a time series of the electrical activity of the brain. The nonlinear dynamic analysis with time series entails two main steps: (i) reconstruction of the dynamics in state space from observations; (ii) characterization of the resulting attractor by nonlinear dynamic measures. Once these measures have been computed, this information can be used as characteristic features of the analyzed EEG signals in the corresponding application.

The aim of this section is to give a brief and intuitive explanation of these two steps of nonlinear dynamic analysis. A more extensive and detailed study can be found in [28,29]. With respect to available software implementations, the TISEAN project (http://www.mpipks-dresden.mpg.de/~tisean) and the TSTOOL package (http://www.physik3.gwdg.de/~tstool/) can be outlined.

4.1 Embedding: reconstruction of the state space

The technique of representing a state space of a dynamic system from a single time series is called *state space reconstruction*, or *embedding of the time series*. There are two main approaches for reconstructing the state space: (i) *time-delay embedding* and (ii) *spatial embedding*. We first describe the time-delay approach, which is the most extended procedure in practice for nonlinear dynamical analysis of EEG.

In the case of the *time-delay embedding*, let x_t be an instantaneous measure of the dynamical system, i.e., a sample of the time series obtained by sampling a given variable of the system. Note that, for our interests, the dynamical system is the neural networks of the brain and the time series is given by the EEG signal. An m-dimensional state space reconstruction with the time-delay approach is

$$\mathbf{x}_t = \left(x_t, x_{t+\tau}, \cdots, x_{t+(m-1)\tau}\right) \quad (1)$$

The *lag* or *delay time*, τ, is the time difference between the successive components of the state vector $\mathbf{x}_t$, and m is the *embedding dimension*. The sequence of the embedding vectors given by (1) forms the reconstructed attractor in the state space as t increases. Thus, time-delay embedding is characterized by two parameters: the time lag, τ, and the embedding dimension, m. The selection of both parameters is a key and difficult step in nonlinear analysis. Since an inappropriate election could lead to wrong results, several criteria have been proposed in practice. With respect to τ, in one hand, if it is very small, the m components of $\mathbf{x}_t$ will be very close and the geometry of the attractor could be lost. On the other hand, if τ is very high, the components of each embedding vector will become totally unrelated to each other. In practice, τ is usually chosen as the first minimum of the mutual information between the components of the vectors in the state space or as the first zero of the time-domain autocorrelation of the data [30]. The value of m has to be chosen in order to the dynamics of the system in the state space are preserved. According to Taken's theorem [31], if the underlying state space of a system has d 'true' dimensions, the embedding dimension should be chosen at least twice the dimension of the attractor, i.e., $m > 2d$. In this case, one first criteria is to take $m > 2D_2$ [31], but it assumes a previous estimation of the correlation dimension, D_2. A possible and pragmatic solution is to repeat the computation of D_2 for increasing values of m until the Taken's criterion is fulfilled. Nevertheless, m and τ are interdependent and, then, the estimations of each parameter depend on the combination of both [32]. For solving it, the *false nearest neighbors* method [33] provides an estimation of a minimum m. The main idea is that the calculation must be repeated if for a given m nearest neighbors in the state space still remain close for a dimension $m+1$. Otherwise, the attractor is not correctly reconstructed and m must be higher. This procedure is repeated until neighbors remain close.

In contrast to the time-lag approach, the *spatial embedding* procedure can be realized when m time series of independent EEG signals are available instead of a single one. In this approach, the m components of each vector in the state space are given by the m values of each time series at a particular time [34,35]. Then, the embedding dimension m is equal to the number of EEG channels and there is an equivalence between the inter electrode distance and the time lag, τ. Using the spatial embedding procedure, it constructs an unique attractor representing the neuronal dynamics [36]. Other option would be to perform an individual time-delay embedding on each of the m time series, i.e., a different attractor for each EEG signal. The major drawback of the spatial embedding approach is the 'spatial lag' (i.e., the distance between EEG channels), which is typically fixed depending on the application and, then, it cannot be optimally selected [23]. In general, it is not possible to establish which embedding approach is better. However, it should be emphasized that the scientific community usually prefers the time-delay approach because it allows to study the interactions between different brain regions [37] and, also, the use of spatial embedding has been debated in literature [34,38,39].

