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Reliable Wavelet based Approximation Method for Some Nonlinear Differential Equations

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Abstract: In this paper, we have developed a Chebyshev wavelet based approximation method to solve some nonlinear differential equations (NLDEs) arrising in science and engineering. To the best of our knowledge, until now there is no rigorous shifted second kind Chebyshev wavelet (S2KCWM) solution has been addressed for the nonlinear differential equations. With the help of shifted second kind Chebyshev wavelets operational matrices, the linear and nonlinear differential equations are converted into a system of algebraic equations. The convergence of the proposed method is established. Finally, we have given some numerical examples to demonstrate the validity and applicability of the proposed wavelet method.

Keywords: Shifted second kind Chebyshev wavelets; Operational matrices; Boundary value problems; Nonlinear ODEs

1 Introduction

In recent years, wavelet based methods have been applied for solving nonlinear differential equations (NLDEs) [1, 2, 3, 4]. In the last few decades the wavelet transform methods of solution for such problems have attracted excellent attention and numerous papers about this topic have been published. Wavelets analysis possesses many useful properties, such as compact support, orthogonality, dyadic, orthonormality and multi-resolution analysis (MRA). An excellent discussion on wavelet transforms and the Fourier transforms presented by Gilbert Strang in the year 1993. In the numerical analysis, wavelet based methods and hybrid methods become important tools because of the properties of localization. In wavelet based techniques, there are two important ways of improving the approximation of the solutions: increasing the order of the wavelet family and the increasing the resolution level of the wavelet. There is a growing interest in using wavelets to study problems, of greater computational complexity. Wavelet methods have proved to be very effective and efficient tool for solving problems of mathematical calculus. Among the wavelet transform families the Haar, Legendre wavelets and Chebyshev wavelets deserve much attention [5, 6, 7, 8, 9, 10, 11]. Chen and Hsiao [12] had implemented the Haar wavelets method for solving lumped and distributed-parameter systems. Lepik [13] introduced the Haar wavelets method (HWM) for solving the integral and differential equations.

Hariharan et al. [14] introduced the Haar wavelet method for solving Fisher's equation arising in population dynamics. The same authors [15] introduced an overview of Haar wavelet method for solving integral and differential equations. Siraj-ul-Islam et al. [16] introduced the Haar wavelet method for second order boundary value problems in which the performance of the Haar wavelets has been compared with other methods like Walsh wavelets, semi-orthogonal B-Spline wavelets, spline functions, Adomain decomposition method (ADM), Runge-Kutta (RK) method and nonlinear shooting method. Recently, Doha et al. [17] introduced the integrals of Bernstein polynomials: an application for the solution of high even-order differential equations.

Moreover, wavelet method establishes a connection with fast approximation algorithms. In the last two decades the wavelet solutions have been attracted great attention and numerous papers about this area have been published. Hariharan and his coworkers [4] introduced the Haar wavelet method for solving linear and non linear differential equations arising in science and engineering.

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In recent years, there are numerous papers have been published about the Chebyshev wavelets for solving ordinary differential equations. Tavassoli Kajani et al. [18] had established the Comparison between the homotopy perturbation method (HPM) and the sine-cosine wavelet method (SCWM) for solving linear integro differential equations. Doha et al. [19,20] had addressed the Chebyshev spectral method for solving the initial and boundary value problems and fractional order differential equations. Zhu et al. [21] had established the second kind Chebyshev wavelets for solving integral equations. Zhu and Fan [22] used the second kind Chebyshev wavelets for solving the fractional nonlinear Fredholm integro-differential equations. Sohrabi [23] established the Chebyshev wavelet methods (CWM) with BPFs method for solving Abels integral equations. Li [24] introduced the Chebyshev wavelet method for solving nonlinear fractional differential equation (NLFDEs). Hojatollah Adibi and Pouria Assari [25] had implemented the CWM for the numerical solutions of Fredholm integral equations of the first kind .Yanxin Wang and Qibin Fan [26] had solved the fractional differential equations by using the second kind Chebyshev wavelet method. Recently, Doha et al. [1] introduced the second kind Chebyshev operational matrix algorithm for solving differential equations of LaneEmden type. Heydari et al. [27] established the CWM for partial differential equations with boundary conditions of the telegraph type. Babolian and Fattahzadeh [6] had used the Chebyshev wavelet operational matrix of integration for solving the differential equations. Ghasemi and. Tavassoli Kajani [28] had introduced the CWM for solving the time-varying delay systems. Recently, Pirabaharan et.al [29] applied the shifted second kind chebyshev wavelet method for Estimating the Concentration of Species and Effectiveness Factors in Porous Catalysts. Wavelets permit the accurate representation of a variety of functions and operators.

