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Bayes Estimation of the Logistic Distribution Parameters Based on Progressive Sampling

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Abstract: In this paper we develop approximate Bayes estimators of the two parameters logistic distribution. Lindley's approximation and importance sampling techniques are applied. The Gaussian-gamma prior distribution and progressively type-II censored samples are assumed. Quadratic, linex and general entropy loss functions are used. The statistical performances of the Bayes estimates under quadratic, linex and general entropy loss functions are compared with those of the maximum likelihood estimators based on simulation study.

Keywords: Logistic distribution, progressively type-II censoring, Gaussian-gamma prior distribution, loss functions, Lindley's approximation, importance sampling technique.

1 Introduction

The logistic function is one of the most popular and widely used for growth models in demographic studies. The logistic distribution has been applied in studies of population growth, physicochemical phenomena, bio-assay and a life test data [2], and of biochemical data [6]. [8] used the logistic function as a model for agricultural production data. [11] compared between the logistic distribution and weibull distribution for modeling wind speed data. [12] proposed askew logistic distribution then they derived some properties for this distribution. Many researchers have used asymmetric loss function applied to several statistical models ([4] and [13]). The normal-gamma distribution (Gaussian-gamma distribution) is a bivariate four-parameter family of continuous probability distributions. It is the conjugate prior of a normal distribution with unknown mean and precision [5]. The Gaussian-gamma distribution has been applied in inventory control problems, the choice of a distribution to describe the demand during the lead time (time between placement and delivery of an order) is an important problem which has generated considerable research activity. This lead time demand may be considered a mixture of two (or even three) components, namely,

$$\left.\begin{array}{l}\text{flow of orders per unit time}\\ \text{size of orders}\\ \text{length of lead time}\end{array}\right\}\text{demand per unit time}$$

[9]. The normal-gamma distribution is a generalization of normal distribution, also applied for fitting real data [1], and for the measurement of efficiency in life insurance [14]. Censoring is a common phenomenon in life-testing and reliability studies. The experimenter may be unable to obtain complete information on failure times for all experimental units. For example, individuals in a clinical trial may withdraw from the study, or the study may have to be terminated for lack of funds. In an industrial experiment, units may break accidentally. In many situations, however, the removal of units prior to failure is preplanned in order to provide savings in terms of time and cost associated with testing. Progressive Type-II censoring scheme can be described as follows: Suppose n units are placed on a life test and the experimenter decides before hand the quantity m, the number of failures to be observed. Now at the time of the first failure, R_1 of the remaining $n-1$ surviving units are randomly removed from the experiment. At the time of the second failure, R_2 of the remaining $n-R_1-2$

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units are randomly removed from the experiment. Finally, at the time of the $m-^{th}$ failure, all the remaining surviving units $R_m = n - m - R_1 - \ldots - R_{m-1}$ are removed from the experiment. Progressive Type-II censoring scheme consists of m, and $R_1, \ldots, R_m$, such that $R_1 + \ldots + R_m = n - m$. The m failure times obtained from a progressive Type-II censoring scheme will be denoted by $x_1, \ldots, x_m$.

In this paper, we propose different methods to estimate the parameters of logistic distribution with Gaussian-gamma prior distribution based on progressive type-II censoring scheme. The paper consists of five sections: In section 1, we present some basic concepts which will be used through out this paper. Also it shows the historical survey on some studies in theoretical and application which have been made on progressive censoring. Finally, it contains a description of under-study problem. In section 2, we use the Maximum Likelihood Estimators (MLEs) of the unknown parameters based on progressively type-II censoring samples. In section 3, we provide a Bayesian method to estimate these parameters. Also the reliability function and hazard rate function, using progressive type-II censoring samples is discussed. Based on the square error loss function, linear-exponential loss function, and general entropy loss function. In the Bayesian method we propose two approaches to approximate the posterior: Lindley's approximation and importance sampling technique. In section 4, to demonstrate the importance of the results obtained in the preceding sections, simulation studies are conducted. Using Monte Carlo method, with fixed sample size n (the total items put in a life test), with constant censoring scheme. In section 5, concluding remarks on simulation study.

2 Maximum Likelihood Estimators (MLEs)

In this section, we derive the MLEs of the unknown parameters based on progressively type-II censoring samples. Assume the failure time distribution to be the logistic distribution with probability density function (pdf)

$$f(x;\mu,\beta) = \frac{e^{-\frac{(x-\mu)}{\beta}}}{\beta\left(1+e^{-\frac{(x-\mu)}{\beta}}\right)^2}; -\infty < x < \infty, \quad -\infty < \mu < \infty, \beta > 0, \tag{2.1}$$

and the corresponding cumulative distribution function (cdf) is given by

$$F(x;\mu,\beta) = \frac{1}{1+e^{-\frac{(x-\mu)}{\beta}}}. \tag{2.2}$$

Based on the observed sample $x_1 < \ldots < x_m$ from a progressive type-II censoring scheme, $(R_1, \ldots, R_m)$, the likelihood function can be written as

$$\mathrm{L}(\underline{x};\mu,\beta) = \mathrm{c}\prod_{i=1}^{m} \mathrm{f}(\mathrm{x_i})\left[1-\mathrm{F}(\mathrm{x_i})\right]^{\mathrm{R_i}}, \tag{2.3}$$

where $c = n(n-1-R_1)\ldots(n-R_1-\ldots-R_{m-1}-m+1)$, $f(.)$ and $F(.)$ are given by (2.1) and (2.2) respectively. Then

$$\mathrm{L}(\underline{x};\mu,\beta) = \frac{\mathrm{c}}{\beta^m} e^{-\sum_{i=1}^{m}\frac{(x_i-\mu)(\mathrm{R_i}+1)}{\beta}} \prod_{i=1}^{m}\left(1+e^{-\frac{(x_i-\mu)}{\beta}}\right)^{-(\mathrm{R_i}+2)}.$$

The log-likelihood function can be written as

$$\log[L] = \ell = \log[c] - m\log[\beta] - \frac{1}{\beta}\sum_{i=1}^{m}(R_i+1)(x_i-\mu) + \log\left[\prod_{i=1}^{m}\left(1+e^{-\frac{(x_i-\mu)}{\beta}}\right)^{-(\mathrm{R_i}+2)}\right]$$

$$\ell = \log[c] - m\log[\beta] - \frac{1}{\beta}\sum_{i=1}^{m}(R_i+1)(x_i-\mu) - \sum_{i=1}^{m}(R_i+2)\log\left[1+\mathrm{e}^{\frac{-(x_i-\mu)}{\beta}}\right]. \tag{2.4}$$