4.2 Characterization of the attractor: nonlinear measures

Once the equivalent attractor in the state space has been reconstructed, the next step is to characterize it using nonlinear measures of its complexity and its stability. Here, these nonlinear measures are described by assuming the time-delay approach.

4.2.1 Correlation dimension

The correlation dimension (D_2) is a measure of complexity of a dynamical system related with the topological dimension of its attractor. It is an estimation of the fractal dimension of the attractor. D_2 is based upon the correlation integral, $C(r)$, which is a function of variable distances r defined as:

$$C(r) = \lim_{N\to\infty} \frac{1}{N(N-1)} \sum_{i=0}^{N-1} \sum_{j=i+1}^{N-1} \Theta\left(r - \left|\mathbf{x}_i - \mathbf{x}_j\right|\right) \quad (2)$$

where N is the number of data points (i.e. the length of the reconstructed attractor) and Θ is the Heaviside function. Thus, $C(r)$ is a measure of the probability that pairwise points ($\mathbf{x}_i$ and $\mathbf{x}_j$) in the attractor will be separated is less than or equal to a distance r. In [40], it is proposed that the vectors to be compared when $C(r)$ is computed should be separated at least w data points ($|i-j| > w$) in order to correct for autocorrelation effects in the time series.

According to [41,42], $C(r)$ follows this relation:

$$C(r) \propto r^{D_2}, \quad (3)$$

and, then, the correlation dimension, D_2, can be estimated

$$D_2 = \lim_{r\to 0} \frac{\log(C(r))}{\log(r)}, \quad (4)$$

if the number of points and the embedding dimension are sufficiently large. As the topology of the attractor is usually unknown, it is necessary to calculate $C(r)$ for different values of m and, for deterministic signals, the convergence of the computation of D_2 can be reached. Different enhancements have been proposed in order to compute D_2 in a faster way [43,44,45,46] and to reduce the amount of noise in the signals [47,48,49].

4.2.2 Additional measures for computing fractal dimensions

Besides D_2, many other methods have been also proposed for computing the fractal dimension of the attractor [50, 51,52]. Among them, the following measures can be highlighted:

–*Katz dimension*. According to [53], the fractal dimension of the EEG signal can be computed as

$$D_K = \frac{\log_{10}(s)}{\log_{10}(s) + \log_{10}(a/L)}, \tag{5}$$

where L is the length of the EEG time series, a is the planar extent of the signal and $s = L/\bar{A}$ is the number of steps of the waveform, being $\bar{A}$ is the average distance between successive points. Given that $dist_{a,b}$ denotes the distance measured between $\mathbf{x}_a$ and $\mathbf{x}_b$, the length of the waveform (L) is given by the sum of the distances between two consecutive points, $L = sum(dist_{i,i+1})$; and the planar extent is $a = max(dist_{1,i})$.

–*Higuchi dimension*. Higuchi's method [54] estimates the fractal dimension of a sample as follows: first, data subsets are constructed from the time series data composed of N samples:

$$\mathbf{x}_k^j = \{x_{j+ik}\}_{i=0}^{\lfloor (N-j)/k \rfloor}, \tag{6}$$

where $j \in [1,k]$ is the initial time and $k \in [1,k_{max}]$ the delay between points. Note that k_{max} is a parameter to be experimentally chosen (p.e. Higuchi originally fixed $k_{max} = 8$). Then, the length of each subset is computed by:

$$L_j(k) = \frac{\sum_{i=1}^{\left\lfloor \frac{N-j}{k} \right\rfloor} |x_{j+ik} - x_{j+(i-1)k}| (N-1)/(\lfloor (N-j)/k \rfloor k)}{k \left\lfloor \frac{N-j}{k} \right\rfloor}, \tag{7}$$

