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Supra Semi Open Soft Sets and Associated Soft Separation Axioms

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Abstract: In this paper, we introduce and investigate some weak soft separation axioms by using the notion of supra semi open soft sets. We study the relationships between these new soft separation axioms and their relationships with some other properties. As a consequence the relations of some supra soft separation axioms are shown in a diagram. We show that, some classical results in general supra topology are not true if we consider supra soft topological spaces instead. For instance, if (X,μ,E) is supra soft semi T_1-space need not every soft singleton x_E is supra semi closed soft.

Keywords: Soft sets, Soft topological spaces, Supra soft topological spaces, Supra semi open soft sets, Supra soft semi T_i spaces $(i = 1,2,3,4)$, Supra soft continuity, Supra semi irresolute open soft function.

1 Introduction

Soft set theory is one of the emerging branches of mathematics that could deal with parameterization inadequacy and vagueness that arises in most of the problem solving methods. It is introduced [18] in 1999 by the Russian mathematician Molodtsov with its rich potential applications in divergent directions. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1,6,15,16,17,19].

It got some stability only after the introduction of soft topology [20] in 2011. In [11], Kandil et al. introduced some soft operations such as semi open soft, pre open soft, $\alpha-$open soft and $\beta-$open soft and investigated their properties in detail. Kandil et al. [14] introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces.

The notion of supra soft topological spaces was initiated for the first time by El-Sheikh and Abd El-Latif [4,8]. They also introduced new different types of subsets of supra soft topological spaces and study the relations between them in detail. The main purpose of this paper, is to generalize the notion of supra soft separation axioms [3] by using the notions of supra semi open soft sets.

2 Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in this paper.

Definition 2.1.[18] Let X be an initial universe and E be a set of parameters. Let $\mathscr{P}(X)$ denote the power set of X and A be a non-empty subset of E. A pair F denoted by F_A is called a soft set over X , where F is a mapping given by $F : A \to \mathscr{P}(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X.

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For a particular $e \in A$, $F(e)$ may be considered the set of $e-$approximate elements of the soft set (F,A) and if $e \notin A$, then $F(e) = \emptyset$ i.e $F_A = \{F(e) : e \in A \subseteq E,\ F : A \to \mathscr{P}(X)\}$.

Definition 2.2.[7] A soft set F over X is a set valued function from E to $\mathscr{P}(X)$. It can be written a set of ordered pairs $F = \{(e,F(e)) : e \in E\}$. Note that if $F(e) = \emptyset$, then the element $(e,F(e))$ is not appeared in F. The set of all soft sets over X is denoted by $S_E(X)$.

Definition 2.3.[7] Let $F,G \in S_E(X)$. Then,

(1)If $F(e) = \emptyset$ for each $e \in E$, F is said to be a null soft set, denoted by $\tilde{\emptyset}$.
(2)If $F(e) = X$ for each $e \in E$, F is said to be absolute soft set, denoted by $\tilde{X}$.
(3)F is soft subset of G, denoted by $F \tilde{\subseteq} G$, if $F(e) \subseteq G(e)$ for each $e \in E$.
(4)$F = G$, if $F \tilde{\subseteq} G$ and $G \tilde{\subseteq} F$.
(5)Soft union of F and G, denoted by $F \tilde{\cup} G$, is a soft set over X and defined by $F \tilde{\cup} G : E \to \mathscr{P}(X)$ such that $(F \tilde{\cup} G)(e) = F(e) \cup G(e)$ for each $e \in E$.
(6)Soft intersection of F and G, denoted by $F \tilde{\cap} G$, is a soft set over X and defined by $F \tilde{\cap} G : E \to \mathscr{P}(X)$ such that $(F \tilde{\cap} G)(e) = F(e) \cap G(e)$ for each $e \in E$.
(7)Soft difference of F and G, denoted by $F \tilde{\setminus} G$, is a soft set over U whose approximate function is defined by $F \tilde{\setminus} G : E \to \mathscr{P}(X)$ such that $(F \tilde{\setminus} G)(e)$=$F(e) \setminus G(e)$.
(8)Soft complement of F is denoted by $F^{\tilde{c}}$ and defined by $F^{\tilde{c}} : E \to \mathscr{P}(X)$ such that $F^{\tilde{c}}(e) = X \setminus F(e)$ for each $e \in E$.

We will consider Definition 2.2 and Definition 2.3. the rest of paper.

Definition 2.4.[21] The soft set $F \in S_E(X)$ is called a soft point if there exist an $e \in E$ such that $F(e) \neq \emptyset$ and $F(e') = \emptyset$ for each $e' \in E \setminus \{e\}$, and the soft point F is denoted by e_F. The soft point e_F is said to be in the soft set G, denoted by $e_F \tilde{\in} G$, if $F(e) \subseteq G(e)$ for the element $e \in E$. Also, we say that $x \in F$ read as x belongs to the soft set F whenever $x \in F(e)$ for each $e \in E$.

Definition 2.5.[14,20] The soft set F over X such that $F(e) = \{x\}$ $\forall e \in E$ is called singleton soft point and denoted by x_E or (x,E).

Definition 2.6.[5] Let $S_E(X)$ and $S_K(Y)$ be families of soft sets, $u : X \to Y$ and $p : E \to K$ be mappings. Therefore $f_{pu} : S_E(X) \to S_K(Y)$ is called a soft function.

