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A New Fuzzy Linear Regression Model for a Special Case of Interval Type-2 Fuzzy Sets

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Abstract: This paper presents a regression model for a special case of interval type-2 fuzzy sets based on the least squares estimation technique. Unknown coefficients are assumed to be triangular fuzzy numbers. The basic idea is to determine aggregation intervals for triangular fuzzy numbers membership functions of whose are low membership function and upper membership function of interval type-2 fuzzy set and to determine an affinity measure for two interval type-2 fuzzy sets based on these intervals.

Keywords: interval type-2 fuzzy sets, fuzzy regression, weighted interval.

1 Introduction

Currently, there are three different approaches in fuzzy regression analysis. Let us dwell briefly on each of them:

(a) Methods proposed by H. Tanaka [1] and investigated by H. Tanaka, et al. [2,3,4], A. Celmins [5, 6], D. Savic and W. Pedrycz [7], Y.-H.O. Chang [8,9,10] Y.-H.O. Chang and B.M. Ayyub [11] in current literature, where the coefficients of input variables are assumed to be fuzzy numbers. These fuzzy regression models are based on the possibility theory instead of the probability theory or they are based on both possibility and probability theories.

(b) Method proposed by R.J. Hathaway and J.C. Bezdek [12] where first the fuzzy clusters determined by an fuzzy c-means clustering (FCM) algorithm define how many ordinary regressions are to be constructed, one for each cluster. Next each fuzzy cluster is used essentially for switching purposes to determine the most appropriate ordinary regression that is to be applied for a new input from amongst a number of ordinary regressions determined in the first place.

(c) Methods proposed by I.B.Turksen [13] and A. Celikyilmaz [14], where the fuzzy functions (FFs) approach to system modeling was developed. The new FFs approach augments the membership values together with their transformations to form a new input variable to find local functions. First the given system domain is fuzzy partitioned into c clusters using fuzzy c-means clustering (FCM) algorithm. Then, one regression function is calculated to model the behavior of each partition. In [13] linear regression function to estimate the parameters of each function is proposed. A new fuzzy system modeling (FSM) approach that identifies the fuzzy functions using support vector machines (SVM) is proposed in [15]. This new approach is structurally different from the fuzzy rule base approaches and fuzzy regression methods. Method SVM is applied to determine the support vectors for each fuzzy cluster obtained by fuzzy c-means (FCM) clustering algorithm. Original input variables, the membership values obtained from the FCM together with their transformations form a new augmented set of input variables. Methods proposed in [13,14], were investigated in [16].

The methods of fuzzy regression from group (a) have received a lot of developing in the past years [1,2,3,4,17, 18,19,20,21] A major difference between fuzzy regression and ordinary regression is in dealing with errors as fuzzy variables in fuzzy regression modeling, and in dealing with errors as random variables in ordinary regression modeling. The researchers have tried to integrate both fuzziness and randomness into regression model. As a result of this the hybrid fuzzy least-squares regressions were developed [8,9,10,11,22,23,24].

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However, the methods of fuzzy regression analysis are limited by consideration of type-1 fuzzy sets (T1 FSs). Moreover, the fuzzy regression analysis must provide a way to model the observed fuzzy data such as words (for example linguistic descriptions of the type: “good”, “very good”, “excellent”) models of whose may be interval type-2 fuzzy sets (IT2 FSs), proposed by L.Zadeh and developed by J. M. Mendel [25]. IT2 FSs have enough degrees of freedom to save individual data of subjects about a word (“words mean different things to different people, and so are uncertain” [25]) and to obtain a collective model for this word. In order to include IT2 FSs into a regression, a need for developing a new method exists. The basic idea of this method is to determine aggregation intervals for triangular fuzzy numbers, membership functions of whose are low membership function (LMF) and upper membership function (UMF) of IT2 FS, to determine an affinity measure for two IT2 FSs based on these intervals and to use the least squares estimation technique.