The MLEs of the unknown parameters can be obtained by differentiating the log-likelihood function (2.4) with respect to the unknown parameters and equating to zero, we get

$$\left.\begin{aligned} &\frac{\sum_{i=1}^{m}(R_i+1)}{\hat{\beta}} - \sum_{i=1}^{m}\frac{\mathrm{e}^{-\frac{(x_i-\hat{\mu})}{\hat{\beta}}}(R_i+2)}{\left(1+\mathrm{e}^{-\frac{(x_i-\hat{\mu})}{\hat{\beta}}}\right)\hat{\beta}} = 0,\\ &-\frac{m}{\hat{\beta}} + \frac{\sum_{i=1}^{m}(R_i+1)(x_i-\hat{\mu})}{\hat{\beta}^2} - \sum_{i=1}^{m}\frac{\mathrm{e}^{-\frac{(x_i-\hat{\mu})}{\hat{\beta}}}(R_i+2)(x_i-\hat{\mu})}{\left(1+\mathrm{e}^{-\frac{(x_i-\hat{\mu})}{\hat{\beta}}}\right)\hat{\beta}^2} = 0. \end{aligned}\right\} \tag{2.5}$$

The solution of the non-linear equations (2.5) is $\hat{\mu}$, $\hat{\beta}$. The MLEs of the reliability function, and the hazard rate function are given as

$$\hat{R}(t) = \frac{1}{1+e^{\frac{t-\hat{\mu}}{\hat{\beta}}}}, \quad \hat{H}(t) = \frac{1}{\hat{\beta}\left(1+e^{-\frac{(t-\hat{\mu})}{\hat{\beta}}}\right)}.$$

3 Bayes Estimates for the Unknown Parameters μ and β

In this section Bayesian estimation of the parameters of the logistic distribution is obtained. Also the reliability function and hazard rate function, using progressive type-II censoring samples is discussed. Quadratic, linex, and general entropy loss functions are used.

Assuming that the joint informative prior distribution for μ and β is a Gaussian-gamma distribution, given by

$$\varphi(\mu,\beta)=\frac{\gamma^{\alpha}\sqrt{\lambda}}{\Gamma(\alpha)\sqrt{2\pi}}\beta^{\alpha-\frac{1}{2}}e^{-\gamma\beta}e^{-\frac{\lambda\beta(\mu-\delta)^2}{2}};\mu\in(-\infty,\infty),\ \beta\in(0,\infty),$$
$$-\infty<\delta<\infty,\lambda>0,\alpha>0,\gamma>0. \tag{3.1}$$

By using equations (2.3) and (3.1) we get the joint posterior distribution for μ and β as follows

$$\varphi(\mu,\beta|\underline{x})=\frac{\varphi(\mu,\beta)\,\mathrm{L}(\underline{x}|\mu,\beta)}{\int_0^\infty\int_{-\infty}^\infty \varphi(\mu,\beta)\,\mathrm{L}(\underline{x}|\mu,\beta)\,d\mu d\beta}$$

$$=\left(\beta^{\alpha-m-\frac{1}{2}}e^{-\gamma\beta}e^{-\frac{\lambda\beta(\mu-\delta)^2}{2}}e^{-\sum_{i=1}^{m}\frac{(x_i-\mu)(\mathrm{R_i}+1)}{\beta}}\times\prod_{i=1}^{m}\left(1+e^{-\frac{(x_i-\mu)}{\beta}}\right)^{-(\mathrm{R_i}+2)}\right)\times\left(\int_{-\infty}^{\infty}\int_0^\infty\left(\beta^{\alpha-m-\frac{1}{2}}e^{-\gamma\beta}e^{-\frac{\lambda\beta(\mu-\delta)^2}{2}}e^{-\sum_{i=1}^{m}\frac{(x_i-\mu)(\mathrm{R_i}+1)}{\beta}}\times\prod_{i=1}^{m}\left(1+e^{-\frac{(x_i-\mu)}{\beta}}\right)^{-(\mathrm{R_i}+2)}\right)d\beta d\mu\right)^{-1}. \tag{3.2}$$

Integration in equation (3.2) cannot be obtained in a closed form, so we solve it numerically. In the following subsections we derive Bayesian estimators for location and scale parameters, the reliability function, and the hazard rate function under some loss functions.

3.1 Bayesian Estimators Under Square Error Loss Function

1. Bayesian estimator for location parameter μ

$$\hat{\mu}_{\mathrm{sq}}=\mathrm{E}(\mu)=\int_0^\infty\int_{-\infty}^{\infty}\left(\mu\times\left(\beta^{\alpha-m-\frac{1}{2}}e^{-\gamma\beta}e^{-\frac{\lambda\beta(\mu-\delta)^2}{2}}e^{-\sum_{i=1}^{m}\frac{(x_i-\mu)(\mathrm{R_i}+1)}{\beta}}\times\prod_{i=1}^{m}\left(1+e^{-\frac{(x_i-\mu)}{\beta}}\right)^{-(\mathrm{R_i}+2)}\right)\times\left(\int_0^\infty\int_{-\infty}^{\infty}\left(\beta^{\alpha-m-\frac{1}{2}}e^{-\gamma\beta}e^{-\frac{\lambda\beta(\mu-\delta)^2}{2}}\times e^{-\sum_{i=1}^{m}\frac{(x_i-\mu)(\mathrm{R_i}+1)}{\beta}}\times\prod_{i=1}^{m}\left(1+e^{-\frac{(x_i-\mu)}{\beta}}\right)^{-(\mathrm{R_i}+2)}\right)d\mu d\beta\right)^{-1}\right)d\mu d\beta. \tag{3.3}$$

Provided that $E(\mu)$ exists and is finite. This integration cannot be solved analytically, so we use Lindley's Bayes approximation [7]. Let $u(\mu,\beta)$ be a function of μ and β, and we want to find Bayes estimator for it, based on $\varphi(\mu,\beta)$ as a prior distribution. The log-likelihood function for the logistic distribution based on progressive type II censored samples is given by (2.4), Bayes estimate of $u(\mu,\beta)$ using Lindley approximation is obtained as follows:

$$E(u(\mu,\beta)|\underline{x})=\frac{\int_0^\infty\int_{-\infty}^\infty u(\mu,\beta)\varphi(\mu,\beta)\mathrm{L}(\underline{x}|\mu,\beta)d\mu d\beta}{\int_0^\infty\int_{-\infty}^\infty \varphi(\mu,\beta)\mathrm{L}(\underline{x}|\mu,\beta)d\mu d\beta}.$$

