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Numerical Study of Heat Exchange in Human Body Through Clothes and Thermal Stability of Biological Tissues

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Abstract: The change in ambient temperature plays a key role in body thermoregulation. The use of protective layer (clothes) has not only ethical values but also acts as a shield to combat with the severe environmental conditions. A mathematical model on bioheat transfer has been formulated with appropriate diffusing and matching conditions at the dermal layers and the protective layer. The finite difference scheme has been employed to solve the model and the conditions for the thermal stability of tissue temperatures were illustrated using MATLAB and FlexPDE software's. The results obtained are applicable to a wide range of problems to maintain body core temperature irrespective of the outside temperature.

Keywords: matching condition; mathematical model; diffusion.
Mathematical Subject Classification (2010)**:** $92BXX; 92CXX; 92C35; 92C50; 46N60$

1 Introduction

The metabolic and cellular processes in human body converts energy provided by the food into work and heat. To maintain the constant balance of temperature within the body and at the surface of the skin, an amount of heat is dissipated through skin to the outside environment. The thermoreceptors and hypothalamus are responsible for the homeostasis of the body through vasodilation, vasoconstriction, shivering, perspiration etc. Since, the body is usually covered with the protective layer by means of clothes, it influences the heat transfer from the skin, acting as an insulating layer or absorbant of moisture or heat exchanger. In human body diffusion has a vital role in many important processes like oxygen transfer, drug diffusion, homeostasis etc. Diffusion through multilayers is again an important process as the human body is made up of different materials with multiple properties. It has many applications in other branches of engineering [9,10,11], geological profiles [7] etc. Henry [1] developed one of the theories of coupled heat and moisture transfer through clothing considering accumulation effects. A steady-state model was studied by Ogniewicz and Tien [2] by incorporating the convective and diffusive transport mechanisms along with phase change due to condensation and evaporation. Heat transfer in biological systems is relevent in many diagnostic and therapeutic applications that involve changes in temperature. For example, in hyperthermia, the temperature in tissue may rise to $42^0C - 43^0C$. Khanday and his co-workers [13,14,15,16,17,18] also studied the diffusion of heat and mass in the biological tissues particularly in dermal regions and human head. Some recent works in this area have been studied by Khanday et al [17] through Variational Finite Element Methods. They studied the fluid distribution pattern in human dermal layers by taking into account similar type of mass diffusion equation. Investigation of thermal properties of skin ([5,6,8,12]) leading to thermal injuries are usually studied through the classical equation of Pennes' bioheat equation [3]. The most recent work using an explicit form of finite difference method for estimating the temperature variation in human dermal regions has been studied by Khanday and Fida [18]. The present paper studies the temperature distribution in multilayered skin with protective layer. A finite difference scheme with

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a suitable boundary and interface conditions is used for predicting the temperature pattern inside the skin tissue layers. Also the role of protective layer as a heat exchanger between the skin and the surroundings has been studied.

The advantage of using Pennes' bioheat equation is that it accounts for the ability of tissue to remove heat by both passive conduction (diffusion) and perfusion of tissue by blood. The bioheat equation used in this study is given by

$$\rho c\frac{\partial U}{\partial t} = div(kgradU) - c_b w(U - T_b) + Q_m + P \quad (1)$$

where ρ, c, k are the density, specific heat and thermal conductivity of tissue respectively, T_b is arterial blood temperature, t is time, c_b is the specific heat of blood, w is the perfusion rate per unit volume of the blood, Q_m is the metabolic heat generation per unit volume and P is a protective layer of clothes at the surface of the skin.
The heat balance at the clothing layer can be described by

$$C_{cl}\frac{dT_{cl}}{dt} = Q \quad (2)$$

where $Q = Q_1 + Q_2 - Q_3 - Q_4 - Q_5 + Q_6$, $C_{cl} = m_{cl}q_{cl}$ is the heat capacity of the garments (m_{cl} is the mass of clothes and q_{cl} is the specific heat capacity of the garments) and

Q_1 = Heat transfer from the skin to the clothes through convection.
Q_2 = Heat transfer from the skin to the clothes through evaporation.
Q_3 = Heat loss through convection with the surrounding environment.
Q_4 = Heat loss through conduction with the surrounding environment.
Q_5 = Heat loss through evaporation with the surrounding environment.
Q_6 = Heat gain due to solar radiation.