(1)If $F \in S_E(X)$, then the image of F under f_{pu}, written as $f_{pu}(F)$, is a soft set in $S_K(Y)$ such that

$$f_{pu}(F)(k) = \begin{cases} \bigcup_{e \in p^{-1}(k)} u(F(e)), & p^{-1}(k) \neq \emptyset \\ \emptyset, & \text{otherwise.} \end{cases}$$

for each $k \in Y$.
(2)If $G \in S_K(Y)$, then the inverse image of G under f_{pu}, written as $f_{pu}^{-1}(G)$, is a soft set in $S_E(X)$ such that

$$f_{pu}^{-1}(G)(e) = \begin{cases} u^{-1}(G(p(e))), & p(e) \in Y \\ \emptyset, & \text{otherwise.} \end{cases}$$

for each $e \in E$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Theorem 2.7.[5]. Let $S_E(X)$ and $S_K(Y)$ be families of soft sets. For the soft function $f_{pu} : S_E(X) \to S_K(Y)$, for each $F, F_1, F_2 \in S_E(X)$ and for each $G, G_1, G_2 \in S_K(Y)$ the following statements hold,

(1)$f_{pu}^{-1}(G^{\tilde{c}}) = (f_{pu}^{-1}G)^{\tilde{c}}$.
(2)$f_{pu}(f_{pu}^{-1}(G)) \tilde{\subseteq} G$. If f_{pu} is surjective, then the equality holds.
(3)$F \tilde{\subseteq} f_{pu}^{-1}(f_{pu}(F))$. If f_{pu} is injective, then the equality holds.
(4)$f_{pu}(\tilde{X}) \tilde{\subseteq} \tilde{Y}$. If f_{pu} is surjective, then the equality holds.
(5)$f_{pu}^{-1}(\tilde{Y}) = \tilde{X}$ and $f_{pu}(\tilde{\emptyset}) = \tilde{\emptyset}$.
(6)If $F_1 \tilde{\subseteq} F_2$, then $f_{pu}(F_1) \tilde{\subseteq} f_{pu}(F_2)$.
(7)If $G_1 \tilde{\subseteq} G_2$, then $f_{pu}^{-1}(G_1) \tilde{\subseteq} f_{pu}^{-1}(G_2)$.
(8)$f_{pu}^{-1}(G_1 \tilde{\cup} G_2) = f_{pu}^{-1}(G_1) \tilde{\cup} f_{pu}^{-1}(G_2)$ and $f_{pu}^{-1}(G_1 \tilde{\cap} G_2) = f_{pu}^{-1}(G_1) \tilde{\cap} f_{pu}^{-1}(G_2)$.
(9)$f_{pu}(F_1 \tilde{\cup} F_2) = f_{pu}(F_1) \tilde{\cup} f_{pu}(F_2)$ and $f_{pu}(F_1 \tilde{\cap} F_2) \tilde{\subseteq} f_{pu}(F_1) \tilde{\cap} f_{pu}(F_2)$. If f_{pu} is injective, then the equality holds.

Definition 2.8.[20] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq S_E(X)$ is called a soft topology on X if

(1)$\tilde{X}, \tilde{\emptyset} \in \tau$, where $\tilde{\emptyset}(e) = \emptyset$ and $\tilde{X}(e) = X$, for each $e \in E$,
(2)The soft union of any number of soft sets in τ belongs to τ,
(3)The soft intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X. A soft set F over X is said to be open soft set in X if $F \in \tau$, and it is said to be closed soft set in X, if its relative complement F' is an open soft set. We denote the set of all open soft sets over X by $OS(X,\tau,E)$, or when there can be no confusion by $OS(X)$ and the set of all closed soft sets by $CS(X,\tau,E)$, or $CS(X)$.

Definition 2.9.[20] Let (X,τ,E) be a soft topological space over X and $F \in S_E(X)$. Then, the soft interior and soft closure of F, denoted by $int(F)$ and $cl(F)$, respectively, are defined as,

$int(F) = \tilde{\bigcup}\{G : G \text{ is open soft set and } G\tilde{\subseteq}F\}$

$cl(F) = \tilde{\bigcap}\{H : H \text{ is closed soft set and } F\tilde{\subseteq}H\}$.

Definition 2.10.[21] Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces and $f_{pu} : S_E(X) \to S_K(Y)$ be a function. Then, the function f_{pu} is called,

(1)Continuous soft if $f_{pu}^{-1}(G) \in \tau_1$ for each $G \in \tau_2$.
(2)Open soft if $f_{pu}(F) \in \tau_2$ for each $F \in \tau_1$.

Definition 2.11.[8] Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\mu \subseteq S_E(X)$ is called supra soft topology on X with a fixed set E if

(1)$\tilde{X},\tilde{\emptyset} \in \mu$,
(2)The soft union of any number of soft sets in μ belongs to μ.

The triplet (X,μ,E) is called supra soft topological space (or supra soft spaces) over X.

Definition 2.12.[8] Let (X,τ,E) be a soft topological space and (X,μ,E) be a supra soft topological space. We say that, μ is a supra soft topology associated with τ if $\tau \subseteq \mu$.

Definition 2.13.[8] Let (X,μ,E) be a supra soft topological space over X, then the members of μ are said to be supra open soft sets in X. We denote the set of all supra open soft sets over X by $supra-OS(X,\mu,E)$, or when there can be no confusion by $supra-OS(X)$ and the set of all supra closed soft sets by $supra-CS(X,\mu,E)$, or $supra-CS(X)$.