Let $Q(\mu,\beta)=\log[\varphi(\mu,\beta)]$

$$E(u(\mu,\beta)|)\approx\left(u(\mu,\beta)+\frac{1}{2}\left[\sum_i\sum_j(u_{ij}+2u_iQ_j)\tau_{ij}+\sum_i\sum_j\sum_k\sum_w L_{ijk}u_w\tau_{ij}\tau_{kw}\right]\right)_{(\mu,\beta)_{ML}}, \tag{3.4}$$

$$\forall i,j,k,w=1,2,Q_1=\frac{\partial Q(\mu,\beta)}{\partial\mu},Q_2=\frac{\partial Q(\mu,\beta)}{\partial\beta},u_1=\frac{\partial u(\mu,\beta)}{\partial\mu},$$

$$u_2=\frac{\partial u(\mu,\beta)}{\partial\beta},u_{11}=\frac{\partial^2u(\mu,\beta)}{\partial\mu^2},u_{22}=\frac{\partial^2u(\mu,\beta)}{\partial\beta^2},u_{12}=\frac{\partial^2u(\mu,\beta)}{\partial\mu\partial\beta},$$

$$L_{11}=\frac{\partial^2\ell}{\partial\mu^2},L_{12}=\frac{\partial^2\ell}{\partial\mu\partial\beta},L_{22}=\frac{\partial^2\ell}{\partial\beta^2},L_{111}=\frac{\partial^3\ell}{\partial\mu^3},L_{112}=\frac{\partial^3\ell}{\partial\mu^2\partial\beta},$$
$$L_{122}=\frac{\partial^3\ell}{\partial\mu\partial\beta^2},L_{222}=\frac{\partial^3\ell}{\partial\beta^3}.$$

Calculate the elements of matrix $\{-\mathrm{L_{ij}}\}$

$\Sigma=\begin{bmatrix}-\frac{\partial^2\ell}{\partial\mu^2} & -\frac{\partial^2\ell}{\partial\mu\partial\beta}\\ -\frac{\partial^2\ell}{\partial\mu\partial\beta} & -\frac{\partial^2\ell}{\partial\beta^2}\end{bmatrix}^{-1}=\begin{bmatrix}\tau_{11} & \tau_{12}\\ \tau_{21} & \tau_{22}\end{bmatrix}$, by using Mathematica program we can calculate the inverse matrix, and find the values of τ_{ij}. Substitution in equation (3.4), $Q_1=-\lambda\beta(\mu-\delta),Q_2=\frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}$, $u=\mu$, the Bayesian estimator for location parameter μ is given as

$$\hat{\mu}_{sq}\simeq\mu+Q_1\tau_{11}+Q_2\tau_{12}+\frac{1}{2}\begin{bmatrix}L_{111}\tau^2{}_{11}+3L_{112}\tau_{12}\tau_{11}+\\ L_{122}\left(\tau_{22}\tau_{11}+2\tau^2{}_{12}\right)+\\ L_{222}\tau_{22}\tau_{12}\end{bmatrix}$$

2. Bayesian estimator for scale parameter β

Substitution in equation (3.4), $Q_1=-\lambda\beta(\mu-\delta),Q_2=\frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta},u=\beta$, the Bayesian estimator for scale parameter β is given as

$$\hat{\beta}_{sq}\simeq\beta+Q_1\tau_{21}+Q_2\tau_{22}+\frac{1}{2}\begin{bmatrix}L_{111}\tau_{11}\tau_{12}+\\ L_{112}\left(\tau_{11}\tau_{22}+2\tau^2{}_{12}\right)+\\ 3L_{122}\tau_{22}\tau_{12}+L_{222}\tau^2{}_{22}\end{bmatrix}$$

3. Bayesian estimator for reliability function $R(t)$

Substitution in equation (3.4), $Q_1 = -\lambda\beta(\mu-\delta), Q_2 = \frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}, u = R(t)$,the Bayesian estimator for reliability function $R(t)$ is given by

$$\hat{R}_{sq} \simeq R(t)+Q_1(u_1\tau_{11}+u_2\tau_{21})+Q_2(u_1\tau_{12}+u_2\tau_{22})+ \frac{1}{2}\begin{bmatrix} u_{11}\tau_{11}+2u_{21}\tau_{12}\\ +u_{22}\tau_{22}\end{bmatrix}+ \frac{1}{2}\begin{bmatrix} L_{111}(u_1\tau^2{}_{11}+u_2\tau_{11}\tau_{12})+\\ L_{112}(u_2(\tau_{11}\tau_{22}+2\tau^2{}_{12})+3u_1\tau_{21}\tau_{11})\\ +L_{122}(u_1(\tau_{22}\tau_{11}+2\tau^2{}_{21})+3u_2\tau_{12}\tau_{22})+\\ L_{222}(u_1\tau_{22}\tau_{21}+u_2\tau^2{}_{22})\end{bmatrix}$$

4. Bayesian estimator for hazard rate function $H(t)$

Substitution in equation (3.4), $Q_1 = -\lambda\beta(\mu-\delta), Q_2 = \frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}, u = H(t)$, the Bayesian estimator for hazard rate function $H(t)$ is given by

$$\hat{H}_{sq} \simeq H(t)+Q_1(u_1\tau_{11}+u_2\tau_{21})+Q_2(u_1\tau_{12}+u_2\tau_{22})+ \frac{1}{2}\begin{bmatrix} u_{11}\tau_{11}+2u_{21}\tau_{12}\\ +u_{22}\tau_{22}\end{bmatrix}+ \frac{1}{2}\begin{bmatrix} L_{111}(u_1\tau^2{}_{11}+u_2\tau_{11}\tau_{12})+L_{112}(u_2(\tau_{11}\tau_{22}+2\tau^2{}_{12})\\ +3u_1\tau_{21}\tau_{11})+L_{122}(u_1(\tau_{22}\tau_{11}+2\tau^2{}_{21})+\\ 3u_2\tau_{12}\tau_{22})+L_{222}\\ (u_1\tau_{22}\tau_{21}+u_2\tau^2{}_{22})\end{bmatrix}$$