The aim of the present work is to develop the shifted second kind Chebyshev wavelet method (S2KCWM) with the operational matrices of integration and differentiation mutually for solving nonlinear differential equations. By several nonlinear boundary value problems, it is shown that the shifted second kind Chebyshev wavelet method (S2KCWM) is very efficient and suitable tool for solving differential equations.

The paper is organized as follows. In section 2, we describe properties of shifted second kind Chebyshev wavelets also we presented the convergence analysis of the proposed algorithm. In section 3 the proposed shifted second kind Chebyshev wavelet method (S2KCWM) is used to solve linear and nonlinear differential equations. In section 4 some numerical examples are solved by applying the method of this paper. Finally a conclusion is drawn in section 5.

2 Properties of second kind Chebyshev polynomials and their shifted forms

2.1 Second kind Chebyshev polynomials

(S2KCP)[19]

It is well known the second kind Chebyshev polynomials are defined on [-1,1] by

$$U_n(x) = \frac{sin(n+1)\theta}{sin\theta}, x = cos\theta \tag{1}$$

These polynomials are orthogonal on [-1,1]

$$\int_{-1}^{1} \sqrt{1-x^2}U_m(x)U_n(x)dx = \begin{cases} 0 & m \neq n \\ \frac{\pi}{2} & m = n \end{cases} \tag{2}$$

The following properties of second kind Chebyshev polynomialsn are of fundamental importance in the sequel. They are eigen functions of the follwing singular Sturm-Liouville equation.

$$(1-x^2)D^2\phi_k(x) - 3xD\phi_k(x) + k(k+2)\phi_k(x) = 0 \tag{3}$$

Where $D \equiv \frac{d}{dx}$ and may be generated by using the recurrence relation

$$U_{k+1}(x) = 2xU_k(x) - U_{k-1}(x),\ k = 1,2,3,\cdots. \tag{4}$$

Starting from $U_0(x) = 1$ and $U_1(x) = 2x$, or from Rodrigues formula

$$U_n(x) = \frac{(-2^n)(n+1)!}{(2n+1)!\sqrt{(1-x^2)}} D^n[(1-x^2)^{n+\frac{1}{2}}] \tag{5}$$

Theorem 2.1.1[19] The first derivative of second kind Chebyshev polynomials is of the form

$$DU_n(x) = 2 \sum_{k=0,(k+n)odd}^{n-1} (k+1)U_k(x). \tag{6}$$

Definition 2.1.1 [19] The shifted second kind Chebyshev polynomials are defined on [0,1] by $U_n^*(x) = U_n(2x-1)$. All results of second kind Chebyshev polynomials can be easily transformed to give the corresponding results for their shifted forms. The orthogonally relation with respect to the weight function $\sqrt{x-x^2}$ is given by

$$\int_{0}^{1} \sqrt{x-x^2}U_n^*(x)U_m^*(x)dx = \begin{cases} 0 & m \neq n \\ \frac{\pi}{8} & m = n \end{cases} \tag{7}$$

Corollay 2.1.1: The first derivative of the shifted second kind Chebyshev polynomial is given by

$$DU_n^*(x) = 4 \sum_{k=0,(k+n)odd} (k+1)U_k^*(x) \tag{8}$$

2.2 Shifted Second kind Chebyshev operational matrix of derivatives

[19]
Second kind Chebyshev wavelets are denoted by $\psi_{n,m}(t) = \psi(k,n,m,t)$, where k,n are positive integers and m is the order of second kind Chebyshev polynomials. Here t is the normalized time. They are defined on the interval [0,1] by

$$\psi_{n,m}(t) = \begin{cases} \frac{2^{\frac{k+3}{2}}}{\sqrt{\pi}} U_m^*(2^k t - n),\ t \in [\frac{n}{2^k}, \frac{n+1}{2^k}] \\ 0 \qquad otherwise \end{cases} \tag{9}$$