Definition 2.14.[8] Let (X,μ,E) be a supra soft topological space over X and $F \in S_E(X)$. Then the supra soft interior of F, denoted by $Sint(F)$ is the soft union of all supra open soft subsets of F. Clearly $Sint(F)$ is the largest supra open soft set over X which contained in F i.e

$$Sint(F) = \tilde{\bigcup}\{G : G \text{ is supra open soft set and } G\tilde{\subseteq}F\}.$$

Definition 2.15.[8] Let (X,μ,E) be a supra soft topological space over X and $F \in S_E(X)$. Then the supra soft closure of F, denoted by $Scl(F)$ is the soft intersection of all supra closed super soft sets of F. Clearly $Scl(F)$ is the smallest supra closed soft set over X which contains F i.e

$$Scl(F) = \tilde{\bigcap}\{H : H \text{ is supra closed soft set and } F\tilde{\subseteq}H\}.$$

Definition 2.16.[8] Let (X,μ,E) be a supra soft topological space and $F \in S_E(X)$. Then, F is called supra semi open soft set if $F\tilde{\subseteq}Scl(Sint(F))$. We denote the set of all supra semi open soft sets by $SSOS(X,\mu,E)$, or $SSOS_E(X)$ and the set of all supra semi closed soft sets by $SSCS(X,\mu,E)$, or $SSCS_E(X)$.

Definition 2.17.[2] Let (X,μ,E) be a supra soft topological space over X and $F \in S_E(X)$. Then, the supra semi soft interior and supra semi soft closure of F, denoted by $Sint_s(F)$ and $Scl_s(F)$, respectively, are defined as

$$Sint_s(F) = \tilde{\bigcup}\{G : G \text{ is supra semi open soft set and } G\tilde{\subseteq}F\}$$

$$Scl_s(F) = \tilde{\bigcap}\{H : H \text{ is supra semi closed soft set and } F\tilde{\subseteq}H\}.$$

Definition 2.18.[3,8] Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. The soft function $f_{pu} : S_E(X) \to S_K(Y)$ is called

(1)Supra continuous soft function if $f_{pu}^{-1}(G) \in \mu_1$ for each $F \in \tau_2$.
(2)Supra open soft if $f_{pu}(F) \in \mu_1$ for each $F \in \tau_1$.
(3)Supra irresolute soft if $f_{pu}^{-1}(F) \in \mu_1$ for each $F \in \mu_2$.
(4)Supra irresolute open soft if $f_{pu}(F) \in \mu_2$ for each $F \in \mu_1$.
(5)Supra semi-continuous soft if $f_{pu}^{-1}(F) \in SSOS(X,\mu_1,E)$ for each $F \in \mu_2$.

3 Supra soft semi separation axioms

In this section, we introduce and investigate some weak soft separation axioms by using the notion of supra semi open soft sets, which is a generalization of the supra soft separation axioms mentioned in [3].

Definition 3.1. Let (X,τ,E) be a soft topological space and μ be an associated supra soft topology with τ. Let $x,y \in X$ such that $x \neq y$. Then, (X,μ,E) is called

(1)Supra soft semi T_0-space (Supra soft s-T_0, for short) if there exists a μ-supra semi open soft set F containing one of the points x,y but not the other.
(2)Supra soft semi T_1-space (Supra soft s-T_1, for short) if there exist μ-supra semi open soft sets F and G such that $x \in F$, $y \notin F$ and $y \in G$, $x \notin G$.
(3)Supra soft semi Hausdorff space (Supra soft s-T_2, for short) if there exist μ-supra semi open soft sets F and G such that $x \in F$, $y \in G$ and $F\tilde{\cap}G = \tilde{\emptyset}$.

Proposition 3.2. Let (X,τ,E) be a soft topological space and $x,y \in X$ such that $x \neq y$. If there exist μ-supra semi open soft sets F and G such that either $x \in F$ and $y \in F^{\tilde{c}}$

or $y \in G$ and $x \in G^{\tilde{c}}$. Then, (X, μ, E) is supra soft semi T_0-space.

Proof. Let $x, y \in X$ such that $x \neq y$. Let F and G be μ-supra semi open soft sets such that either $x \in F$ and $y \in F^{\tilde{c}}$ or $y \in G$ and $x \in G^{\tilde{c}}$. If $x \in F$ and $y \in F^{\tilde{c}}$. Then, $y \in (F(e))^{\tilde{c}}$ for each $e \in E$. This implies that, $y \notin F(e)$ for each $e \in E$. Therefore, $y \notin F$. Similarly, if $y \in G$ and $x \in G^{\tilde{c}}$, then $x \notin G$. Hence, (X, μ, E) is supra soft semi T_0-space.

Theorem 3.3. A supra soft topological space (X, μ, E) is supra soft semi T_0-space if and only if the supra closures of each distinct points x and y are distinct.

Proof. Necessity: Let (X, μ, E) be a supra soft semi T_0-space and $x, y \in X$ such that $x \neq y$. Then, there exists a μ-supra semi open soft set F such that $x \in F$ and $y \notin F$. Hence, $F^{\tilde{c}}$ is supra semi closed soft set containing y but not x. It follows that, $Scl_s(y_E) \tilde{\subseteq} F^{\tilde{c}}$. Therefore, $x \notin Scl_s(y_E)$. Thus, $Scl_s(x_E) \neq Scl_s(y_E)$.
Sufficient: Let x, y be two distinct points in X such that $Scl_s(x_E) \neq Scl_s(y_E)$. Then, there exists a point z belongs to one of the sets $Scl_s(x_E), Scl_s(y_E)$ but not the other. Say, $z \in Scl_s(x_E)$ and $z \notin Scl_s(y_E)$. Now, if $x \in Scl_s(y_E)$. Then, $Scl_s(x_E) \tilde{\subseteq} Scl_s(y_E)$, which is a contradiction with $z \notin Scl_s(y_E)$. So, $x \notin Scl_s(y_E)$. Hence, $[Scl_s(y_E)]^{\tilde{c}}$ is supra semi open soft set containing x but not y. Thus, (X, μ, E) is supra soft semi T_0-space.