2 Method

Due to complex structure of the dermal layers, the heat distribution models in these layers are spatially varied and highly heterogeneous. The analytical solution to the equation-(1) together with equation-(2) is cumbersome. Therefore, the numerical solution based on finite difference approximations has been taken into account at the appropriate situations.
First we re-write equation-(1) as

$$\frac{\partial U}{\partial t} = D\frac{\partial^2 U}{\partial x^2} - \alpha U + \beta \quad (3)$$

where $D = \frac{k}{\rho c}$, $\alpha = \frac{c_b w}{\rho c}$ and $\beta = \frac{Q_m + P + c_b w T_b}{\rho c}$
Now, using $U = V + \frac{\beta}{\alpha}$, equation-(3) reduces to

$$\frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial x^2} - \alpha V \quad (4)$$

Finally making the transformation, $V = Te^{-\alpha t}$, equation-(4) reduces to the following standard form

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} \quad (5)$$

The layerwise heat transfer through different dermal and insulating layers having different physiological properties, based on equation-(5) is given by

$$\frac{\partial T_i}{\partial t} = D\frac{\partial^2 T_i}{\partial x^2}; i = 1,2,3,4 \quad (6)$$

where T_i is the concentration of diffusing substance in layer i at time t, D_i is the diffusivity of layer i.
The humans and other mammals are warm blooded animals, therefore, the core temperature is maintained by the thermoregulatory system upto large extent even in extreme environmental conditions. Moreover, the clothes are used for insulation to combat with the external climate interference to the thermal stability of the tissues. Thus the boundary conditions associated with the model are

$$\left.\begin{aligned} T = 37^0C \text{ for the internal core} \\ \frac{\partial T}{\partial t} = 0 \text{ for outer temperature surrounding the clothes} \end{aligned}\right\} \quad (7)$$

Due to the roughness of the materials in contact at the interfaces, the use of appropriate continuity conditions of the diffusing material are given by equations (8)-(10).

$$T_i(x_i,t) = T_{i+1}(x_i,t) \quad (8)$$
$$k_i\frac{\partial T_i}{\partial x} = k_{i+1}\frac{\partial T_{i+1}}{\partial x} \quad (9)$$

where, $k_i = \rho_i c_i D_i$.
The more general matching condition at the interfaces is given as

$$\left.\begin{aligned} k_i\frac{\partial T_i}{\partial x} = H_i(T_{i+1} - T_i) \\ k_{i+1}\frac{\partial T_{i+1}}{\partial x} = H_i(T_{i+1} - T_i) \end{aligned}\right\} \quad (10)$$

where H_i is the heat transfer coefficient. If H_i is sufficiently large, then the contact between the layers is perfect for the smooth flow of the diffusing material.

3 Numerical Solution

To study the heat distribution in human dermal regions through three layers as shown in Figure-1, we consider the size of three layers as

Subcutaneous Tissue $(L_0 \le x \le L_1)$ of length l_1
Dermis $(L_1 \le x \le L_2)$ of length l_2 and
Epidermis $(L_2 \le x \le L_3)$ of length l_3

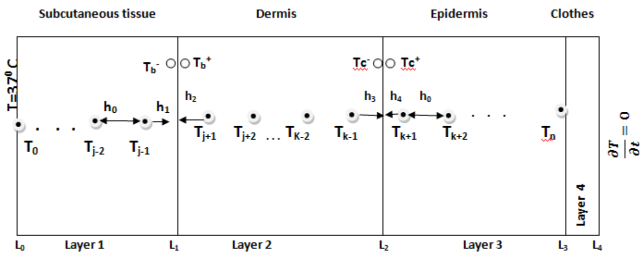

Fig. 1: Domain of study with different layers and the boundary conditions.

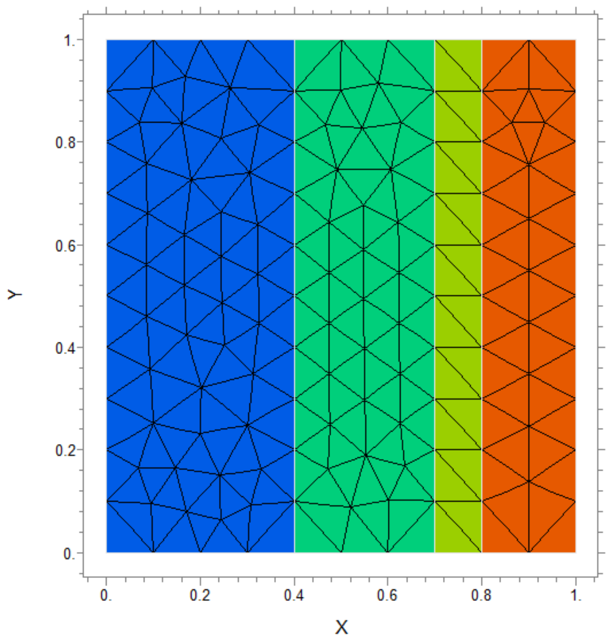

Fig. 2: Mesh of the domain of study for obtaining solution of the problem through FEM.