3.2 Bayesian Estimators Under Linear-Exponential Loss Function (LINEX)

1. Bayesian estimator for location parameter μ

$\hat{\mu}_{LINEX} = -\frac{1}{c}\log\left[E\left(e^{-c\mu}\right)\right]$

Provided that $E\left(e^{-c\mu}\right)$ exists and is finite. Substitution in equation (3.4), $Q_1 = -\lambda\beta(\mu-\delta), Q_2 = \frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}, u = e^{-c\mu}$, the Bayesian estimator for location parameter μ is given as

$$\hat{\mu}_{LINEX} \simeq -\frac{1}{c}\log\begin{bmatrix} e^{-c\mu}-cQ_1e^{-c\mu}\tau_{11}-cQ_2e^{-c\mu}\tau_{12}+\\ \frac{c^2e^{-c\mu}\tau_{11}}{2}-\\ \frac{1}{2}\begin{bmatrix} ce^{-c\mu}L_{111}\tau^2{}_{11}+\\ 3ce^{-c\mu}L_{112}\tau_{12}\tau_{11}+\\ ce^{-c\mu}L_{122}(\tau_{22}\tau_{11}+2\tau^2{}_{12})+\\ ce^{-c\mu}L_{222}\tau_{22}\tau_{12}\end{bmatrix}\end{bmatrix}$$

2. Bayesian estimator for scale parameter β

Substitution in equation (3.4), $Q_1 = -\lambda\beta(\mu-\delta), Q_2 = \frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}, u = e^{-c\beta}$, the Bayesian estimator for scale parameter β is given as

$$\hat{\beta}_{LINEX} \simeq -\frac{1}{c}\log\begin{bmatrix} e^{-c\beta}-cQ_1e^{-c\beta}\tau_{21}-cQ_2e^{-c\beta}\tau_{22}+\\ \frac{c^2e^{-c\beta}\tau_{22}}{2}-\frac{1}{2}\begin{bmatrix} ce^{-c\beta}L_{111}\tau_{11}\tau_{12}+\\ ce^{-c\beta}L_{112}\\ \times\begin{pmatrix}\tau_{22}\tau_{11}+\\ 2\tau^2{}_{12}\end{pmatrix}+\\ 3ce^{-c\beta}L_{122}\tau_{12}\tau_{22}+\\ ce^{-c\beta}L_{222}\tau^2{}_{22}\end{bmatrix}\end{bmatrix}$$

3. Bayesian estimator for reliability function $R(t)$

Substitution in equation (3.4), $Q_1 = -\lambda\beta(\mu-\delta), Q_2 = \frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}, u = e^{-cR(t)}$, the Bayesian estimator for reliability function $R(t)$ is given by

$$\hat{R}_{LINEX} \simeq -\frac{1}{c}\log\begin{bmatrix} e^{-cR(t)}+Q_1(u_1\tau_{11}+u_2\tau_{21})+\\ Q_2\begin{pmatrix}u_1\tau_{12}+\\ u_2\tau_{22}\end{pmatrix}+\\ \frac{1}{2}\begin{bmatrix} u_{11}\tau_{11}+2u_{21}\tau_{12}\\ +u_{22}\tau_{22}\end{bmatrix}+\\ \frac{1}{2}\begin{bmatrix} L_{111}(u_1\tau^2{}_{11}+u_2\tau_{11}\tau_{12})+\\ L_{112}(u_2(\tau_{11}\tau_{22}+2\tau^2{}_{12})+\\ 3u_1\tau_{21}\tau_{11})+\\ L_{122}(u_1(\tau_{22}\tau_{11}+2\tau^2{}_{21})+\\ 3u_2\tau_{12}\tau_{22})+L_{222}\\ (u_1\tau_{22}\tau_{21}+u_2\tau^2{}_{22})\end{bmatrix}\end{bmatrix}$$

4. Bayesian estimator for hazard rate function $H(t)$

substitution in equation (3.4), $Q_1 = -\lambda\beta(\mu-\delta), Q_2 = \frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}, u = e^{-cH(t)}$, the Bayesian estimator for hazard rate function $H(t)$ is given by

$$\hat{H}_{LINEX} \simeq -\frac{1}{c}\log\begin{bmatrix} e^{-cH(t)}+Q_1(u_1\tau_{11}+u_2\tau_{21})+\\ Q_2\begin{pmatrix}u_1\tau_{12}+\\ u_2\tau_{22}\end{pmatrix}+\\ \frac{1}{2}\begin{bmatrix} u_{11}\tau_{11}+2u_{21}\tau_{12}\\ +u_{22}\tau_{22}\end{bmatrix}+\\ \frac{1}{2}\begin{bmatrix} L_{111}(u_1\tau^2{}_{11}+u_2\tau_{11}\tau_{12})+\\ L_{112}(u_2(\tau_{11}\tau_{22}+2\tau^2{}_{12})\\ +3u_1\tau_{21}\tau_{11})+L_{122}(u_1(\tau_{22}\tau_{11}+\\ 2\tau^2{}_{21})+\\ 3u_2\tau_{12}\tau_{22})+L_{222}\\ (u_1\tau_{22}\tau_{21}+u_2\tau^2{}_{22})\end{bmatrix}\end{bmatrix}$$

3.3 Bayesian Estimators Under General Entropy Loss Function

1. Bayesian estimator for location parameter μ

$\hat{\mu}_{Gentropy} = \left[E\left(\mu^{-q}\right)\right]^{-\frac{1}{q}}$

Provided that $E\left(\mu^{-q}\right)$ exists and is finite. Substitution in equation (3.4), $Q_1 = -\lambda\beta(\mu-\delta)$,

$Q_2 = \frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}, u = \mu^{-q}$, the Bayesian estimator for location parameter μ is given as

$$\hat{\mu}_{Gentropy} \simeq \begin{bmatrix} \mu^{-q} - qQ_1\mu^{-q-1}\tau_{11} - qQ_2\mu^{-q-1}\tau_{12} + \\ \frac{q(q+1)\mu^{-q-2}\tau_{11}}{2} - \\ \frac{1}{2}\begin{bmatrix} q\mu^{-q-1}L_{111}\tau^2{}_{11} + \\ 3q\mu^{-q-1}L_{112}\tau_{12}\tau_{11} + \\ L_{122}\left(q\mu^{-q-1}\left(\tau_{22}\tau_{11} + 2\tau^2{}_{12}\right)\right) + \\ q\mu^{-q-1}L_{222}\tau_{22}\tau_{12} \end{bmatrix} \end{bmatrix}^{-\frac{1}{q}}$$