Proposition 3.4. Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. If there exist μ-supra semi open soft sets F and G such that $x \in F$ and $y \in F^{\tilde{c}}$ and $y \in G$ and $x \in G^{\tilde{c}}$. Then (X, μ, E) is supra soft semi T_1-space.

Proof. It is similar to the proof of Proposition 3.2.

Theorem 3.5. Every supra soft semi T_i-space is supra soft semi T_{i-1} for each $i = 1, 2$.

Proof. Obvious from Definition 3.1.

The converse of the above theorem is not true in general, as following examples shall show.

Examples 3.6.

(1)Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}, F\}$ where F is s soft sets over X defined as follows:

$$F(e_1) = \{h_1\},\ F(e_2) = X.$$

Then, τ defines a soft topology on X. Consider the associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X}, \tilde{\emptyset}, G_1, G_2, G_3\}$, where G_1, G_2 and G_3 are soft sets over X defined as follows:

$$G_1(e_1) = X, \quad G_1(e_2) = \{h_2\},$$
$$G_2(e_1) = \{h_1\},\ G_2(e_2) = X,$$
$$G_3(e_1) = \{h_1\},\ G_3(e_2) = \{h_1\}.$$

Therefore, (X, μ, E) is supra soft semi T_1-space, but it is not supra soft semi T_2-space, for $h_1, h_2 \in X$ and $h_1 \neq h_2$, but there are no μ-supra semi open soft sets F and G such that $h_1 \in F$, $h_2 \in G$ and $F \tilde{\cap} G = \tilde{\emptyset}$.

(2)Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}, F\}$ where F is soft set over X defined as follows by

$$F(e_1) = \{h_1, h_2\},\ F(e_2) = \{h_1, h_2\}.$$

Then,τ defines a soft topology on X. The associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X}, \tilde{\emptyset}, G_1, G_2, G_3\}$, where $G_1, G_2,$ *and* G_3 are soft sets over X defined as follows:

$$G_1(e_1) = \{h_1\}, \quad G_1(e_2) = \{h_1\},$$
$$G_2(e_1) = \{h_1, h_2\},\ G_2(e_2) = \{h_1, h_2\},$$
$$G_3(e_1) = \{h_2, h_3\},\ G_3(e_2) = \{h_2, h_3\}.$$

Hence, (X, μ, E) is supra soft semi T_0-space, but it is not supra soft semi T_1-space, since $h_2, h_3 \in X$, and $h_2 \neq h_3$, but every supra semi open soft set which contains h_3 also contains h_2.

Theorem 3.7. Let (X, μ, E) be a supra soft topological space. If x_E is supra semi closed soft set in μ for each $x \in X$, then (X, μ, E) is supra soft semi T_1-space.

Proof. Let $x \in X$ and x_E be supra semi closed soft set in μ, then, $x_E^{\tilde{c}}$ is supra semi open soft set in μ. Let $x, y \in X$ such that $x \neq y$. For $x \in X$ and $x_E^{\tilde{c}}$ is supra semi open soft set such that $x \notin x_E^{\tilde{c}}$ and $y \in x_E^{\tilde{c}}$. Similarly $y_E^{\tilde{c}}$ is supra semi open soft set in μ such that $y \notin y_E^{\tilde{c}}$ and $x \in y_E^{\tilde{c}}$. Thus, (X, μ, E) is supra soft semi T_1-space over X.

The converse of Theorem 3.7 is not true in general, as following example shall show.

Example 3.8. Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}, F\}$ where F is a soft sets over X defined as follows:

$$F(e_1) = X,\ F(e_2) = \{h_2\}.$$

Then, τ defines a soft topology on X. Consider the associated supra soft topology μ with τ is defined as $\mu = \{\tilde{X}, \tilde{\emptyset}, G_1, G_2, G_3\}$ where G_1, G_2 and G_3 are soft sets over X defined as follows:

$$G_1(e_1) = X, \quad G_1(e_2) = \{h_2\},$$
$$G_2(e_1) = \{h_1\},\ G_2(e_2) = X$$
$$G_3(e_1) = \{h_1\},\ G_3(e_2) = \{h_1\}.$$

Then, μ defines a supra soft topology on X. Therefore, (X,μ,E) is a supra soft semi T_1-space. On the other hand, we note that for the singleton soft points h_{1E} and h_{2E}, where

$$h_1(e_1)=\{h_1\},\ h_1(e_2)=\{h_1\},$$
$$h_2(e_1)=\{h_2\},\ h_2(e_2)=\{h_2\}.$$

The relative complement $h^{\tilde{c}}_{1E}$ and $h^{\tilde{c}}_{2E}$, where

$$h^{\tilde{c}}_1(e_1)=\{h_2\},\ h^{\tilde{c}}_1(e_2)=\{h_2\},$$
$$h^{\tilde{c}}_2(e_1)=\{h_1\},\ h^{\tilde{c}}_2(e_2)=\{h_1\}.$$

Thus, $h^{\tilde{c}}_{2E}$ is not μ-supra semi open soft set. This shows that, the converse of the above theorem does not hold.
Also, we have
$\mu_{e_1}=\{X,\emptyset,\{h_1\}\}$, and
$\mu_{e_2}=\{X,\emptyset,\{h_1\},\{h_2\}\}$. Therefore,$(X,\mu_{e_1})$ is not a supra semi T_1-space, at the time that (X,μ,E) is a supra soft semi T_1-space.

Definition 3.9. Let (X,τ,E) be a soft topological space and μ be an associated supra soft topology with τ. Let G be a μ-supra semi closed soft set in X and $x\in X$ such that $x\notin G$. If there exist μ-supra semi open soft sets F_1 and F_2 such that $x\in F_1$, $G\tilde{\subseteq}F_2$ and $F_1\tilde{\cap}F_2=\tilde{\emptyset}$, then (X,μ,E) is called supra soft semi regular space. A supra soft semi regular semi T_1-space is called supra soft semi T_3-space (Supra soft s-T_3, for short).

