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Electromagnetic Field and Rotation Effects on S-waves Propagation in a Non-homogeneous Anisotropic Incompressible Medium under Initial Stress and Gravity Field

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Abstract: In this paper, shear waves propagation in a non-homogeneous anisotropic incompressible medium under influence of the electromagnetic field, gravity field, rotation and initially stressed medium has been studied. Analytical analysis reveals that the velocity of propagation of the shear waves depends upon the direction of propagation, the anisotropy, magnetic field, rotation, gravity field, non-homogeneity of the medium, and the initial stress. The frequency equation that determines the velocity of the shear waves has been obtained. The dispersion equations have been obtained and investigated for different cases. In fact, these equations are an agreement with the corresponding classical results when the medium is isotropic. The results obtained are discussed and presented graphically. The results indicate that the effects of gravity field, initial stress, magnetic field, electric field, non-homogeneous, anisotropy and rotation are very pronounced.

Keywords: Incompressible medium, initial stress, anisotropic, gravity field, rotation, electromagnetic field, non-homogeneous.

1 Introduction

The study of shear wave propagation is very important because of its extensive application in various branches of Science and Technology, especially, Geophysics, Plasma, Physics, Earthquake science, and Optics. Shear waves propagation over the surface of homogeneous and inhomogeneous elastic half-spaces are a well known and prominent feature of wave theory. Effect of the rotation on propagation of thermoelastic waves in a non-homogeneous infinite cylinder of isotropic material by studied by [2]. Magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model has been investigated by [3]. Rayleigh waves in a magneto-elastic initially stresses conducting medium with the gravity field is pointed out by [17]. Some observations on interactions of Rayleigh waves in an elastic solid medium with the gravity field investigated by [14]. Surface waves in the influence of gravity is discussed by [15]. Surface waves in an inhomogeneous elastic medium under the influence of gravity has been discussed by [13]. [10] discussed the influence of gravity on propagation of waves in a composite layer. Rayleigh waves in granular medium over an initially stressed elastic half-space is investigated by [11]. Effect of anisotropy on surface wave under the influence of gravity is pointed out by [16]. [23] mentioned that initial stresses have remark able effect on the propagation of elastic waves in a medium. Shear waves in acoustic anisotropic media is studied by [19]. Effect of the non- homogenity on the composite infinite cylinder of isotropic material is explained by [6]. Propagation of S-wave in a non-homogeneous anisotropic incompressible and initially stressed medium under influence of gravity field is studied by [5]. Effect of initial stress and magnetic field on shear wave propagation discussed by [18]. Propagation of shear wave in anisotropic medium is pointed out by [20]. Shear waves propagation in an irregular magnetoelastic monoclinic layer sandwiched between two isotropic half-spaces is

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investigated by [12]. Propagation of S-waves in a non-homogeneous anisotropic incompressible and initially stressed medium is discussed by [21]. Influences of rotation, magnetic field, initial stress, and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space has been discussed by [4]. [1] investigated propagation of Rayleigh waves in generalized magneto-thermoelastic orthotropic material under initial stress and gravity field. [9] investigated effects of voids and rotation on plane waves in generalized thermoelasticity. Recently, [7,8,22], investigated a new features on different waves propagation under influence of multi parameters.

In this work, the effects of gravity field, rotation, electromagnetic field, and initial stress on the propagation of S-waves in a nonhomogeneous anisotropic incompressible and initially stressed medium have been discussed using the wave equations which satisfied by the displacement potentials φ. The frequency equation that determines the velocity of shear waves have been obtained. The dispersion equations have been obtained. Some special cases are investigated. In fact, these equations are in agreement with the corresponding classical results when the medium is isotropic. The results obtained are calculated numerically and presented graphically.

2 Formulation of the problem

Most materials behave as incompressible media and the velocities of longitudinal waves in them are very high. The varieties of hard rocks present in the earth are also almost incompressible. Due to the factors like external pressure, slow process of creep, difference in temperature, manufacturing processes, nitriding, pointing etc., the medium stay under high stresses. These stresses are regarded as initial stresses. Owing to the variation of elastic properties and the presence of these initial stresses, the medium becomes isotropic as well. We consider an unbounded incompressible anisotropic medium under initial stresses s_{11} and s_{22} along the $x-, y-$directions respectively. When the medium is slightly distribute, (u, v), the incremental stresses state become s_{11}, s_{12} and s_{22} are developed, and the equations of motion in incremental state become

$$\frac{\partial s_{11}}{\partial x}+\frac{\partial s_{12}}{\partial y}-P\frac{\partial w_3}{\partial y}-\rho g\frac{\partial v}{\partial x}+F_x = \rho\left[\ddot{u}+\left(\vec{\Omega}\times\vec{\Omega}\times\vec{u}\right)_x+\left(2\vec{\Omega}\times\dot{\vec{u}}\right)_x\right], \quad (1)$$

$$\frac{\partial s_{12}}{\partial x}+\frac{\partial s_{22}}{\partial y}-P\frac{\partial w_3}{\partial x}-\rho g\frac{\partial u}{\partial x}+F_y = \rho\left[\ddot{v}+\left(\vec{\Omega}\times\vec{\Omega}\times\vec{u}\right)_y+\left(2\vec{\Omega}\times\dot{\vec{u}}\right)_y\right], \quad (2)$$

Where, $F_i=\left(\vec{J}\times\vec{B}\right)_i$.

Where, F_x and F_y are components of the magnetic field in x and y directions, respectively. Initial stress $P=s_{22}-s_{11}$, the rotational components about z- axis $W_3=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$, ρ represents the density of the medium, g is the acceleration due to gravity, (u, v) are incremental deformation, and $\vec{\Omega}$ is the angular rotation. For slowly moving medium, the variation of magnetic field and electric field are given by Maxwell's equation as the following form:

$$curl\,\vec{h}=\vec{J}+\varepsilon_0\dot{\vec{E}},\ curl\,\vec{E}=\mu_e\dot{\vec{h}},div\,\vec{h}=0,div\,\vec{E}=0, \quad (3)$$

$$\vec{E}=\mu_e\left(\dot{\vec{u}}\times\vec{E}\right),\vec{h}=curl\left(\vec{u}\times\vec{H_0}\right),$$

Where, $\vec{H}=\vec{H_0}+\vec{h}(x,y,t)$, $\vec{H_0}=(H_0,0,0)$. Where, $\vec{B}$ is a magnetic induction vector, $\vec{E}$ is electric intensity vector, $\vec{F}$ is Lorentz's body forces vector, $\dot{\vec{u}}$ is the velocity vector, $\vec{h}$ is perturbed magnetic field vector, $\vec{H}$ is magnetic field vector, $\vec{H_0}$ is primary constant magnetic field vector, H_0 is the absolute magnetic field, $\vec{J}$ is an electric current density vector, and μ_e is magnetic permeability, is the electric permeability. The incremental stress-strain relations for an incompressible medium may be taken as:

$$s_{11}=2Ne_{xx}+s\,,\ s_{22}=2Ne_{yy}+s \text{ and } s_{12}=2Qe_{xy} \quad (4)$$

Where, $s=\frac{s_{11}+s_{22}}{2}$, e_{ij} are incremental strain components, and N and Q are rigidities of medium.

Maxwell' s stress equation

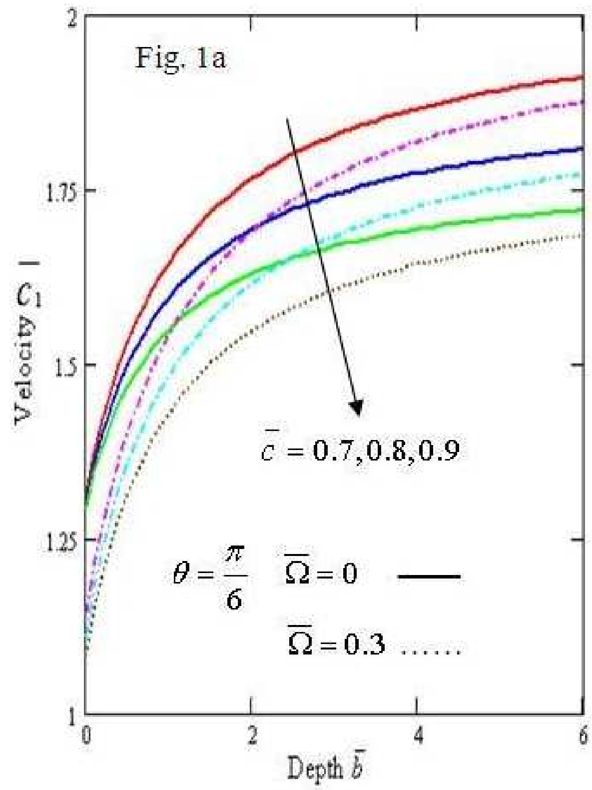
$$\tau_{ij}=\mu_e\left[H_ih_j+H_jh_i-H_kh_k\delta_{ij}\right] \quad (5)$$