2 Weighted intervals for interval type-2 fuzzy sets

Let us consider a special case of IT2 FS $\tilde{A}$ shown in Fig. 1.

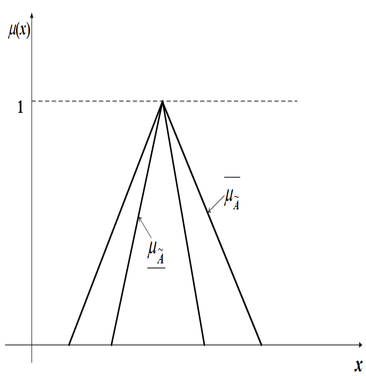

Fig. 1. IT2 FS $\tilde{A}$ with LMF $\underline{\mu_{\tilde{A}}}$ and UMF $\overline{\mu_{\tilde{A}}}$.

This IT2 FS is defined by LMF and UMF , which are denoted by $\underline{\mu_{\tilde{A}}}$ and $\overline{\mu_{\tilde{A}}}$ respectively, $\underline{\mu_{\tilde{A}}} = \left(a^L, a_l^L, a_r^L\right)$, $\overline{\mu_{\tilde{A}}} = \left(a^U, a_l^U, a_r^U\right)$. The first parameter in bracket is abscissa of the apex of the triangle that is a graph of the corresponding membership function, while the last two parameters are the lengths of the left and right triangle wings correspondingly.

The definition of weighted point B for a triangular number $\tilde{B} = (b, b_l, b_r)$ was given in [10]:

$$B = \frac{\int_0^1 \left(\frac{B_\alpha^1 + B_\alpha^2}{2}\right) 2\alpha d\alpha}{\int_0^1 2\alpha d\alpha} = \int_0^1 \left(B_\alpha^1 + B_\alpha^2\right) \alpha d\alpha =$$

$$\int_0^1 \left(b - (1-\alpha) b_l + b + (1-\alpha) b_r\right) \alpha d\alpha = b + \tfrac{1}{6}\left(b_r - b_l\right)$$

According to this definition, two normalized symmetrical triangular numbers with different fuzzy widths are converted into one crisp number. For example, let consider two triangular fuzzy numbers: $\tilde{A} = (2, 2, 2)$, $\tilde{B} = (2, 1, 1)$. The weighted points for numbers $\tilde{A}$,$\tilde{B}$ we shall designate accordingly as A, B, then:

$$A = \int_0^1 \left(4 - 2(1-\alpha) + 2(1-\alpha)\right) \alpha d\alpha = 2,$$

$$B = \int_0^1 \left(4 - (1-\alpha) + (1-\alpha)\right) \alpha d\alpha = 2.$$

While this may not present a problem to solve a number of practical tasks, however, for example, in decision-making problems and some other problems the necessity arises to find aggregative indexes that will possibly accumulate different bounds of input fuzzy numbers.

That is why we propose to use the definition of weighted point for a triangular number in order to determine a weighted interval for this number.

Let define the weighted set for the triangular l fuzzy number $\tilde{A} \equiv (a, a_l, a_r)$ as the set of weighted points of all triangular numbers $\tilde{B} \equiv (b, b_l, b_r)$ that belong to the number $\tilde{A}$.

Proposition 1 [26]. The weighted set for the triangular fuzzy number $\tilde{A} \equiv (a, a_l, a_r)$ is an interval $[A_1, A_2]$, such as $A_1 = a - \frac{1}{6} a_l$, $A_2 = a + \frac{1}{6} a_r$.

We shall call the interval $[A_1, A_2]$ the weighted interval for the triangular fuzzy number $\tilde{A} \equiv (a, a_l, a_r)$.

