Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/100420

Entropy Analysis of Variable Viscosity Hartmann Flow through a Rotating Channel with Hall Effects

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Received: 2 Apr. 2016, Revised: 19 May 2016, Accepted: 20 May 2016
Published online: 1 Jul. 2016

Abstract: In this work, entropy generations of a variable viscosity Hartmann flow through a rotating channel with Hall effects are investigated numerically. It is assumed that the fluid and the channel rotate in agreement with the angular velocity about y-axis. The governing non-linear partial differential equations are transformed and solved numerically using Runge Kutta - Fehlberg method with shooting technique. Results obtained for velocities and temperature profiles are used to compute the entropy generation rate, skin friction and Nusselt number. The important results are displayed and discussed.

Keywords: Hartmann flow; Rotating Channel; Variable viscosity; Hall effects; Entropy analysis.

1 Introduction

In the field of magnetohydrodynamics (MHD), Hartmann flow is the flow of a conductive liquid in between two parallel plates in the presence of a transverse magnetic field. This flow has attracted the attention of many researchers due to its useful applications in many areas such as propulsion, plasma confinement, liquid metal pumping and microfluidic pumping. Hartmann [1] carried out a pioneer work of Hartmann flow which is regarded as the source of MHD channel flow. He examined flow of a viscous incompressible electrically conducting fluid within a parallel plate channel in the presence of a transverse magnetic field. Many varieties of important experimental, analytical and numerical studies as regards flow of a conductive liquid in between two parallel plates in the presence of a transverse magnetic field are found in the literatures based on the work of Hartmann, Cramer and Pai [2], Ghosh and Bhattacharjee [3], Seth and Singh [4], Michael et a.l [5], Makinde and Onyejekwe [6], Chinyoka and Makinde [7], Makinde [8] and Guria and Jana [9]. Ansari et al. [10] studied unsteady hydromagnetic flow of a viscous incompressible electrically conducting fluid in a rotating channel with finitely conducting walls, induced due to an oscillating pressure gradient, in the presence of a uniform transverse magnetic field. Attia and Aboul-Hassan [11] investigated the influence of temperature dependent viscosity and thermal conductivity on the transient

Hartmann flow with heat transfer. Anwar Bg et al. [12] presented a theoretical study of unsteady magnetohydrodynamic viscous HartmannCouette laminar flow and heat transfer in a Darcian porous medium intercalated between parallel plates, under a constant pressure gradient. Eegunjobi and Makinde [13] investigated theoretically the inherent irreversibility in a steady hydromagnetic permeable channel flow of a conducting fluid with variable electrical conductivity and asymmetric Navier slip at the channel walls in the presence of induced electric field. Meanwhile, entropy plays crucial roles in understanding of many diverse phenomena ranging from cosmology to biology. Its importance is manifested in areas such as engineering, the origins of macroscopic irreversibility from microscopic reversibility and the source of order and complexity in nature. It forms the foundation of most of the formulations of thermodynamics. Many researchers are making progress in delving to the understanding of entropy and entropy generation. Guillermo [14] studied the combined effects of hydrodynamic slip, magnetic field, suction/injection and convective boundary conditions on the global entropy generation in steady flow

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of an incompressible electrically conducting fluid through a channel with permeable plates. Eegunjobi and Makinde [15] examined the effects of the thermodynamic second law on steady flow of an incompressible variable viscosity electrically conducting fluid in a channel with permeable walls and convective surface boundary conditions. Arikoglu et al. [16] investigated the effect of slip on entropy generation in magnetohydrodynamic (MHD) flow over a rotating disk by semi-numerical analytical solution technique. In this present work, we consider the analysis of entropy of variable viscosity Hartmann flow through a rotating channel with Hall effects. In the flow, conducting fluid between two infinite parallel walls in the presence of a uniform transverse magnetic field B0 is applied. Mathematical formulation of the problem is given in section two. The model boundary value problem is tackled numerically using shooting quadrature coupled with Runge-Kutta-Fehlberg integration scheme. Pertinent results are presented graphically and discussed quantitatively for velocities, skin friction, Nusselt number, entropy generation rate and Bejan number in section three while section four gives a concluding remarks.

2 Mathematical Model

Consider the steady flow of a variable viscosity, incompressible, electrically and thermally conducting fluid between two infinite parallel walls $y=0$ and $y=L$ in the presence of a uniform transverse magnetic field B_0 which is applied parallel to y-axis taking Hall current into account. Both the fluid and channel rotate in unison with a uniform angular velocity Ω about y-axis. Fluid flow within the channel is induced due to uniform pressure gradient applied along x-direction. The channel lower wall is maintained at temperature T_0 while the upper wall is maintained at temperature T_1 such that $T_0 < T_1$. Physical model of the problem is presented in Figure 1. Since channel walls are of infinite extent in x and z-directions and the flow is fully developed, all physical quantities, except pressure depend on y only.