The incompressibility condition $e_{xx}+e_{yy}=0$ is satisfied by

$$u=-\frac{\partial\phi}{\partial y},\ v=\frac{\partial\phi}{\partial x}. \quad (6)$$

Substituting from Eqs. (3) and (4) into Eqs. (1) and (2), we get

$$\frac{\partial s}{\partial x}-2N\frac{\partial^3\phi}{\partial x^2\partial y}+\frac{\partial}{\partial y}\left[Q\left(\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^2\phi}{\partial y^2}\right)\right] -\frac{P}{2}\frac{\partial}{\partial y}\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right) =\rho\left[g\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^3\phi}{\partial y\partial t^2}+\Omega^2\frac{\partial\varphi}{\partial y}-2\Omega\frac{\partial^2\phi}{\partial x\partial t}\right] \quad (7)$$



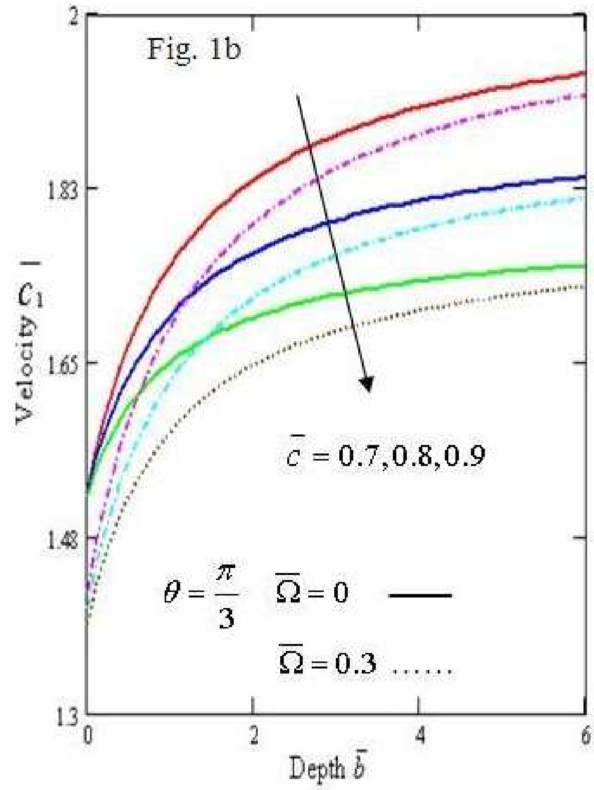


Fig. 1: Variation of velocity $\overline{C_1}$ with respect to the depth with variation of $\bar{c}$.

$$\frac{\partial s}{\partial y}+\frac{\partial}{\partial y}\left(2N\frac{\partial^2\phi}{\partial x\partial y}\right)+Q\left(\frac{\partial^3\phi}{\partial x^3}-\frac{\partial^3\phi}{\partial x\partial y^2}\right) -\frac{P}{2}\left(\frac{\partial^3\phi}{\partial x^3}+\frac{\partial^3\phi}{\partial x\partial y^2}\right) +\mu_e H_0^2\left(\frac{\partial^3\phi}{\partial x^3}+\frac{\partial^2\phi}{\partial x\partial y^2}-\varepsilon_0\mu_e\frac{\partial^3\phi}{\partial x\partial t^2}\right) =\rho\left[g\frac{\partial^2\phi}{\partial x\partial y}+\frac{\partial^3\phi}{\partial x\partial t^2}-\Omega^2\frac{\partial\varphi}{\partial x}-2\Omega\frac{\partial^2\phi}{\partial y\partial t}\right]. \quad (8)$$

Assuming non-homogeneties as

$$Q=Q_0(1+ay),\quad N=N_0(1+by),\quad \rho=\rho_0(1+cy) \quad (9)$$

where, Q_0, N_0 and ρ_0 are rigidities and is the density of the medium at the surface $(y=0)$.

Substituting from Eq. (9) into Eqs. (7) and (8) we get:

$$\left[Q_0(1+ay)-\frac{P}{2}+\mu_e H_0^2\right]\frac{\partial^4\phi}{\partial x^4}+[4N_0(1+by) -2Q_0(1+ay)]\frac{\partial^4\phi}{\partial x^2\partial y^2} +[4N_0b-2aQ_0]\frac{\partial^3\phi}{\partial x^2\partial y}+\left[Q_0(1+ay)+\frac{P}{2}\right]\frac{\partial^4\phi}{\partial y^4} +2aQ_0\frac{\partial^3\phi}{\partial y^3}=\rho_o(1+cy)\times[(1+\mu_e^2H_0^2\varepsilon_0)\frac{\partial^4\phi}{\partial x^2\partial t^2} +\frac{\partial^4\phi}{\partial y^2\partial t^2}-\Omega^2\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)]-\rho_0c[g\frac{\partial^2\phi}{\partial x^2}-\frac{\partial^3\phi}{\partial t^2\partial y} +\Omega^2\frac{\partial\varphi}{\partial y}-2\Omega\frac{\partial^2\phi}{\partial x\partial t}]. \quad (10)$$

3 Method of solution

For propagation of sinusoided waves in any arbitrary direction, we take the solution of Eq. (10) as

$$\varphi(x,y,t)=Ae^{ik\,(p_1x\,+\,p_2y\,-c_1t)} \quad (11)$$

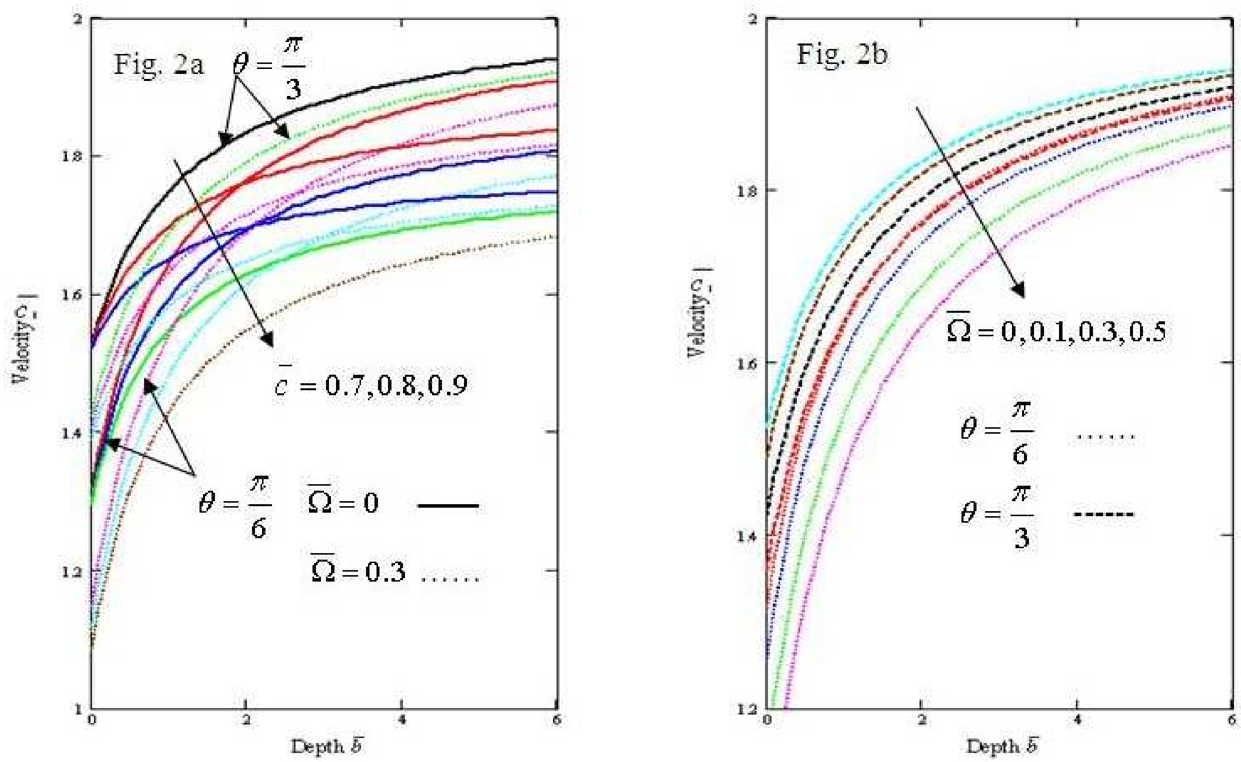


Fig. 2: Variation of velocity $\overline{C_1}$ with respect to the depth with variation of (a)$\bar{c}$ and (b)$\overline{\Omega}$.

where, p_1 and p_2 are cosine of angles made by direction of propagation with $x-$ and $y-$ axes, and c_1 and k are the velocity of propagation and the wave number respectively.