The two dimensional mesh of the domain generated using FlexPDE sotware [22] is given in Figure-2.
On employing central difference method, Taylor series method and the geometry of the dermal regions described in Figure- 1, the following set of equations corresponding to each layer is given by

$$\left.\begin{aligned}
\frac{\partial^2 T_{j-1}}{\partial x^2} &\simeq \frac{h_0 T_{b^-} - (h_0+h_1)\,T_{j-1} + h_1 T_{j-2}}{h_0 h_1 (h_0+h_1)} \\
\frac{\partial^2 T_{j+1}}{\partial x^2} &\simeq \frac{h_0 T_{b^+} - (h_2+h_0)\,T_{j+1} + h_2 T_{j+2}}{h_0 h_2 (h_0+h_2)} \\
\frac{\partial^2 T_{k-1}}{\partial x^2} &\simeq \frac{h_0 T_{c^-} - (h_3+h_0)\,T_{k-1} + h_3 T_{k-2}}{h_0 h_3 (h_0+h_3)} \\
\frac{\partial^2 T_{k+1}}{\partial x^2} &\simeq \frac{h_0 T_{c^+} - (h_4+h_0)\,T_{k+1} + h_4 T_{k+2}}{h_0 h_4 (h_0+h_4)}
\end{aligned}\right\} \tag{11}$$

We now find the values of different nodal point temperatures shown in Figure-1 with the help of Taylor series as follows:

$$\left.\begin{aligned}
T_{j-1} &\simeq T_{b^-} - h_1 \frac{\partial T_{b^-}}{\partial x} + \frac{h_1^2}{2}\frac{\partial T_{b^-}^2}{\partial x^2} \\
T_{j-2} &\simeq T_{b^-} - (h_1+h_0)\frac{\partial T_{b^-}}{\partial x} + \frac{(h_1+h_0)^2}{2}\frac{\partial T_{b^-}^2}{\partial x^2} \\
T_{j+1} &\simeq T_{b^+} - h_2 \frac{\partial T_{b^+}}{\partial x} + \frac{h_2^2}{2}\frac{\partial T_{b^-}^2}{\partial x^2} \\
T_{j+2} &\simeq T_{b^+} - (h_2+h_0)\frac{\partial T_{b^+}}{\partial x} + \frac{(h_2+h_0)^2}{2}\frac{\partial T_{b^+}^2}{\partial x^2} \\
T_{k-1} &\simeq T_{c^-} - h_3 \frac{\partial T_{c^-}}{\partial x} + \frac{h_3^2}{2}\frac{\partial T_{c^-}^2}{\partial x^2} \\
T_{k-2} &\simeq T_{c^-} - (h_3+h_0)\frac{\partial T_{c^-}}{\partial x} + \frac{(h_3+h_0)^2}{2}\frac{\partial T_{c^-}^2}{\partial x^2} \\
T_{k+1} &\simeq T_{c^+} - h_4 \frac{\partial T_{c^+}}{\partial x} + \frac{h_4^2}{2}\frac{\partial T_{c^+}^2}{\partial x^2} \\
T_{k+2} &\simeq T_{c^+} - (h_4+h_0)\frac{\partial T_{c^+}}{\partial x} + \frac{(h_4+h_0)^2}{2}\frac{\partial T_{c^+}^2}{\partial x^2}
\end{aligned}\right\} \tag{12}$$

On solving the above system of equations along with the matching conditions given in equation-(10), the values of the unknowns T_{b^-}, T_{b^+}, T_{c^-} and T_{c^+} obtained are given by the following equations:

$$T_{b^-} = \frac{1}{\Omega_i}\Big\{ \left[-((h_0+h_2)(h_2H_i+k_{i+1})+h_2k_{i+1})h_1^2k_i\right]T_{j-2} + \left[((h_0+h_2)(h_2H_i+k_{i+1})+h_2k_{i+1})(h_0+h_1)^2k_i\right]T_{j-1} + \left[h_1(h_0+h_1)(h_0+h_2)^2H_ik_{i+1}\right] + \left[-h_1h_2^2(h_0+h_1)H_ik_{i+1}\right]T_{j+2}\Big\} \tag{13}$$

