Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/100404

Lagrangian Discretization of Generic Second Order Models: Application to Traffic Control

Asma Khelifi [1,2,3,*], *Habib Haj-Salem* [2], *Jean-Patrick Lebacque*[2] *and Lotfi Nabli* [3]

[1] Ecole Nationale d'Ingenieurs de Tunis (ENIT), Rue Bechir Salem Belkhiria Campus universitaire, BP 37, 1002 Tunis, Tunisie.
[2] Universite Paris-Est, IFSTTAR, GRETTIA, 14-20 Boulevard Isaac Newton, Cite Descartes, Champs sur Marne, 77447 Marne la Valle cedex 2, France.
[3] Ecole Nationale d'Ingenieurs de Monastir (ENIM), LARATSI ; Rue Ibn El Jazzar, 5000, Monastir, Tunisie.

Received: 31 Jan. 2015, Revised: 3 May 2016, Accepted: 4 May 2016
Published online: 1 Jul. 2016

Abstract: The steady increase in car ownership has made essential the development of macroscopic models of fine resolution changing dynamics of vehicles using different theories such as traffic signaling systems in the presence of signaling lights. The central objective of this study is to provide elements of knowledge needed to characterize the road traffic conditions on through the development of macroscopic modeling based on a lagrangian discretisation of the GSOM (generic second order model). Our research is to model the evolution of a line of vehicle packets and a distribution of microscopic variables by a system of differential equations while using an exponential law for the generation of vehicles. It's a simulation study of the behavior of a queue of vehicle packets moving according to the GSOM model in lagrangian coordinates. We focused on a representation of real objects: segment, junction and vehicle. We showed how the distribution of vehicle packets varies according to different driving conditions. To assess the ability of our basic lagrangian GSOM model to correctly reproduce the observations of behavior common traffic, we have developed a traffic control model in the presence of signaling lights, while implementing an analysis of key results. The advantage of these signaling lights is that they can observe the creation of shock waves (deceleration waves) when the lights turn red and rarefaction waves (acceleration waves) when the lights turn green.

Keywords: macroscopic models, Lagrangian GSOM model, traffic control model, traffic signaling systems, shock waves, rarefaction waves

1 Introduction

Traffic flow models have been developed to cope the needs of traffic operations [12], management [27], planning, control and evaluation in order to find a solution to congestion problems [2,4]. Most of these applications require models that are simple, robust, with modest requirements in terms of data processing, also with a low computational cost. Macroscopic models answer those needs in a satisfactorily way[18]. During the last decade, significant researches in traffic flow modeling were devoted to higher order macroscopic models, to improve the description of non-equilibrium flow traffic characteristics. In the literature, Payne [23], proposed the first second order traffic flow model. This model has been criticized in particular in [6], proclaiming for second order models on the grounds that they do not respect the anisotropic nature of traffic [29]. In order to avoid such non-physical solutions, other models have been proposed such as the model of Del Castillo in [23] and Zhang in [36]. These models do not treat the satisfactory anisotropy problem. The main reason is that they have characteristics speeds greater than traffic flow speed.

Most macroscopic models can be stated as conservation laws systems with possible sources terms. LWR model [19,32] is particularly simple: it is expressed by a single conservation law (with the unknown density). Nevertheless, it incorporates many important elements of traffic dynamics (capacity [12], fundamental diagram [30]). In [6], Daganzo showed that the LWR model responds to a principle of variation. The main point is that flow and density derived from a single function (cumulative flows or the so-called Moskowitz function). A natural extension of the LWR model consists of the

* Corresponding author e-mail: asma.khelifi@ifsttar.fr , asma.khelifi@enit.rnu.tn

© 2016 NSP
Natural Sciences Publishing Cor.

GSOM family of models developed by Lebacque & al. [22]. The GSOM family models generalize the LWR model and include many other macroscopic models [1, 35, 3, 23, 5, 11, 34, 15].

The first order model LWR [19, 26] Fixed speed motorway as decreasing function of its density, constituting thus a traffic state law. Then it evolves in time and space the latter by a single scalar equation. The second order traffic models are evolving independently density and speed variables characterizing their macroscopic conceptualization and continuous traffic. After their calling into question in [6], they have been modified by Zhang in [35], Aw and Rascle in [1] and Lebacque and his team in [20, 22, 25]. Dynamics of traffic flow results from the interaction of two processes. The first is hydrodynamic in nature, and yields behavioral characteristics of traffic that can be observed on a regular basis, namely the acceleration waves (rarefaction) and deceleration (shock waves), congestion, decreasing speed with the density, etc. The second process is the result of the impact of specific driver attributes (behavior, origin-destination, class vehicle/driver) on the fluidity of traffic. These two processes can be integrated into a single macroscopic traffic flow model, ie the GSOM family. Generally, a typical GSOM model combines the density conservation equation (representing kinematic wave of traffic) with a system of conservation laws for individual vehicle attributes and behavior of drivers, such as the type of vehicle, aggressiveness, the destination or the information flow to and from a vehicle. The particular structure of GSOM models, in a way very similar to that of the LWR model, allows reaffirming the GSOM as a problem of optimality and therefore GSOM models satisfy a variational formulation. GSOM family traffic flow models combine the LWR model with the dynamics of specific driver attributes and can be expressed as a system of conservation laws.

The purpose of this paper focuses on GSOM (Generic Second Order model) models of traffic flow. They are generally continuous in time and space, and dynamic, they are defined by a system of differential equations. In the literature, Payne [31] was the first to propose a second order traffic flow model. The dynamic flow results from the interaction of two processes. The first is of hydrodynamic type, and yields behavioral characteristics of traffic that can be observed on a regular basis, namely the acceleration wave (rarefaction waves) and deceleration (shock waves), the congestion, the reduction of the speed density, etc. The second process results from the impact of the driver specific attributes (behavior, origin, destination, vehicle/driver class) on the flow of traffic. These two processes can be integrated into a single macroscopic model of traffic flow, namely the GSOM family according to the selected attribute [19, 1, 36, 5, 11, 34, 15, 21, 17, 33]. In this paper, we will study the macroscopic GSOM model in Lagrangian coordinates and a discretization numerical scheme using Godunov scheme [8, 10] to solve the macroscopic GSOM model recast in lagrangian coordinates. The objective sought is the numerical resolution in Lagrangian discretization and modeling the trajectory of vehicle packets. It is pertinent to build a numerical resolution of GSOM model in the main purpose of easily conducted runs. The objective assesses also the nature of vehicle packets whose evolution is described by a general class. We also deal with the dynamics of traffic signals through four actions: optimize cycle time, optimize the phase diagram, adjust the gap between intersections and finally optimize the durations of the various phases. The interesting objective of the control model developed is to see the effects of orange and red lights on the dynamics of standard vehicle packets following the discretized GSOM model.