being N the length of the time series and $\left\lfloor \frac{N-j}{k} \right\rfloor$ a normalization factor. Total average length, $L(k)$, is computed for all time series for each k (ranging from 1 to k_{max}): $L(k) = \sum_{j=1}^{k} L_j(k)$. Finally, according to the Higuchi's method [54], the fractal dimension (D_H) is solved from:

$$L(k) \propto k^{-D_H} \tag{8}$$

Thus, in the representation of $\ln(L(k))$ with respect to $\ln(1/k)$, the estimate of D_H is given by the slope of the least-squares linear fit.

–*Petrosian dimension*. In this method [55], the EEG signal is first converted to a binary signal according to a predefined procedure. For example, a common procedure is the following: the differences between consecutive samples are equal to one or zero depending on whether it exceeds or not a standard deviation magnitude. Once the binary signal has been constructed, the fractal dimension is:

$$D_P = \frac{\log_{10}(L)}{\log_{10}(L) + \log_{10}(\frac{L}{L+0.4B})}, \tag{9}$$

where L is the length of the signal and B is the number of bit changes in the resulting binary sequence.

Finally, we would like to outline the *Hurst Exponent* [56,57,58], which is used to evaluate the long-memory dependence and its degree in a time series. The Hurst exponent, H, is a measure of the smoothness of a time series data based on the asymptotic behavior of the rescaled range of the process [59] and it is given by:

$$H = \frac{\log_{10}(R/S)}{\log_{10}(T)}, \tag{10}$$

where T is the duration of the time series data and R/S is the rescaled range, which characterizes the divergence of time series, defined as the range of the mean-centered values for a given duration (T) divided by the standard deviation for that duration. From [56], $R/S \propto T^H$. It should be noted that there is a linear relationship between the fractal dimension (D), a measure of roughness, and the Hurst coefficient ($0 \leq H \leq 1$): $D + H = 1 + E$, where E is the Euclidean dimension. The more jagged the EEG signal, the closer its Hurst coefficient will be to 0 [58].

4.2.3 Lyapunov exponents: Measuring stability

In a chaotic attractor, trajectories typically evolves following two steps: (i) *expansion process*, the trajectories diverge exponentially fast from similar initial conditions (nearby points in the state space); (ii) *folding process*, the trajectories will have to fold back into it as time evolves. The *Lyapunov exponents* measure the average rate of expansion and folding that occurs along the local eigen-directions within an attractor [13]. When an attractor is chaotic, the Largest Lyapunov Exponent (LLE) should be positive. A negative exponent entails that the trajectories tends to common fixed point; and a zero exponent means that the trajectories maintain their positions: they are on a stable attractor. Note that if the state space is m-dimensional, we can theoretically measure up to m Lyapunov exponents.

There are several procedures for computing the LLE from EEG data [13,23,60]. Now, we introduce the well-known Wolf's algorithm [27]. The nearest neighbor to the initial state vector of the attractor ($\mathbf{x}_{t_0} = (x_{t_0}, x_{t_0+\tau}, \cdots, x_{t_0+(m-1)\tau})$) is located, being $L(t_0)$ the distance between these two vectors. At a later time $t_1 = t_0 + T$, this initial length will be $L'(t_1)$, where T is a fixed time known as evolution time. This process is repeated by computing the successive distances until the separation is greater than a certain value (δ_{max}). Then, a new state space vector (replacement vector) is searched as close as possible to the first one. Finally, the Lyapunov exponent, which measures the mean exponential divergence of two initially nearby state space orbits, is characterized by:

$$\lambda = \frac{1}{(t_M - t_0)} \sum_{i=0}^{M} \log \frac{L'(t_i)}{L'(t_{i-1})}, \tag{11}$$

where M is the number of evolution steps. With respect to its implementation, and according to [61], an embedding dimension (m) between 5 and 20 and a delay time (τ) of 1 should be chosen for computing the Lyapunov exponent for EEG data. In addition, note that the parameters δ_{max} and T must be also tuned. Other popular procedure for calculating the Lyapunov exponent is the Rosenstein's method [62].