$$\left.\begin{aligned}
T_{b^+} &= \frac{1}{\Omega_i}\Big\{ \left[-h_1^2h_2(h_0+h_2)H_ik_i\right]T_{j-2} + \left[h_2(h_0+h_1)^2(h_0+h_2)H_ik_i\right]T_{j-1} \\
&\quad + \left[((h_0+h_1)(h_1H_i+k_i)+h_1k_i)(h_0+h_2)^2k_{i+1}\right]T_{j+1} \\
&\quad + [-((h_0+h_1)(h_1H_i+k_i)+h_1k_{i+1})T_{j+2}]\Big\} \\
T_{c^-} &= \frac{1}{\Delta_i}\Big\{ \left[-((h_0+h_4)(h_4H_i+k_{i+1})+h_3k_{i+1})h_3^2k_i\right]T_{j-2} \\
&\quad + \left[((h_0+h_4)(h_4H_i+k_{i+1})+h_3k_{i+1})(h_0+h_3)^2k_i\right]T_{j-1} \\
&\quad + \left[h_1(h_0+h_3)(h_0+h_4)^2H_ik_{i+1}\right] \\
&\quad + \left[-h_3h_4^2(h_0+h_3)H_ik_{i+1}\right]T_{j+2}\Big\} \\
T_{c^+} &= \frac{1}{\Delta_i}\Big\{ \left[-h_3^2h_4(h_0+h_4)H_ik_i\right]T_{j-2} + \left[h_4(h_0+h_3)^2(h_0+h_4)H_ik_i\right]T_{j-1} \\
&\quad + \left[((h_0+h_3)(h_3H_i+k_i)+h_3k_i)(h_0+h_4)^2k_{i+1}\right]T_{j+1} \\
&\quad + [-((h_0+h_3)(h_3H_i+k_i)+h_1k_{i+1})T_{j+2}]\Big\}
\end{aligned}\right\} \tag{14}$$

$$\Omega_i = h_o\Big\{[(h_o+2h_3)(h_o+h_4)h_4k_i+(h_o+h_3)(h_o+2h_4)h_3k_{i+1}]H_i + (h_o+2h_3)(h_o+2h_4)k_ik_{i+1}\Big\}$$

$$\Delta_i = h_o\Big\{[(h_o+2h_1)(h_o+h_2)h_2k_i+(h_o+h_1)(h_o+2h_2)h_1k_{i+1}]H_i + (h_o+2h_1)(h_o+2h_2)k_ik_{i+1}\Big\}$$

The mixed boundary condition at L_3, given by equation-(15), is included in the finite difference scheme by adding a fictitious point T_{n+1} at a distance h_0 to the right of boundary at L_3 with the help of equation-(16)

$$\frac{\partial T}{\partial t} = -h\,(T-T_a) \tag{15}$$

where T_a is the ambient temperature.

$$\frac{\partial T_n}{\partial t} = \frac{1}{h_0^2}\left\{2T_{n-1} - 2T_n\left(h_0^2+1\right)\right\} \tag{16}$$

4 Analytical Solution

To solve equation-(6), the model consisting of bio-heat equation given in equation-(1) and boundary, matching and interface conditions given in (7), (8), (10) and (11), we invoke variables separable technique.

Define the solution of equation-(6) in the form $T_i(x,t) = U_i(x) + V_i(x,t)$; $i = 1, 2, 3$, where $U_i(x)$ is the steady state solution and $V_i(x,t)$ is the transient part of solution with initial condition $T_i(x,0) = f_i(x) = 0$.
Therefore, the solution is given by

$$T_i(x,t) = U_i(x) + \sum_{k=1}^{\infty} \alpha_k e^{-\lambda_k^2 t} X_{i,k}(x) \tag{17}$$

The steady state solutions $U_1(x)$, $U_2(x)$ and $U_3(x)$ corresponding to the three layers shown in Figure-1 are given by

$$\left.\begin{aligned}
U_1(x) &= 37 - \frac{37k_3x}{k_1} \\
U_2(x) &= 37 - \frac{37k_2H_1\left[k_1H_1(x-x_1)+N_1k_2\right]}{k_1H_1} \\
U_3(x) &= 37 - \frac{37\left[k_1H_1H_2k_2(x-x_2)+k_2k_3H_2N_1+k_1k_3H_1N_2\right]}{k_2k_3H_1H_2}
\end{aligned}\right\} \tag{18}$$

where, $N_1 = l_1H_1 + k_1$ and $N_2 = l_2H_2 + k_2$.
Also

$$\alpha_k = \frac{\sum\limits_{i=1}^{3} \rho_i c_i \int\limits_{x_{i-1}}^{x_i} g_i(x) X_{i,k}(x)dx}{\sum\limits_{i=1}^{3} \rho_i c_i \int\limits_{x_{i-1}}^{x_i} X_{i,k}^2(x)dx} \tag{19}$$

where $g_i(x) = f_i(x) - w_i(x)$, $i = 1, 2, 3.$ and

$$X_{i,k}(x) = J_{i,k}sin\left(\frac{\lambda_k}{d_i}(x-x_{i-1})\right) + K_{i,k}cos\left(\frac{\lambda_k}{d_i}(x-x_{i-1})\right) \tag{20}$$