2. Bayesian estimator for scale parameter β

substitution in equation (3.4), $Q_1 = -\lambda\beta(\mu-\delta), Q_2 = \frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}, u = \beta^{-q}$, the Bayesian estimator for scale parameter β is given as

$$\hat{\beta}_{Gentropy} \simeq \begin{bmatrix} \beta^{-q} - qQ_1\beta^{-q-1}\tau_{21} - qQ_2\beta^{-q-1}\tau_{22} + \\ \frac{q(q+1)\beta^{-q-2}\tau_{22}}{2} - \\ \frac{1}{2}\begin{bmatrix} q\beta^{-q-1}L_{111}\tau_{11}\tau_{12} + \\ q\beta^{-q-1}L_{112}\left(\tau_{22}\tau_{11} + 2\tau^2{}_{12}\right) + \\ 3q\beta^{-q-1}L_{122}\tau_{22}\tau_{12} + \\ q\beta^{-q-1}L_{222}\tau^2{}_{22} \end{bmatrix} \end{bmatrix}^{-\frac{1}{q}}$$

3. Bayesian estimator for reliability function $R(t)$

Substitution in equation (3.4), $Q_1 = -\lambda\beta(\mu-\delta), Q_2 = \frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}, u = (R(t))^{-q}$, the Bayesian estimator for reliability function $R(t)$ is given by

$$\hat{R}_{Gentropy} \simeq \begin{bmatrix} (R(t))^{-q} + Q_1(u_1\tau_{11} + u_2\tau_{21}) + \\ Q_2\begin{pmatrix} u_1\tau_{12} + \\ u_2\tau_{22} \end{pmatrix} + \\ \frac{1}{2}\begin{bmatrix} u_{11}\tau_{11} + 2u_{21}\tau_{12} \\ +u_{22}\tau_{22} \end{bmatrix} + \\ \frac{1}{2}\begin{bmatrix} L_{111}(u_1\tau^2{}_{11} + u_2\tau_{11}\tau_{12}) + \\ L_{112}(u_2(\tau_{11}\tau_{22} + 2\tau^2{}_{12}) \\ +3u_1\tau_{21}\tau_{11}) + L_{122}(u_1(\tau_{22}\tau_{11} + \\ 2\tau^2{}_{21}) + \\ 3u_2\tau_{12}\tau_{22}) + L_{222} \\ (u_1\tau_{22}\tau_{21} + u_2\tau^2{}_{22}) \end{bmatrix} \end{bmatrix}^{-\frac{1}{q}}$$

4. Bayesian estimator for hazard rate function $H(t)$

Substitution in equation (3.4), $Q_1 = -\lambda\beta(\mu-\delta), Q_2 = \frac{2\alpha-1-2\beta\gamma-\lambda\beta(\mu-\delta)^2}{2\beta}, u = (H(t))^{-q}$, the Bayesian estimator for hazard rate function $H(t)$ is given by

$$\hat{H}_{Gentropy} \simeq \begin{bmatrix} (H(t))^{-q} + Q_1(u_1\tau_{11} + u_2\tau_{21}) + \\ Q_2\begin{pmatrix} u_1\tau_{12} + \\ u_2\tau_{22} \end{pmatrix} + \\ \frac{1}{2}\begin{bmatrix} u_{11}\tau_{11} + 2u_{21}\tau_{12} \\ +u_{22}\tau_{22} \end{bmatrix} + \\ \frac{1}{2}\begin{bmatrix} L_{111}(u_1\tau^2{}_{11} + u_2\tau_{11}\tau_{12}) + \\ L_{112}(u_2(\tau_{11}\tau_{22} + 2\tau^2{}_{12}) \\ +3u_1\tau_{21}\tau_{11}) + L_{122}(u_1(\tau_{22}\tau_{11} + \\ 2\tau^2{}_{21}) + \\ 3u_2\tau_{12}\tau_{22}) + L_{222} \\ (u_1\tau_{22}\tau_{21} + u_2\tau^2{}_{22}) \end{bmatrix} \end{bmatrix}^{-\frac{1}{q}}.$$

It is worth noting that when the value $q = -1$, the general entropy loss function is the same as the squared error loss function.

3.4 Importance Sampling Technique

Importance sampling is the general technique of sampling from one distribution to estimate an expectation under a different distribution. In Bayesian analyses, given a likelihood $L(\theta)$ for a parameter vector θ, based on data $\underline{X}$ and a prior $\varphi(\theta)$, the posterior is given by $\varphi^*(\theta) = C^{-1}L(\theta)\varphi(\theta)$, where the normalizing constant $C = \int L(\theta)\varphi(\theta)d\theta$ is determined by the constraint that the density integrate to 1. This normalizing constant often does not have an analytic expression. General problems of interest in Bayesian analyses are computing means and variances of the posterior distribution, and also finding quantities of marginal posterior distributions. In general let $g(\theta)$ be a parametric function for which

$$\tilde{g}(\theta) = \int g(\theta)\varphi^*(\theta\,|\underline{X})d\theta, \quad (3.5)$$

needs to be evaluated. In many applications,(3.5) cannot be evaluated explicitly, and it is difficult to sample directly from the posterior distribution, so importance sampling can be applied. Samples can be drawn from a distribution with density $q(\theta)$. In this case, if $\theta_1, \theta_2, ..., \theta_N$ is a random sample from $q(\theta)$ then (3.5) can be estimated with

$$\tilde{g}(\theta) = \frac{\sum_{i=1}^{N} g(\theta_i)w_i}{\sum_{i=1}^{N} w_i}, \quad (3.6)$$

where $w_i = \frac{L(\theta_i)\varphi(\theta_i)}{q(\theta_i)}$ and the sampling density $q(\theta)$ need not be normalized. This technique is described in detail [10].We generate a samples from normal-gamma distribution with parameters $(\delta = 0.2,\ \lambda = 1,\ \alpha = 2,\ and\ \gamma = 1)$. We use the following procedure:

1. Generate $\beta_1 \sim Gamma(\alpha, \gamma)$ and

$\mu_1 | \beta_1 \sim Normal\left(\delta, \frac{1}{\lambda \beta_1}\right)$.
2. Repeat this procedure to obtain $(\beta_1, \mu_1), ..., (\beta_N, \mu_N)$.
3. The approximate value of (3.5) can be obtained by (3.6).