Substituting from Eq. (11) into Eq. (10) and equating real and imaginary parts separately, we get

$$[\frac{1}{\beta}(c_1+\frac{c\Omega}{k^2\Lambda_p}p_1)]^2=\frac{1}{\Lambda_p}[(1+ay-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0})p_1^4 + (1+ay-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0})p_2^4-\frac{gc}{k^2\beta^2}p_1^2+(\frac{4N_0}{Q_0}(1+by) - 2(1+ay)\frac{\mu_e H_0^2}{Q_0})p_1^2p_2^2\frac{c^2\Omega^2}{\beta^2k^4\Lambda_p}p_1^2-(1+cy)\frac{\Omega^2}{k^2\beta^2}] \quad (12)$$

and

$$\left(\frac{c_1}{\beta}\right)^2=\frac{2}{c}\left[\left(\frac{2N_0 b}{Q_0}-a\right)p_1^2+ap_2^2-\frac{c\Omega^2}{2k^2\beta^2}\right] \quad (13)$$

where, $\beta=\left(\frac{Q_0}{\rho_0}\right)^{\frac{1}{2}}$ is the velocity of shear waves in homogeneous isotropic medium. And $\Lambda_p=(1+cy)+\frac{\mu_e^2 H_0^2\varepsilon_0 p_1^2}{\rho_0}$.

Eq. (13) gives the velocity of propagation of shear wave and Eq. (12) gives the damping. From Eq. (13) it is shown that the velocity $\left(\frac{c_1}{\beta}\right)$ depends much on the rotation, magnetic field, anisotropy factor, and the initial stress factor, also on the direction of propagation denoted by (p_1, p_2).

4 Particular cases

In order to gain more insight information the following cases have been discussed: Analysis of Eq. (12) obtained by equating the real part of equation of motion.

Case I: In case Q is homogeneous $(a \to 0)$, *i.e*, rigidity along vertical direction is constant

$$\left[\frac{1}{\beta}\left(c_1+\frac{c\Omega}{k^2\Lambda_p}p_1\right)\right]^2=\frac{1}{\Lambda_p}\left\{[1-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0}\right]p_1^4+\left[\frac{4N_0}{Q_0}(1+by)-2+\frac{\mu_e H_0^2}{Q_0}\right]p_1^2p_2^2+\left[1+\frac{P}{2Q_0}\right]p_2^4 -\frac{gc}{k^2\beta^2}p_1^2+\frac{c^2\Omega^2}{\beta^2k^4\Lambda_p}p_1^2-(1+cy)\frac{\Omega^2}{k^2\beta^2}\right\} \quad (14)$$

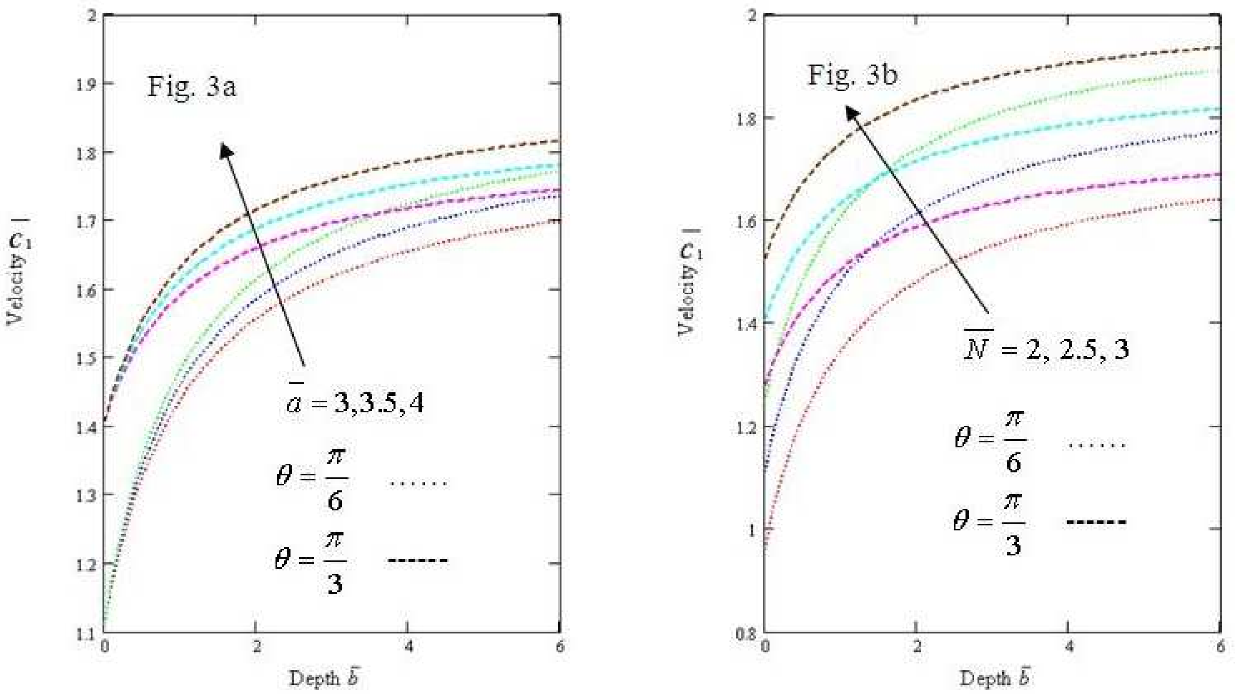


Fig. 3: Variation of velocity $\overline{C_1}$ with respect to the depth with variation of (a)$\overline{a}$ and (b)$\overline{N}$.

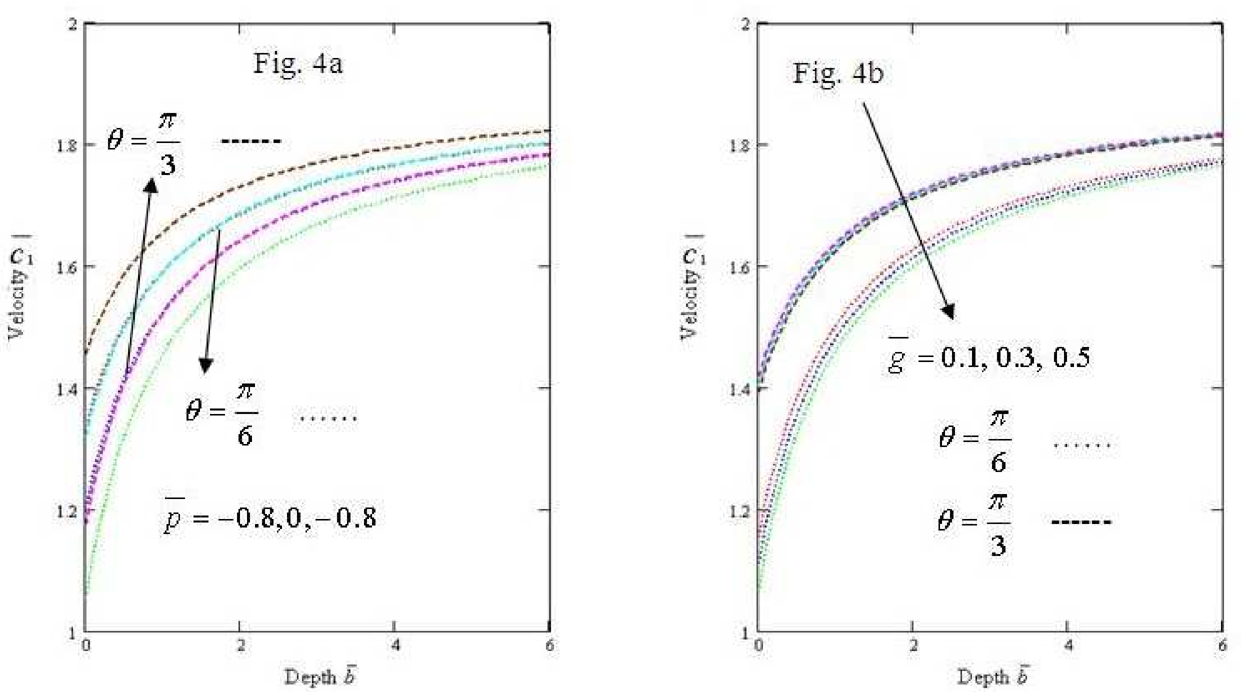


Fig. 4: Variation of velocity $\overline{C_1}$ with respect to the depth with variation of (a) $\overline{p}$ and (b) $\overline{g}$.