4.2.4 Entropy

The entropy of an attractor is the rate of information loss of its dynamics [23]. When the LLE is positive, the rate of expansion is greater than the rate of folding (i.e. a production rather than destruction of information) [13], the Lyapunov exponents are strongly related with concept of entropy. In fact, the Kolmogorov entropy is equal to the sum of all positive Lyapunov exponents [63]:

$$K_2 = \sum_{\lambda>0} \lambda_i, \tag{12}$$

and a positive entropy denotes chaos. Entropy has been computed in different formals [13], such as (i) *Kolmogorov entropy* [42] and (ii) *approximate entropy* (ApEn) [64], both are descriptors of the changing complexity in embedding space; (iii) *spectral entropy*, which evaluates the energy distribution in wavelet subspace [65] or uniformity of spectral components [66]; and (iv) *amplitude entropy*, a direct uncertainty measure of the signal in the time domain [67].

5 Review of neuroscience applications with nonlinear methods

The research of EEG signals with nonlinear methods is able to open a new window to understand the brain function, i.e, to analyze the dynamical properties underlying the acquired EEG in subjects. This nonlinear analysis is applied in healthy subjects in different scenarios, during no-task resting states, perceptual processing, performance of cognitive tasks and different sleep stages [23]. Also it has been used to detect abnormal function of the brain like seizures, dementia, schizophrenic, depression, autism, Alzheimer's disease, Creutzfeldt-Jakob's disease and to detect and quantify toxic states [23]. Besides medical applications, the information extracted in this type of analysis has been used for non-medical purposes [68,69,70]. This section gives to the reader an overview of the state of the art of the neuroscience applications with nonlinear methods. It is important to remark that there are applications that have been developed along the last twenty years; while others have been recently developed and, therefore, there are few works published about them. In this work, we have classified some of the most common applications in medical and non-medical contexts.

5.1 Medical applications

It is assumed that the EEG must reflect the dynamic of the brain and, of course, psychiatric disorders and pathological states. Additionally, it is widely accepted the use of EEG analysis for early detection of several brain disorders and diseases, such as epilepsy, autism, depression and Alzheimer, and for measuring the depth of anesthesia [71].

5.1.1 Epilepsy

Epilepsy is a neurological disorder in which patients suffer spontaneous seizures. In each seizure, brain produces unexpected electrical discharges in a oscillatory state [72]. It is a common neurological disorder: about 60 million people worldwide are affected and suffered recurrent seizures [12,73]. The most common and traditional analysis is still the visual inspection of the EEG signals by experienced professionals. Fortunately, in recent years, there have appeared scientific papers that present results by applying signal processing techniques to predict epileptic seizures in a efficient, automatic and objective way [12].

From the different proposed methods, linear approaches do not allow to detect the previous changes in EEG to seizures due to fact that the the brain activity is a dynamic system and the epileptic neuron is inside nonlinear networks with nonlinear responses. The nonlinear analysis and quantification of EEG signals could detect changes in the brain activity and, then, get enough information to predict seizures [12]. Different techniques have been proposed along the last years to improve the prediction and detection of seizures. One of the most used methods to detect seizures is to compute the LLE. It was introduced in 1991 to predict seizures by Iasemedis *et al.* [72]. Along the last years, LLE have been used alone or combined with other methods. In 2007, Adeli *et al.* [74] presented a 'wavelet-chaos methodology' for analysis five sub-bands of EEGs for detection of seizure and epilepsy. The nonlinear dynamics of the original EEGs were quantified in the form of the D_2 and the LLE. It was concluded that, in the higher frequency beta and gamma subbands, the D_2 differentiates among three different groups: healthy subjects, epileptic subjects during a seizure-free interval and epileptic subjects during a seizure (ictal EEG); whereas in the lower frequency alpha subband, the LLE differentiates between the three groups. Other nonlinear measure have been applied to epilepsy, such as fractal dimension. As an example, in 2011, Easwaramoorthy *et al.* [75] proposed an improved version of generalized fractal dimension to discriminate between healthy and epileptic subjects. Le Van Quyen *et al.* introduces the correlation density [76] and, besides that, his research group proposed new techniques, such as the 'dynamical similarity index' [77], that compares and gives a measure of similarity between two signals captures in different times. In 2005, Kannathal *et al.* [78] compared different entropy estimators when applied to EEG data from normal and epileptic subjects.