2 Characterization of GSOM family

2.1 GSOM model equations

The GSOM model is a model that generalizes the ARZ model [1, 35]. It combines the conservation equation with the fundamental diagram specific to the driver behavior attribute. It can be expressed as follows:

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = 0 \quad \text{Conservation of vehicles} \\ \frac{\partial \rho I}{\partial t} + \frac{\partial \rho I v}{\partial x} = -\rho f(I) \quad \text{Conservation attribute driver I} \\ v = \Im(\rho, I) \quad \text{Driver dependent fundamental diagram} \end{cases} \tag{1}$$

The function $f(I)$ is a relaxation function. I is a Lagrangian driver attribute that characterizes the behavior of each driver. It is preserved along the trajectories of vehicles, a result in harmony with the fact that contact discontinuities waves propagate discontinuities of I to the speed of traffic. The fundamental diagram is expressed as follows:

$$v = \Im(\rho, I) \text{ and } \rho v \overset{def}{=} \Re(\rho, I) \tag{2}$$

$\Re$ is assumed to be concave with respect to ρ, for all values of I . Note that the equation of I can be reformulated as an advection equation as follows:

$$\dot{I} = \frac{\partial I}{\partial t} + v\frac{\partial I}{\partial x} = \phi(I) \text{ and } \phi(I) = f(I) \tag{3}$$

In this context, the function $\phi(I)$ can express the relaxation of I to a reference or an equilibrium value (specific to the driver), or can express a disturbance process, in the case of stochastic attribute. The eigenvectors W_1 and W_2 respectively associated with the eigenvalues λ_1 and λ_2 of the GSOM model are:

$$W(\rho, v)_1 = \begin{pmatrix} -\partial_v I \\ \partial_\rho I \end{pmatrix} \text{ and } W(\rho, v)_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4}$$

© 2016 NSP
Natural Sciences Publishing Cor.

The functions of demand and supply of GSOM model are defined by:

$$\begin{aligned}\Delta(\rho,I;x) &= \text{Max}_{\ 0\leq r\leq\rho}\Re\left(\rho,I;x^{-}\right)\forall(x,t)\\ \Sigma(\rho,I;x) &= \text{Max}_{\ r\geq\rho}\Re\left(\rho,I;x^{+}\right)\forall(x,t)\end{aligned} \quad (5)$$

2.2 Examples

The GSOM family recovers a wide range of existing models:

- The LWR model [19,32] itself is a GSOM model with no specific driver attribute, expressed as follows:

$$\begin{cases}\partial_t\rho+\partial_x(\rho v)=0 & \text{Conservation of vehicles}\\ v=\Im(\rho,x) & \text{Fundamental diagram}\end{cases} \quad (6)$$

  The fundamental diagram for the LWR model states that traffic flow is always at an equilibrium state. It is commonly assumed that the flow is an increasing function of density between zero and a critical density and then the flow decreases until the maximal density. However the fundamental diagram shape is always a subject of debates [7] and there exists a wide variety in the literature encompassing concave and triangular flow functions [9].
- The LWR model with bounded acceleration proposed in [24,25,28] is also a GSOM model in which the propagated driver attribute is simply the speed of vehicles.
- The ARZ model (standing for [1] and [35]) for which the driver attribute is taken as $I=v-V_e(\rho)$ that is $\Im(\rho,I)=I+V_e(\rho)$.
- The Generalized ARZ model proposed in [7] that can be also seen as a particular case of the model described in [34]. These models introduce an interaction mechanism between two different fundamental diagrams for distinguish equilibrium and non-equilibrium states.
- Multi-commodity models of Lebacque & al. [23] or Herty & al. in [13].
- The Colombo 1-phase model deduced in [22] from the 2-phase model of Colombo in [5]. In this case, the driver attribute I is a scalar which is non-trivial in congested situation. In fluid area, the model follows the classical LWR model.
- The stochastic GSOM model of Khoshyaran and Lebacque in [16]. The driver attribute I is a random variable depending on the vehicle index N and on the random event ω such that $I=I(N,t,\omega)$.
  The random perturbations do not affect the vehicle dynamics but affect the driver perception and its behavior.

3 Numerical Resolution of the GSOM Model by Lagrangian Discretization Method : Application of the Godunov Scheme

The basic idea of the Lagrangian discretization of the GSOM model is the following: first we rewrite the GSOM model in its Lagrangian form, second, we integrate the equations for the behavioral attributes and integrate the results in the kinematic wave equation. The result of this procedure will be a single conservation law for vehicle spacing. A very useful feature of the GSOM model is its simple expression in Lagrangian coordinates.
The first fundamental variable of GSOM model as Lagrange is the number of cumulated vehicles $N\overset{def}{=}\int_x^{+\infty}\rho(\xi,t)\,d\xi$. This variable is supplemented by a second variable which is the time $T\overset{def}{=}tN$ can be interpreted intuitively as an index of the vehicle.
From these definitions we can identify the following two relationships:

$$\begin{cases}\partial_x N=-\rho\\ \partial_t N=q\end{cases} \quad (7)$$

A little algebra gives the following expressions for the change of coordinates:

$$\begin{cases}\partial_x=-\rho\partial_N\\ \partial_t=\partial_T+q\partial_N\end{cases}\Leftrightarrow\begin{cases}\partial_N=-r\partial_x\\ \partial_T=\partial_t+v\partial_x\end{cases}\ \text{with}\ r\overset{def}{=}\frac{1}{\rho}\ \text{and}\ v=rq \quad (8)$$

By rewriting the original GSOM model, the conservation equation can be rewritten as follows:

$$\partial_t\rho+\partial_x(\rho v)=0\Leftrightarrow\partial_T r+\partial_N v=0\ \text{ with }\ v=\Im(\rho,I)\overset{def}{=}V(r,I) \quad (9)$$

This result is logical and I is preserved along the trajectories.
Then, it can be shown that the following transformation is admissible, in the sense that it represents the shock waves (shock waves of coordinates (x,t) and (N,T) are equivalent):

Thus, the system (1) of the GSOM model is equivalent to the following system of Lagrangian coordinates:

$$\begin{cases}\partial_T r+\partial_N v=0\\ \partial_T I=0\\ v=\vartheta(r,I)\overset{def}{=}\Im(1/r,I)\end{cases} \quad (10)$$

Discretization is an abstraction that represents the projection methods and recovery of continuous spaces to the discrete spaces, and conversely.
Our primary objective in this paper is to solve numerically the Lagrangian discretization of GSOM model and to modelling the trajectory of vehicle packets as shown in the Fig. 1.