Taking into consideration the assumptions made above, the governing equations for steady flow of a viscous, incompressible, electrically and thermally conducting fluid in a rotating system taking Hall current into account are presented in the following form:

$$2\Omega w=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\sigma B_0^2(u+mw)}{\rho(1+m^2)} \quad (1)$$

$$-2\Omega u=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right)-\frac{\sigma B_0^2(w-mu)}{\rho(1+m^2)} \quad (2)$$

$$\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}\left(\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right)+\frac{\sigma B_0^2}{\rho C_P}\left(\frac{(u+mw)^2+(w-mu)^2}{(1+m^2)^2}\right)=0 \quad (3)$$

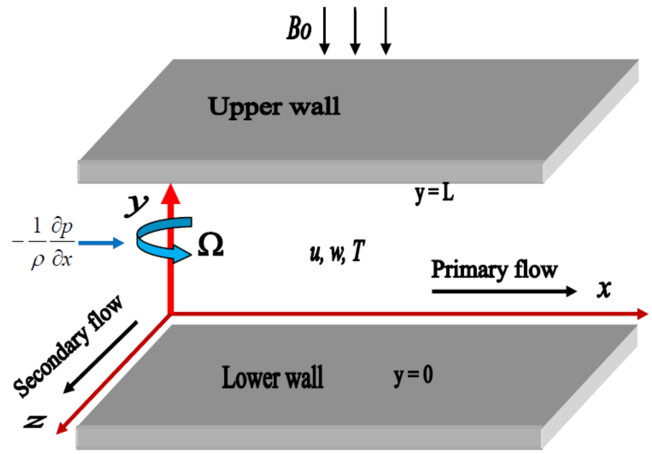

Fig. 1: Physical model of the problem

$$E_g=\frac{k}{T_0^2}\left(\frac{\partial T}{\partial y}\right)^2+\frac{\mu}{T_0}\left(\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right)+\frac{\sigma B_0^2}{T_0}\left(\frac{(u+mw)^2+(w-mu)^2}{(1+m^2)^2}\right) \quad (4)$$

where $u,w,T,\sigma,\rho,m=\omega_e T_e,\omega_e,T_e,k,C_P$ and E_g are respectively, the fluid velocity in x-direction, fluid velocity in z-direction, fluid temperature, fluid electrical conductivity, fluid density, Hall current parameter, cyclotron frequency, electron collision time, thermal conductivity coefficient, specific heat at constant pressure and the volumetric entropy generation rate. The fluid dynamical viscosity is assumed to be an exponential decreasing function of temperature given by

$$\mu(T)=\mu_0 e^{-\beta(T-T_0)} \quad (5)$$

where μ is the fluid dynamic viscosity at the lower wall and β is viscosity parameter variation. The boundary conditions for the fluid velocities and temperature are given as

$$u(0)=0,\quad w(0)=0\quad T(0)=T_0$$
$$u(L)=0,\quad w(L)=0\quad T(L)=T_1. \quad (6)$$

We introduce the dimensionless variables and parameters as follows:

$$\eta=\frac{y}{L},\quad X=\frac{x}{L},\quad \theta=\frac{T-T_0}{T_1-T_0},\quad \delta=\beta(T_1-T_0),$$
$$\nu=\frac{\mu_0}{\rho},\quad U=\frac{uL}{\nu},\quad W=\frac{wL}{\nu},\quad Pr=\frac{\nu_0 C_p}{k},$$
$$A=-\frac{\partial P}{\partial X},\quad \gamma=\frac{T_1-T_0}{T_0},\quad Ec=\frac{\nu^2}{C_P(T_1-T_0)L^2},$$
$$M=\frac{\sigma B_0^2 L^2}{\nu},\quad R=\frac{\Omega L^2}{\nu},\quad Ns=\frac{E_g T_0^2 L^2}{k(T_1-T_0)^2},\quad \bar{P}=\frac{L^2 P}{\rho\nu^2}. \quad (7)$$

Substituting equation (7) into equations (1)-(6), we obtain,

$$2RW = Ae^{-\delta\theta}\frac{d^2U}{d\eta^2} - \delta e^{-\delta\theta}\frac{d\theta}{d\eta}\frac{dU}{d\eta} - \frac{M(U+mW)}{1+m^2}, \quad (8)$$

$$-2RU = e^{-\delta\theta}\frac{d^2W}{d\eta^2} - \delta e^{-\delta\theta}\frac{d\theta}{d\eta}\frac{dW}{d\eta} - \frac{M(U+mW)^2+(W-mU)^2}{1+m^2}, \quad (9)$$

$$\frac{d^2\theta}{d\eta^2} + PrEc\left[\left(\frac{dU}{d\eta}\right)^2 + \left(\frac{dW}{d\eta}\right)^2\right]e^{-\delta\theta} + PrEcM\left[\frac{(U+mW)^2+(W-mU)^2}{(1+m^2)^2}\right] = 0, \quad (10)$$

$$N_s = \left(\frac{d\theta}{d\eta}\right)^2 + \frac{Br}{\gamma}\left[\left(\frac{dU}{d\eta}\right)^2 + \left(\frac{dW}{d\eta}\right)^2\right]e^{-\delta\theta} + \frac{BrM}{\gamma}\left[\frac{(U+mW)^2+(W-mU)^2}{(1+m^2)^2}\right] = 0, \quad (11)$$

with

$$U(0)=0, \quad W(0)=0, \quad \theta(0)=0$$
$$U(1)=0, \quad W(1)=0, \quad \theta(1)=1 \quad (12)$$