4 Simulation studies

To demonstrate the importance of the results obtained in the preceding sections, simulation studies are conducted. For this purpose, by using Monte Carlo method, with fixed sample size n (the total items put in a life test), with constant censoring scheme, where $R_1 = R_2 = R_3 = ... = R_m$, where m is the sample size of progressively censored from the sample of size n. For example if the R's are ones, n must be even and m is half the value of n. The following algorithm is used to generate sample based on progressive type-II censoring scheme, based on any continuous df F, see [3].

1. Generate m independent Uniform (0,1) observations $W_1, ..., W_m$.
2. Set $V_i = W_i^{1/\gamma_i}$, $\gamma_i = \left(i + \sum_{j=m-i+1}^{m} R_j\right)$ for $i = 1, 2, ..., m$.
3. $U_i = 1 - V_m V_{m-1} ... V_{m-i+1}$, $i = 1, 2, ..., m$.
4. Set $X_i = F^{-1}(U_i)$, then X_i, for $i = 1, 2, ..., m$, is the progressive type-II censoring scheme based on the df F.
5. We repated steps 1,2,3 and 4 (1000) times, for different values of n and m.

Estimation average $= \frac{\sum_{i=1}^{1000} \widehat{\theta}_i}{1000}$, mean square error $= \frac{\sum_{i=1}^{1000} \left(\hat{\theta}_i - \theta_i\right)^2}{1000}$, where, θ is the parameter and $\widehat{\theta}$ is the estimator.
All the computations are prepared by Mathematica 9.
Since the non-linear equations (2.5) are not solvable analytically, numerical methods can be used, as Newton Raphson method with initial values closed to real values of the parameters.

Throughout this section we will use the following abbreviations:

1. ML : means that the estimate by using the (MLE),
2. B_{Sq} : means that the estimate under squared error loss function,
3. $B_{Lx,c=15}$: means that the estimate under linex loss function at $c = 15$,
4. $B_{Lx,c=18}$: means that the estimate under linex loss function at $c = 18$,
5. $B_{Lx,c=20}$: means that the estimate under linex loss function at $c = 20$,
6. $B_{Ge,q=10}$: means that the estimate under general entropy loss function at $q = 10$,
7. $B_{Ge,q=18}$: means that the estimate under general entropy loss function at $q = 18$,
8. $B_{Ge,q=20}$: means that the estimate under general entropy loss function at $q = 20$.

From the simulation studies we noted that:

1. In general, the Bayesian estimators have mean square error less than that of the MlE.
2. Increasing the sample size leads to decrease mean square error and increase the accuracy of estimators.
3. The estimate of μ under general entropy loss function is the best, and the importance sampling technique is more accurate than Lindley's Bayes approximation. Also by decreasing the value of the parameter β, the accuracy of estimates increases and mean square error decreases.
4. For the parameter β, the estimate under squared error loss function is the best, and it followed by the general entropy loss function.
5. The estimate of the reliability function $R(t)$ under linex loss function is the best at $\beta = 0.8$, and the general entropy loss function is the best at $\beta = 0.7, 0.9$, especially when the value $q = 18$, and it followed by the MLE.
6. For the hazard rate function $H(t)$, the estimate under linex loss function is the best at $\beta = 0.8$, and the general entropy loss function is the best at $\beta = 0.7, 0.9$. Also the importance sampling technique is more accurate than Lindley's Bayes approximation especially in case of the estimate under squared error loss function.

5 Concluding remarks

In this paper, MLE and Bayesian estimation of the two parameters, reliability, and hazard rate functions for the logistic distribution using Lindley's approximation and importance sampling technique, based on progressively type-II censoring samples are obtained. We assumed Gaussian-gamma prior distributions for the parameters. Computer simulation study is performed, and show that increasing the sample size leads to decrease mean square error and increase the accuracy of estimators. The estimate of μ under general entropy loss function is the best, and the importance sampling technique is more accurate than Lindley's Bayes approximation. For the parameter β, the estimate under squared error loss function is the best. The estimate of the reliability function $R(t)$ under linex loss function is the best at $\beta = 0.8$, and the general entropy loss function is the best at $\beta = 0.7, 0.9$, especially when the value $q = 18$, and it followed by the MLE. For the hazard rate function $H(t)$, the estimate under linex loss function is the best at $\beta = 0.8$, and the general entropy loss function is the best at $\beta = 0.7, 0.9$. Also the importance sampling technique is more accurate than Lindley's Bayes approximation especially in case of the estimate under squared error loss

Table 1: The average, mean square error, when n=200, m=100, scheme(100*1) and $\mu = 0$.