Proposition 3.10. Let (X,τ,E) be a soft topological space and μ be an associated supra soft topology with τ. Let G be a μ-supra semi closed soft set in X and $x\in X$ such that $x\notin G$. If (X,μ,E) is supra soft semi regular space, then there exists a μ-supra semi open soft set F such that $x\in F$ and $F\tilde{\cap}G=\tilde{\emptyset}$.

Proof. Obvious from Definition 3.9.

Theorem 3.11. Every supra soft T_i-space is supra soft semi T_i for each $i=0,1,2,3$.

Proof. It is clear from the fact that, every supra open soft set is supra semi open soft set [8].

Proposition 3.12. Let (X,μ,E) be a supra soft topological space, $F\in S_E(X)$ and $x\in X$. Then:

(1)$x\in F$ if and only if $x_E\tilde{\subseteq}F$.
(2)If $x_E\tilde{\cap}F=\tilde{\emptyset}$, then $x\notin F$.

Proof. Obvious.

Theorem 3.13. Let (X,μ,E) be a supra soft topological space and $x\in X$. If (X,μ,E) is supra soft semi regular space, then

(1)$x\notin F$ if and only if $x_E\tilde{\cap}F=\tilde{\emptyset}$ for every μ-supra semi closed soft set F.
(2)$x\notin G$ if and only if $x_E\tilde{\cap}G=\tilde{\emptyset}$ for every μ-supra semi open soft set G.

Proof.

(1)Let F be a μ-supra semi closed soft set such that $x\notin F$. Since (X,μ,E) is supra soft semi regular space. By Proposition 3.10, there exists a μ-supra semi open soft set G such that $x\in G$ and $F\tilde{\cap}G=\tilde{\emptyset}$. It follows that, $x_E\tilde{\subseteq}G$ from Proposition 3.12 (1). Hence, $x_E\tilde{\cap}F=\tilde{\emptyset}$. Conversely, if $x_E\tilde{\cap}F=\tilde{\emptyset}$, then $x\notin F$ from Proposition 3.12 (2).
(2)Let G be a μ-supra semi open soft set such that $x\notin G$. If $x\notin G(e)$ for each $e\in E$, then we get the proof. If $x\notin G(e_1)$ and $x\in G(e_2)$ for some $e_1,e_2\in E$, then $x\in G^{\tilde{c}}(e_1)$ and $x\notin G^{\tilde{c}}(e_2)$ for some $e_1,e_2\in E$. This means that, $x_E\tilde{\cap}G\neq\tilde{\emptyset}$. Hence, $G^{\tilde{c}}$ is μ-supra semi closed soft set such that $x\notin G^{\tilde{c}}$. It follows by (1) $x_E\tilde{\cap}G^{\tilde{c}}=\tilde{\emptyset}$. This implies that, $x_E\tilde{\subseteq}G$ and so $x\in G$, which is contradiction with $x\notin G(e_1)$ for some $e_1\in E$. Therefore, $x_E\tilde{\cap}G=\tilde{\emptyset}$. Conversely, if $x_E\tilde{\cap}G=\tilde{\emptyset}$, then it is obvious that $x\notin G$. This completes the proof.

Corollary 3.14. Let (X,μ,E) be a supra soft topological space and $x\in X$. If (X,μ,E) is supra soft semi regular space, then the following statements are equivalent:

(1)(X,μ,E) is supra soft semi T_1-space.
(2)$\forall\, x,y\in X$ such that $x\neq y$, there exist μ-supra semi open soft sets F and G such that $x_E\tilde{\subseteq}F$ and $y_E\tilde{\cap}F=\tilde{\emptyset}$ and $y_E\tilde{\subseteq}G$ and $x_E\tilde{\cap}G=\tilde{\emptyset}$.

Proof. Obvious from Theorem 3.13.

Theorem 3.15. Let (X,μ,E) be a supra soft topological space and $x\in X$. Then, the following statements are equivalent:

(1)(X,μ,E) is supra soft semi regular space.
(2)For every μ-supra semi closed soft set G such that $x_E\tilde{\cap}G=\tilde{\emptyset}$, there exist μ-supra semi open soft sets F_1 and F_2 such that $x_E\tilde{\subseteq}F_1$, $G\tilde{\subseteq}F_2$ and $F_1\tilde{\cap}F_2=\tilde{\emptyset}$.

Proof.

(1) $\Rightarrow$ (2) Let G be a μ-supra semi closed soft set such that $x_E\tilde{\cap}G=\tilde{\emptyset}$. Then, $x\notin G$ from Theorem 3.13 (1). It follows by (1), there exist μ-supra semi open soft sets F_1 and F_2 such that $x\in F_1$, $G\tilde{\subseteq}F_2$ and $F_1\tilde{\cap}F_2=\tilde{\emptyset}$. This means that, $x_E\tilde{\subseteq}F_1$, $G\tilde{\subseteq}F_2$ and $F_1\tilde{\cap}F_2=\tilde{\emptyset}$.
(2) $\Rightarrow$ (1) Let G be a μ-supra semi closed soft set such that $x\notin G$. Then, $x_E\tilde{\cap}G=\tilde{\emptyset}$ from Theorem 3.13 (1). It follows by (2), there exist μ-supra semi open soft sets F_1 and F_2 such that $x_E\tilde{\subseteq}F_1$, $G\tilde{\subseteq}F_2$ and $F_1\tilde{\cap}F_2=\tilde{\emptyset}$.

Hence, $x \in F_1$, $G \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$. Thus, (X,μ,E) is supra soft semi regular space.