Let consider two triangular fuzzy numbers: $\tilde{A} = (2, 2, 2)$, $\tilde{B} = (2, 1, 1)$ again and define the weighted intervals $[A_1, A_2]$,$[B_1, B_2]$ for numbers $\tilde{A}$, $\tilde{B}$.

$$A_1 = \int_0^1 \left(4 - 2(1-\alpha)\right) \alpha d\alpha = 2 - 2 \times \frac{1}{6} = 1\frac{2}{3},$$

$$A_2 = \int_0^1 \left(4 + 2(1-\alpha)\right) \alpha d\alpha = 2 + 2 \times \frac{1}{6} = 2\frac{1}{3},$$

$$B_1 = \int_0^1 (4-(1-\alpha))\,\alpha d\alpha = 2-\frac{1}{6} = 1\frac{5}{6},$$

$$B_2 = \int_0^1 (4+2(1-\alpha))\,\alpha d\alpha = 2+\frac{1}{6} = 2\frac{1}{6},$$

$$[A_1, A_2] = \left[1\frac{2}{3}, 2\frac{1}{3}\right], [B_1, B_2] = \left[1\frac{5}{6}, 2\frac{1}{6}\right].$$

It can be observed that the weighted points of $\tilde{A}$ and $\tilde{B}$are the same while the weighted intervals for these fuzzy numbers are different. The greater wings of the triangle, the greater the weighted interval.

The weighted intervals are suggested to be used in situations where it is necessary to accumulate more information about fuzzy numbers than aggregative point crisp indexes contain when there is no requirement to get only aggregative numbers.

Proposition 2 [26]. The weighted interval for number $\tilde{A}+\tilde{B}$ can be obtained as $[A_1+B_1, A_2+B_2]$, where $[A_1, A_2], [B_1, B_2]$ are weighted intervals for triangular numbers$\tilde{A}$, $\tilde{B}$.

Proposition 3 [26]. The boundaries of weighted interval for number $\tilde{D} = \tilde{A}\times\tilde{B}$ are defined by linear combinations of parameters $\tilde{A}$, $\tilde{B}$.

Let us consider nonnegative $\tilde{A} \equiv (a, a_l, a_r)$ and a triangular number $\tilde{a} \equiv \left(b, b_l, b_r\right)$.

Proposition 4 [27]. Boundaries of the weighed interval $\left[\theta^1_{\tilde{a}\tilde{A}},\, \theta^2_{\tilde{a}\tilde{A}}\right]$ of product of fuzzy numbers $\tilde{a}$ and $\tilde{A}$ look like

$$\theta^1_{\tilde{a}\tilde{A}} = b\left(a+(-1)^q\frac{1}{6}a_{M_q}\right) - b_l\left(\frac{1}{6}a+(-1)^q\frac{1}{12}a_{M_q}\right),$$

$$\theta^2_{\tilde{a}\tilde{A}} = b\left(a+(-1)^p\frac{1}{6}a_{M_p}\right) + b_r\left(\frac{1}{6}a+(-1)^q\frac{1}{12}a_{M_p}\right),$$

$$q = \begin{cases} 1, b-b_l \geq 0 \\ 2, b+b_r < 0 \end{cases}, M_q = \begin{cases} l, q=1 \\ r, q=2 \end{cases},$$

$$p = \begin{cases} 2, b-b_l \geq 0 \\ 1, b+b_r < 0 \end{cases}, M_p = \begin{cases} l, p=1 \\ r, p=2 \end{cases}.$$

Let determine aggregation intervals $\left[A^L_1, A^L_2\right], \left[A^U_1, A^U_2\right]$ for LMF $\underline{\mu_{\tilde{A}}} = \left(a^L, a^L_l, a^L_r\right)$ and UMF $\overline{\mu_{\tilde{A}}} = \left(a^U, a^U_l, a^U_r\right)$ of IT2 FS $\tilde{A}$:

$$A^L_1 = a^L - \frac{1}{6}a^L_l, A^L_2 = a^L + \frac{1}{6}a^L_r,$$

$$A^U_1 = a^U - \frac{1}{6}a^U_l, A^U_2 = a^U + \frac{1}{6}a^U_r.$$

Let us define an affinity measure for two IT2 FSs $\tilde{A}, \tilde{B}$ with weighed intervals $\left[A^L_1, A^L_2\right], \left[A^U_1, A^U_2\right], \left[B^L_1, B^L_2\right], \left[B^U_1, B^U_2\right]$

$$f^2\left(\tilde{A}, \tilde{B}\right) = \left(A^L_1 - B^L_1\right)^2 + \left(A^L_2 - B^L_2\right)^2 + \left(A^U_1 - B^U_1\right)^2 + \left(A^U_2 - B^U_2\right)^2.$$

3 Problem formulation and solution

Let $\tilde{Y}_i i = \overline{1,n}$ are output IT2 FSs , defined by LMFs $\underline{\mu_{\tilde{Y}_i}} = \left(y^{iL}, y^{iL}_l, y^{iL}_r\right), i = \overline{1,n}$ and UMFs $\overline{\mu_{\tilde{Y}_i}} = \left(y^{iU}, y^{iU}_l, y^{iU}_r\right), y^{iU} - y^{iU}_l \geq 0 i = \overline{1,n}$.

Let $\tilde{X}^i_j$, $j = \overline{1,m}$, $i = \overline{1,n}$ input IT2 FSs, defined by LMFs $\underline{\mu_{\tilde{X}^i_j}} = \left(x^{jiL}, x^{jiL}_l, x^{jiL}_r\right)$ and UMFs $\overline{\mu_{\tilde{X}^i_j}} = \left(x^{jiU}, x^{jiU}_l, x^{jiU}_r\right), x^{jiU} - x^{jiU}_l \geq 0, j = \overline{1,m}, i = \overline{1,n}$. LMFs and UMFs of output and input IT2 FSs are triangular fuzzy numbers.

The linear fuzzy regression model relates $\tilde{Y}$(with meanings $\tilde{Y}_i i = \overline{1,n}$) to $\tilde{X}_j, j = \overline{1,m}$ (with meanings $\tilde{X}^i_j$, $j = \overline{1,m}$, $i = \overline{1,n}$) as follows:

$$\tilde{Y} = \tilde{a}_0 + \tilde{a}_1\tilde{X}_1 + \ldots + \tilde{a}_m\tilde{X}_m.$$

$\tilde{a}_j \equiv \left(b^j, b^j_l, b^j_r\right), j = \overline{0,m}$ - unknown coefficients, which are defined as triangular numbers (not necessarily symmetrical).

The method of regression's creation is based on the transformation of the LMFs and UMFs of input and output IT2 FSs into weighted intervals.

Let us determine the weighed intervals $\left[\theta^{1L}_{\hat{Y}_i},\, \theta^{2L}_{\hat{Y}_i}\right]$, $\left[\theta^{1U}_{\hat{Y}_i},\, \theta^{2U}_{\hat{Y}_i}\right]$ for LMFs and UMFs of model output data $\hat{Y}_i = \tilde{a}_0 + \tilde{a}_1\tilde{X}^i_1 + \ldots + \tilde{a}_m\tilde{X}^i_m$ using propositions 1-4:

$$\theta^{1L}_{\hat{Y}_i} = b^0 - \frac{1}{6}b^0_l + \sum_{j=1}^m \theta^{1L}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,\, b^j_l,\, b^j_r\right),$$

$$\theta^{2L}_{\hat{Y}_i} = b^0 - \frac{1}{6}b^0_l + \sum_{j=1}^m \theta^{2L}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,\, b^j_l,\, b^j_r\right),$$

$$\theta^{1U}_{\hat{Y}_i} = b^0 - \frac{1}{6}b^0_l + \sum_{j=1}^m \theta^{1U}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,\, b^j_l,\, b^j_r\right),$$

$$\theta^{2U}_{\hat{Y}_i} = b^0 - \frac{1}{6}b^0_l + \sum_{j=1}^m \theta^{2U}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,\, b^j_l,\, b^j_r\right),$$