where Pr is the Prandtl number, R is the rotation parameter, Ec is the Eckert number,δ is the viscosity variation parameter, M is the magnetic field parameter, $Br(=EcPr)$ is the Brinkmann number,γ is the temperature difference parameter and A is the pressure gradient parameter. Other quantities of interest are the skin friction coefficients $(Cf_1 and Cf_2)$, Nusselt number (Nu) and the Bejan number (Be) which are given as

$$Cf_1 = \frac{L^2\tau_1}{\rho\mu^2} = e^{-\delta\theta}\frac{dU}{d\eta}\Big|_{\eta=0,1}, \quad Cf_2 = \frac{L^2\tau_2}{\rho\mu^2} = e^{-\delta\theta}\frac{dW}{d\eta}\Big|_{\eta=0,1},$$
$$Nu = -\frac{Lq_m}{k(T_1-T_0)} = -\frac{d\theta}{d\eta}\Big|_{\eta=0,1}, \quad Be = \frac{N_1}{N_s} = \frac{1}{1+\phi} \quad (13)$$

where

$$\tau_1 = \mu\frac{du}{dy}, \quad \tau_1 = \mu\frac{dw}{dy}, \quad q_m = -k\frac{\partial T}{\partial y}, N_1 = \left(\frac{d\theta}{d\eta}\right)^2, \quad \phi = \frac{N_2}{N_1},$$
$$N_2 = \frac{Br}{\gamma}e^{-\delta\theta}\left[\left(\frac{dU}{d\eta}\right)^2 + \left(\frac{dW}{d\eta}\right)^2\right] + \frac{BrM}{\gamma}\left[\frac{(U+mW)^2+(W-mU)^2}{(1+m^2)^2}\right] \quad (14)$$

It is very important to note that N_1 represents the thermodynamic irreversibility due to heat transfer while N_2 corresponds to the combined effects of fluid friction and magnetic field irreversibility. When $Be=0.5$ both N_1 and N_2 contribute equally to the entropy generation in the flow process. The model equations (8)-(13) are tackled numerically using a shooting technique coupled with Runge-Kutta-Fehlberg integration scheme. The procedure involved transforming the model boundary value problem (BVP) to initial value problem (IVP) and employing shooting method to obtain the unknown initial values while the Runge-Kutta-Fehlberg integration scheme is utilized for the solution up to the prescribed boundary conditions.

3 Results and Discussion:

The variation effects of the key parameters on the velocities profiles are shown in figures 2-11. Generally, we see that these figures are parabolic in natures. Figure 2 shows the effect of increasing pressure gradient (A) on velocity profile in x-direction. It is noticed that as A increases, the velocity also increases. Similarly, figure 3 shows the effect of increasing pressure gradient (A) on velocity profile in z-direction. Increase in A leads to increase in the velocity on the channel. Figure 4 shows the effect on increasing magnetic field parameter (M) on velocity profile. As M is increasing, decrease in velocity profile is noticed in the channel. Figure 5 depicts the effect of M on z-direction velocity profile. In the figure, the velocity profile increases as M is increasing. Figure 6 presents the effect of Hall current parameter (m) on velocity profile in x-direction. It is noticed that the velocity in x-direction increases as m is increasing and velocity in z-direction decreases as m increases as shown in figure 7. Figure 8 and 9 show the effect of dimensionless viscosity parameter variation (δ) on both velocities profiles. We noticed that both velocities profiles increase as δ increases. Figure 10 and 11 show the effect of rotation parameter (R) on the velocities profiles. Increase in R, decreases the velocity profile in x-direction as shown in figure 10 and increases the velocity profile as shown in figure11.

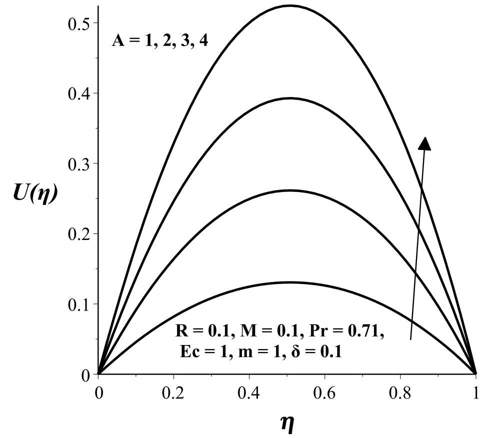

Figure 2: Velocity in x-direction with increasing pressure gradient

Figures 12-17 show the variation effects of key parameter on the skin friction coefficients and Nusselt number. Figure 12 presents the effect of increasing M versus m on the skin friction (Cf_1). It is noticed that both skin friction coefficient (Cf_1) at lower and the upper wall decreases with increase in M versus m while figure 13 shows the effect of increasing δ versus m. Here, (Cf_1) at the lower wall increases and decreases at the upper wall with increasing δ and m. Figures 14 and 15 show the effects of increasing δ versus m and M versus m on the skin friction (Cf_2). It is noticed that as these parameters are increasing, the skin friction coefficient (Cf_2) both at lower and upper wall increased. Figure 16 depicts the effect on increasing A versus δ on Nusselt number. As these parameters are increasing, the nusselt number on the lower wall increases while that of the upper wall