Theorem 3.16. Let (X,μ,E) be a supra soft topological space. If (X,μ,E) is supra soft T_3-space, then $\forall\, x \in X$, x_E is μ-supra semi closed soft set.

Proof. We want to prove that x_E is μ-supra semi closed soft set, which is sufficient to prove that $x_E^{\tilde{c}}$ is μ-supra semi open soft set for each $y \in \{x\}^{\tilde{c}}$. Since (X,μ,E) is supra soft semi T_3-space. Then, there exist μ-supra semi open soft sets F_y and G such that $y_E \tilde{\subseteq} F_y$ and $x_E \tilde{\cap} F_y = \tilde{\emptyset}$ and $x_E \tilde{\subseteq} G$ and $y_E \tilde{\cap} G = \tilde{\emptyset}$. It follows that, $\bigcup_{y \in \{x\}^{\tilde{c}}} F_y \tilde{\subseteq} x_E^{\tilde{c}}$. Now, we want to prove that $x_E^{\tilde{c}} \tilde{\subseteq} \bigcup_{y \in \{x\}^{\tilde{c}}} F_y$. Let $\bigcup_{y \in \{x\}^{\tilde{c}}} F_y = H$, where $H(e) = \bigcup_{y \in \{x\}^{\tilde{c}}} F(e)_y$ for each $e \in E$. Since $x_E^{\tilde{c}}(e) = \{x\}^{\tilde{c}}$ for each $e \in E$ from Definition 2.5. So, for each $y \in \{x\}^{\tilde{c}}$ and $e \in E$, $x_E^{\tilde{c}}(e) = \{x\}^{\tilde{c}} = \bigcup_{y \in \{x\}^{\tilde{c}}} \{y\} = \bigcup_{y \in \{x\}^{\tilde{c}}} y_E(e) \tilde{\subseteq} \bigcup_{y \in \{x\}^{\tilde{c}}} F(e)_y = H(e)$. Thus, $x_E^{\tilde{c}} \tilde{\subseteq} \bigcup_{y \in \{x\}^{\tilde{c}}} F_y$, and so $x_E^{\tilde{c}} = \bigcup_{y \in \{x\}^{\tilde{c}}} F_y$. This means that, $x_E^{\tilde{c}}$ is μ-supra semi open soft set for each $y \in \{x\}^{\tilde{c}}$. Therefore, x_E is μ-supra semi closed soft set.

Theorem 3.17. Every supra soft semi T_3-space is supra soft semi T_2-space.

Proof. Let (X,μ,E) be a supra soft semi T_3-space and $x, y \in X$ such that $x \neq y$. By Theorem 3.16, y_E is μ-supra semi closed soft set and $x \notin y_E$. It follows from the supra soft semi regularity, there exist μ-supra semi open soft sets F_1 and (F_2) such that $x \in F_1$, $y_E \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$. Thus, $x \in F_1$, $y \in y_E \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$. Therefore, (X,μ,E) is supra soft semi T_2-space.

Corollary 3.18. The following implications hold from Theorem 3.5, Theorem 3.11 and [[3], Corollary 3.2] for a supra soft topological space (X,μ,E).

soft T_3 $\longrightarrow$ soft T_2 $\longrightarrow$ soft T_1 $\longrightarrow$ soft T_0
$\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$
supra soft T_3 $\rightarrow$ supra soft T_2 $\rightarrow$ supra soft T_1 $\rightarrow$ supra soft T_0
$\Downarrow$ $\Downarrow$ $\Downarrow$ $\Downarrow$
supra soft s-T_3 $\rightarrow$ supra soft s-T_2 $\rightarrow$ supra soft s-T_1 $\rightarrow$ supra soft s-T_0

Definition 3.19. Let (X,μ,E) be a supra soft topological space, F and G be μ-supra semi closed soft sets in X such that $F \tilde{\cap} G = \tilde{\emptyset}$. If there exist μ-supra semi open soft sets F_1 and F_2 such that $F \tilde{\subseteq} F_1$, $G \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$, then (X,μ,E) is called supra soft semi normal space. A supra soft semi normal semi T_1-space is called a supra soft semi T_4-space.

Theorem 3.20. Let (X,μ,E) be a supra soft topological space and $x \in X$. Then, the following statements are equivalent:

(1) (X,μ,E) is supra soft semi normal space.

(2) For every μ-supra semi closed soft set F and μ-supra semi open soft set G such that $F \tilde{\subseteq} G$, there exists a μ-supra semi open soft set F_1 such that $F \tilde{\subseteq} F_1$, $Scl_s(F_1) \tilde{\subseteq} G$.

Proof.

(1) $\Rightarrow$ (2) Let F be a μ-supra semi closed soft set and G be a μ-supra semi open soft set such that $F \tilde{\subseteq} G$. Then, $F, G^{\tilde{c}}$ are μ-supra semi closed soft sets such that $F \tilde{\cap} G^{\tilde{c}} = \tilde{\emptyset}$. It follows by (1), there exist μ-supra semi open soft sets F_1 and F_2 such that $F \tilde{\subseteq} F_1$, $G^{\tilde{c}} \tilde{\subseteq} F_2$ and $F_1 \tilde{\cap} F_2 = \tilde{\emptyset}$. Now, $F_1 \tilde{\subseteq} (F_2)^{\tilde{c}}$, so $Scl_s(F_1) \tilde{\subseteq} Scl_s(F_2)^{\tilde{c}} = (F_2)^{\tilde{c}}$, where G is μ-supra semi open soft set. Also, $(F_2)^{\tilde{c}} \tilde{\subseteq} G$. Hence, $Scl_s(F_1) \tilde{\subseteq} (F_2)^{\tilde{c}} \tilde{\subseteq} G$. Thus, $F \tilde{\subseteq} F_1$, $Scl_s(F_1) \tilde{\subseteq} G$.