$m = 0,1,\cdots,M, n = 0,1,\cdots,2^k - 1$. A function $f(t)$ is defined over [0,1] may be expanded in terms of second kind Chebyshev wavelets as

$$f(t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nm} \psi_{nm}(t). \tag{10}$$

Where

$$c_{nm} = (f(t), \psi_{nm}(t))_w = \int_0^1 \sqrt{t - t^2} f(t) \psi_{nm}(t) dt. \tag{11}$$

If the infinite series is truncated, then it can be written as

$$f(t) = \sum_{n=0}^{2^k-1} \sum_{m=0}^{M} c_{nm} \psi_{nm}(t) = C^T \psi(t) \tag{12}$$

Where C and $\psi(t)$ are $2^k(M+1) \times 1$ defined by

$$\begin{aligned} C &= [c_{0,0}, c_{0,1}, \cdots, c_{0,M}, \cdots, c_{2^k-1,M}, \cdots, c_{2^k-1,1}, \cdots, c_{2^k-1,M}]^T \\ \psi(t) &= [\psi_{0,0}, \psi_{0,1}, \cdots, \psi_{0,M}, \cdots, \psi_{2^k-1,M}, \cdots, \psi_{2^k-1,1}, \cdots, \psi_{2^k-1,M}]^T \end{aligned} \tag{13}$$

Theorem 2.2.1 [19] Let $\Psi(t)$ be the second kind Chebyshev wavelets vector. Then the first derivative of the vector $\Psi(t)$ can be expressed as

$$\frac{d\psi(t)}{dt} = D\psi(t) \tag{14}$$

Where D is $2^k(M+1)$ square matrix of derivatives and is defined by

$$D = \begin{bmatrix} F & O & \cdots & O \\ O & F & \cdots & O \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ O & O & \cdots & F \end{bmatrix}$$

in which F is an $(M+1)$ square matrix and its $(r,s)^{th}$ element is defined by

$$F_{r,s} = \begin{cases} 2^{k+2} s \quad r \geq 2, r > s \text{ and } (r+s) odd \\ 0 \qquad otherwise \end{cases} \tag{15}$$

Corollary 2.2.1 The operational matrix for the n^{th} derivative can be obtained from

$$\frac{d^n \psi(t)}{dt^n} = D^n \psi(t),\ n = 1,2,\cdots where\ D^n\ is\ the\ n^{th}\ power\ of\ D. \tag{16}$$

2.3 Convergence Analysis

We state and prove a theorem ascertaining that the second kind Chebyshev wavelet expansion of a function $f(x)$, with bounded second derivative, converges uniformly to $f(x)$.

Theorem 2.3.1 A function $f(x) \in L^2_\omega[0,1]$, with $|f''(x)| \leq L$ can be expanded as an infinite sum of Chebyshev wavelets and the series converges uniformly to $f(x)$. Explicitly the expansion coefficients in (11) satisfying the following in equality:

$$|c_{nm}| < \frac{8\sqrt{2\pi}L}{(n+1)^{\frac{5}{2}}(m+1)^2},\ \forall\ m > 1, n \geq 0 \tag{17}$$

Proof: From (11) it follows that

$$c_{nm} = \frac{2^{\frac{(k+3)}{2}}}{\sqrt{\pi}} \int_{\frac{n}{2^k}}^{\frac{n+1}{2^k}} f(x) U_m^*(2^k x - n) \omega(2^k x - n) dx \tag{18}$$

If we set $2^k x - n = cos\theta$ in (17), then we get

$$c_{nm} = \frac{2^{\frac{(-k+3)}{2}}}{\sqrt{\pi}} \int_0^{\pi} f\left(\frac{cos\theta + n}{2^k}\right) sin(m+1)\theta sin\theta d\theta$$

$$= \frac{2^{\frac{(-k+1)}{2}}}{\sqrt{\pi}} \int_0^{\pi} f\left(\frac{cos\theta + n}{2^k}\right) [cosm\theta - cos(m+2)\theta] d\theta \tag{19}$$

Which gives after integration by parts two times

$$C_{nm} = \frac{2}{2^{\frac{5k}{2}} \sqrt{2\pi}} \int_0^{\pi} f''\left(\frac{cos\theta + n}{2^k}\right) \lambda_m(\theta) d\theta \tag{20}$$