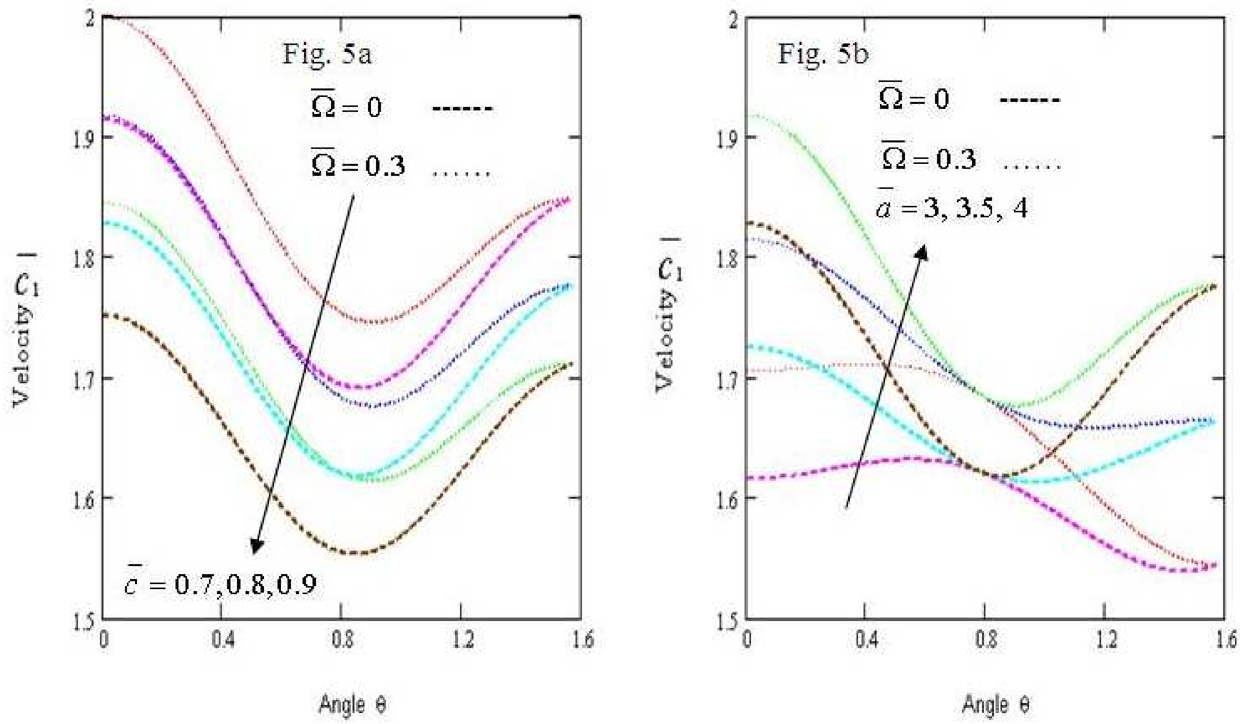


Fig. 5: Variation of velocity $\overline{C_1}$ with respect to the angle θ with variation of (a) $\bar{c}$ and (b) $\bar{a}$.

$$\left(\frac{c_1}{\beta}\right)^2=\frac{4N_0b}{cQ_0}p_1^2-\frac{\Omega^2}{k^2\beta^2} \tag{15}$$

The velocity on x- direction is

$$(p_1=1,\quad p_2=0,\quad c_1=c_{11})$$

$$[\frac{1}{\beta}(c_{11}+\frac{c\Omega}{k^2\Lambda})]^2=\frac{1}{\Lambda}[1-\frac{P}{2Q_0}+\frac{\mu_eH_0^2}{Q_0}-\frac{gc}{k^2\beta^2}+\frac{c^2\Omega^2}{k^4\beta^2\Lambda}-(1+cy)\frac{\Omega^2}{k^2\beta^2}] \tag{16}$$

$$\left(\frac{c_{11}}{\beta}\right)^2=\frac{4N_0b}{cQ_0}-\frac{\Omega^2}{k^2\beta^2} \tag{17}$$

In case the medium is free from initial stress $(P\to 0, c\to 0)$ and $c_{11}=\beta$.

Similarly the velocity of propagation along $y-$ direction $(p_1=0,\ p_2=1,\ c_1=c_{22})$, is obtained as

$$c_{22}^2=\frac{\beta^2}{1+cy}\left[1+\frac{P}{2Q_0}\right]-(1+cy)\frac{\Omega^2}{k^2} \tag{18}$$

Substracting Eq. (18) from Eq. (16) we get

$$\frac{c_{22}^2-\left(c_{11}+\frac{c\Omega}{k^2\Lambda}\right)^2}{\beta^2}=\frac{1}{\Lambda}\left(\frac{P}{Q_0}+\frac{gc}{k^2\beta^2}-\frac{\mu_eH_0^2}{Q_0}-\frac{c^2\Omega^2}{k^4\beta^2\Lambda}\right)+\frac{\Omega^2}{k^2Q_0} \tag{19}$$

Where, $\Lambda=(1+cy)+\frac{\mu_e^2H_0^2\varepsilon_0}{\rho_0}$

which a function of initial stress, gravity, magnetic field, initial stress, rotation and density, this may also be observed that if $P=s_{22}-s_{11}>0$, the effect of initial stresses on the body is compressive along x- direction and tensile along y- direction. The compressive initial stress reduces while tensile stress increases the velocity of shear wave along x- direction. A reverse effect is obtained along direction y.

Case II In case N is homogeneous $(b\to 0)$, i.e., rigidity along horizontal direction is constant.

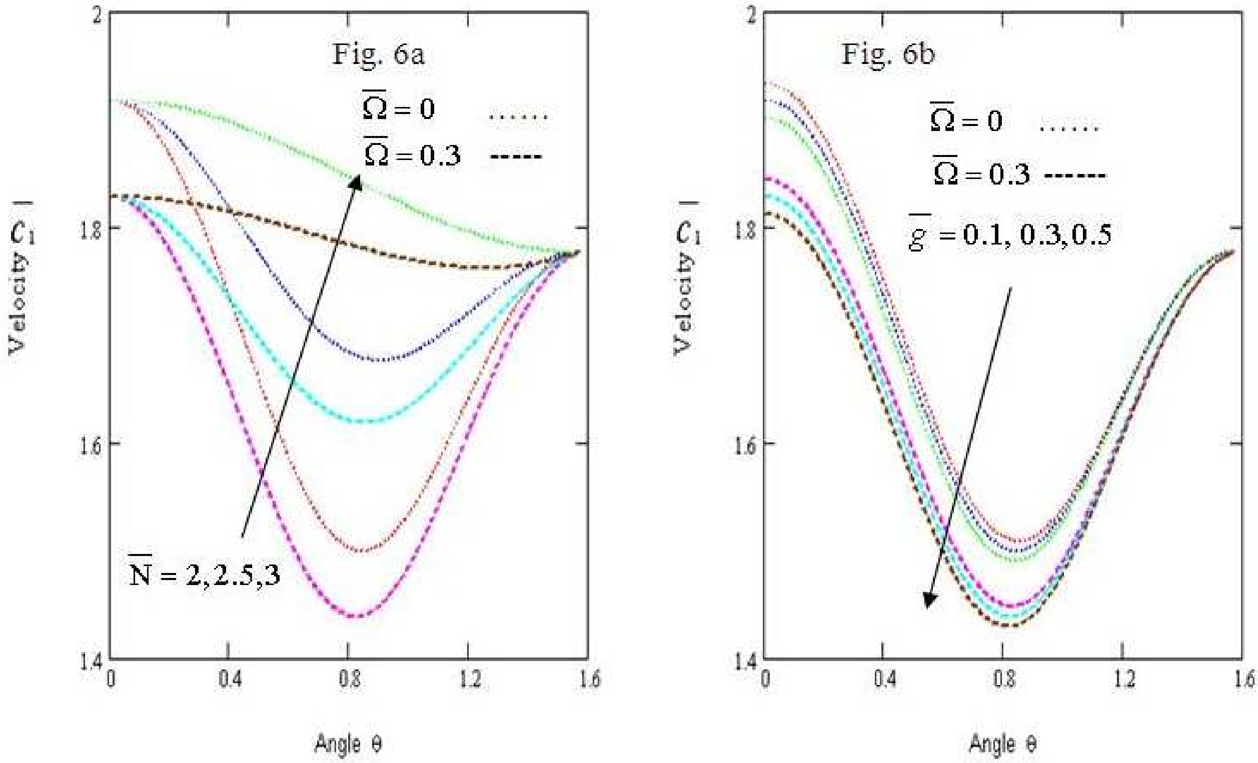


Fig. 6: Variation of velocity $\overline{C_1}$ with respect to the depth with variation of (a) $\overline{N}$ and (b) $\overline{g}$.