where $J_{1,k} = 1$, $K_{1,k} = 0$ and

$$\begin{aligned}
J_{2,k} &= \frac{k_1\sqrt{D_2}}{\sqrt{D_1}k_2}cos\left(\lambda_k\frac{l_1}{\sqrt{D_1}}\right) \\
K_{2,k} &= \frac{k_1\lambda_k}{\sqrt{D_1}H_1}cos\left(\lambda_k\frac{l_1}{\sqrt{D_1}}\right) + sin\left(\lambda_k\frac{l_1}{\sqrt{D_1}}\right) \\
J_{3,k} &= \frac{k_2\sqrt{D_3}}{\sqrt{D_2}k_3}\left[J_{2,k}cos\left(\lambda_k\frac{l_2}{\sqrt{D_2}}\right) - K_{2,m}sin\left(\frac{\lambda_kl_2}{\sqrt{D_2}}\right)\right] \\
K_{3,k} &= J_{2,k}\left[sin\left(\lambda_k\frac{l_2}{\sqrt{D_2}}\right) + \frac{k_2\lambda_k}{\sqrt{D_2}H_2}cs\left(\frac{\lambda_kl_2}{\sqrt{D_2}}\right)\right] \\
&\quad + K_{2,k}\left[cos\left(\lambda_k\frac{l_2}{\sqrt{D_2}}\right) - \frac{k_2\lambda_k}{\sqrt{D_2}H_2}sin\left(\frac{\lambda_kl_2}{\sqrt{D_2}}\right)\right]
\end{aligned}$$

The eigenvalues λ_k, are defined by the following equation

$$J_{3,k}\left[\frac{\lambda_k}{\sqrt{D_3}}cos\left(\lambda_k\frac{l_3}{\sqrt{D_3}}\right)\right] - K_{3,k}\left[\frac{\lambda_k}{\sqrt{D_3}}sin\left(\lambda_k\frac{l_2}{\sqrt{D_3}}\right)\right] = 0 \tag{21}$$

4.1 Numerical Computation

To know the efficiency of the finite difference method used in this study, we substituted the relevant values of the parameters given in Table-1. The step length in the spatial discretization is taken as $\Delta x = 0.01$ at which the

Table 1: Physiological parameters and their numerical values.

Parameter	Unit	Value
Thickness of subcutaneous tissue(l_1)[5]	μm	1800
Thickness of Dermis(l_2)[5]	μm	2000
Thickness of epidermis(l_3)[5]	μm	80
Thermal conductivity of subcutaneous tissue(k_1)[5]	$Wm^{-1}\,{}^0C^{-1}$	0.19
Thermal conductivity of dermis(k_2)[5]	$Wm^{-1}\,{}^0C^{-1}$	0.45
Thermal conductivity of epidermis(k_3)[5]	$Wm^{-1}\,{}^0C^{-1}$	0.23
Heat transfer coefficient(h)[4]	$Cal.s^{-1}m^{-2}\,{}^0C^{-1}$	0.70
Specific heat of subcutaneous tissue (c_1)[5]	$Jkg^{-1}\,{}^0C^{-1}$	2675
Specific heat of dermis (c_2)[5]	$Jkg^{-1}\,{}^0C^{-1}$	3300
Specific heat of epidermis (c_1)[5]	$Jkg^{-1}\,{}^0C^{-1}$	3590
Density of subcutaneous tissue (ρ_1)[5]	$Jkgm^{-3}$	1000
Density of dermis (ρ_2)[5]	$Jkgm^{-3}$	1200
Density of epidermis (ρ_3)[5]	$Jkgm^{-3}$	1200
Diffusivity of subcutaneous tissue(D_1)[23]	m^2min^{-1}	204×10^{-9}
Diffusivity of dermis(D_2)[23]	m^2min^{-1}	203×10^{-9}
Diffusivity of epidermis(D_3)[23]	m^2min^{-1}	2×10^{-9}

running time of the program is 0.4 seconds. Figure-3 gives a 3-dimensional view of the temperature profile through the defined layers which have been plotted using FlexPDE software [22]. Figures-5, 6 and 7 respectively show the behaviour in the temperature profiles at different ambient temperatures across the layers of the skin when the protective layer allows the exchange of heat from the skin surface with the surroundings. Moreover, the behaviour of temperature variation at some located points in the domain at different ambient temperatures are shown by the Figures-8, 9, 10. At $H_1 = H_2 = 0.5$, the graphs in the Figure-12, show jumps at the interfaces as expected. By increasing the value of the transfer coefficients H_i in the matching condition defined by the pair of equations in equation-(11), the resulting temperature profiles and its surface have been illustrated in Figure-11. The graphs in the Figure-11 clearly show negligible jumps at the interfaces which demonstrates that the matching conditions given by equation-(11) are a more general form of the heat flux continuity equation given by equation-(10). At smaller times the matching condition gives more accurate results than using equation-(10).