We can solve numerically the GSOM model by the method of particle or Lagrangian discretization method. The objective of this discretization is to consider the

© 2016 NSP
Natural Sciences Publishing Cor.

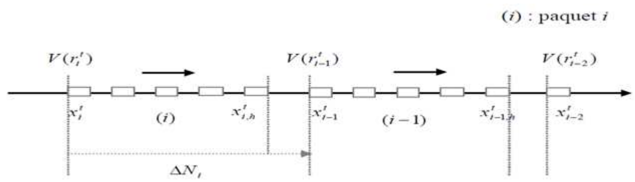


Fig. 1: The Lagrangian discretization based on vehicle packets.

cumulative function, which we denote by $N(x,t)$ and express the model as follows:

$$\begin{cases} \dot{x} = \Im(\rho, I) \\ \dot{I} = -S(I) \end{cases} \tag{11}$$

In fact, considering that $x_n(t)$ the trajectory of the nth vehicle $(N(x,t) = n)$, the discretized system (11) is then given by:

$$\begin{cases} \dot{x}_n(t) = \Im\left(\dfrac{1}{x_{n-1}(t) - x_n(t)}, I_n(t)\right) \\ \dot{I}_n(t) = -S(I_n(t)) \end{cases} \tag{12}$$

The model (1) discretized in time can be written as follows:

$$\begin{cases} x_n(t+\Delta t) = x_n(t) + \Delta t \Im\left(\dfrac{x_{n-1}(t) - x_n(t)}{\Delta N}, I_n(t)\right) \\ I_n(t+\Delta t) = \Psi(I_n(t), \Delta t); \text{ with } \Delta t = dt \end{cases} \tag{13}$$

With $\Psi(I_0, \tau)$ is the solution to the moment τ of $\frac{dt}{d\tau} = -S(I)$ and $I_{\tau=0} = I_0$. Therefore the discretized GSOM model is:

$$\begin{cases} x_n(t+\Delta t) = x_n(t) + \Delta t v_n(t) \\ v_n(t) = \Im\left(\dfrac{x_{n-1}(t) - x_n(t)}{\Delta N}, I_n(t)\right) \\ I_n(t+\Delta t) = I_n(t) + \Delta t S(I_n(t)) \\ a_n(t) = \dfrac{v_n(t+\Delta t) - v_n(t)}{\Delta t} \end{cases} \tag{14}$$

Note that$S(I_n(t)) = -\dot{I}_n(t)$, $I_n(t)$ represents the invariant associated to the nth vehicle and $a_n(t)$ is the acceleration.

We have chosen the 1-phase Colombo model [5] to represent the speed function. To ensure that vehicles comply the minimal spacing, the following CFL stability condition must be respected:

$$\Delta t \leq \frac{\Delta N}{\rho_{max}(I_+) W_{max}(I_+)} \tag{15}$$

Note that ρ_{max} is the maximum density for $\Im(\rho, I) \geq 0$ and $W_{max}(I) = \partial_\rho \Im(\rho, I)_{|\rho=\rho_{max}(I)}$.

I_+ is defined by the Godunov scheme as the largest value taken for the invariant I and this value will depend on initial conditions, the input and the function $S(I_n(t), \Delta t)$.

4 Application to Traffic Control on the Descretized GSOM Model

4.1 Characteristics of signaling light network

Traffic control is a vast area in which several techniques and signal forms are used to increase the safety of passengers. Among the most important issues of traffic control, we find the management of intersections [14].

Around 1920 appeared at the intersections of major cities a material to orchestrate conflicts between vehicles. Signaling lights are a set of signals, devices and regulations to ensure the safety of traffic. It is a means of giving orders to the driver of a vehicle and aims to avoid risks inherent in the traffic. Indeed, during the last decades, major advances have been made in the field of automatic traffic control. There are many ways to act on traffic but the most important remains the regulation via the signaling lights.

They play an important role in the traffic control process by allowing mainly; to ensure the safety of vehicles and pedestrians sharing in time using the same space between the conflicting flows; to ensure to the management system based on lights plans a significant adaptation to traffic and a real effectiveness in wide operating ranges, since the monitoring and maintenance of the system are provided; and to minimize the time spent in the transport system as well as the waiting time for files.

There are two intersections control modes for signaling lights: light plan and adaptive control. The first is older and less expensive, while the second is more complex and powerful. The light plan consists to construct and organize lights phases. These ones are defined by their duration and by the award of a light color for each mobile power involved in the considered intersection. The simplest light plan consist to repeat indefinitely the same sequence of fixed time phases, always arranged in the same order, in a way to constitute a fixed cycle; this is the case chosen for our practical application to manage the signaling lights.

A signaling light is characterized by its color (green, yellow or red). Each signalized intersection is associated with a light plan that defines the sequence of states of all control lines of the intersection signals.

A signal plan is described by the following variables:

- A directional movement:
  It is a set of vehicles that are from the same entry in an intersection and headed for the same output (direct movement, turn left, turn right).
- A phase diagram:
  A phase is the period during which one or more compatible movements are allowed in a crossroads. This is the operating state of the intersection while some traffic flows are allowed. Note that any phase change results in the failover of at least one light. The phase diagram specifies all phases with passing opportunities for each other. For our application, the

network operates on a single phase diagram, chosen on the one hand to prevent the simultaneous passage of conflicting movements, on the other hand, in accordance with the importance of the traffic load on certain movements.

- A release time:
It is a minimum safety period that must be respected in the transition of two incompatible lights. Between two phases passes necessarily a safety time: orange and red full (all branches of the intersection are at the red light in order to clear the inside of the junction). For our application, lights plans are programmed with a minimum number of phases in order to minimize this loss of time.
- A cycle:
This is the duration of green lights, orange and red. It is constituted by the phase chaining. Its duration is equal to the time between two consecutive events counterparts of the same phase. Cycle time refers to the time between two successive passages of all the lights of an intersection by the same phase, in the case where these phases are not retracted.
- An offset:
This is duration relative to a reference time which is used for synchronization of the various network controllers. This is the time that separates the green beginnings of two adjacent phases. Indeed, if they belong to two intersections with equal constant cycles, the offset is constant and is expressed modulo the cycle. The set of offsets is then the network coordination plan. Different methods and a multitude of tools exist to regulate traffic with signaling lights. The fixed light plan in time; used for our application in this paper; remains the basis of a light management system.