$B_{lx,c=20}$	$B_{lx,c=18}$	$B_{lx,c=15}$	$B_{Ge,q=20}$	$B_{Ge,q=18}$	$B_{Ge,q=10}$	B_{sq}	Technique	ML	β
The average, (mean square error) of the estimators of parameter μ									
-0.5333	-0.5452	-0.5668	0.1268	0.1176	0.0884	-0.1526	Lindley	-0.1597	0.7
(0.2937)	(0.3090)	(0.3378)	(0.0160)	(0.0138)	(0.0036)	(0.0233)		(1.0005)	
-0.0799	-0.0687	-0.0478	0.0017	0.0018	0.0029	-0.1430	Importance		
(0.0351)	(0.0342)	(0.0331)	(0.0031)	(0.0053)	(0.0091)	(0.0331)	Sampling		
-0.7212	-0.7270	-0.7361	0.2221	0.2196	0.1724	-0.2726	Lindley	-0.2807	0.8
(0.5270)	(0.5372)	(0.5541)	(0.0471)	(0.0482)	(0.0140)	(0.0744)		(2.1261)	
-0.3527	-0.3517	-0.3496	0.0023	0.0025	0.0044	-0.2692	Importance		
(0.3921)	(0.4211)	(0.3341)	(0.0132)	(0.0339)	(0.0129)	(0.1339)	Sampling		
-0.8872	-0.8964	-0.9114	0.3004	0.2893	0.2372	-0.3491	Lindley	-0.3495	0.9
(0.7944)	(0.8124)	(0.8438)	(0.0902)	(0.0799)	(0.0265)	(0.1219)		(2.4622)	
-0.4141	-0.4112	-0.4054	0.0066	0.0083	0.0510	-0.2876	Importance		
(0.8821)	(0.6620)	(0.6204)	(0.0112)	(0.0631)	(0.1134)	(0.1124)	Sampling		
The average, (mean square error) of the estimators of parameter β									
0.6762	0.6791	0.6838	0.6717	0.6819	0.6982	0.7060	Lindley	0.6818	0.7
(0.0005)	(0.0004)	(0.0002)	(0.0008)	(0.0003)	(0.0001)	(0.0001)		(0.0127)	
0.6664	0.6713	0.6798	0.6701	0.6770	0.6856	0.6831	Importance		
(0.3102)	(0.1241)	(0.0922)	(0.0810)	(0.0422)	(0.0120)	(0.0230)	Sampling		
0.7631	0.7674	0.7710	0.7642	0.7705	0.7715	0.7817	Lindley	0.7584	0.8
(0.0027)	(0.0024)	(0.0021)	(0.0034)	(0.0019)	(0.0011)	(0.0003)		(0.0332)	
0.7473	0.7502	0.7550	0.7413	0.7558	0.7679	0.7694	Importance		
(0.4321)	(0.2225)	(0.1213)	(0.1201)	(0.0235)	(0.0102)	(0.0125)	Sampling		
0.8421	0.8611	0.8665	0.8464	0.8631	0.8760	0.8742	Lindley	0.8496	0.9
(0.0045)	(0.0023)	(0.0011)	(0.0029)	(0.0017)	(0.0013)	(0.0007)		(0.0453)	
0.8329	0.8521	0.8636	0.8456	0.8589	0.8643	0.8606	Importance		
(0.4211)	(0.3254)	(0.2312)	(0.3321)	(0.2322)	(0.1211)	(0.1125)	Sampling		
The average, (mean square error) of the estimators of reliability function $R(t)$									
R(t=2)=0.0543									0.7
0.0589	0.0589	0.0588	0.0503	0.0547	0.0581	0.0586	Lindley	0.0528	
(0.0003)	(0.0003)	(0.0002)	(0.0004)	(5×10^{-6})	(1×10^{-5})	(0.0002)		(0.0001)	
0.0615	0.0616	0.0616	0.0404	0.0568	0.0598	0.0618	Importance		
(0.0024)	(0.0013)	(0.0013)	(0.0089)	(0.0021)	(0.0022)	(0.0031)	Sampling		
R(t=2)=0.0759									0.8
0.0782	0.0780	0.0778	0.0393	0.0489	0.0650	0.0765	Lindley	0.0719	
(1×10^{-6})	(8×10^{-6})	(7×10^{-6})	(0.0013)	(0.0007)	(0.0002)	(2×10^{-6})		0.0003	
0.0491	0.0495	0.0503	0.0412	0.0435	0.0548	0.0518	Importance		
(0.0619)	(0.0608)	(0.0416)	(0.0623)	(0.0622)	(0.0533)	(0.0619)	Sampling		
R(t=2)=0.0978									0.9
0.1020	0.1019	0.1018	0.0913	0.0971	0.0999	0.1008	Lindley	0.0941	
(3×10^{-5})	(2×10^{-5})	(1×10^{-5})	(3×10^{-5})	(0.0001)	(7×10^{-6})	(2×10^{-5})		(0.0004)	
0.0992	0.1023	0.1015	0.0817	0.1084	0.1096	0.1063	Importance		
(0.0503)	(0.0506)	(0.0502)	(0.0533)	(0.0503)	(0.0540)	(0.0503)	Sampling		
The average, (mean square error) of the estimators of hazard rate function $H(t)$									
H(t=2)=1.3510									0.7
1.3383	1.3406	1.3476	1.3497	1.3563	1.3675	2.7140	Lindley	2.8132	
(0.0003)	(0.0003)	(0.0002)	(0.0002)	(0.0003)	(0.00088)	(4.3012)		(4.6120)	
1.3446	1.3621	1.3697	1.3554	1.3614	1.3674	1.3824	Importance		
(0.0052)	(0.0061)	(0.0072)	(0.0051)	(0.0062)	(0.0063)	(0.0236)	Sampling		
H(t=2)=1.1552									0.8
1.2039	1.2079	1.2184	1.2134	1.2202	1.3072	2.5402	Lindley	2.6301	
(0.0024)	(0.0029)	(0.0042)	(0.0035)	(0.0044)	(0.0238)	(3.6108)		(3.8043)	
1.1972	1.1996	1.2036	1.2003	1.2026	1.2129	1.2383	Importance		
(0.0210)	(0.0322)	(0.0442)	(0.0332)	(0.0421)	(0.0451)	(0.0553)	Sampling		
H(t=2)=1.0025									0.9
1.0283	1.0329	1.0448	1.0306	1.0358	1.1036	2.1364	Lindley	2.4623	
(0.0007)	(0.0011)	(0.0019)	(0.0009)	(0.0013)	(0.0108)	(3.3021)		(3.6230)	
0.9844	1.0147	1.0398	0.9605	0.9937	1.0091	0.9817	Importance		
(0.0056)	(0.0044)	(0.0065)	(0.0251)	(0.0042)	(0.0012)	(0.0041)	Sampling		

Table 2: The average, mean square error, when n=100, m=50, scheme(50*1) and $\mu = 0$.