Where

$$\lambda_m(\theta) = \frac{\sin\theta}{m}\left[\frac{sin(m-1)\theta}{m-1} - \frac{sin(m+1)\theta}{m+1}\right] - \frac{\sin\theta}{m+2}\left[\frac{sin(m+1)\theta}{m+1} - \frac{sin(m+3)\theta}{m+3}\right] \tag{21}$$

Therefore we have

$$\begin{aligned}|c_{nm}| &= \left|\frac{1}{2^{\frac{5k-1}{2}}\sqrt{\pi}}\int_0^{\pi} f''\left(\frac{cos\theta+n}{2^k}\right)\lambda_m(\theta)d\theta\right| \\ &= \frac{1}{2^{\frac{5k-1}{2}}\sqrt{\pi}}\left|\int_0^{\pi} f''\left(\frac{cos\theta+n}{2^k}\right)\lambda_m(\theta)d\theta\right| \\ &\leq \frac{L}{2^{\frac{5k-1}{2}}\sqrt{\pi}}\int_0^{\pi}|\lambda_m(\theta)|d\theta \\ &\leq \frac{L\sqrt{\pi}}{2^{\frac{(5k-1)}{2}}} \\ &\left[\frac{1}{m}\left(\frac{1}{m-1}+\frac{1}{m+1}\right)+\frac{1}{m+2}\left(\frac{1}{m+1}+\frac{1}{m+3}\right)\right] \\ &= \frac{L\sqrt{\pi}}{2^{\frac{(5k-1)}{2}}(m^2+2m-3)} \\ &< \frac{2L\sqrt{\pi}}{2^{\frac{(5k-5)}{2}}(m+1)^2}\end{aligned}$$

Since $n \leq 2^k-1$, we have

$$|c_{nm}| < \frac{8\sqrt{2\pi}L}{(n+1)^{\frac{5}{2}}(m+1)^2}$$

This completes the proof of theorem.

2.4 Linear second-order two-point boundary value problems

[19]
Consider the linear second-order differential equation

$$y''(x)+g_1(x)y'(x)+g_2(x)y(x) = G(x), \quad x \in [0,1] \tag{22}$$

Subject to the initial conditions

$$y(0) = \alpha,\ y'(0) = \beta \tag{23}$$

or the boundary conditions

$$y(0) = \alpha\ y(1) = \beta \tag{24}$$

or the most general mixed boundary conditions

$$a_1y(0)+a_2y'(0) = \alpha,\ \ b_1y(1)+b_2y'(1) = \beta. \tag{25}$$

If we approximate the functions $y(x), g_1(x), g_2(x)$ and $G(x)$ in terms of the second kind Chebyshev wavelet basis, one can write

$$\begin{aligned} y(x) &\approx \sum_{n=0}^{2^k-1}\sum_{m=0}^{M} c_{nm}\psi_{nm}(x) = C^T\psi(x) \\ g_1(x) &\approx \sum_{n=0}^{2^k-1}\sum_{m=0}^{M} g_{nm}\psi_{nm}(x) = G_1^T\psi(x)\end{aligned} \tag{26}$$

$$\begin{aligned} g_2(x) &\approx \sum_{n=0}^{2^k-1}\sum_{m=0}^{M} g_{nm}\psi_{nm}(x) = G_2^T\psi(x) \\ g(x) &= \sum_{n=0}^{2^k-1}\sum_{m=0}^{M} g_{nm}\psi_{nm}(x) = G^T\psi(x)\end{aligned} \tag{27}$$

$$Then\ y'(x) \approx C^TD\psi(x),\ y''(x) = C^TD^2\psi(x) \tag{28}$$

Now substitution of relations Eq.(26), Eq.(27) and Eq.(28) into Eq.(22), enable us to define the residual $R(x)$, of this equation as

$$\begin{aligned}R(x) = C^TD^2\psi(x)+G_1^T\psi(x)(\psi(x))^TD^TC+ \\ G_2^T\psi(x)(\psi(x))^TC-G^T\psi(x).\end{aligned} \tag{29}$$

and application of the tau method, yields the following $(2^k(M+1)-2)$ linear equations in the unknown expansion coefficients, c_{nm}, namely

$$\int_0^t \sqrt{x-x^2}\psi_j(x)R(x)dx = 0, \quad j = 1,2,\cdots,2^k(M+1)-2 \tag{30}$$