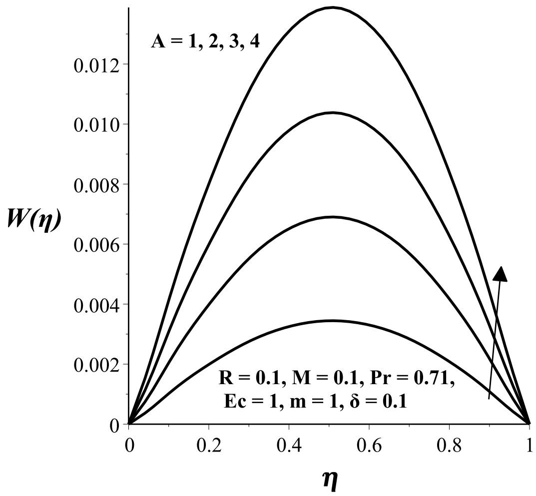

Figure 3: Velocity in z -direction with increasing pressure gradient

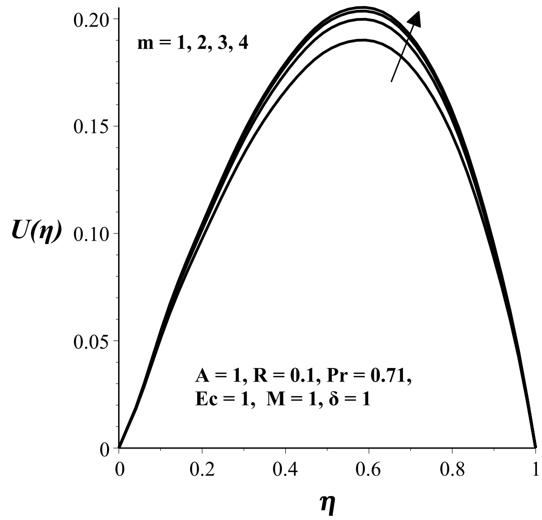

Figure 6: Velocity in x-direction with increasing m

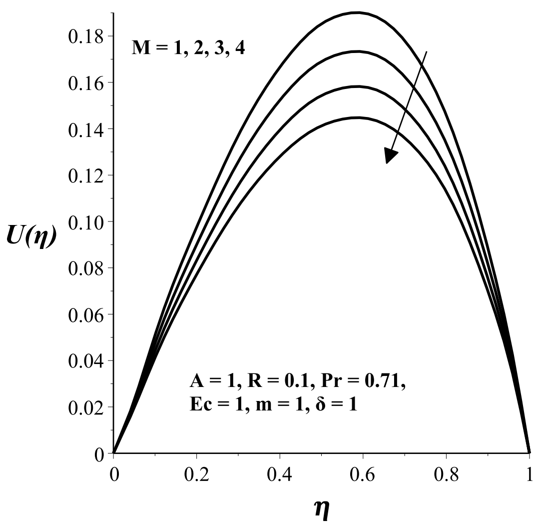

Figure 4: Velocity in x-direction with increasing M

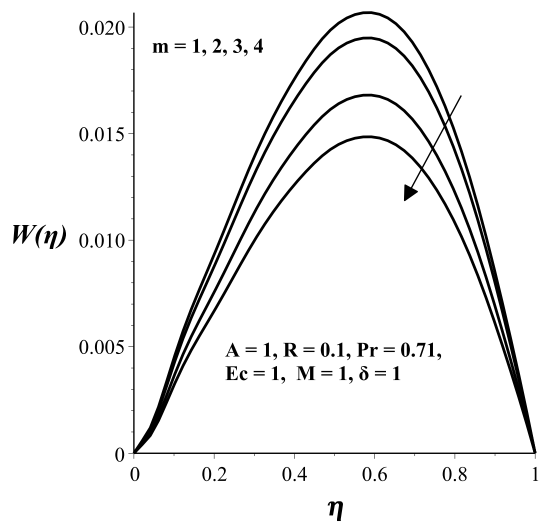

Figure 7: Velocity in z-direction with increasing m

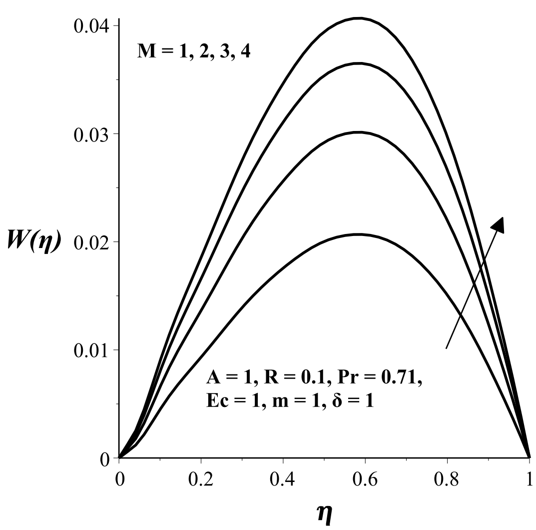

Figure 5: Velocity in z-direction with increasing M

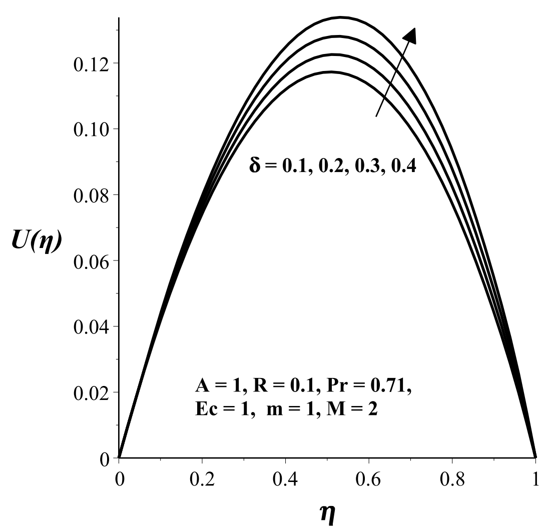

Figure 8: Velocity in x-direction with increasing δ

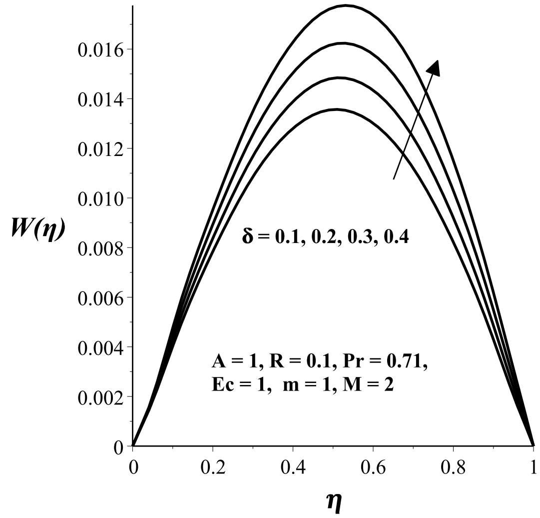

Figure 9: Velocity in z-direction with increasing δ

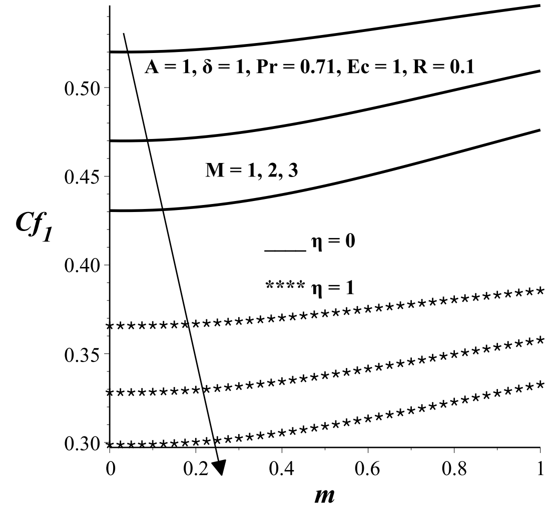

Figure 12: Cf_1 with increasing M versus m

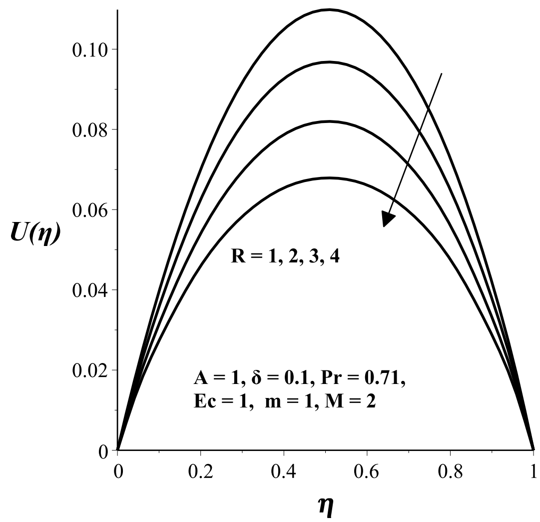

Figure 10: Velocity in x-direction with increasing R

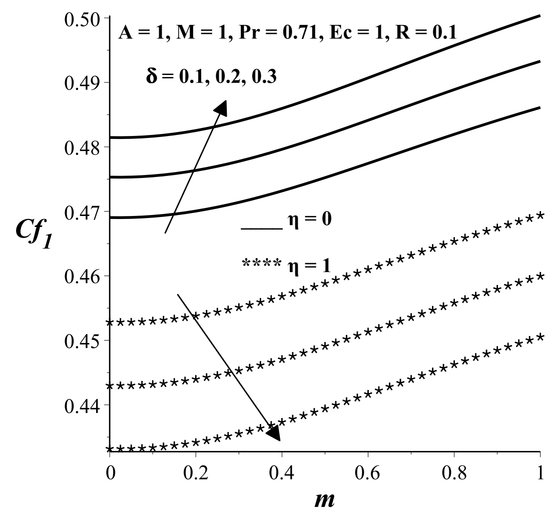

Figure 13: Cf_1 with increasing δ versus m

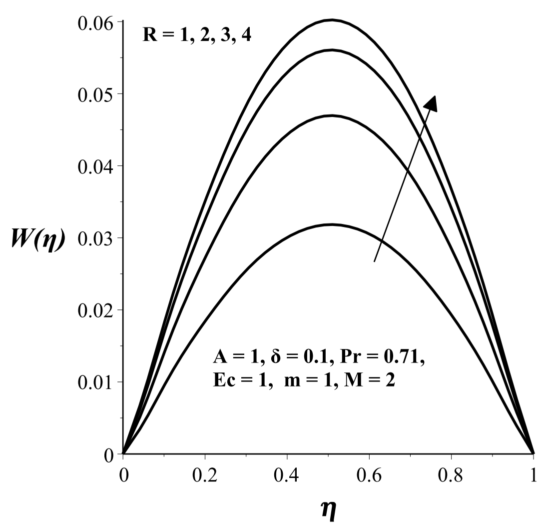

Figure 11: Velocity in z-direction with increasing R

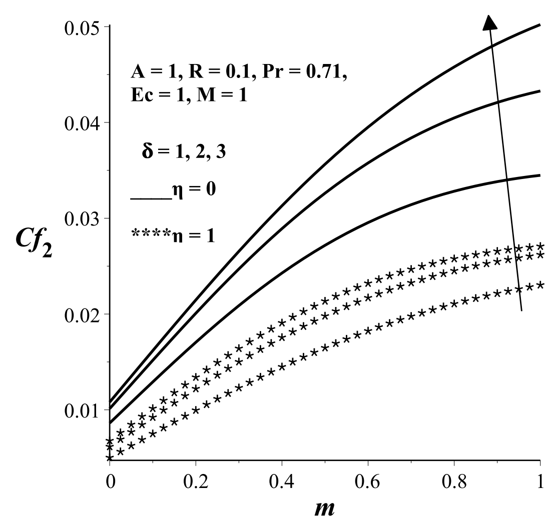

Figure 14: Cf_2 with increasing δ versus m