4.2 Modeling Approach

4.2.1 Simulation Characteristics

To develop our model, the use of object-oriented programming is a promising solution. We set our model on a network. In this section, we show how we have implemented these concepts to obtain a set of simulation components, which will be used in a wide variety of applications.
The GSOM simulation model developed comprises logic to implement: generating vehicles in the system to be simulated, the vehicles move through the system, interactions vehicle model. So our application is mainly based on simulation as a tool to retrieve results that will optimize evaluation strategies. But before we have prepared the ground for determining the characteristics of the road network: the network topology used for the simulation. The development is based mainly on the architecture part.
For discretized GSOM model, the model description must be accompanied by an individual injection model vehicles based on a static distribution law. The generation vehicles model used in this paper is based on an exponential distribution function (Poisson Process). With a necessary and sufficient condition: $y \in [0\ 1]$, and knowing that λ is the maximum flow; the generation model based on the statistical exponential function of individual injection distribution vehicle is:

$$\phi(y) = -\frac{1}{\lambda}\log(1-y) \tag{16}$$

We give in the following figure the network used in simulation:

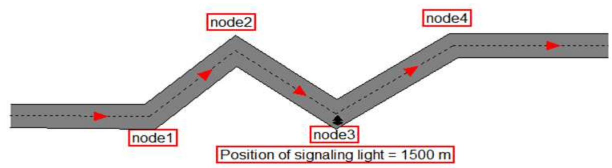


Fig. 2: Description of the network used in simulation.

Traffic control is a broad field in which several techniques and forms of signaling are used to increase user safety. In this paper, the applied signal lights control strategy is discussed. The impact of signal lights on traffic dynamic is the creation of shock waves (deceleration waves) when the lights is red and rarefaction waves when the lights is green (acceleration waves). Kinematic waves are reflected by the upstream or downstream propagation of the abrupt variations in acceleration and correspondingly the speed. Kinematic waves include start or output capacity restriction waves (downstream propagation), the formation of queues (upstream propagation).
The presence of a signaling light is simulated as follows: we allow the link controlled by a signaling light to allow passing vehicles only during the actual green cycle. When the light is red, vehicles cannot cross the end of junction and join the queue.
Various signaling light control strategies can be represented, ranging from fixed lights plans to adaptive strategies on isolated intersection or network.
The methodology applied for our case is to repeat indefinitely (until the end of the simulation) the same sequence of fixed time phases, always arranged in the same order. We mentioned also, because of the importance in regulation by signaling lights of traffic acceleration phenomena, at first at signaling lights but also in output congested areas.
The considered simulation site corresponds to a road network with two lanes. We studied the evolution of traffic on a homogeneous network (no lanes changing); with a length equal to 1500 meters in the presence of a signaling light spaced with a length equal to 15 meters

© 2016 NSP
Natural Sciences Publishing Cor.

and with a position equal to 900 meters, as is shown in Figure 2.

Simulation data used in the simulation are: network length = $3000m$, time step: $\Delta t = 0.1s$, maximum speed : $v_{max} = 25ms$ $(90kmh)$, maximum acceleration : $acc_{max} = 2ms^2$, maximum deceleration :$dec_{max} = -2ms^2$, maximum density $= 0.2veh/m$,$\Delta N = 5$, maximum attribute $= 3,6veh/m$, minimum attribute $= -3,6veh/m$, vehicle length : $long_{veh} = 5m$, position of signaling lights : $posi_{light} = 1500m$, width of the intersection : $width_{inters} = 15m$, duration of green:$T_{green} = 35s$, duration of orange: $T_{orange} = 5s$, duration of red: $T_{red} = 35s$, cycle : $T = T_{green} + T_{orange} + T_{red} = 75s$.

To simplify modeling, we examine the traffic behavior of the network used in Figure 2 given above in the presence of only one signaling light located at the third junction before the third section.

We suppose that the signaling light operates in phase; it is the question of repeating indefinitely the same sequence of fixed time phases. The interesting objective of this control model is to see the effects of orange and red lights on the standard dynamics of vehicle packets according to the discretized GSOM model.

4.2.2 Implementation and Development

The control of road traffic through signaling lights requires 4 actions generally: to optimize the cycle time, to optimize the phase diagram, to adjust the offsets between intersections and finally to optimize the durations of the different phases.

The strategy that we implement is interested in optimizing durations of the different phases for the network. For our application, and to simplify programming, we will not consider the phase diagram and offsets; cycle and phases are assumed to be known and fixed.

Microscopic and macroscopic variables are calculated by the discretized GSOM model. The standard vehicle dynamics for our application based on the discretized GSOM model described in the previous section.

The model that we have chosen to represent the speed $v_n(t)$ is the 1-phase Colombo model.

Thus, during the green light, there are no constraints on the acceleration, so the individual speeds satisfy our standard discretized GSOM model according Lagrangian discretization:

$$\begin{cases} \ddot{x}_k = a_n \\ \dot{x}_k = v_n \end{cases} \tag{17}$$

During the red light, we must be able to stop before the stop line, which implies a minimal braking. Finally when it is orange, there are two possibilities: brake enough to stop or accelerate enough to pass the intersection before the orange time.

By combining the three points (in front of the vehicle, from behind, light), we arrives at a field of acceleration that can be chosen to increase comfort. If this field was empty (incompatible conditions) it mean that it's necessary to apply another policy, and that it is in a normal situation.

In this context, we will try to change the policy of acceleration according to simulation time and cycle.

We define two variables shown as follows:

$$\begin{cases} T_m = modulo\,(t, cycle) \\ T_{mm} = modulo\,((t+1)\,, cycle) \end{cases} \tag{18}$$

Therefore, the time t is expressed modulo the cycle, and therefore the variable *rest* that we calculated gives information on the status of the signaling light (red, orange or green).

So during the time interval $T_m \leq t \leq T_m + T_{green}$ each vehicle packets checks the standard equations of our discretized GSOM model. Define another variable: $T_y = T_m + T_{green}$ when the light changes from green to orange.