Moreover, the initial conditions Eq.(23), the boundary conditions Eq.(24), and the mixed boundary conditions Eq.(25) lead respectively, to the following equations

$$C^T\psi(0) = \alpha,\ C^TD\psi(0) = \beta,$$

$$C^T\psi(0) = \alpha,\ C^T\psi(1) = \beta \tag{31}$$

$$a_1C^T\psi(0)+a_2D\psi(0) = \alpha,\ b_1C^T\psi(1)+b_2C^TD\psi(1) = \beta \tag{32}$$

Thus Eq.(29) with the two equations of Eq.(31) or Eq.(32) generate $2^k(M+1)$ a set of linear equations which can be solved for the unknown components of the vector C, and hence an approximate spectral wavelets solution to $y(x)$ can be obtained.

3 Numerical Examples

Example 3.1 Consider the following nonlinear second order differential equation [34]

$$y''+\ln x\, y'+y^2 = 2+2x\ln x+x^4 \tag{33}$$

with the initial condition $y(0) = 0, y'(0) = 0, x \in (0,1)$

$$which\ has\ the\ exact\ solution\ y(x) = x^2. \tag{34}$$

We solve Eq.(33) using the algorithm described in section.3

We solve this problem by using the shifted second kind Chebyshev wavelets (S2KCWM) with values of $k = 0$ and $M = 2$.

Put $y(x) = C^T\psi(x), y'(x) = C^T D\psi(x), y''(x) = C^T D^2\psi(x)$ then the Eq.(33) becomes

$$[C^T D^2\psi(x)] + \ln x[C^T D\psi(x)] + [C^\psi(x)]^2 = 2 + 2x\ln x + x^4 \quad (35)$$

Use the initial conditions $C^T\psi(0) = 1$ and $C^T D\psi(0) = 0$ leads to the two equations

$$2C_0 - 4C_1 + 6C_2 = 1 \quad (36)$$

$$8C_1 - 32C_2 = 0 \quad (37)$$

Using equation (37), we get the following relations between the unknown coefficients

$$C_1 = 4C_2 \quad (38)$$

Substitute (38) in the equation (36) we obtain

$$C_0 = 5C_2 \quad (39)$$

Applying (38) and (39) in $C^T D^2\psi(x), C^T D\psi(x), C^T\psi(x)$, equation (35) becomes

$$64C_2 + \ln x(9.372C_2) + (0.470596C_2)^2 = 2 + 2x\ln x + x^4 \quad (40)$$

We only need to satisfy this equation at the first root of $U_3^*(x)$. That is $x = \frac{2-\sqrt{2}}{4}$

Equation (40) reduced to

$$(0.470596)C_2^2 + 45.9926392C_2 - 1.4379 = 0 \quad (41)$$

Solving the equation (41) we obtain the root $C_2 = 0.03125$

Substitute these into (38) and (39) we get $C_0 = 0.15625$ and $C_1 = 0.125$

Thus the solution can be written as

$$y(x) = C^T\psi(x) = (C_0, C_1, C_2)\begin{pmatrix} 2 \\ 8x-4 \\ 32x^2-32x+6 \end{pmatrix}$$

Thus $y(x) = x^2$, which is the exact soution.

It is clear that in Example 3.1, our proposed method is speedily convergent to the exact solution, for a small value of $M = 2$, we attain the exact solution. This proves the efficiency of our shifted second kind Chebyshev wavelets (S2KCWM) method.

Example 3.2 Let us consider the another problem [35]

$$-y'' - \frac{2}{x}y' + (1-x)^2 y = x^4 - 2x^2 + 7 \quad (42)$$

$$\textit{With the conditions } y'(0) = 0 \textit{ and } y(1) = 0 \quad (43)$$

The exact solution of Eqs.(42)-(43) is $1 - x^2$.

By applying the shifted second kind Chebyshev wavelet method (S2KCWM) to the given boundary condition $y'(0) = 0$ we get $8C_1 - 32C_2 = 0$, it gives the relation

$$C_1 = 4C_2 \quad (44)$$

Table 1: Errors of the proposed methd compared with results in [35] for the example 3.2

x	Spline h = 0.05[35]	Spline h = 0.0125[35]	our solution
0.1	2e-06	2e-06	0
0.2	2e-06	2e-06	0
0.3	2e-06	1e-06	0
0.4	2e-06	1e-06	0
0.5	1e-06	1e-06	0
0.6	1e-06	1e-06	0
0.7	1e-06	1e-06	0
0.8	1e-06	1e-06	0
0.9	0	0	0
1	0	0	0

Similarly using another condition $y(1) = 0$, we obtain

$$C_0 = -11C_2 \quad (45)$$

Applying the proposed algorithm to the equation (42) and using (44) and (45) we have the following result $C_2 = -0.03125$.