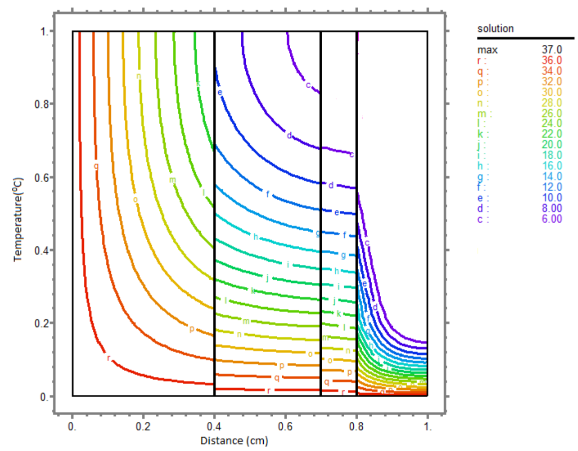

Fig. 3: Temperature profile across the four layers when the heat exchanges through the protective layers when the ambient temperature is 25^0C, $H_1 = 0.6$ **and** $H_2 = 0.8$**.**

5 Discussion and Conclusion

The basic Pennes' bio-heat equation has been suitably reformulated in which the role of protective layer at the skin surface is incorporated. The appropriate interface, matching and boundary conditions have been suitably defined to form a boundary value problem. The model has been transformed into the standard heat transfer model with the help of certain transformations. The solution obtained to the transformed equation-(6) is then substituted back to get the solution of the original model equation-(1). The variation of temperature profiles across the defined layers in presence of different protective layers of clothes at various climatic conditions were calculated at the nodal points of the layered skin with the help of Taylor series and finite difference method.

The finite difference method discussed in this paper has been compared with the the analytical solution of equation-(6) described by equation-(17). The series in equation-(17) is truncated at $k = 40$. Equation-(17) is then plotted using MATLAB software along with the numerical solution computed through the system of

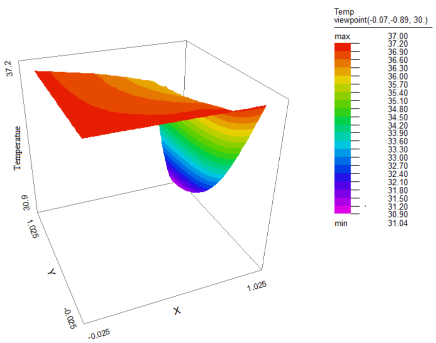

Fig. 4: 3-D view of the temperature profile variation across the layers.

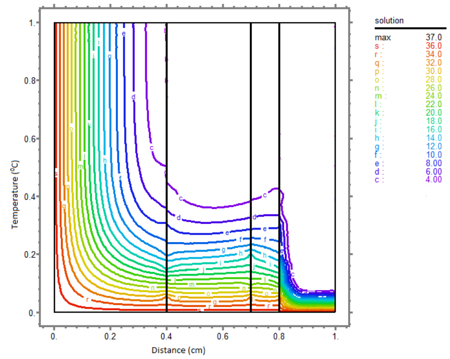

Fig. 5: Variation of temperature with distance through the four layers when the heat exchanges through the protective layers at the ambient temperature of 10^0C and $H_1 = H_2 = 20$.

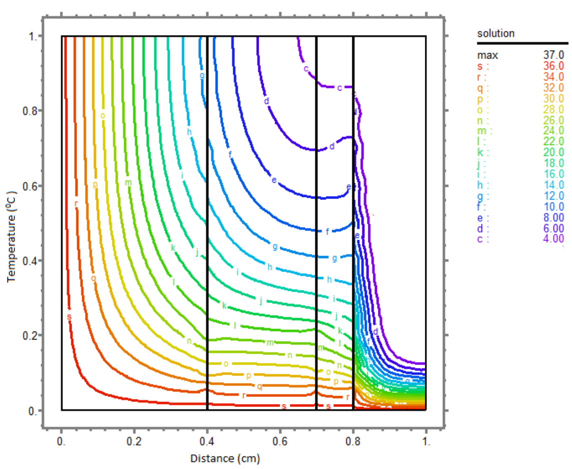

Fig. 6: Variation of temperature with distance through the four layers when the heat exchanges through the protective layers at the ambient temperature of 20^0C and $H_1 = H_2 = 20$.