In addition, many studies and experiments about epilepsy are developed using ECoG signals [12]. ECoG signals are captured using intracraneal sensors that provide clean signals with high amplitude. In contrast, EEG signals are captured placing sensors over scalp and collect noise from the environment and low amplitude signal but they allow to measure the electrical brain activity without clinical intervention. This is the reason that there are different works that clean the noise of EEG signals in order to enhance the seizure and epilepsy detection [12]. More recently, Quang *et al.* [73] combined the ICA (Independent Component Analysis) method and the LLE to clean the EEG signal.

5.1.2 Depth of anesthesia

The monitoring of anesthesia is very important to provide an adequate level that ensures a safety and comfortable scenario to work during a medical intervention. We also note that a high level of anesthesia could produce an over dosing effects, but a low level of substance is contraindicated because patients could suffer intra-operative awareness [79]. Therefore, it is needed to obtain an objective and quantitative measure of the depth of anesthesia (DA) in the operating room.

In the last 10 years, several linear and nonlinear methods have been applied to this matter. This work is focused in nonlinear methods and, then, they are now exposed in some examples. In 2006, Jordan *et al.* [80] introduced approximated entropy combined with the weighted spectral median frequency and got an EEG indicator based on fuzzy logic that let to separated wakefulness from unconsciousness patients. In 2007, Ferenets *et al.* [81] introduced spectral entropy combined with approximated entropy, Higuchi's fractal dimension, Lempel-Ziv complexity, relative β ratio, and the SyncFastSlow measure to evaluate the effect of remifentanil in EEG measures to detect the DA. In 2008, Roca et al. [82] evaluated Lyapunov exponents to get a short-term predictability from EEG signals. In 2012, Klockars et al. [83] used spectral entropy as a measure of depth of hypnosis and the hypnotic drug effect in children during total intravenous anesthesia. They reported a an age influence and recommended to use an other complementary indicators to the doctors.

The industry of medical equipment have developed different standard indexes, such as the Patient State Index (PSI) [84] or the Narcotrend monitoring [85], to help in the operating room, being the Bispectral Index Score (BIS) the reference procedure [79]. As a example to estimate the BIS index, Ahmadi *et al.* [86] compared the correlation dimension and the Higuchi's fractal dimension, by obtaining in general better results using the second approach. However, Errando *et al.* [87] published that, despite the use of these medical equipments, incidence of awareness continues. Due to these drawbacks, research about EEG to monitoring the DA has a long way to run along the next years.

5.1.3 Autism

Autism disease (ASD) is defined as a disorder of neural development characterized by impaired social interaction and verbal and non-verbal communication, and by restricted, repetitive or stereotyped behavior [79].

The use of chaos theory to extract features to analyze ASD is relatively recent [79]. A very active group in this field is Ahmadlou's group. In 2010, Ahmadlou *et al.* [88] introduced a new methodology called 'Fractality and a Wavelet-Chaos-Neural Network' for ASD diagnosis. In [88], they evaluated the use of Fractal dimension computed by Higuchi or Katz's method and got better results with the Katz's aproach. More recently, in 2012, [89] proposed a diagnosis system more robust which improved performance of the power of scale-freeness of visibility graph. As a final example of published works about analysis of EEG signal in ASD, in 2011, the group of Catarino [90] applied a multiscale entropy analysis in order to test difference in complexity between people with ASD and healthy subjects, obtaining a positive response.