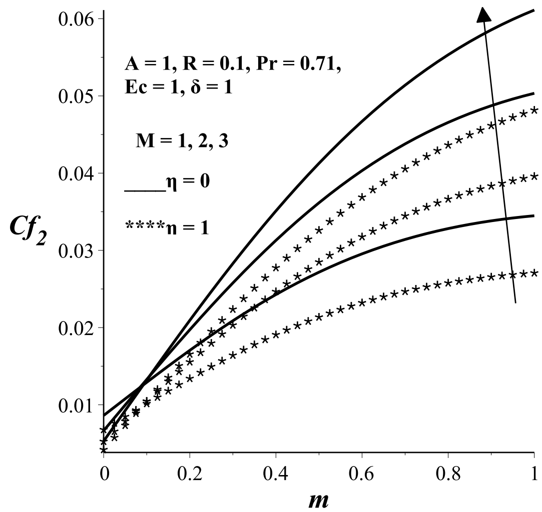

Figure 15: Cf_2 with increasing M versus m

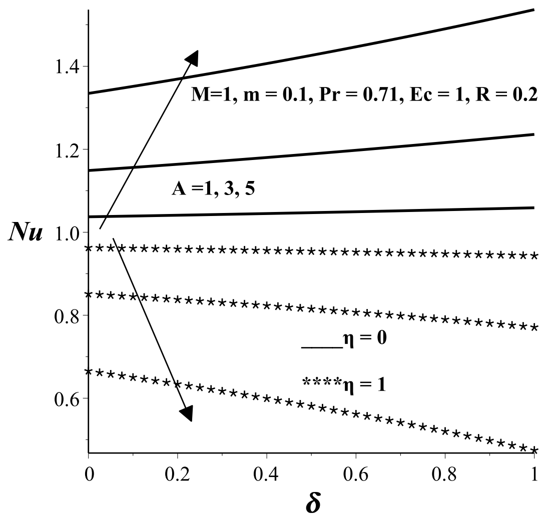

Figure 16: Nusselt number with increasing A versus δ

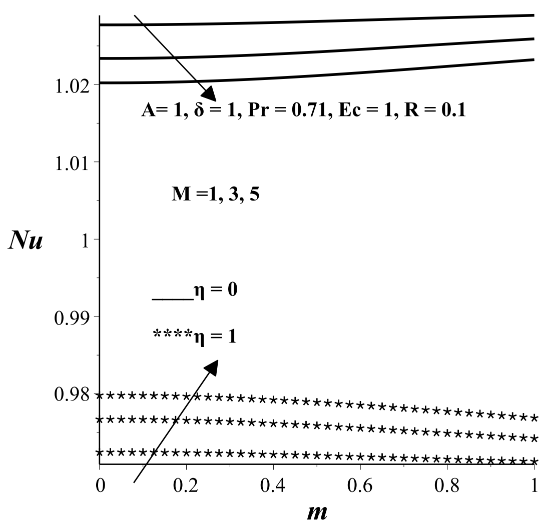

Figure 17: Nusselt number with increasing M versus m

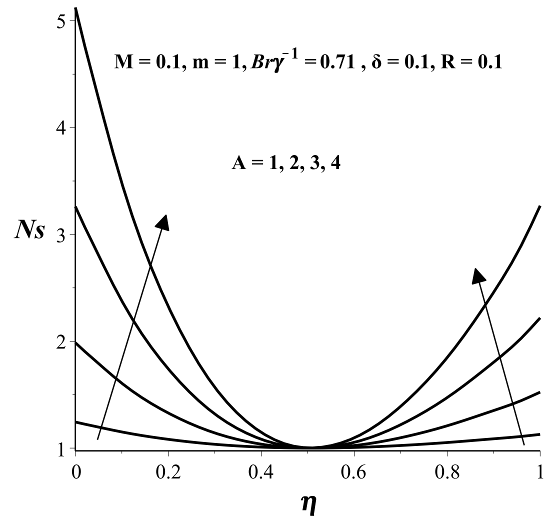

Figure 18: Entropy generation with increasing A

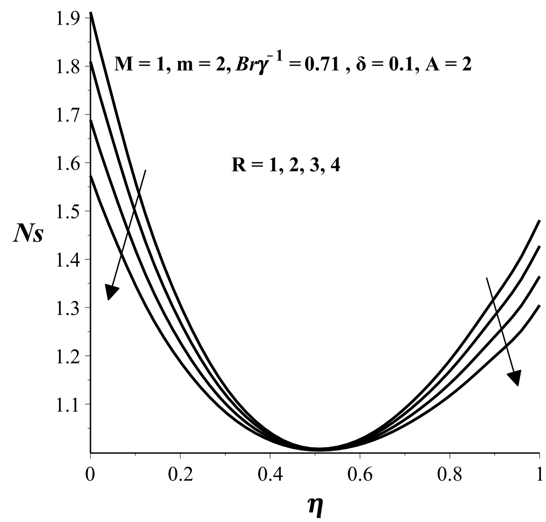

Figure 19: Entropy generation with increasing R

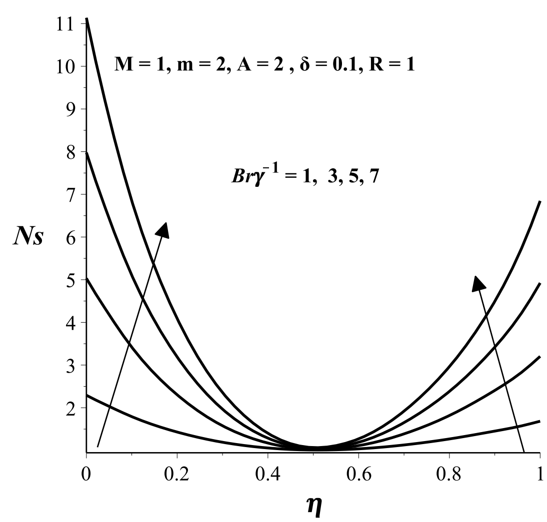

Figure 20: Entropy generation with increasing $Br\gamma^{-1}$

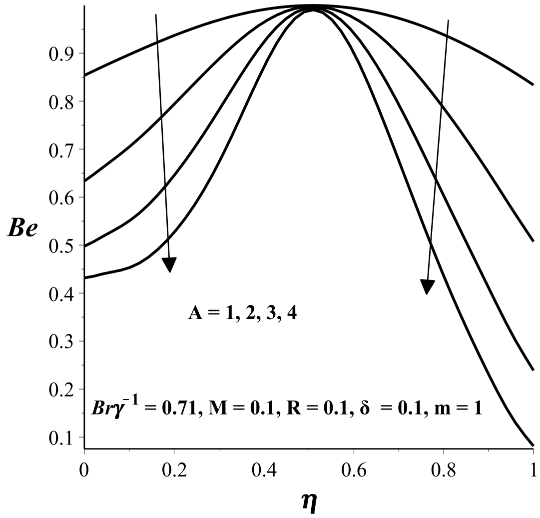

Figure 21: Bejan number with increasing A

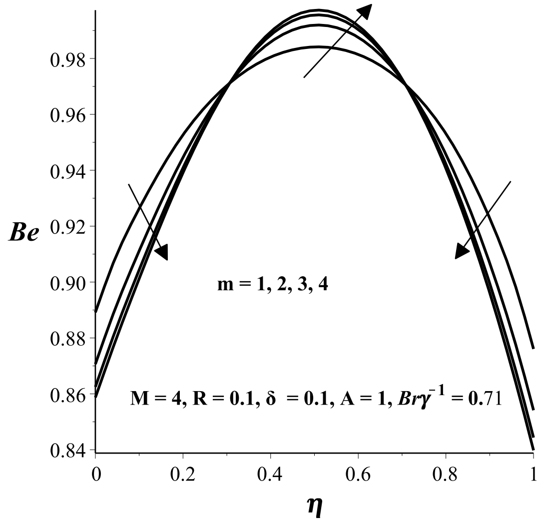

Figure 22: Bejan number with increasing m

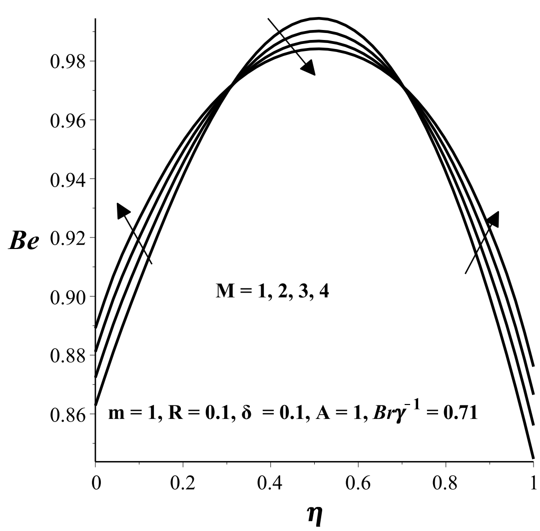

Figure 23: Bejan number with increasing M

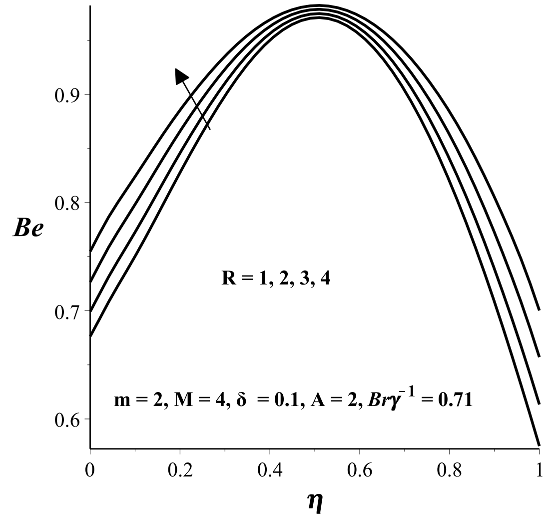

Figure 24: Bejan number with increasing R

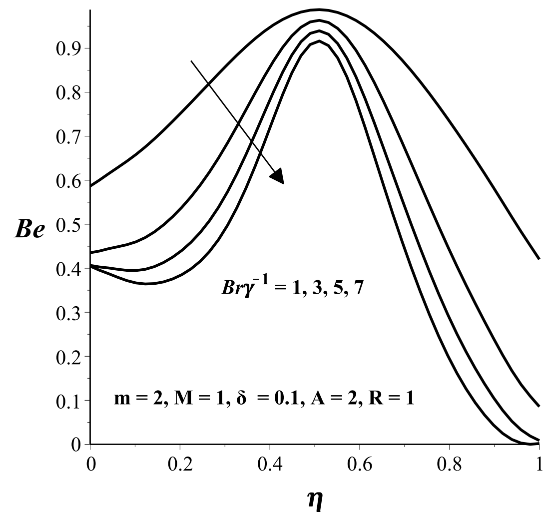

Figure 25: Bejan number with increasing $Br\gamma^{-1}$