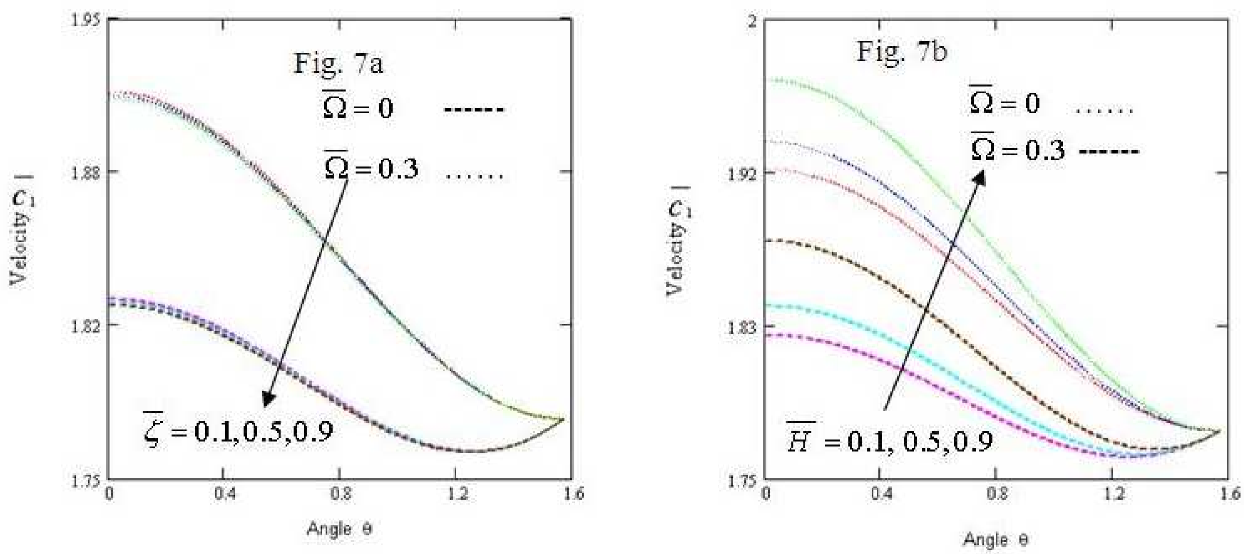


Fig. 7: Variation of velocity $\overline{C_1}$ with respect to the depth with variation of (a) $\overline{\zeta}$ and (b) $\overline{H}$.

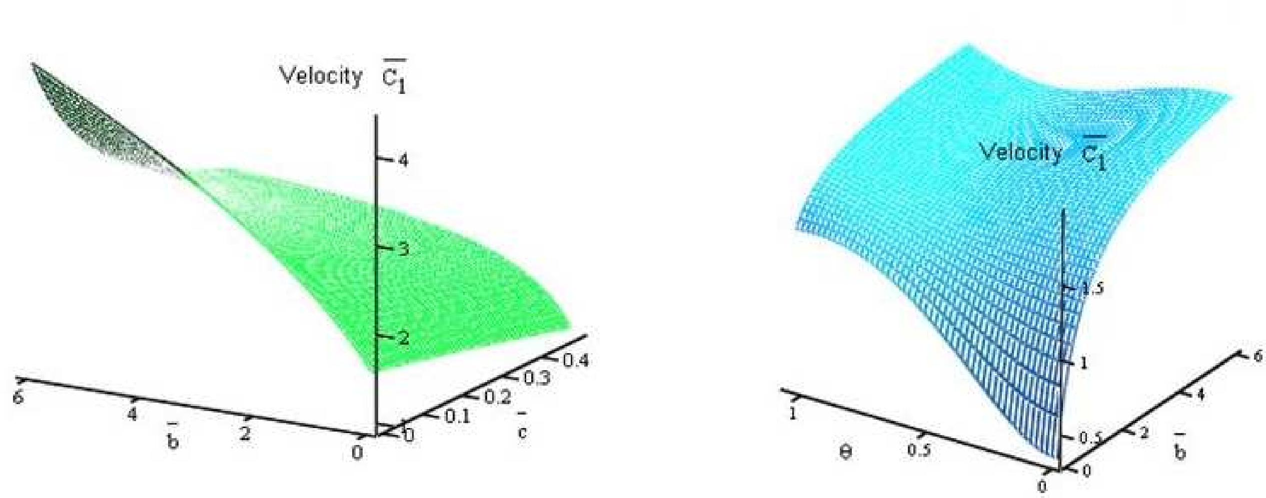


Fig. 8: Variation of velocity $\overline{C_1}$ with respect to $(\bar{b},\ \bar{c})$ and $(\theta,\ \bar{b})$.

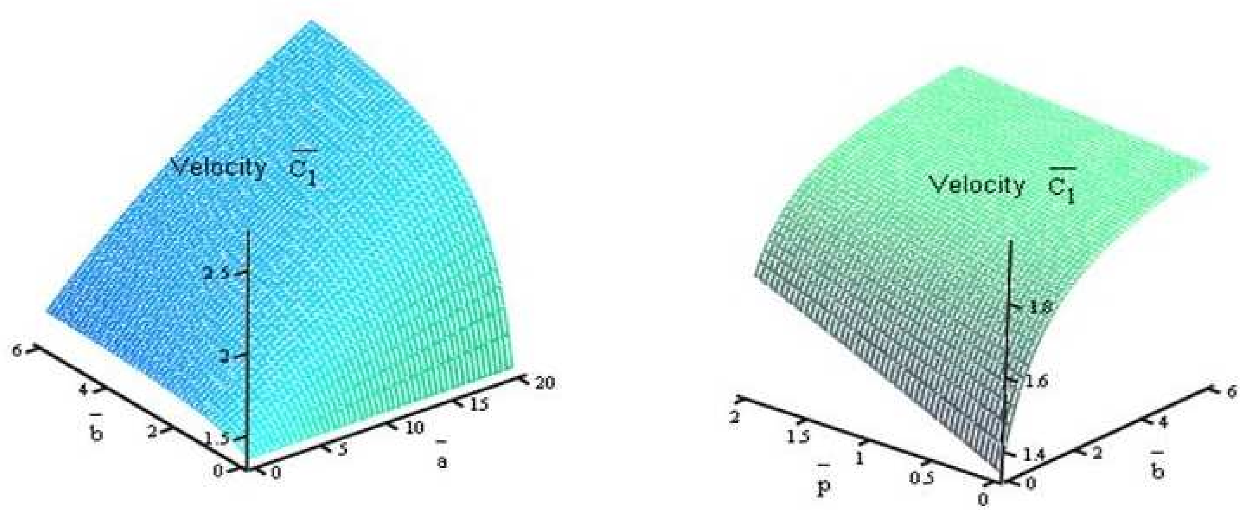


Fig. 9: Variation of velocity $\overline{C_1}$ with respect to $(\bar{b},\ \bar{a})$ and $(\bar{p},\ \bar{b})$.

$$[\frac{1}{\beta}(c_1+\frac{c\Omega}{k^2\Lambda_p}p_1)]^2=\frac{1}{\Lambda_p}[(1+ay-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0})p_1^4 +(\frac{4N_0}{Q_0}-2(1+ay)+\frac{\mu_e H_0^2}{Q_0})p_1^2p_2^2+(1+ay+\frac{P}{2Q_0})p_2^4 -\frac{gc}{k^2\beta^2}p_1^2+\frac{c^2\Omega^2}{\beta^2k^4\Lambda_p}p_1^2-\frac{\Omega^2(1+cy)}{k^2\beta^2}] \quad (20)$$

the velocity along x- direction $(p_1=1,\ p_2=0, c_1=c_{11})$ is given by

$$[\frac{1}{\beta}(c_{11}+\frac{c\Omega}{k^2\Lambda})]^2=\frac{1}{\Lambda}[(1+ay-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0}) -\frac{gc}{k^2\beta^2}+\frac{c^2\Omega^2}{k^4\Lambda}-\frac{\Omega^2(1+cy)}{k^2\beta^2}] \quad (21)$$

which depends on the depth y magnetic field, rotation, gravity and the wave is dispersive, the velocity along y-direction is $(p_1=0, p_2=1, c_1=c_{22})$

$$\left[\frac{c_{22}}{\beta}\right]^2=\frac{1}{1+cy}\left[1+ay+\frac{P}{2Q_0}\right]-\frac{\Omega^2}{k^2\beta^2} \quad (22)$$

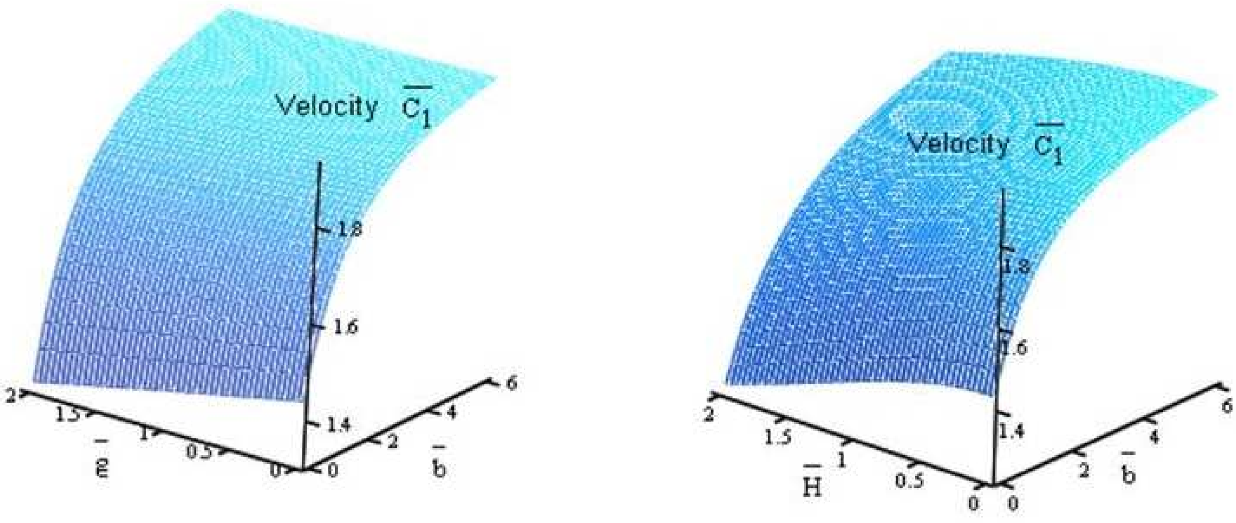


Fig. 10: Variation of velocity $\overline{C_1}$ with respect to $(\overline{g},\ \overline{b})$ and $(\overline{H},\ \overline{b})$.