$$\theta^{1L}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j, b^j_l, b^j_r\right) = b^j\left(x^{jiL} + (-1)^q\tfrac{1}{6}x^{jiL}_{M_q}\right) - b^j_l\left(\tfrac{1}{6}x^{jiL} + (-1)^q\tfrac{1}{12}x^{jiL}_{M_q}\right),$$

$$\theta^{2L}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_l,b^j_r\right)=b^j\left(x^{jiL}+(-1)^p\tfrac{1}{6}x^{jiL}_{M_p}\right)$$
$$+b^j_r\left(\tfrac{1}{6}x^{jiL}+(-1)^q\tfrac{1}{12}x^{jiL}_{M_p}\right),$$

$$\theta^{1U}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_l,b^j_r\right)=b^j\left(x^{jiU}+(-1)^q\tfrac{1}{6}x^{jiU}_{M_q}\right)$$
$$-b^j_l\left(\tfrac{1}{6}x^{jiU}+(-1)^q\tfrac{1}{12}x^{jiU}_{M_q}\right),$$

$$\theta^{2U}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_l,b^j_r\right)=b^j\left(x^{jiU}+(-1)^p\tfrac{1}{6}x^{jiU}_{M_p}\right)$$
$$+b^j_r\left(\tfrac{1}{6}x^{jiU}+(-1)^q\tfrac{1}{12}x^{jiU}_{M_p}\right),$$

$$q=\begin{cases}1, b-b_l\geq 0\\ 2, b+b_r<0\end{cases}, M_q=\begin{cases}l, q=1\\ r, q=2\end{cases},$$

$$p=\begin{cases}2, b-b_l\geq 0\\ 1, b+b_r<0\end{cases}, M_p=\begin{cases}l, p=1\\ r, p=2\end{cases}.$$

LMFs and UMFs of model output data will not be triangular fuzzy numbers. While multiplying fuzzy numbers it is not always possible to set an analytical form for membership function of a fuzzy number which is a result out of the multiplication. But we can always determine model output data with the help of α-cuts.

For example if $\tilde{a}\equiv\left(b,b_l,b_r\right)$ is a negative fuzzy number $(b+b_r<0)$, $\tilde{A}=(a,a_l,a_r)$ is a nonnegative number $(a-a_l\geq 0)$ then according to multiplication operation for fuzzy numbers, the α-cut of $\tilde{a}\tilde{A}$ looks like $\left[C^1_\alpha,C^2_\alpha\right]$, where:

$$C^1_\alpha=ba+(1-\alpha)ba_r-(1-\alpha)b_la-(1-\alpha)^2b_la_r,$$

$$C^2_\alpha=ba+(1-\alpha)ba_l+(1-\alpha)b_ra-(1-\alpha)^2b_ra_l.$$

If $\tilde{a}\equiv\left(b,b_l,b_r\right)$ is a nonnegative fuzzy number $\left(b-b_l\geq 0\right)$ and $\tilde{A}=(a,a_l,a_r)$ is a nonnegative number $(a-a_l\geq 0)$ then according to multiplication operation for fuzzy numbers, the α-cut of $\tilde{a}\tilde{A}$ looks like $\left[C^1_\alpha,C^2_\alpha\right]$, where:

$$C^1_\alpha=ba-(1-\alpha)ba_l-(1-\alpha)b_ra+(1-\alpha)^2b_ra_l,$$

$$C^2_\alpha=ba+(1-\alpha)ba_r+(1-\alpha)b_la+(1-\alpha)^2b_la_r.$$

Let us determine the weighed intervals $\left[\theta^{1L}_{\tilde{Y}_i},\theta^{2L}_{\tilde{Y}_i}\right]$, $\left[\theta^{1U}_{\tilde{Y}_i},\theta^{2U}_{\tilde{Y}_i}\right]$ for LMFs and UMFs of initial output data $\tilde{Y}_i i=\overline{1,n}$:

$$\theta^{1L}_{\tilde{Y}_i}=y^{iL}-\frac{1}{6}y^{iL}_l,\theta^{2L}_{\tilde{Y}_i}=y^{iL}+\frac{1}{6}y^{iL}_r,$$

$$\theta^{1U}_{\tilde{Y}_i}=y^{iU}-\frac{1}{6}y^{iU}_l,\theta^{2U}_{\tilde{Y}_i}=y^{iU}+\frac{1}{6}y^{iU}_r.$$