(2) $\Rightarrow$ (1) Let G_1, G_2 be μ-supra semi closed soft sets such that $G_1 \tilde{\cap} G_2 = \tilde{\emptyset}$. Then $G_1 \tilde{\subseteq} G_2^{\tilde{c}}$, then by hypothesis, there exists a μ-supra semi open soft set F_1 such that $G_1 \tilde{\subseteq} F_1$, $Scl_s(F_1) \tilde{\subseteq} G_2^{\tilde{c}}$. So, $G_2 \tilde{\subseteq} [Scl_s(F_1)]^{\tilde{c}}$, $G_1 \tilde{\subseteq} F_1$ and $[Scl_s(F_1)]^{\tilde{c}} \tilde{\cap} F_1 = \tilde{\emptyset}$, where F_1 and $[Scl_s(F_1)]^{\tilde{c}}$ are μ-supra semi open soft sets. Thus, (X,μ,E) is supra soft semi normal space.

Theorem 3.21. Let (X,μ,E) be a supra soft topological space. If (X,μ,E) is supra soft semi normal space and x_E is μ-supra semi closed soft set for each $x \in X$, then (X,μ,E) is supra soft semi T_3-space.

Proof. Since x_E is μ-supra semi closed soft set for each $x \in X$, then (X,μ,E) is supra soft semi T_1-space from Theorem 3.7. Also, (X,μ,E) is supra soft semi regular space from Theorem 3.15 and Definition 3.19. Hence, (X,μ,E) is supra soft semi T_3-space.

Proposition 3.22. Not every supra soft semi open soft subspace of supra soft semi T_i-space is supra soft semi T_i-space for each $i = 0,1,2,3,4$.

Proof. Obvious from the fact that, the soft intersection of two supra semi open soft sets need not to be supra semi open soft.

4 Supra semi irresolute soft functions

Definition 4.1. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. The soft function $f_{pu} : S_E(X) \rightarrow S_K(Y)$ is called

(1) Supra semi open soft if $f_{pu}(F) \in SSOS_E(\mu_1)$ for each $F \in \tau_1$.

(2)Supra semi irresolute soft if $f_{pu}^{-1}(F) \in SSOS_E(\mu_1)$ for each $F \in SSOS_K(\mu_2)$.

(3)Supra semi irresolute open soft if $f_{pu}(F) \in SSOS_K(\mu_2)$ for each $F \in SSOS_E(\mu_1)$.

Theorem 4.2. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively and $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective and supra semi irresolute open soft. If (X,μ_1,E) is supra soft semi T_0-space, then (Y,μ_2,K) is also a supra soft semi T_0-space.

Proof. Let $y_1,y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then there exist $x_1,x_2 \in X$ such that $u(x_1) = y_1$, $u(x_2) = y_2$ and $x_1 \neq x_2$. By hypothesis, there exist μ_1-supra semi open soft sets F and G in X such that either $x_1 \in F$ and $x_2 \notin F$, or $x_2 \in G$ and $x_1 \notin G$. So, either $x_1 \in F_E(e)$ and $x_2 \notin F_E(e)$ or $x_2 \in G_E(e)$ and $x_1 \notin G_E(e)$ for each $e \in E$. This implies that, either $y_1 = u(x_1) \in u[F_E(e)]$ and $y_2 = u(x_2) \notin u[F_E(e)]$ or $y_2 = u(x_2) \in u[G_E(e)]$ and $y_1 = u(x_1) \notin u[G_E(e)]$ for each $e \in E$. Hence, either $y_1 \in f_{pu}(F)$ and $y_2 \notin f_{pu}(F)$ or $y_2 \in f_{pu}(G)$ and $y_1 \notin f_{pu}(G)$. Since f_{pu} is supra semi irresolute open soft function, then $f_{pu}(F), f_{pu}(G)$ are supra semi open soft sets in Y. Hence, (Y,μ_2,K) is also a supra soft semi T_0-space.

Theorem 4.3. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective and supra semi irresolute open soft. If (X,μ_1,E) is supra soft semi T_1-space, then (Y,μ_2,K) is also a supra soft semi T_1-space.

Proof. It is similar to the proof of Theorem 4.2.

Theorem 4.4. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective and supra semi irresolute open soft. If (X,μ_1,E) is supra soft semi T_2-space, then (Y,μ_2,K) is also a supra soft semi T_2-space.

Proof. Let $y_1,y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then there exist $x_1,x_2 \in X$ such that $u(x_1) = y_1$, $u(x_2) = y_2$ and $x_1 \neq x_2$. By hypothesis, there exist μ_1-supra semi open soft sets F and G in X such that $x_1 \in F$, $x_2 \in G$ and $F \tilde{\cap} G = \tilde{\emptyset}_E$. So, $x_1 \in F_E(e)$, $x_2 \in G_E(e)$ and $F_E(e) \tilde{\cap} G_E(e) = \emptyset$ for each $e \in E$. This implies that, $y_1 = u(x_1) \in u[F_E(e)]$, $y_2 = u(x_2) \in u[G_E(e)]$ for each $e \in E$. Hence, $y_1 \in f_{pu}(F)$, $y_2 \in f_{pu}(G)$ and $f_{pu}(F) \tilde{\cap} f_{pu}(G) = f_{pu}[F \tilde{\cap} G] = f_{pu}[\tilde{\emptyset}_E] = \tilde{\emptyset}_K$ from Theorem 2.7. Since f_{pu} is supra semi irresolute open soft function, then $f_{pu}(F), f_{pu}(G)$ are supra semi open soft sets in Y. Thus, (Y,μ_2,K) is also a supra soft semi T_2-space.