Also we get $C_0 = 0.34375$ and $C_1 = -0.125$

Therefore the wquation (42) gives the solution by

$$y(x) = C^T\psi(x) = (C_0, C_1, C_2)\begin{pmatrix} 2 \\ 8x-4 \\ 32x^2-32x+6 \end{pmatrix}$$

$$= (0.34375, -0.125, -0.03125)\begin{pmatrix} 2 \\ 8x-4 \\ 32x^2-32x+6 \end{pmatrix}$$

That is $y(x) = 1 - x^2$ which is the exact solution.

Result of the proposed method compared with results in the Cubic spline method [35]

Comparision of Errors between the proposed method and spline method [35] for the problem 3.2 documented in Table.1. From the table, obiviously we understand our method produces best result, since our method gives the exact solution.

Example 3.3 Next we consider the nonlinear problem [33]

$$y'' = \frac{1}{2}[y + x + 1]^3, 0 < x < 1 \quad (46)$$

Subject to the boundary conditions $y(0) = 0$ and $y(1) = 0$

whose exact solution is $y(x) = \frac{2}{2-x} - x - 1$.

we solve the equation (46) using the algorithm described in Section 3 for the case corresponds to $M = 2$ and $k = 0$, we obtain $C_0 = -0.0315, C_1 = 0, C_2 = 0.0105$ Consequently we have

$$y = C^T\psi(x) = (-0.0315, 0, 0.0105)\begin{pmatrix} 2 \\ 8x-4 \\ 32x^2-32x+6 \end{pmatrix}$$

$$\textit{We get } y(x) = (0.3360)x^2 - (0.3360)x \quad (47)$$

The numerical results are presented in the following Table

Table 2: Comparison between exact and S2KCWM solutions for example 3.3

x	Exact	S2KCWM k =0, M=2
0	0	0
0.1	-0.0474	-0.0302
0.2	-0.0889	-0.0538
0.3	-0.1235	-0.0706
0.4	-0.1500	-0.0806
0.5	-0.1667	-0.0840
0.6	-0.1714	-0.0806
0.7	-0.1615	-0.0706
0.8	-0.1333	-0.0538
0.9	-0.0818	-0.0302
1	0	0

Table 3: Absolute errors for Example 3.3

x	S2KCWM k =0, M=2
0	0
0.1	0.0172
0.2	0.0351
0.3	0.0529
0.4	0.0694
0.5	0.0827
0.6	0.0908
0.7	0.0909
0.8	0.0795
0.9	0.0516
1	0

Example 3.4 Consider the singular boundary value problem [6]

$$y''(x)+\frac{1}{x}y'(x)=\left(\frac{8}{8-x^2}\right)^2 y'(0)=0 \; and \; y(1)=0 \quad (48)$$

The exact solution in a closed form is

$$y(x)=2\log_e\left(\frac{7}{8-x^2}\right) \quad (49)$$

Using the aforesaid method with $k=0$ and $M=2$, we gain

Thus we can write $y=(0.2528)x^2-(0.0376)x-0.25$

Example 3.5 Consider the Bessel differential equation of order zero [30,31]

$$xy''(x)+y'(x)+xy(x)=0 \quad (50)$$

$y(0)=1, y'(0)=0, x\in(0,1)$

For $M=2$ and $k=0$ we get

$C_0=0.4611; C_1=-0.0311; C_2=-0.0078$

Thus we can write

$$y(x)=C^T\psi(x)=(-0.2496)x^2+0.0008x+0.9998 \quad (51)$$

Table 4: Comparison beteen exact and S2KCWM solutions for Example 3.4

x	Exact	S2KCWM M=2
0.1	-0.2646	-0.2512
0.2	-0.2570	-0.2474
0.3	-02444	-0.2385
0.4	-0.2267	-0.2246
0.5	-0.2036	-0.2056
0.6	-0.1750	-0.1816
0.7	-0.1407	-0.1524
0.8	-0.1003	-0.1183
0.9	-0.0536	-0.0791