Let us consider a functional $F\left(b^j,b^j_l,b^j_r\right)=\sum\limits_{i=1}^{n}f^2\left(\hat{Y}_i,\tilde{Y}_i\right)=$ $\sum\limits_{i=1}^{n}\left[\left(\theta^{1L}_{\hat{Y}_i}-\theta^{1L}_{\tilde{Y}_i}\right)^2+\left(\theta^{2L}_{\hat{Y}_i}-\theta^{2L}_{\tilde{Y}_i}\right)^2\right]+$ $\sum\limits_{i=1}^{n}\left[\left(\theta^{1U}_{\hat{Y}_i}-\theta^{1U}_{\tilde{Y}_i}\right)^2+\left(\theta^{2U}_{\hat{Y}_i}-\theta^{2U}_{\tilde{Y}_i}\right)^2\right]$, which characterizes an affinity measure between initial and model output data. It is easy to demonstrate that

$$F\left(b^j,b^j_l,b^j_r\right)=$$
$$\sum_{i=1}^{n}\left[b^0-\tfrac{1}{6}b^0_l-y^{iL}+\tfrac{1}{6}y^{iL}_l+\sum_{j=1}^{m}\theta^{1L}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_l,b^j_r\right)\right]^2+$$
$$\sum_{i=1}^{n}\left[b^0+\tfrac{1}{6}b^0_r-y^{iL}-\tfrac{1}{6}y^{iL}_r+\sum_{j=1}^{m}\theta^{2L}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_L,b^j_R\right)\right]^2+$$
$$\sum_{i=1}^{n}\left[b^0-\tfrac{1}{6}b^0_l-y^{iU}+\tfrac{1}{6}y^{iU}_l+\sum_{j=1}^{m}\theta^{1U}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_l,b^j_r\right)\right]^2+$$
$$\sum_{i=1}^{n}\left[b^0+\tfrac{1}{6}b^0_r-y^{iU}-\tfrac{1}{6}y^{iU}_r+\sum_{j=1}^{m}\theta^{2U}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_l,b^j_r\right)\right]^2.$$

The optimization problem is set as follows:

$$F\left(b^j,b^j_l,b^j_r\right)=\sum_{i=1}^{n}f^2\left(\hat{Y}_i,\tilde{Y}_i\right)\rightarrow\min,$$
$$b^j_l\geq 0, b^j_r\geq 0,\ j=\overline{0,m}.$$

As $\theta^{1L}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_l,b^j_r\right)$, $\theta^{2L}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_l,b^j_r\right)$, $\theta^{1U}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_l,b^j_r\right)$ and $\theta^{2U}_{\tilde{a}_j\tilde{X}^i_j}\left(b^j,b^j_l,b^j_r\right)$ are piecewise linear functions in the field $b^j_l\geq 0, b^j_r\geq 0,\ j=\overline{0,m}$, then F is piecewise differentiable function, and solutions of an optimization problem are found by means of known methods [28].