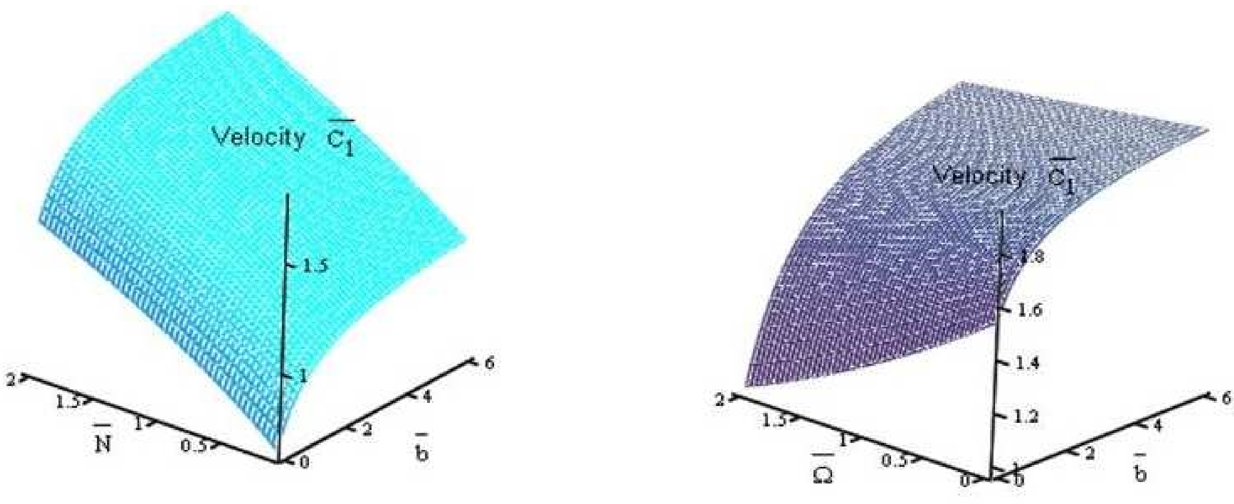


Fig. 11: Variation of velocity $\overline{C_1}$ with respect to $(\overline{N},\ \overline{b})$ and $(\overline{\Omega},\ \overline{b})$.

for $P > 0$, the velocity along y- direction may increase considerably at distance y from free surface and the wave becomes dispersive.

Case III: In case N, Q and c are homogeneous i.e., $a \to 0, b \to 0, c \to 0)$

$$(\frac{c_1}{\beta})^2 = \frac{1}{\Lambda_{p_1}}[(1-\frac{P}{2Q_0}+\frac{\mu_e H_0^2}{Q_0})\,p_1^4 + (\frac{4N_0}{Q_0}-2+\frac{\mu_e H_0^2}{Q_0})\,p_1^2 p_2^2 + (1+\frac{P}{2Q_0})\,p_2^4 - \frac{\Omega^2}{k^2\beta^2}] \quad (23)$$

where $\Lambda_{p_1} = 1+\frac{\mu_e^2 H_0^2 \varepsilon_0 p_1^2}{\rho_0}$

Case IV:

If the initial stress is absence (i.e., $P \to 0$), the velocity is obtained as

$$[\frac{1}{\beta}(c_1+\frac{c\Omega}{k^2\Lambda_p}p_1^2)]^2 = \frac{1}{\Lambda_p}[(1+ay+\frac{\mu_e H_0^2}{Q_0})\,p_1^4 + (\frac{4N_0}{Q_0}(1+by)-2(1+ay)+\frac{\mu_e H_0^2}{Q_0})\,p_1^2 p_2^2 + (1+ay)\,p_2^4 - \frac{gc}{k^2\beta^2}p_1^2 + \frac{c^2\Omega^2}{k^4\beta^2\Lambda_p}p_1^2 - \frac{\Omega^2(1+cy)}{k^2 Q_0}] \quad (24)$$

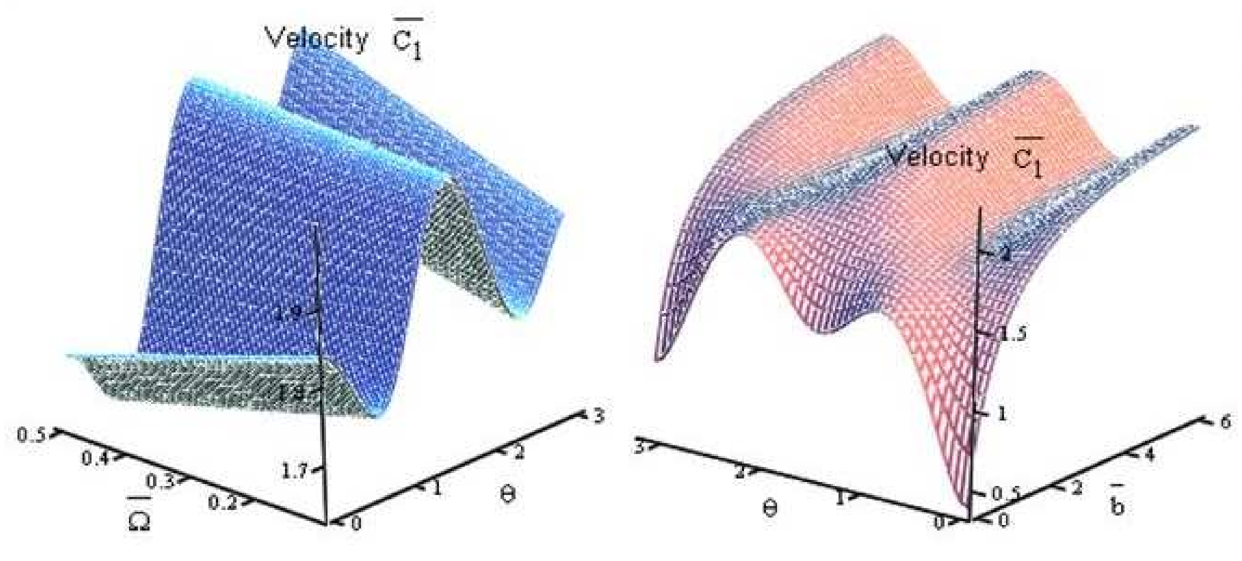


Fig. 12: Variation of velocity $\overline{C_1}$ with respect to $(\overline{\Omega},\ \overline{\theta})$ and $(\overline{h},\ \overline{b})$.

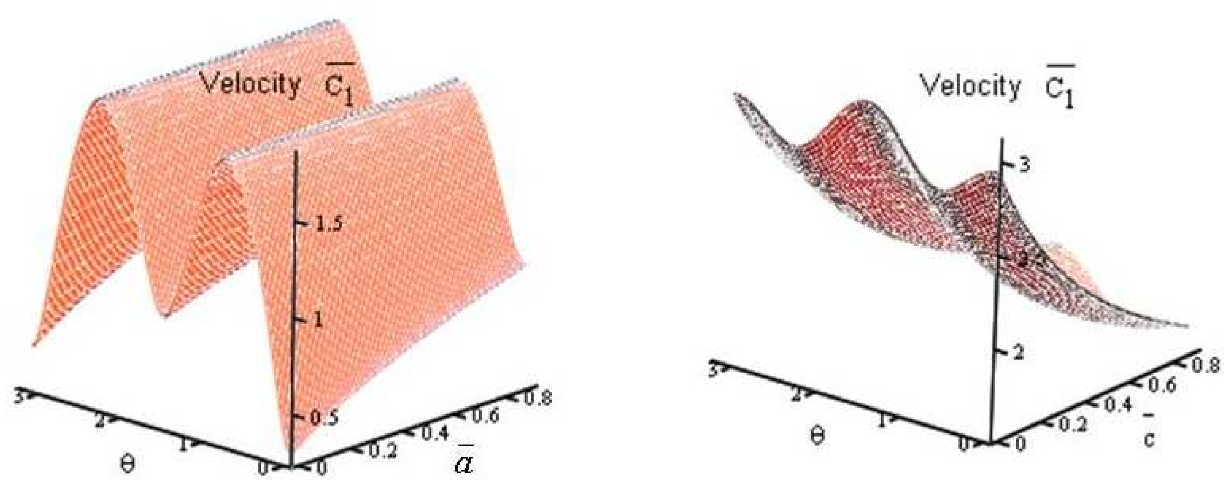


Fig. 13: Variation of velocity $\overline{C_1}$ with respect to $(\overline{a},\ \theta)$ and $(\theta,\ \overline{c})$.