Also we can calculate respectively x_{T_y} and v_{T_y} ; the position and speed that correspond to the time $T_y < posi_{light}$ will be $\begin{cases} x_k(T_y) = x_{T_y} \\ \dot{x}_k(T_y) = v_{T_y} \end{cases}$

- If : $x_{T_y} + v_{T_y}(T_{orange}) \geq posi_{light} + width_{inters} + long_{veh}$ each vehicle packets will cross the intersection following the standard dynamic and so it will accelerate crossing the intersection.
- However, if : $x_{T_y} + v_{T_y}(T_{orange}) < posi_{light} + width_{inters} + long_{veh}$ it will be impossible for this vehicle packet to liberate the intersection during the Orange light Phase if it continues with the same acceleration and the same speed. Then, two possibilities arise here:
  1. If $x_{T_y} + v_{T_y}(T_{orange} + T_{red}) \leq posi_{light}$: we imposes on the vehicle packets to follow the changed dynamics ie: $\begin{cases} \ddot{x}_k = 0 \\ \dot{x}_k = v_n \end{cases}$, that is it must keep its current speed with an acceleration equal to 0
  2. If now $x_{T_y} + v_{T_y}(T_{orange} + T_{red}) > posi_{light}$, the vehicle packet must decelerate; there are then two other situations:

  a) If $x_{T_y} + \frac{v_{T_y}(T_{orange}+T_{red})}{2} > posi_{light}$; the vehicle will follow:

$$\begin{cases} \ddot{x}_k = \begin{cases} 0 \text{ if } T_y + \dfrac{2\left(posi_{light} - x_{T_y}\right)}{v_{T_y}} \leq t \leq T_{mm} \\ \dfrac{\left(-v_{T_y}\right)^2}{2\left(posi_{light} - x_{T_y}\right)} \text{ if} \\ \qquad T_y \leq t \leq T_y + \dfrac{2\left(posi_{light} - x_{T_y}\right)}{v_{T_y}} \end{cases} \\ \dot{x}_k = v_n \end{cases} \tag{19}$$

© 2016 NSP
Natural Sciences Publishing Cor.

This deceleration strategy will cause the vehicle packet to stop at $x = posi_{light}$ at time $t = T_y + \frac{2(posi_{light} - x_{T_y})}{v_{T_y}}$

b)If $x_{T_y} + \frac{v_{T_y}(T_{orange} + T_{red})}{2} \leq posi_{light}$; the vehicle packet must follow:

$$\begin{cases} \ddot{x}_k = \dfrac{-2\left(x_{T_y} + v_{T_y}\left(T_{orange} + T_{ref}\right) - posi_{light}\right)}{\left(T_{orange} + T_{red}\right)^2} \\ \quad \text{for } T_y \leq t \leq T_{mm} \\ \dot{x}_k = v_n \end{cases} \tag{20}$$

This strategy will lead the vehicle packet to light $x = posi_{light}$ at a given time T_{mm} with the following speed:

$$\dot{x}_k\left(T_{mm}\right) = \frac{2\left(posi_{light} - x_{T_y}\right)}{T_{orange} + T_{red}} - v_{T_y} > 0 \tag{21}$$

We applied these requirements for all vehicles packets (leader and follower) at each time step. In this context computationally speaking, we developed two new methods: in proceedings where we advance our vehicle packets on the entire network using the discretized GSOM model.

So we adopted the same strategy as for the application without the presence of signaling lights. We used the exponential distribution for the injection of vehicle packets and the discretized GSOM model for advancing these vehicle packets following the modified approach.

After and at each time step and just after to advance the vehicle packet, we needs to compare his new position with the maximum length of the current segment and thus test the possibility of its inclusion on the section.

As an illustration, we can see that our model is not based on a division into time slots. On the contrary, vehicle packets are gradually introduced to the network and their movement to their destination is followed continuously. Each vehicle packet has an origin, a destination, a departure time and a route selection strategy (fixed for all vehicle packets) at each junction along its route; a decision will be made on the following link to go. Each vehicle packets circulating on the modeled network is found in interaction with other vehicle packets and vehicles subject to the effective control strategy.

As outputs of the simulation model developed, values are available at the end of the simulation, such as average speed, travel time, acceleration and position. We will analyze the simulation results in the next section.

5 Simulation Results

The purpose of the simulations is to observe the progress of vehicle packets on the network. We will present in this section our simulation results. We will conclude that the behavior of the position and velocity to variations of acceleration is consistent with the physical phenomena that we want to reproduce. Indeed, the purpose of simulations is to observe the progress of vehicle packets on the network based depending on the state of the signaling light passing through these vehicle packets. We was respectively plotted changes of only some outputs vehicle packets of network outputs in a plane of simulation time/position, simulation time/speed and simulation time/acceleration. The results are the following figures:

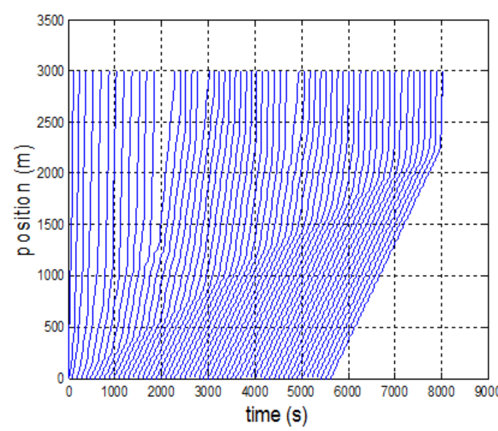


Fig. 3: Trajectories of vehicle packets (Without signaling light).

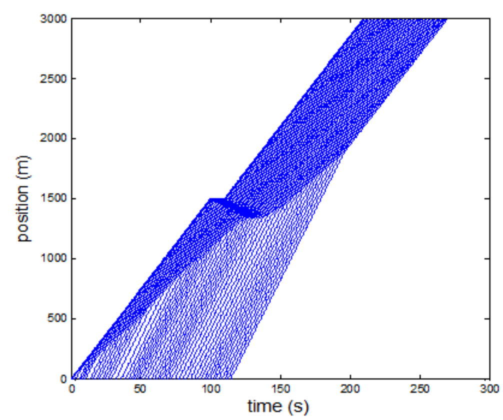


Fig. 4: Trajectories of vehicle packets in the presence of 1 signaling light.

If we observe the trajectories in Fig. 4, it can therefore be seen that compared to Fig. 3, the behavior vehicle packets model to a signaling light has rounded trajectories. Note that the area where positions are

© 2016 NSP
Natural Sciences Publishing Cor.

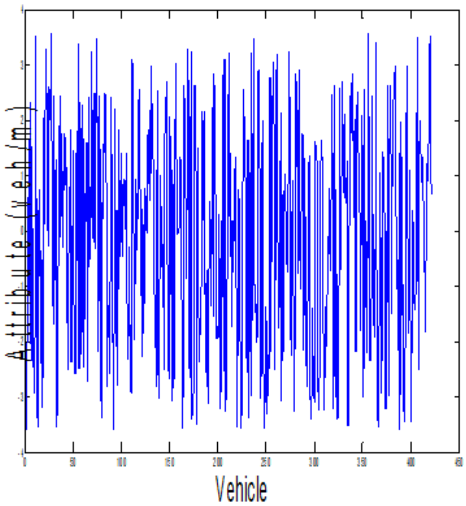


Fig. 5: I dynamics.