After obtaining the regression coefficients, it is of interest to evaluate the hybrid regression equation. For reliability evaluation, the standard deviation ($S_{\tilde{y}}$), a hybrid correlation coefficient (HR), a hybrid standard error of estimates (HS_e) are defined as follows:

$$S_{\tilde{y}}=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}f^2\left(\tilde{Y}_i,\overline{\tilde{Y}}\right)},\overline{\tilde{Y}}=\frac{\sum\limits_{i=1}^{n}\tilde{Y}_i}{n},$$

$$HR^2=\frac{\sum\limits_{i=1}^{n}f^2\left(\hat{Y}_i,\overline{\tilde{Y}}\right)}{\sum\limits_{i=1}^{n}f^2\left(\tilde{Y}_i,\overline{\tilde{Y}}\right)},$$

$$HS = \sqrt{\frac{1}{n-m-1}\sum_{i=1}^{n} f^2\left(\hat{Y}_i, \tilde{Y}_i\right)}.$$

Let $\hat{Y}_i i = \overline{1,n}$ are model output IT2 FSs, defined by LMFs $\mu_{\hat{Y}_i} = \left(v^{iL}, v_l^{iL}, v_r^{iL}\right), i = \overline{1,n}$ and UMFs $\overline{\mu_{\hat{Y}_i}} = \left(v^{iU}, v_l^{iU}, v_r^{iU}\right), i = \overline{1,n}$. After obtaining $\hat{Y}_i, i = \overline{1,n}$ a problem of identifying them with initial collection of words $Y_k, k = \overline{1,p}$, that formalized with the help of IT2 FSs $\tilde{\tilde{Y}}_k, k = \overline{1,p}$ defined by LMFs $\mu_{\tilde{\tilde{Y}}_k} = \left(y^{kL}, y_l^{kL}, y_r^{kL}\right), k = \overline{1,p}$ and UMFs $\overline{\mu_{\tilde{\tilde{Y}}_k}} = \left(y^{kU}, y_l^{kU}, y_r^{kU}\right), k = \overline{1,p}$ appears.

The weighted intervals for LMF and UMF of model $\hat{Y}_i$, $i = \overline{1,n}$ are designated by $\left[C_1^{iL}, C_2^{iL}\right]$, $\left[C_1^{iL}, C_2^{iL}\right]$, $i = \overline{1,n}$accordingly. The weighted intervals for LMF and UMF of $\tilde{\tilde{Y}}_k$, $k = \overline{1,p}$are designated by $\left[D_1^{iL}, D_2^{iL}\right]$, $\left[D_1^{iL}, D_2^{iL}\right]$, $k = \overline{1,p}$accordingly.

Let

$f^2\left(\hat{Y}_i, \tilde{\tilde{Y}}_k\right) = \left(C_1^{iL} - D_1^{kL}\right)^2 + \left(C_2^{iL} - D_2^{kL}\right)^2 + \left(C_1^{iU} - D_1^{iU}\right)^2 + \left(C_2^{iU} - D_2^{iU}\right)^2$, $i = \overline{1,n}$, $k = \overline{1,p}$The model$\hat{Y}_i$is identified to Y_s, if

$$f^2\left(\hat{Y}_i, \tilde{\tilde{Y}}_s\right) = \min_k f^2\left(\hat{Y}_i, \tilde{\tilde{Y}}_k\right),\ k = \overline{1,p}.$$

4 Conclusions

A method for a multiple fuzzy linear regression was developed in this paper. The input and output data of the regression model are interval type-2 fuzzy sets. The basic idea of this paper is to determine aggregation intervals for triangular fuzzy numbers, membership functions of whose are low membership function and upper membership function of interval type-2 fuzzy sets, to determine an affinity measure for two interval type-2 fuzzy sets based on these intervals and to use the least squares estimation technique. The proposed method extends a group of initial data membership functions, as it can be applied not only to type-1 fuzzy sets, but also to are interval type-2 fuzzy sets. For reliability evaluation, the standard deviation, the hybrid correlation coefficient, the hybrid standard error of estimates are defined.

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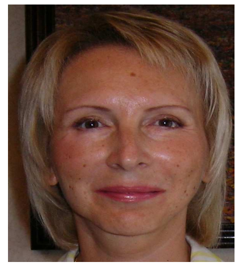

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