Theorem 4.5. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra semi irresolute soft and supra semi irresolute open soft. If (X,μ_1,E) is supra soft semi regular space, then (Y,μ_2,K) is also a supra soft semi regular space.

Proof. Let G be a supra semi closed soft set in Y and $y \in Y$ such that $y \notin G$. Since f_{pu} is surjective and supra semi irresolute soft, then there exists $x \in X$ such that $u(x) = y$ and $f_{pu}^{-1}(G)$ is supra semi closed soft set in X such that $x \notin f_{pu}^{-1}(G)$. By hypothesis, there exist μ_1-supra semi open soft sets F and H in X such that $x \in F$, $f_{pu}^{-1}(G) \tilde{\subseteq} H$ and $F \tilde{\cap} H = \tilde{\emptyset}_E$. It follows that, $x \in F_E(e)$ for each $e \in E$ and $G = f_{pu}[f_{pu}^{-1}(G)] \tilde{\subseteq} f_{pu}(H)$ from Theorem 2.7. So, $y = u(x) \in u[F_E(e)]$ for each $e \in E$ and $G \tilde{\subseteq} f_{pu}(H)$. Hence, $y \in f_{pu}(F)$ and $G \tilde{\subseteq} f_{pu}(H)$ and $f_{pu}(F) \tilde{\cap} f_{pu}(H) = f_{pu}[F \tilde{\cap} H] = f_{pu}[\tilde{\emptyset}_E] = \tilde{\emptyset}_K$ from Theorem 2.7. Since f_{pu} is supra semi irresolute open soft function. Then, $f_{pu}(F), f_{pu}(H)$ are supra semi open soft sets in Y. Thus, (Y,μ_2,K) is also a supra soft regular space.

Theorem 4.6. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra semi irresolute soft and supra semi irresolute open soft. If (X,μ_1,E) is supra soft semi T_3-space, then (Y,μ_2,K) is also a supra soft semi T_3-space.

Proof. Since (X,μ_1,E) is supra soft semi T_3-space, then (X,μ_1,E) is supra soft semi regular semi T_1-space. It follows that, (Y,μ_2,K) is also a supra soft semi T_1-space from Theorem 4.2, and supra soft semi regular space from Theorem 4.5. Hence, (Y,μ_2,K) is also a supra soft semi T_3-space.

Theorem 4.7. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra semi irresolute soft and supra semi irresolute open soft. If (X,μ_1,E) is supra soft semi normal space, then (Y,μ_2,K) is also a supra soft semi normal space.

Proof. Let F,G be supra semi closed soft sets in Y such that $F \tilde{\cap} G = \tilde{\emptyset}_K$. Since f_{pu} is supra semi irresolute soft, then $f_{pu}^{-1}(F)$ and $f_{pu}^{-1}(G)$ are supra semi closed soft set in

X such that $f_{pu}^{-1}(F)\tilde{\cap} f_{pu}^{-1}(G) = f_{pu}^{-1}[F\tilde{\cap}G] = f_{pu}^{-1}[\tilde{\emptyset}_K] = \tilde{\emptyset}_E$ from Theorem 2.7. By hypothesis, there exist supra semi open soft sets M and H in X such that $f_{pu}^{-1}(F)\tilde{\subseteq}M$, $f_{pu}^{-1}(G)\tilde{\subseteq}H$ and $F\tilde{\cap}H = \tilde{\emptyset}_E$. It follows that, $F = f_{pu}[f_{pu}^{-1}(F)]\tilde{\subseteq} f_{pu}(M)$,$G = f_{pu}[f_{pu}^{-1}(G)]\tilde{\subseteq} f_{pu}(H)$ from Theorem 2.7 and $f_{pu}(M)\tilde{\cap} f_{pu}(H) = f_{pu}[M\tilde{\cap}H] = f_{pu}[\tilde{\emptyset}_E] = \tilde{\emptyset}_K$ from Theorem 2.7. Since f_{pu} is supra semi irresolute open soft function. Then, $f_{pu}(M), f_{pu}(H)$ are supra semi open soft sets in Y. Thus, (Y,μ_2,K) is also a supra soft semi normal space.

Corollary 4.8. Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2, respectively. Let $f_{pu} : S_E(X) \to S_K(Y)$ be a soft function which is bijective, supra semi irresolute soft and supra semi irresolute open soft. If (X,μ_1,E) is supra soft semi T_4-space, then (Y,μ_2,K) is also a supra soft semi T_4-space.

Proof. It is obvious from Theorem 4.3 and Theorem 4.7.

5 Conclusion

The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [8]. In this paper, we introduce and investigate some weak soft separation axioms by using the notion of supra semi open soft sets, which is a generalization of the supra soft separation axioms mentioned in [3]. We study the relationships between these new soft separation axioms and their relationships with some other properties. As a consequence the relations of some supra soft separation axioms are shown in a diagram. We show that, some classical results in general supra topology are not true if we consider supra soft topological spaces instead. For instance, if (X,μ,E) is supra soft semi T_1-space need not every soft singleton x_E is supra semi closed soft. Our next work, is to generalize this paper by using the notion of b-open soft sets. We hope that, the results in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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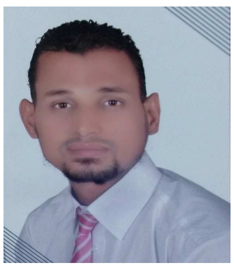

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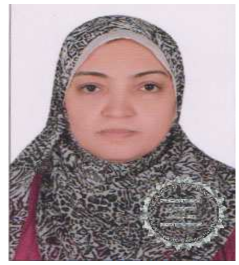

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