$B_{lx,c=20}$	$B_{lx,c=18}$	$B_{lx,c=15}$	$B_{Ge,q=20}$	$B_{Ge,q=18}$	$B_{Ge,q=10}$	B_{sq}	Technique	ML	β
The average, (mean square error) of the estimators of parameter μ									
-1.7306	-1.7613	-1.8192	0.6044	0.5891	0.5060	-0.7012	Lindley	-0.6750	0.7
(2.9984)	(3.1060)	(3.3151)	(0.3669)	(0.3664)	(0.1210)	(0.4985)		(2.9253)	
0.02777	0.0655	-0.0742	0.0021	0.0027	0.0140	-0.0567	Importance		
(0.0631)	(0.0932)	(0.2531)	(0.0112)	(0.0135)	(0.0726)	(0.9874)	Sampling		
-1.9212	-1.9499	-2.0042	0.6974	0.6961	0.6796	-0.7976	Lindley	-0.7786	0.8
(3.6941)	(3.8061)	(4.0222)	(0.4864)	(0.4845)	(0.4410)	(0.6365)		(4.0368)	
-0.1653	-0.0995	0.1221	0.0229	0.0264	0.0352	-0.2316	Importance		
(0.5124)	(0.5221)	(0.4725)	(0.1213)	(0.1436)	(0.2265)	(0.3628)	Sampling		
-2.0645	-2.0918	-2.1431	0.7638	0.7619	0.7432	-0.8706	Lindley	-0.8569	0.9
(4.2652)	(4.3791)	(4.5979)	(0.5835)	(0.5807)	(0.5274)	(0.7584)		(5.2232)	
-0.2316	-0.2316	-0.2315	0.0119	0.0882	0.0696	0.2878	Importance		
(0.9921)	(0.4236)	(0.2118)	(0.0657)	(0.1302)	(0.1020)	(0.8547)	Sampling		
The average, (mean square error) of the estimators of parameter β									
0.5651	0.5679	0.5727	0.5664	0.5879	0.5912	0.6025	Lindley	0.5782	0.7
(0.0182)	(0.0174)	(0.0162)	(0.0178)	(0.0127)	(0.0121)	(0.0118)		(0.0852)	
0.5778	0.5910	0.6103	0.5673	0.5828	0.6317	0.6598	Importance		
(0.1251)	(0.1631)	(0.1328)	(0.0610)	(0.0412)	(0.0124)	(0.0121)	Sampling		
0.6405	0.6435	0.6597	0.6458	0.6619	0.6733	0.6839	Lindley	0.6608	0.8
(0.0254)	(0.0245)	(0.0197)	(0.0288)	(0.0190)	(0.0161)	(0.0134)		(0.1113)	
0.6744	0.6763	0.6772	0.6730	0.6760	0.6780	0.6782	Importance		
(0.2154)	(0.2240)	(0.2251)	(0.1201)	(0.1124)	(0.1102)	(0.1112)	Sampling		
0.7301	0.7546	0.7714	0.7315	0.7615	0.7706	0.7883	Lindley	0.7632	0.9
(0.0289)	(0.0251)	(0.0140)	(0.0298)	(0.0192)	(0.0167)	(0.0125)		(0.1231)	
0.7661	0.7662	0.7665	0.7655	0.7686	0.7710	0.7699	Importance		
(0.2130)	(0.2033)	(0.2012)	(0.2204)	(0.2019)	(0.1332)	(0.1923)	Sampling		
The average, (mean square error) of the estimators of reliability function $R(t)$									
R(t=2)=0.0543									0.7
0.0483	0.0481	0.0478	0.0465	0.0484	0.0490	0.0467	Lindley	0.0448	
(0.0001)	(0.0002)	(0.0004)	(0.0021)	(0.0010)	(0.0007)	(0.0001)		(0.0005)	
0.0460	0.0461	0.0463	0.0392	0.0452	0.0472	0.0424	Importance		
(0.0136)	(0.0134)	(0.0113)	(0.1430)	(0.1411)	(0.1322)	(0.3251)	Sampling		
R(t=2)=0.0759									0.8
0.0680	0.0675	0.0669	0.0314	0.0324	0.0538	0.0649	Lindley	0.0626	
(6×10^{-5})	(7×10^{-5})	(8×10^{-5})	(0.0038)	(0.0031)	(0.0016)	(0.0002)		(0.0010)	
0.0589	0.0592	0.0611	0.0324	0.0331	0.0521	0.0645	Importance		
(0.0135)	(0.0131)	(0.0122)	(0.0182)	(0.0162)	(0.0132)	(0.0241)	Sampling		
R(t=2)=0.0978									0.9
0.0852	0.0841	0.0833	0.0916	0.0906	0.0894	0.0859	Lindley	0.0829	
(0.0016)	(0.0002)	(0.0002)	(5×10^{-5})	(6×10^{-5})	(7×10^{-5})	(0.0002)		(0.0014)	
0.0861	0.0850	0.0843	0.0903	0.0891	0.0851	0.0794	Importance		
(0.0722)	(0.0732)	(0.0711)	(0.0533)	(0.0543)	(0.0556)	(0.1264)	Sampling		
The average, (mean square error) of the estimators of hazard rate function $H(t)$									
H(t=2)=1.3510									0.7
1.6099	1.6351	1.7952	1.6217	1.6287	1.7291	2.9302	Lindley	2.9921	
(0.0674)	(0.0818)	(0.2021)	(0.0741)	(0.0781)	(0.1463)	(4.5102)		(4.7213)	
1.6131	1.6211	1.7561	1.6133	1.6293	1.7496	1.4228	Importance		
(0.0422)	(0.0462)	(0.0512)	(0.0356)	(0.0371)	(0.0422)	(0.1066)	Sampling		
H(t=2)=1.1552									0.8
1.4026	1.4038	1.4089	1.4369	1.5117	1.9102	2.7301	Lindley	2.8210	
(0.0615)	(0.0621)	(0.0648)	(0.0803)	(0.1294)	(0.5851)	(3.7113)		(3.9621)	
1.5182	1.5918	1.6755	1.6414	1.8411	2.1798	2.1821	Importance		
(0.0433)	(0.0463)	(0.0482)	(0.0533)	(0.0562)	(0.0611)	(0.1076)	Sampling		
H(t=2)=1.0025									0.9
1.2153	1.2292	1.2924	1.2027	1.2051	1.2127	2.3442	Lindley	2.5210	
(0.0457)	(0.0521)	(0.0858)	(0.0403)	(0.0413)	(0.0446)	(3.4210)		(3.7120)	
1.0876	1.1126	1.1801	1.1006	1.1162	1.1413	1.2162	Importance		
(0.0321)	(0.0332)	(0.0346)	(0.0211)	(0.0241)	(0.0298)	(0.1211)	Sampling		

function. The simulation also stresses the importance of linex and general entropy loss functions as shown in the case studied.

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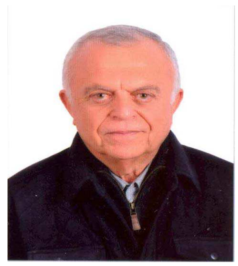

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