Table 5: Absolute errors for Example 3.4

x	S2KCWM k =0, M=2
0.1	0.0134
0.2	0.0096
0.3	0.0059
0.4	0.0021
0.5	0.0020
0.6	0.0066
0.7	0.0117
0.8	0.0180
0.9	0.0255

Table 6: Comparison between exact and S2KCWM solutions for Example 3.5

x	Exact	S2KCWM k = 0, M=2
0.1	0.997502	0.997384
0.2	0.990025	0.989976
0.3	0.977626	0.977576
0.4	0.960398	0.960184
0.5	0.938470	0.937800
0.6	0.912005	0.910424
0.7	0.881201	0.878056
0.8	0.846287	0.840696
0.9	0.807524	0.798344

Table 7: Absolute errors for Example 3.5

x	S2KCWM k =0, M=2
0.1	0.000118
0.2	0.000049
0.3	0.000005
0.4	0.000124
0.5	0.000670
0.6	0.001581
0.7	0.003145
0.8	0.005591
0.9	0.009180

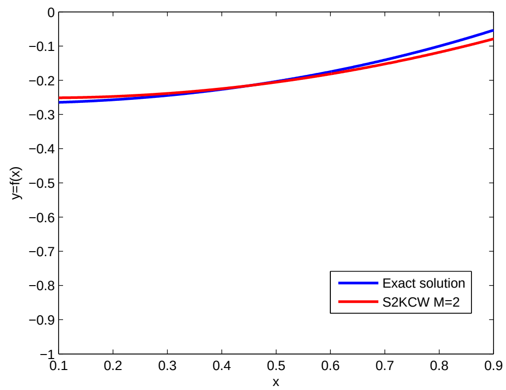

Fig. 1: Comparison between exact and S2KCWM solutions with k =0, M= 2 for Example 3.4

Table 8: Comparison between exact and S2KCWM solutions for Example 3.6

x	Exact	S2KCWM k = 0, M=2
0	0	0
0.1	0.0984	0.1650
0.2	0.1871	0.2934
0.3	0.2575	0.3851
0.4	0.3027	0.4401
0.5	0.3183	0.4584
0.6	0.3027	0.4401
0.7	0.2575	0.3851
0.8	0.1871	0.2934
0.9	0.0984	0.1650
1	0	0

Example 3.6 Consider the singular boundary value problem [32]

$$y''(x)+\pi^3\frac{[y(x)]^2}{sin(\pi x)}=0,\ \ 0<x<1 \tag{52}$$

$y(0)=0, y(1)=0,\ x\in[0,1]$
which has the exact solution

$$y(x)=\frac{1}{\pi}sin(\pi x) \tag{53}$$

We solve Eq.(52) using S2KCWM with values of $k=0$ and $M=2$
Solving the system of equations, we obtain

$C_0=0.013993; C_1=0; C_2=-0.00466$

Thus we can write

$$y(x)=C^T\psi(x)=(-0.14912)x^2+(0.14912)x \tag{54}$$

Table 9: Absolute errors for Example 3.6

x	S2KCWM k =0, M=2
0	0
0.1	0.0666
0.2	0.1063
0.3	0.1276
0.4	0.1374
0.5	0.1401
0.6	0.1374
0.7	0.1276
0.8	0.1063
0.9	0.0666
1	0

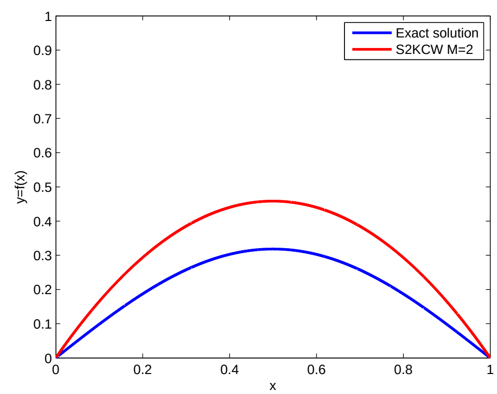

Fig. 2: Comparison between exact and S2KCWM solutions with k =0, M= 2 for Example 3.6