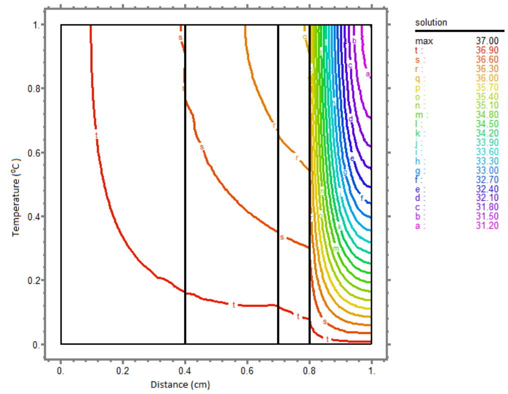

Fig. 7: Variation of temperature with distance through the four layers when the heat exchanges through the protective layers at the ambient temperature of 37^0C and $H_1 = H_2 = 20$.

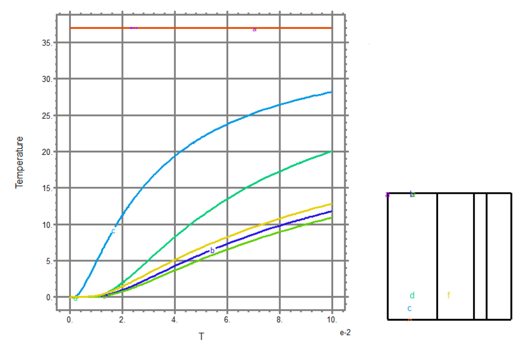

Fig. 8: Variation of temperature at different locations (shown on the right side of the graph) of the four layers when the heat exchanges through the protective layer at the ambient temperature of 10^0C, seconds and $H_1 = H_2 = 20$.

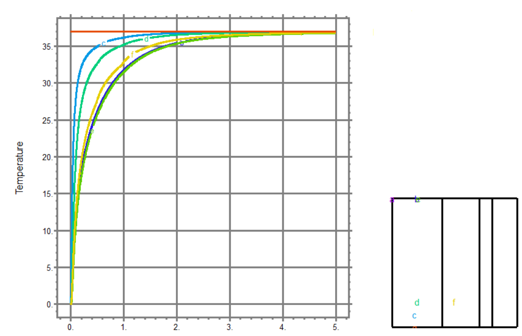

Fig. 9: Variation of temperature at different locations (shown on the right side of the graph) of the four layers when the heat exchanges through the protective layers at the ambient temperature of 20^0C and $H_1 = H_2 = 20$.

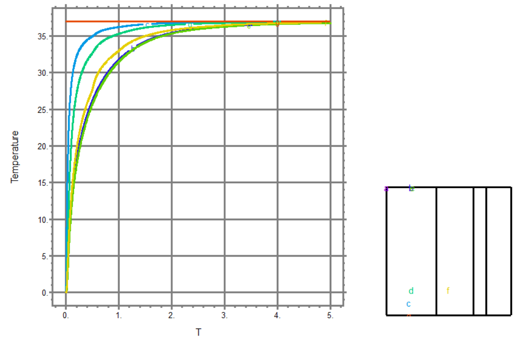

Fig. 10: Variation of temperature at different locations (shown on the right side of the graph) of the four layers when the heat exchanges through the protective layers at the ambient temperature of 25^0C and $H_1 = H_2 = 20$.

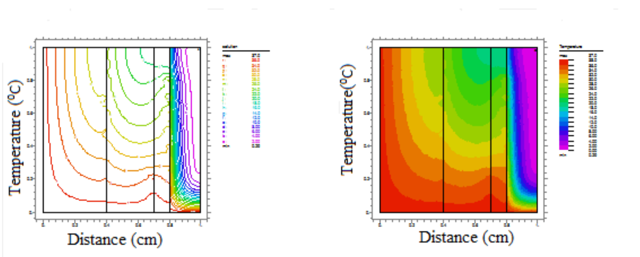

Fig. 11: Temperature variation and its surface across the four layers of the domain of study when $H_1 = H_2 = 50$.

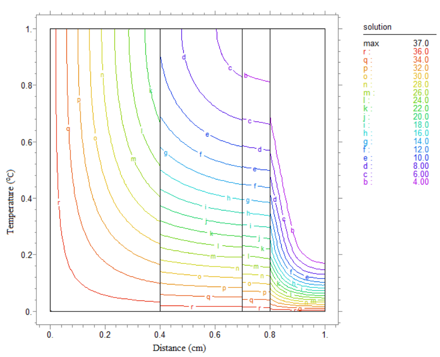

Fig. 12: Temperature variation across the four layers of the domain of study when $H_1 = 0.5$, $H_2 = 0.5$.

equations in (18),(13),(14) and the results were illustrated in Figure-13. The graphs clearly show the efficiency of the finite difference technique invoked in the paper.