5.1.4 Depression

Depression is a mental disorders that produces low mood, lack of interest, low self-esteem, and poor concentration. Sometimes depression could even produce the suicide of the patients. It is one of the most common brain disorder and affects about 121 million people worldwide: it is expected that this number will be increased in the future [91]. The quantitative analysis of EEG signals reflects objective information about the changes in brain activity produced by depression [91,92].

Mostly of results have been obtained using linear methods, such as band power features in order to to detect changes in frontal interhemispheric asymmetries [93,94, 95]. Although the study of frontal asymmetry is one of the most used methods, there are some doubts about it as a marker for depression [96]. Some research groups recently have applied nonlinear methods to diagnosis depression disease [91,92,97] but the number of published works is still very low. In 2012, Ahmadlou *et al.* [97] presented an investigation of the frontal brain of major depressive disorder patients using the wavelet-chaos methodology and Katz's and Higuchi's fractal dimensions as measures of nonlinearity and complexity. [97] reported that Higuchi's fractal dimension gets the better results. The study by Hosseinifard *et al.* [91] performed a nonlinear analysis of EEG signal for discriminating depression patients and normal controls.

To develop this interesting work, the power of four EEG bands and four nonlinear features (e.g. fractal dimensions and LLE) were extracted from EEG signals. Finally, Bachmann *et al.* [92] compared two EEG analysis methods for detection of depression: a linear approach, spectral asymmetry index; and a nonlinear approach, Higuchi's fractal dimension. Their results indicated that both methods got a good sensitivity for detection of characteristic features of depression.

5.1.5 Alzhehimer's disease

Alzheimer's disease (AD) is suffered by 35 million people worldwide and it is expected that the number will be increased to 115 million by the year 2050 [71]. This is a type of dementia and it is characterized by the gradual destruction of the brain cells of the patient, neurofibrillary tangles, and senile plaques in different widespread brain regions [98]. There is an intermediate step between a healthy subject and Alzheimer's disease called 'Mild cognitive impairment' (MCI) which presents symptoms. The most usual symptom is short term memory loss, but this not enough to disturb routine in an adult and sometimes people no need to go to the doctor. It is important to remark that the MCI do not have to finish in AD, because it may revert to a normal state, develop into any of several forms of dementia or even revert back to a normal state. In order to avoid the AD progression, research effort have been done to get an early detection of MCI [71].

Several studies have showed the usefulness of nonlinear methods to analyze the EEG in patients with AD [23]. Fractal dimension was introduced to detect AD in 1997 by Besthorn *et al.* [99]. After that, Jeong *et al.* [100,101]applied the LLE and the D_2 to detect AD. They claimed that the D_2 measured in the occipital region was very useful for detecting AD because it presented a lower level in patients with AD than in healthy subjects [100, 101]. Finally, entropy is one of the most used method's to diagnosis AD and different groups have reported satisfactory results in this area [98,71,102,103,104].

5.2 Non-medical applications

5.2.1 Brain computer interface

A Brain Computer Interface (BCI) system acquires and analyzes EEG signals in order to provide a direct communication and control pathway from the human brain to a computer/machine [105]. BCI research is a growing area of research and this technology has been already extended from assistive care to other non-medical uses, such as gaming, assessment of driving performance and safety/security applications [106]. According to Bashashati *et al.* [107], linear features have been widely applied in many BCI systems but there are several works that use nonlinear features. As we have no found medical BCI applications using nonlinear analisys, BCIs have been categorized in this work as a non-medical application.