Example 3.7 Finally we consider the equation

$$y''-y'+xy=x^2 \tag{55}$$

$y(0)=1, y'(0)=-1,\ x\in[0,1]$
The exact solution in a closed form is

$$y(x)=1-x-\frac{x^2}{2}-\frac{x^3}{3}+\frac{x^4}{12}-\frac{x^5}{24}+\cdots \tag{56}$$

For $M=2$ and $k=0$, we have the linear system of equations and solving this system we gain
$C_0=0.1668; C_1=-0.1916; C_2=-0.0166$
Thus we can write

$$y(x)=C^T\psi(x)=(-0.5312)x^2-(1.0016)x+0.9992 \tag{57}$$

The numerical results for the absolute errors are given in Table.11. Obviously we identified the errors are close to zero. This proves the efficiency of our proposed algorithm

Table 10: Comparison between exact and S2KCWM solutions for Example 3.7

x	Exact	S2KCWM M=2
0.1	0.894675	0.893728
0.2	0.777467	0.777632
0.3	0.646676	0.650912
0.4	0.500802	0.513568
0.5	0.338546	0.365600
0.6	0.158807	0.207008
0.7	-0.039314	0.037792
0.8	-0.256518	-0.142048
0.9	-0.493303	-0.332512

Table 11: Absolute errors for Example 3.7

x	S2KCWM k =0, M=2
0.1	0.000947
0.2	0.000165
0.3	0.004236
0.4	0.012766
0.5	0.027054
0.6	0.048201
0.7	0.077106
0.8	0.114470
0.9	0.160791

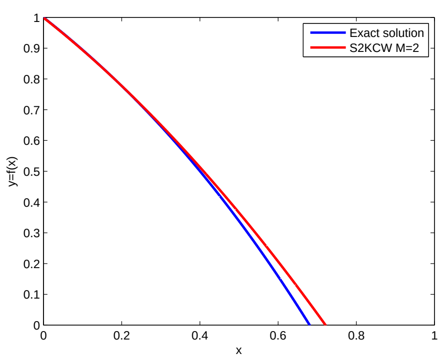

Fig. 3: Comparison between exact and S2KCWM solutions with k =0, M= 2 for Example 3.7

4 Results and Discussion

The accuracy of the results is estimated by error function $E = |C_{exact} - C_{CWM}|$. The results are shown in Tables (See Tables 1,3,5,7,9 and 11). In order to assess the advantages, efficiency and the accuracy of the shifted second kind Chebyshev wavelets method (S2KCWM) for solving the nonlinear differential equations, we use our method to solve nonlinear differential equation, whose exact solutions are known. Results in the Tables 1,3,5,7,9 and 11 show that the Chebyshev wavelet method agrees with the results obtained in. When solving the non-periodic problems, the second kind Chebyshev wavelet has the superiorities (the calculation is easy implementation, and the approximation effect is better). Figures (1-3) show the comparison between the exact solutions and the S2KCWM solutions for various values of x. In this paper, the second kind Chebyshev wavelet method (S2KCWM) has been compared with the exact solution. Results in the Tables 2, 4,6,8 and 10 show the comparison between the exact solutions and the Chebyshev wavelet solutions for various values of x and M.

In Example 3.1 and 3.2, for a small value of $k = 0, M = 2$, the shifted second kind Chebyshev wavelets method (S2KCWM) solutions have attained the exact solutions. Errors of the remaining examples are also very close to zero. This proves the efficiency of our algorithm.

All the numerical experiments presented in this section were computed in double precision with some MATLAB codes on a personal computer System Vostro 1400 Processor x86 Family 6 Model 15 Stepping 13 Genuine Intel 1596 Mhz.

5 Concluding remarks

This paper provides shifted second kind Chebyshev wavelet methods (S2KCWM) for solving a few nonlinear differential equations arising in engineering. It offers a state-of-the-art in several active areas of research where numerical methods for solving nonlinear differential equations have proved particularly effective. The proposed schemes are the capability to overcome the difficulty arising in calculating the integral values while dealing with nonlinear problems. This method shows higher efficiency than the traditional methods for solving nonlinear ODEs. Also the proposed method has a simple implementation process. It may be concluded that S2KCWM is very powerful and efficient in finding analytical as well as numerical solutions for a wide class of linear and nonlinear differential equations. It provides more realistic series solutions that converge very rapidly in real physical problems.

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