in x- direction ($p_1 = 1, p_2 = 0, c_1 = c_{11}$) Eq. (22) reduces to

$$[\frac{1}{\beta}(c_{11}+\frac{c\Omega}{k^2\Lambda})]^2 = \frac{1}{\Lambda}[(1+ay+\frac{\mu_e H_0^2}{Q_0}) - \frac{gc}{k^2\beta^2} + \frac{c^2\Omega^2}{k^4\Lambda} - \frac{\Omega^2(1+cy)}{k^2\beta^2}] \quad (25)$$

and along y- direction $(p_1 = 0,\ p_2 = 1, c = c_{22})$ Eq. (27) tends to

$$\left(\frac{c_{22}}{\beta}\right)^2 = \frac{1+ay}{1+cy} - \frac{\Omega^2}{k^2\beta^2}. \quad (26)$$

Case V:

In the absence of gravity field, magnetic field and rotation, the velocity is obtained from the following equation

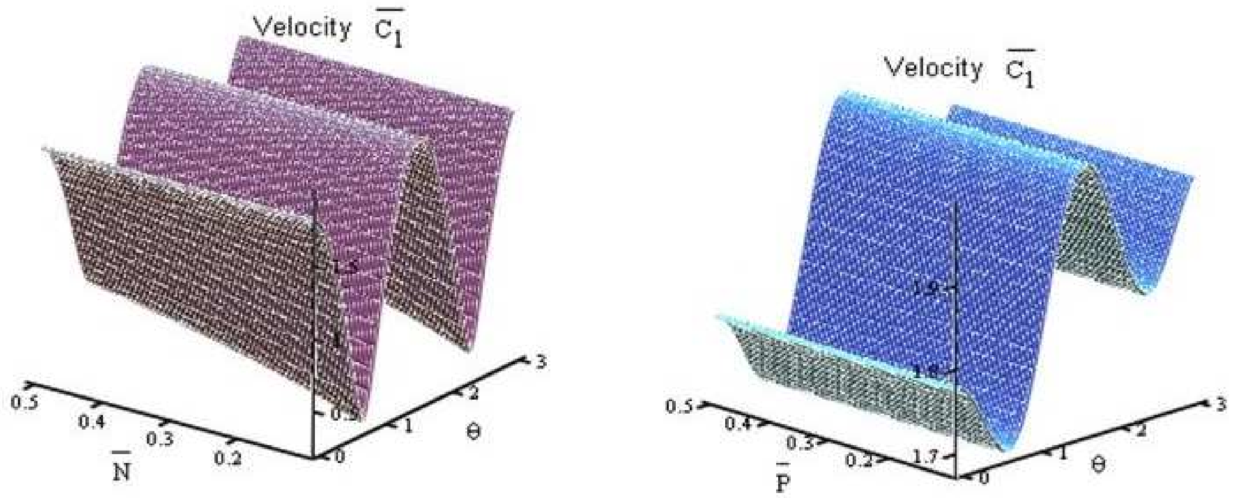


Fig. 14: Variation of velocity $\overline{C_1}$ with respect to $(\overline{N},\ \theta)$ and $(\overline{P},\ \theta)$.

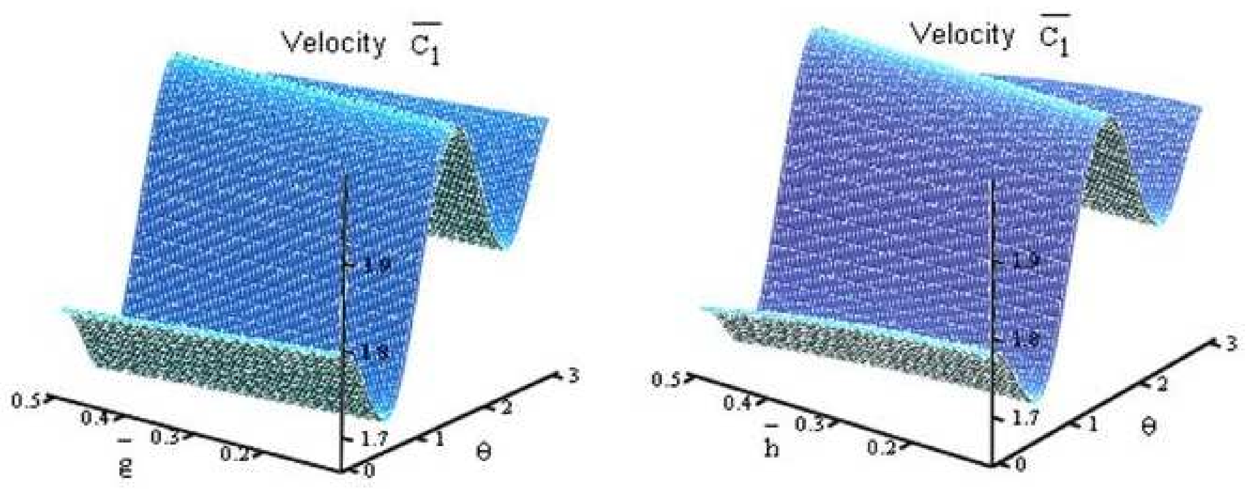


Fig. 15: Variation of velocity $\overline{C_1}$ with respect to $(\overline{g},\ \theta)$ and $(\overline{h},\ \theta)$.

$$(\frac{c_1}{\beta})^2 = \frac{1}{1+cy}\{(1+ay-\frac{P}{2Q_0})\,p_1^4 + 2[2\frac{N_0}{Q_0}(1+by) -(1+ay)]\,p_1^2p_2^2 + (1+ay+\frac{P}{2Q_0})\,p_2^4 \quad (27)$$

Analysis of Eq. (13) In absence of P in equation (13), following three cases to have been analyzed as follows:

Case I: In case Q is homogeneous $(a \to 0)$ *i.e*, rigidity along vertical direction is constant, one may obtain

$$\left(\frac{c_1}{\beta}\right)^2 = 2\left(\frac{2N_0}{Q_0}\frac{b}{c}\right)p_1^2 - \frac{\Omega^2}{k^2\beta^2} \quad (28)$$

this shows that the velocity of shear wave is always damped. The velocity of wave along x- direction $(p_1 = 1, p_2 = 0, c = c_{11})$ is obtained as

$$\left(\frac{c_{11}}{\beta}\right)^2 = 2\left(2\frac{N_0}{Q_0}\frac{b}{c}\right) - \frac{\Omega^2}{k^2\beta^2} \quad (29)$$

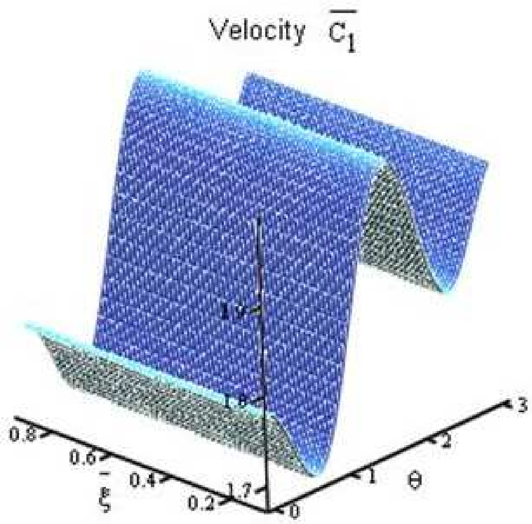


Fig. 16: Variation of velocity $\overline{C_1}$ with respect to $\left(\overline{\zeta},\ \theta\right)$.

this shows that actual velocity in x- direction is damped by $2\left(\frac{2N_0}{Q_0}\frac{b}{c}\right)$ and nodamping takes place along direction.

Case II: In case N is homogeneous $(b \to 0)$ *i.e*, rigidity along horizontal direction is constant

$$\left(\frac{c_1}{\beta}\right)^2 = 2\left(-\frac{a}{c}\right) p_1^2 + 2\left(\frac{a}{c}\right) p_2^2 - \frac{\Omega^2}{k^2\beta^2} \tag{30}$$

the velocity of wave along x- direction $(p_1 = 1, p_2 = 0, c = c_{11})$ is given by:

$$\left(\frac{c_{11}}{\beta}\right)^2 = 2\left(-\frac{a}{c}\right) - \frac{\Omega^2}{k^2\beta^2} \tag{31}$$

Existence of negative sign shows that damping doesn't takes place along x- direction for $(b \to 0)$, the velocity along direction is given by

$$\left(\frac{c_{22}}{\beta}\right)^2 = 2\left(\frac{a}{c}\right) - \frac{\Omega^2}{k^2\beta^2} \tag{32}$$

indicating damping of magnitude $[\left(\frac{2a}{c}\right)$ takes place along y- direction.