One of the most used nonlinear methods is the LLE, which has been combined with other procedures in different works [108]. For example, in 2009 Banitalebi *et al.* [109] used LLE, mutual information, $D2$ and the minimum embedding dimension as the features for the classification of EEG signals to identify motor imagery of the hands and foot. Esfahani *et al.* [110] combined LLE with the power spectral density of each EEG frequency band to detect human satisfaction in human-robot interaction. In the same year, Wang *et al.* [69] proposed a new nonlinear fractal dimension based approach to neuro-feedback implementation aimed at EEG-based games design. Note that the use of fractal dimension in BCI design was previously introduced in [111].

5.2.2 Emotion recognition

A computer that recognize human's emotions could improve the communication and get a more affective environment for the user [112,113,114]. Emotion recognition (ER) and affective computing are now two growing research areas. In this fields, researchers have used different methods (such as face expression and speech analysis), and even biosignals measures (such as EEG, ECG, EMG, skin conductance, peripheral temperature [115,116].

There are works to get ER using linear and nonlinear methods. Linear method usually computed power spectrum features while nonlinear method applied several different computations. In 2009, Khalili *et al.* [115] used the International Affective Picture System (IAPS), that is a database of pictures used to elicit a range of emotions, to produce three different emotions (calm, positively excited, and negatively excited) and decided to combines EEG with galvanic skin resistance, temperature, blood pressure and respiration for ER. They used correlation dimension as a strong nonlinear feature for EEG that seem to perform better than other physiological signals. Liu *et al.* [70,117] proposed a fractal dimension-based algorithm for quantifying basic emotions and, also, described its implementation as a feedback in 3D virtual environments. Note that different sound clips from the International Affective Digitized Sounds (IADS) were used to elicit emotions. Recently, in 2013, Bajaj *et al.* [118] compute ratio of the norms based measure, Shannon entropy measure, and normalized Renyi entropy measure as features of the EEG signal obtained during audio-video stimulus with good results.

5.2.3 Mental fatigue

Mental fatigue (MF) is an usual sense that is characterized by a decreasing level of attention, sense of weariness, drowsiness and a low mental performance [119]. These symptoms could produce performance decrements in the productivity or even accidents in industrial or commercial environments. There are several scenarios in which an early detection of MF could be very interesting, such as nuclear power industry, flight pilots or car drivers [120]. There is not an unique method to measure MF and it is usually recommended to combine different biosignals, such as EEG, ECG, EOG and EMG.

In particular, analysis of EEG must reflect the changes of the brain activity produced by MF. The most extended technique is to compare the power spectrum between the frontal and the occipital lobes to get a classification of mental fatigue at different levels [121]. Recently, nonlinear methods have been applied to MF but there are a few works focused in this area. As a example, in 2010, Sibsambhu *et al.* [122] proposed a new entropy-based method for relative quantification of MF during driving tasks. Liu *et al.* [68] used approximate entropy and Kolmogorov complexity (K_2) to characterize the complexity and irregularity of EEG data under the different mental fatigue states. Both parameters were very useful due to significant drop in value with increasing MF.

6 Conclusions

As it is widely accepted, this work has considered the brain as a chaotic system and, then, we have exposed how the nonlinear methods have been successfully applied in biomedical applications. We have categorized some of the most common applications in medical and non-medical contexts. In particular, this paper analyzes epilepsy, depth of anesthesia, autism, depression and Alzheimer's disease, mental fatigue, brain computer interfaces and emotion recognition. The most used nonlinear dynamics methods for these EEG-based applications are correlation and fractal dimension as complexity measures, and Lyapunov exponents and entropy as stability measures. Finally, it is important to remark that progress in nonlinear dynamics and nonlinear time series analysis has reached a level in which fruitful EEG-based applications have become a reality for users.

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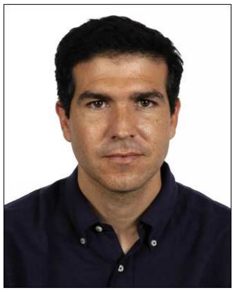

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