decreases. Meanwhile, in figure 17 as M versus m are increasing, the Nusselt number at lower wall decreases and increases at upper wall. The variations of some of the important key parameters on the entropy generation rate are displayed in figures 18-20. Figure 18 shows as the pressure gradient, A, increases, the entropy generation at both lower and upper walls are increasing. Figure 19 put into consideration the effect of rotation parameter, R, on the entropy generation. As the rotation parameter is increasing, the entropy generation at both walls is decreasing but at the centre of the channel, there seems to be no effect. The effect of group parameter $(Br\gamma^{-1})$ on the entropy generation rate is shown in figure 20. An increase in $Br\gamma^{-1}$, leads to increase in the entropy generation rate at both walls.

The analyses of some of the key parameters on the Bejan number are presented in figures 21-25. Figure 21 shows that, as pressure gradient is increasing, the Bejan number at the both walls is decreasing. This shows that the thermodynamic irreversibility due to heat transfer dominate at both walls but at the centre of the channel both thermodynamic irreversibility due to heat transfer and the combined effects of fluid friction and magnetic field irreversibility contributed equally. Figure 22 depicts the effect of Hall current parameter on the Bejan number. As *m* increases, it is noticed that the Bejan number at both walls is decreasing and at the centre of the channel is increasing. This means that the thermodynamic irreversibility due to heat transfer dominates at the walls but the combined effects of fluid friction and magnetic field irreversibility dominates at the centre of the channel while figure 23 indicates a reverse of scenario in figure 22 as magnetic field parameter, *M*, is increasing. The combined effect of fluid friction and magnetic field irreversibility dominates at the walls while thermodynamic irreversibility due to heat transfer dominates at the centre of the channel. In figure 24, as rotation parameter is increasing, the Bejan number is increasing. This shows that the combined effect of fluid friction and magnetic field irreversibility dominates the entire flow while thermodynamic irreversibility due to heat transfer dominates the entire flow, as shown in figure 25, as group parameter is increasing

4 Conclusion

We have theoretically considered the entropy analysis of variable viscosity Hartmann flow through a rotating channel with Hall effects. Generally, the velocities profiles are parabolic in nature and some of the results obtained can be summarized as follows;

– Velocity profile in the *z*-direction increases with increase in *A*, *M*, δ and *R* but decrease with increase in *m*.
– Velocity profile in the *x*-direction increases with increase in *A*, *M* and δ but decrease with increase in *M* and *R*.
– Skin friction coefficient decreases at both walls with increase in *M* versus *m* while with increase in δ versus *m* increases at lower wall and decreases at upper wall.
– Skin friction coefficient (Cf_1) increases at both walls with increase in δ versus *m* and *M* versus *m*.
– Entropy generation increase with increase in $Br\gamma^{-1}$ and *A* but decrease with increase in *R*.
– The Bejan number increases in the entire flow with increase in *R* and decreases in the entire flow with increase in $Br\gamma^{-1}$ and *A*.

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