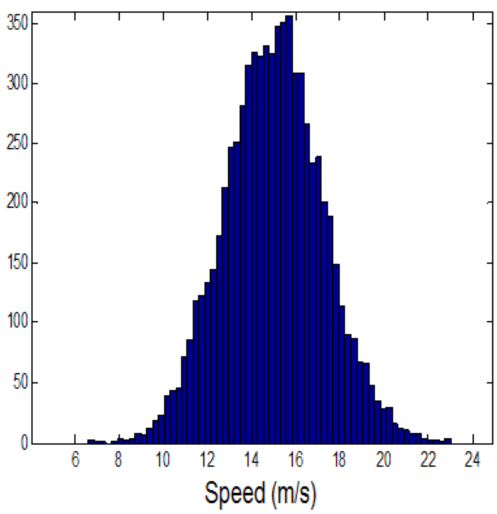


Fig. 6: Velocity profile of the first leader.

constant corresponds to the formation of the queue at red light. The characteristics issued right line of light includes information on the duration of red light : $(T_{red} = 35s)$. The speed is therefore zero in the entire area. The latter is delimited upstream of light by the shock wave between the end of the queue and the fluid traffic, unperturbed by the presence of signaling light.

For the startup area of vehicle packets, this area is between the trajectory of the first vehicle packet facing the red light and the shock wave modeling the behavior of the rear of the queue. Indeed, the queue is formed during the red phase of signaling light.

The trajectory analysis also shows that the first vehicle packet of this scenario cross the area slowly (see Fig. 6).

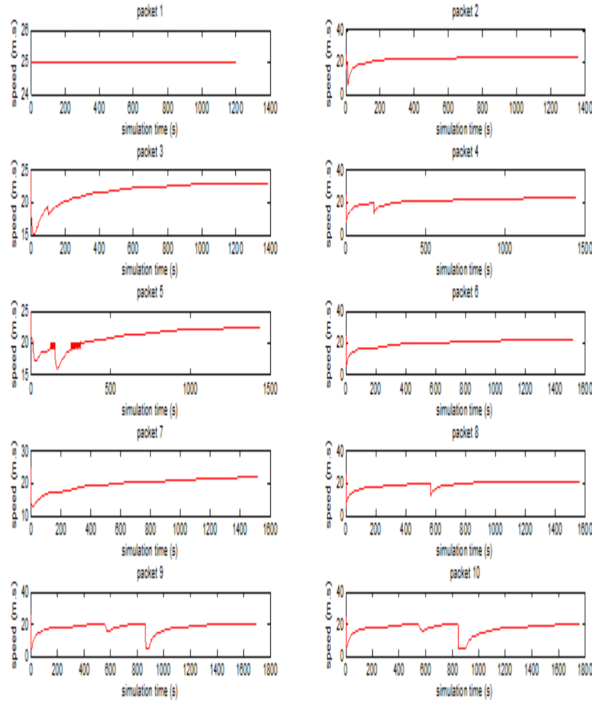


Fig. 7: Evolution speeds of the first 10 vehicle packets (without signaling light).

Fig. 5 shows the attribute dynamics.

So as we can see, the evolution of fundamental variables of the discretized GSOM model in the presence of signaling light; i.e. acceleration (see Fig. 10 compared to Fig. 8) and speed (see Fig. 9 compared to Fig. 7), is in the form of an accordion. This form is due to the presence of stop waves when the signaling light is red, and restarts waves when the signaling light turns green.

In fact, the figures show that for positions less than $1500m$, speeds and acceleration have amplitudes of high fluctuations. They are moderately influenced by the phases of the signaling light, with a triangular or trapezoid profile whose apexes do not reach the bearings for the speed v_{max}, for the acceleration acc_{max} and for the deceleration dec_{max}.

However, for sufficiently large time and superior to positions of light position, we note that microscopic variables regain their equilibrium again and back to normal since there is no obstacle beyond $X = 1500m$.

The curves confirm the significant impact of the durations of red phases specifically on the progress of vehicle packets throughout the network.

The simulation test also shows that the discretized GSOM model for vehicle packets is coherent and consistent. The low time calculation of simulations stimulates the idea that the discretized GSOM model can be used as a decision support in real-time traffic control strategy.

© 2016 NSP
Natural Sciences Publishing Cor.

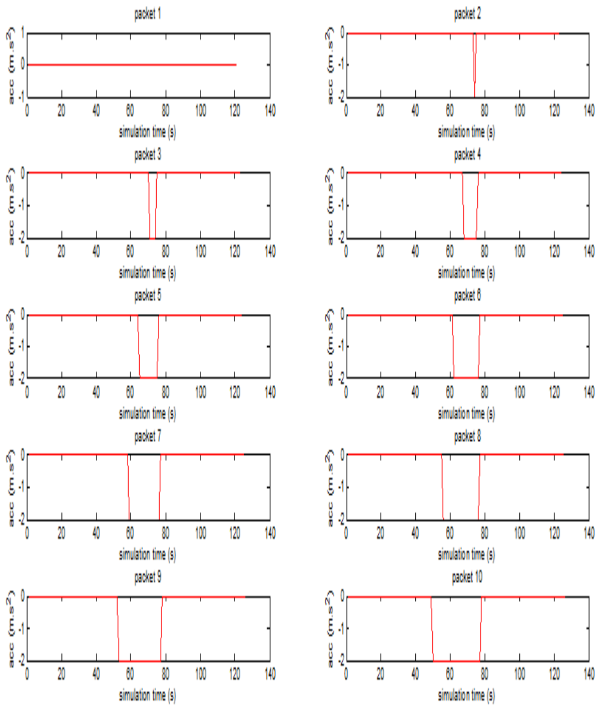


Fig. 8: Evolution accelerations of the first 10 packets of vehicles (without signaling light).

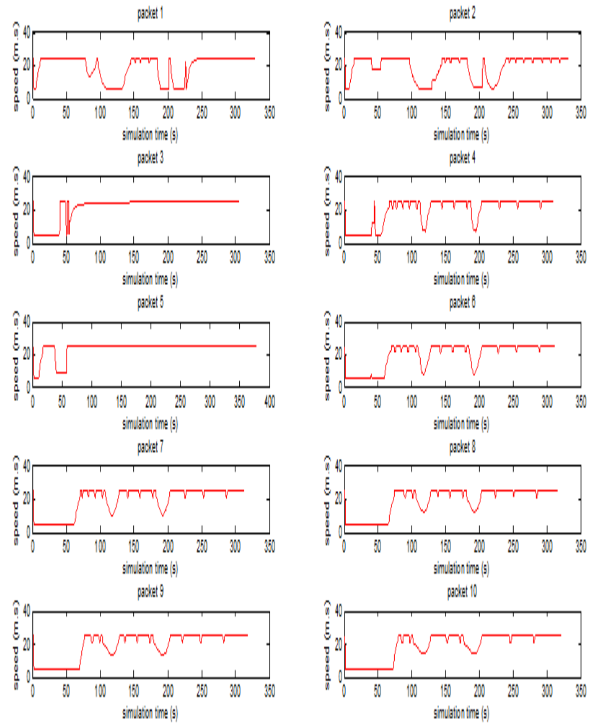


Fig. 9: Evolution speeds of the first 10 vehicle packets in the presence of 1 signaling light.