Case III: In case N and Q are homogeneous but density is linearly varying with depth

$$\left(\frac{c_1}{\beta}\right) = -\frac{\Omega^2}{k^2\beta^2} \tag{33}$$

i.e; no damping takes place.

5 Numerical results and discussion

To get numerical information on the velocity of shear waves in the non-homogeneous initially stressed medium we introduce the following non-dimensional parameters:

$$\bar{a} = \frac{a}{b};\quad \bar{b} = by;\quad \bar{c} = \frac{c}{b};\quad \overline{C_1} = \frac{c_1}{\beta};\quad \bar{N} = \frac{N_0}{Q_0};\quad \bar{P} = \frac{P_0}{2Q_0};$$
$$\bar{g} = \frac{bg}{k^2\beta^2};\quad \bar{H} = \sqrt{\frac{\mu_e}{Q_0}}H_0;\quad \overline{\Omega} = \frac{\Omega b}{k^2\beta};\quad \overline{\zeta} = \beta^2\mu_e\varepsilon_0. \tag{34}$$

Using these parameters in equation (12) after neglecting the term includes the high degree of k, β and Ω we obtain:

$$\left(\overline{C_1} + \frac{\bar{c}\overline{\Omega}}{(1+\bar{c}\bar{b}) + \overline{H}^2\overline{\zeta}p_1^2}p_1\right)^2$$
$$= \frac{1}{(1+\bar{c}\bar{b}) + \overline{H}^2\overline{\zeta}p_1^2}\left\{\left[1+\bar{a}\bar{b} - \bar{P} + \overline{H}^2\right] p_1^4 + \left[4\bar{N}(1+\bar{b})\right.\right.$$
$$\left. -2(1+\bar{a}\bar{b}) + \overline{H}^2\right] p_1^2p_2^2 + \left[1+\bar{a}\bar{b}+\bar{P}\right] p_2^4 - \overline{cg}p_1^2$$
$$\left. + \frac{\bar{c}^2\overline{\Omega}^2}{(1+\bar{c}\bar{b}) + \overline{H}^2\overline{\zeta}p_1^2}p_1^2\right\}. \tag{35}$$

The numerical values of $\overline{C_1}$ has been calculated for different values of $\bar{c}$,$\bar{a}$, $\bar{N}$, $\bar{P}$,$\bar{g}$, $\bar{H}$, $\overline{\Omega}$, $\bar{b}$ and θ respect b and θ respectively, and the results are presented in Figs. 1 and 2 for constant values

$$\bar{c} = 8,\ \bar{a} = 3,\ \bar{N} = 2.5,\ \bar{P} = 0.8,\ \bar{g} = 0.3,\ \bar{H} = 0.3,$$
$$\overline{\Omega} = 0.5,\ \ \bar{b} = 0.3 \text{ and } \theta = \frac{\pi}{3}$$

Fig. 1 displays the variation of velocity $\overline{C_1}$ respect to the depth with a variation of $\bar{c}$ if $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{3}$ with and without rotation, it obvious that $\overline{C_1}$ increases with an increasing of depth, decreases with an increasing of $\bar{c}$ and takes the largest values in the absence of rotation compared with its values in the presence of rotation, also, it seen that increases with an increasing of the angle θ. Fig. 2 plots the variation of velocity $\overline{C_1}$ with respect to the depth with variation of $\bar{c}$ and Ω . From Fig. 2a, it clears that $\overline{C_1}$ takes the largest values with the largest values of θ that indicate to the influence of the angle θ on the velocity, from Fig. 2b, it appears that $\overline{C_1}$ decreases with an increasing of the rotation Ω.

Fig. 3 clears the influence of the parameters $\bar{a}$ and $\overline{N}$ on S-waves velocity with respect to the depth, it shown that $\overline{C}_1$ increases with an increasing of $\bar{a}$ and $\overline{N}$. Fig. 4 displays the variation of wave velocity with with variation of $\bar{p}$ and $\bar{g}$, it appears that $\overline{C}_1$ increasing with an increasing of $\bar{p}$ if $\theta = \frac{\pi}{3}$ but decreases with the increasing values of $\bar{p}$ if $\theta = \frac{\pi}{6}$ (see, Fig. 4a) and decreases with an increasing of the gravity $\bar{g}$.

Fig. 5 plots the variation of velocity with respect to angle θ with variation of $\overline{c}$ and $\overline{a}$, respectively, it clears that S-waves velocity decreases with an increasing of angle θ arrive to its minimum values if $\theta = \frac{\pi}{4}$ and then increasing if $\frac{\pi}{4} < \theta < \frac{\pi}{2}$, also, it shown that it decreases with an increasing of $\overline{c}$, vice versa, respect to the influence of $\overline{a}$. From Fig. 6, we can conclude that $\overline{C_1}$ increases with an increasing of $\overline{N}$ but decreases with the increased values of $\overline{g}$. Fig. 7 appears the variation of velocity $\overline{C_1}$ with respect to the angle θ with varies values of $\overline{\zeta}$ and $\overline{H}$, it obvious that $\overline{C_1}$ decreases with an increasing of $\overline{\zeta}$ but increases with an increasing of $\overline{H}$.

Fig. 8 shows the variation of velocity $\overline{C_1}$ with respect to $\left(\overline{b},\ \overline{c}\right)$ and $\left(\theta,\ \overline{b}\right)$, it seens that $\overline{C_1}$ increases with the increased values of $\overline{b}$ and θ but decreases with an increasing of $\overline{c}$. Fig. 9 displays the variation of velocity $\overline{C_1}$ with respect to $\left(\overline{b},\ \overline{a}\right)$ and $\left(\overline{p},\ \overline{b}\right)$, it concludes that $\overline{C_1}$ influence positively with the increasing of all parameters. Fig. 10 plots the variation of velocity $\overline{C_1}$ with respect to $\left(\overline{g},\ \overline{b}\right)$ and $\left(\overline{h},\ \overline{b}\right)$, it appears that the wave velocity increases with an increasing of $\overline{b}$ but decrease with the increased values of $\overline{g}$ and $\overline{H}$. From Fig. 11, it appears that the wave velocity $\overline{C_1}$ influences negatively with the increasing of $\overline{\Omega}$ but increases with the variation positively of $\overline{N}$ and $\overline{b}$. Fig. 12 displays the variation of velocity $\overline{C_1}$ with respect to $\left(\overline{\Omega},\ \theta\right)$ and $\left(\overline{h},\ \overline{b}\right)$, it appears that the largest values of $\overline{b}$ affect positively in the waves velocity, also, any one can see that $\overline{C_1}$ increases and decreases periodically with an increasing of the angle θ. Fig. 13 shows the variation of velocity $\overline{C_1}$ with respect to $(\overline{a},\ \theta)$ and $(\theta,\ \overline{c})$, it is shown that $\overline{C_1}$ increases with an increasing of $\overline{a}$ but decreases with the increased values of $\overline{c}$. Fig. 14 clears the variation of velocity $\overline{C_1}$ with respect to $\left(\overline{N},\ \theta\right)$ and $(\overline{p},\ \theta)$, it is seen that $\overline{C_1}$ increases with an increasing of $\overline{N}$ and $\overline{p}$. Fig. 15 shows the variation of velocity $\overline{C_1}$ with respect to $(\overline{g},\ \theta)$ and $\left(\overline{h},\ \theta\right)$, it appears that $\overline{g}$ and $\overline{H}$ affect negatively on the waves velocity. Fig. 16 displays the variation of velocity $\overline{C_1}$ with respect to $\left(\overline{\zeta},\ \theta\right)$, it obvious that $\overline{C_1}$ decreases slightly with an increasing of $\overline{\zeta}$.

6 Conclusion

From the results obtained analytically and displaying the numerical results graphically, we concluded the following remarks:

- The depth $\overline{b}$ affects positively on the waves velocity $\overline{C_1}$.
- The angle of incidence θ affects periodically as increasing and decreasing on the velocity.
- The magnetic field $\overline{H}$ and gravity $\overline{g}$ cause interruption on the waves velocity.
- The waves velocity $\overline{C_1}$ affects strongly by the angular velocity (rotation) comparing with the results obtained with absence of the rotation that indicate to the influence of rotation in aircrafts and planes has utilitarian aspects in Geophysics, Geology, Biology, Acoustics, and, Plasma.
- All parameters affect strongly on the waves velocity, expect $\overline{\zeta}$ decreases slightly on $\overline{C_1}$.

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