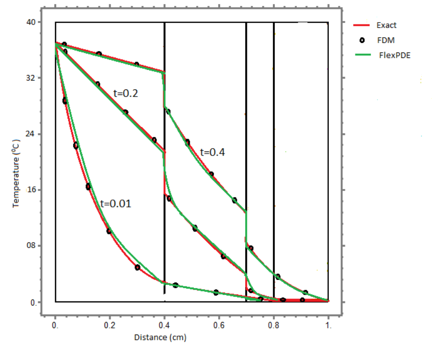

Fig. 13: Comparison of exact, FDM and FlexPDE solutions across the four layers of the domain of study at different times (in seconds) and $H_1 = H_2 = 0.5$.

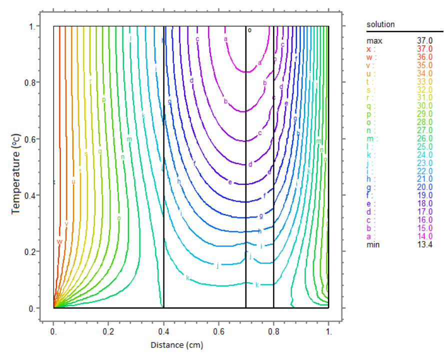

Fig. 14: Temperature variation across the four layers when the temperature outside the protective layer of clothes (Briefs, trousers, suit Jacket) is 15^0C.

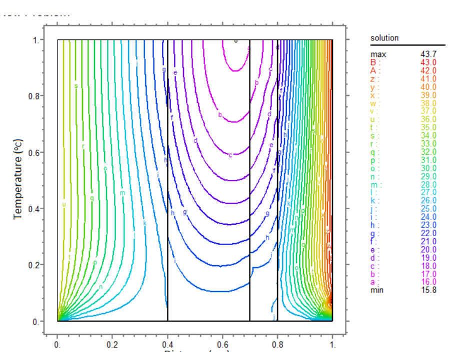

Fig. 15: Temperature variation across the four layers when the temperature outside the protective layer of clothes (Briefs, Shirt, Trouser) is 45^0C.

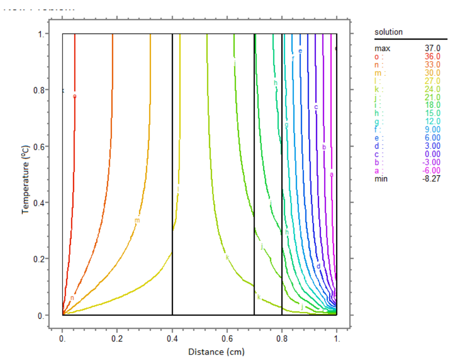

Fig. 16: Temperature variation across the four layers when the temperature outside the protective layer of clothes (Briefs, Two Trousers, Shirt, Jacket) is -10^0C.

5.1 Role of Clothes

The sensible heat transfer from the skin surface to the clothing layer (Q_7) can be obtained using [24]

$$Q_7 = \frac{1}{I_a + I_{cl}} A_{cl}(T_{skin} - T_{cl}) \tag{22}$$

where T_{skin} is the temperature at the skin surface, I_a is the insulation of air, $I_{cl} = \sum_i I_{clu,i}$ is the total effective insulation provided by the number of garments worn and A_{cl} is the area of the body covered by the clothes. The unit of insulation is taken as clo ($1\ clo = 0.155 m^2 K/W$). The average value of mass and insulation of different garments are listed in Table-2. The insulation of the clothing layer and the air in between skin surface and worn clothes may be effected by the air velocity and tissue movement. So, $A_{cl} = A_{skin} . f_{cl}$ with $f_{cl} = 1 + 0.31 I_{cl}$ [24]. The clothes play a vital role in maintaining a suitable temperature at the skin surface. For this purpose, we took different protecting clothes listed in table-1 at different atmospheric temperatures and plotted their graphs as shown in Figures- 14, 15, 16. It is clear from the graphs that the clothing layer maintains a suitable temperature at the skin surface irrespective of the outside temperature.

Finite difference schemes are helpful in exploring various research problems pertaining to the materials diffusing through the different layers having different physiological properties. The finite difference schemes are applicable to a wide range of problems involving multiple layers [5,19,20,21] where numerical integration techniques are often used. The technique of using the finite difference scheme, outlined in this paper, is very easy to implement for obtaining good approximating results to certain degree of accepted errors. The analytical and numerical results obtained in this study have a capability to be applicable to many practical bioheat transfer problems than a few existing analytical solutions.

Table 2: Mass and insulation coefficients of some garments [24]

Garment	Mass(Kg)	$I_{clu}(clo)$
Briefs	0.065	0.04
Shirt	0.196	0.28
Trousers	0.459	0.24
Socks	0.049	0.03
Suit Jacket	0.652	0.04

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