Control strategy, with the simplifying assumptions (constant phases) and a simple network to avoid the influence of exogenous parameters, achieved the objective assigned to it in the beginning. This strategy improves the overall fluidity if the network design allows.
The increase and gradual decrease of the speed, observable in curves, take into account that the capacity of acceleration or deceleration of vehicle packets is variable: they depend on the characteristics of braking system.
The signaling light can be clearly observed creating shock waves that is to say, a spatio-temporal discontinuity representing the front of the queue when the lights turn red and a rarefaction waves representing the back of the queue when the lights turn green.
Here is a spectacular phenomenon that we have a good perception for this application. Kinematic waves are reflected by the propagation to upstream or to downstream of abrupt variations of acceleration and correspondingly speed. The kinematic waves include start waves to light or in capacity restriction release (downstream propagation in general), the queue formation and the traffic bubbles propagation moving in congestion (upstream propagation).
This is the following observation: vehicle packets grouping (platoons) resulting from startups vehicle packets light tend to disperse in the absence of constraints (green light and after the position light).

6 Conclusion and next steps

The objective of the paper is to introduce a numerical solution to the GSOM model using Lagrangian discretization with the Gudonov scheme. The application of such a numerical simulation model is in traffic management. Traffic conditions in a network can be predicted and analyzed by simulating the trajectory of vehicle packets and analyzing their behavior along trajectories.
The discretized GSOM model constitute a tool for knowledge by providing important concepts such as supply and local demand for traffic, static or dynamic equilibrium networks, behavior laws of vehicle and the representation of traffic flows. In this paper, we presented the GSOM model. We proposed a numerical discretization method according to the Lagrangian Godunov scheme. Finally, we developed an algorithm based on a deceleration strategy, and a control application in response to signaling lights based on the discretized GSOM model. This will allow us to conclude that the behavior of the position and velocity to variations in acceleration is consistent with the physical phenomena

© 2016 NSP
Natural Sciences Publishing Cor.

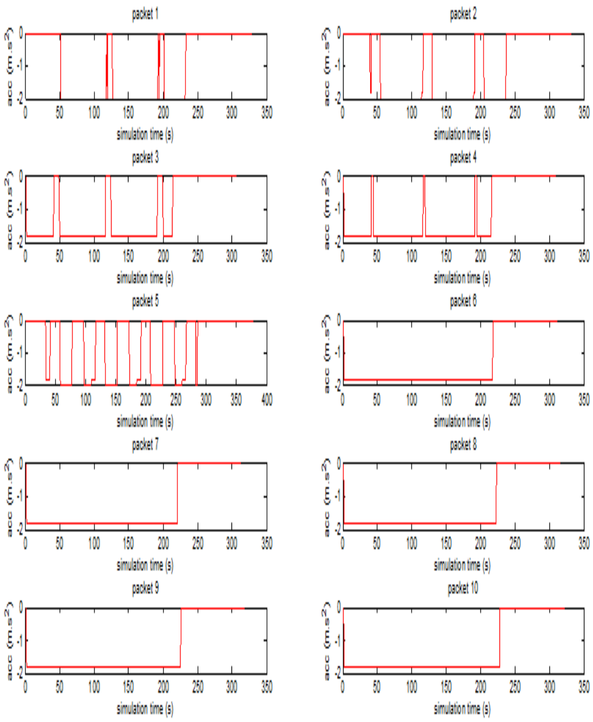


Fig. 10: Evolution accelerations of the first 10 packets of vehicles by GSOM model in the presence of 1 signaling lights.

that we want to reproduce.
The next step would be modeling lanes changing and intersection and the GSOM discretized model. Ultimately, the objective is to construct a simulation model in real time, for predictive purposes. The flow model should be coordinated to an optimization algorithm modeling the route choice of drivers: this is the step of calibration and validation of the model on experimental NGSIM data.
Research on microscopic modeling based on the Lagrangian discretization of GSOM model can be used as part of driver assistance. The concept aims to control acceleration and deceleration of a vehicle packets including interactive following situation.

Acknowledgement

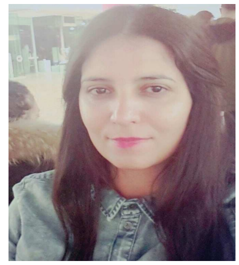
We would like to thank all the Laboratory GRETTIA/IFSTTAR and LARATSI/ENIM members whose work makes this paper possible: conducting the research, review and offering concrete suggestions, participation in the Working Group, writing proposals, creating and running models, designing instruments, managing projects, etc. This paper attempts to summarize and highlight some of your efforts.

References

[1] A. Aw and M. Rascle, Resurrection of second order models of traffic flow, in SIAM journal on applied mathematics, 2000.
[2] M. Bando, Dynamical model of traffic congestion and numerical simulation, in Physical Review, 1995.
[3] H. Bar-Gera and S. Ahn, Empirical macroscopic evaluation of freeway merge-ratios, in Transportation Research Part C: Emerging Technologies, 2010, pp. 457-470.
[4] J. H. Banks, Average time gaps in congested freeway flow, IN Transportation Research Part A, 2003.
[5] R. M. Colombo, Hyperbolic traffic flow model, in Mathematical and computer modeling, 2002.
[6] C. F. Daganzo, The cell transmission model, part ii: network traffic, in Transportation Research Part B: Methodological, 1994, pp. 269 287.
[7] S.H. Fan and B.E. Seibold, A comparison of data-fitted first order traffic models and their second order generalizations via trajectory and sensor data, arXiv preprint, 2012.
[8] S. K. Godunov, A difference method for numerical calculation of discontinuous solutions of hydrodynamic equations, 1959.
[9] M. Garavello and B. Piccoli, Traffic flow on networks, American institute of mathematical sciences Spring field MO, USA, 2006.
[10] S. K. Godunov, A difference method for numerical calculation of discontinuous solutions of the equations of hydrodynamics, in Matematicheskii Sbornik, 1959.
[11] H. A. Greenberg, An analysis of traffic flow, Operations Research, 1959.
[12] B. Greenshields, A study of traffic capacity, in Highway Research Board Proceedings, 1935.
[13] M. Herty and B. Seibold, Comparative model accuracy of a data-fitted generalized Aw-Rascle-Zhang model, in arXiv preprint, 2013.
[14] D. Heidemann, Queuing at unsignalized intersections, in Transportation Research Part B, 1997.
[15] M. M. Khoshyaran and J.-P. Lebacque, Lagrangian modelling of intersections for the gsom generic macroscopic traffic flow model, in Proceedings of the 10th International Conference on Application of Advanced Technologies in Transportation, Athens, Greece, 2008.
[16] M. M. Khoshyaran and J.-P. Lebacque, Internal state models for intersections in macroscopic traffic flow models, in Conference of Traffic and Granular Flow, Shanghai, 2009.
[17] M. M. Khoshyaran and J.-P. Lebacque, A stochastic macroscopic traffic model devoid of diffusion, in Traffic and Granular Flow'07, Springer, 2009.
[18] C. J. Leo and R. L. Pretty, Numerical simulation of macroscopic continuum traffic models, in Transportation Research Part B, 1992.
[19] M. J. Lighthill and G. B. Whitham, On kinematic waves ii : A theory of traffic flow on long crowded roads, in ProRoySoc, 1955.
[20] J.-P. Lebacque, H. Haj-Salem, and S. Mammar, Second order traffic flow modeling: supply-demand analysis of the inhomogeneous Riemann problem and of boundary conditions, in Proceedings of the 10th Euro Working Group on Transportation, 2005.
[21] J.-P. Lebacque, H. Haj-Salem, and S. Mammar, An intersection model based on the GSOM model, in Proceedings of the 17th World Congress, Seoul, Korea, 2008.

© 2016 NSP
Natural Sciences Publishing Cor.

[22] J.-P. Lebacque, H. Haj-Salem, and S. Mammar, Generic second order traffic flow modeling, in Proceeding of International Symposium on Transportation and Traffic Flow theory, London, 2007.
[23] J.-P. Lebacque and M. M. Khoshyaran, A variational formulation for higher order macroscopic traffic flow models of the GSOM family, in Proceeding of International Symposium on Transportation and Traffic Flow theory, 2013.
[24] J.-P. Lebacque and M. M. Khoshyaran, First order macroscopic traffic flow models for networks in the context of dynamic assignment, in Transportation Planning, Springer, 2002.
[25] J.-P. Lebacque, The Godunov scheme and what it means for first order traffic flow models, in International symposium on transportation and traffic theory, 1996.
[26] J.-P. Lebacque, A two phase extension of the LRW model based on the boundedness of traffic acceleration, in Transportation and Traffic Theory in the 21st Century. Proceedings of the 15th International Symposium on Transportation and traffic Theory, 2002.
[27] J.-P. Lebacque, Two-phase bounded-acceleration traffic flow model: analytical solutions and applications, in Transportation Research Record: Journal of the Transportation Research Board, 2003.
[28] L. Leclercq, J. A. Laval and E. Chevallier, The lagrangian coordinates and what it means for first order traffic flow models, in Transportation and Traffic Theory, 2007.
[29] L. Leclercq and S. Moutari, Hybridization of a class of second order models of traffic flow, Simulation modelling practice and theory, 2007.
[30] A. D. May, Traffic Flow Fundamentals, Prentice Hall, Englewood Cliffs, New Jersey, 1990
[31] L. A. Pipes, An operational analysis of traffic dynamics, in Journal of Applied Physics, 1953.
[32] P. I. Richards, Shock waves on the highway, in Operations research, 1956.
[33] F. Van Wageningen-Kessels, Y. Yuan, S. P. Hoogendoorn, L. V. Lint, and Vuik, Discontinuities in the lagrangian formulation of the kinematic wave model, in Transportation Research Part C: Emerging Technologies, 2013.
[34] P. E. Zhang, S. H. Wong and S. Q. Dai, A conserved higher-order anisotropic traffic flow model: description of equilibrium and non-equilibrium flows, in Transportation Research Part B: Methodological, 2009.
[35] H. M. Zhang, A non-equilibrium traffic model devoid of gas-like behaviour, in Transportation Research Part B, 2002.
[36] H. M. Zhang, Phase transition in nonequilibrium traffic theory, in ASCE Journal of Transportation Engineering, 2000.



Asma Khelifi Doctoral researcher at IFSTTAR/GRETTIA France and ENIT/LARATSI on Traffic Flow Modeling Based on Hybrid Model and Junction Modeling.

Habib HAJ-SALEM Research Director and Responsible of the traffic modeling and control research area at IFSTTAR /GRETTIA France; UEVP, IUP France: courses, automatic control and electronics; University of Paris XII France; MASTER EEA: courses of automatic control; EFREI - Villejuif 94 France (2nd year): Numerical analysis and electronics courses; Ecole Central de Paris France (ECP) (3rd year level): Co responsible of the final projects; ENTPE of Lyon France (2nd year level) TD of automatic; Technical coordination and Managements of the French test site in frame of “EURAMP” European project; Traffic flow modeling: Theoretical investigations on the resolution of the second order traffic model and Hybrid modeling; Completion of the missing data: participation to the redaction of the specification of the call of tender SIER document; Participation in the frame work of “OPTIRAIL”: European project (D.G VII); Head of INRETS France Project: “Application of the advanced technique for the traffic control of road networks and the incident management”; Technical coordination and management of the French site and partners in the 4th PCRD European Project “DACCORD”; Main partner in two European projects DRIVE-2-EUROCOR and QUOVADIS; Head of INRETS France Project “Monitoring and Traffic control of motorway corridor”; “Prime contractor” of European Project DRIVE I V1035 “CHRISTIANE”; Head of INRETS France project “Motorway ramp metering control”.

© 2016 NSP
Natural Sciences Publishing Cor.

Jean-Patrick Lebacque Director of GRETTIA/IFSTTAR France; General engineer of Roads and Bridges France; CERMICS France teacher researcher; Center for Teaching and research in mathematics, computer science and scientific computing; ENPC France teacher researcher: Applied Mathematics, Transportation Engineering, Transport Economics

Lotfi Nabli Professor at the National School of Engineers of Monastir Tunisia, departments of Electrical Engineering; Researcher at the Research Laboratory of Automatic, Signal, Processing and Imaging LARATSI Tunisia.

© 2016 NSP
Natural Sciences Publishing Cor.