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Fixed Point Theory for the α-Admissible Meir-Keeler-Type Set Contractions Having KKM* Property on Almost Convex Sets

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Abstract: The purpose of this paper is to study fixed points for the KKM* family satisfying the α-admissible Meir-Keeler-type set contractions with respect to the set measure σ_p of noncompactness in the context of Hausdorff topological vector spaces. Our results generalize or improve many recent fixed point theorems for the KKM family in the literature.
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1 Introduction and preliminaries

In 1929, Knaster, Kuratowski and Mazurkiewicz [7] introduced the KKM mapping and proved the well-known KKM theorem in the setting of the n-simplex. Later, in 1961, Ky Fan [4] generalized the celebrated KKM theorem to an infinite dimensional topological vector space. In 2009, Chen [2] discussed the set measure σ_p of noncompactness and the properties of the almost convex sets on a Hausdorff topological vector space, and then, in the setting of the almost convex sets, he established the fixed point theorems for the KKM* family with the ψ-set contraction, where $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ is upper semicontinuous with $0 < \psi(t) < t$ and $\psi(0) = 0$. In this work, we study fixed points for the KKM* family satisfying the Meir-Keeler-type set contractions with respect to the set measure σ_p of noncompactness in the context of Hausdorff topological vector spaces

For the sake of completeness, we recall basic definitions and fundamental results from the literature. Throughout this paper, by $\mathbb{R}^+$, we denote the set of all nonnegative real numbers, while $\mathbb{N}$ is the set of all natural numbers. Let X and Y be two sets, and let $T : X \to 2^Y$ be a set-valued mapping. We shall use the following notations in the sequel.

(1) $T(x) = \{y \in Y : y \in T(x)\}$;
(2) $T(A) = \cup_{x\in A} T(x)$;
(3) $T^{-1}(y) = \{x \in X : y \in T(x)\}$;
(4) $T^{-1}(B) = \{x \in X : T(x) \cap B \neq \phi\}$;
(5) if D is a nonempty subset of X, then $< D >$ denotes the class of all nonempty finite subsets of D.

Let X and Y be two topological spaces. Then $T : X \to 2^Y$ is said to be closed if its graph $\mathscr{G}_T = \{(x,y) \in X \times Y, y \in T(x)\}$ is closed, and T is said to be compact if the image $T(X)$ of X under T is contained in a compact subset of Y. A subset X of a Hausdorff topological vector space E is said to be almost convex [10], if there is a mapping $f_{A,V} : A \to X$ such that $x \in f_{A,V}(x) + V$ for each $x \in A$ and $co(f_{A,V}(A)) \subset X$. We call $f_{A,V}$ a convex-inducing mapping.

In 2009, Chen proved the following important properties of the almost convex sets.

Proposition 1.[2] *Let X be an almost convex subset of a Hausdorff topological vector space E. Then $\overline{X}$ (the closure of X) is convex.*

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Proposition 2.[2] *Let E be a Hausdorff topological vector space. If X is an almost convex subset of E, and Y is an open convex subset of E, then $X \cap Y$ is also an almost convex subset of E.*

Remark.[2] Let us note that the open condition of the above Proposition 2 is really needed. For instance, if we consider the Euclidean topology in $\mathbb{R}^2$, and we let $A = \{(1,1),(1,-1),(-1,1),(-1,-1)\}$, $B = \{(1,1),(1,-1),(-1,1),(-1,-1)\}$

$$X = int(co(A)) \cup B;$$

and

$$Y = co(\{(-1,1),(-2,1),(-1,-1),(-2,-1)\}).$$

Then $X \cap Y = \{(-1,1),(-1,-1)\}$ is not almost convex.

The generalized KKM property on a convex subset of a Hausdorff topological vector space was introduced by Chang and Yen [1]. Next, Jent et al. [6] extended this class KKM(X,Y) to the class KKM*(X,Y) for the almost convex set X.

In this paper, we also introduce the notion of the measure of noncompactness on Hausdorff topological vector spaces. Let E be a Hausdorff topological vector space, and $\mathscr{B}(E)$ the family of nonempty bounded subsets, and let

$\mathscr{P} = \{P : P$ is a family of seminorms which determines the topology on $E\}$.

A mapping $\Phi : \mathscr{B}(E) \to \mathbb{R}^+$ is called a measure of noncompactness [5] provided that the following conditions hold:

(Φ1) $\Phi(\overline{co}(A)) = \Phi(A)$ for each $A \in \mathscr{B}(E)$, where $\overline{co}(A)$ denotes the closure of the convex hull of A;
(Φ2) $\Phi(A) = 0$ if and only if A is precompact;
(Φ3) $\Phi(A \cup B) = \max\{\Phi(A), \Phi(B)\}$ for each $A, B \in \mathscr{B}(E)$;
(Φ4) $\Phi(\lambda A) = \lambda \Phi(A)$ for each $\lambda \in \mathbb{R}^+$ and $A \in \mathscr{B}(E)$.

The above notion is a generalization of the set-measure of noncompactness; if $\{p : p \in P\}$ with $P \in \mathscr{P}$, is a family of seminorms which determines the topology on E, then for each $p \in P$ and $A \subset E$, we define the set-measure of noncompactness $\sigma_p : \mathscr{B}(E) \to \mathbb{R}^+$ by
$\sigma_p(A) = inf\{\gamma \geq 0 :$ A can be covered by a finite number of sets and each

$$p - \text{diameter of the sets is less than } \gamma\},$$

where the p-diameter of $A = \sup\{p(x-y) : x, y \in A\}$.

In 1969, Meir and Keeler [9] introduced an interesting new contraction in the following way.

Definition 1. *Let (X,d) be a metric space, $f : X \to X$. Then f is called a Meir-Keeler-type contraction whenever for each $\eta > 0$ there exists $\gamma > 0$ such that*

$$\eta \leq d(x,y) < \eta + \gamma \Longrightarrow d(fx, fy) < \eta.$$

Recently, the weaker Meir-Keeler mapping that was introduced in [3].

Definition 2. *(See [3]) Let $\psi : \mathbb{R}^+ \to \mathbb{R}^+$. Then ψ is called a weaker Meir-Keeler mapping whenever for each $\eta > 0$ there exists $\gamma > 0$ such that for each $t \in \mathbb{R}^+$*

$$\eta \leq t < \eta + \gamma \Longrightarrow \exists n_0 \in \mathbb{N}, \psi^{n_o}(t) < \eta.$$

Very recently, Samet et. al. [11] introduced the notion of an α-admissible function in the following way (see also [8]).

Definition 3. *Let $f : X \to X$ be a self-mapping of a set X and $\alpha : X \times X \to \mathbb{R}^+$. Then f is called an α-admissible function if*

$$x, y \in X, \ \alpha(x,y) \geq 1 \Longrightarrow \alpha(fx, fy) \geq 1.$$

In this paper, we first introduce the α-admissibility for set-valued mapping which is an extension of Definition 3.

Definition 4. *Let X be a nonempty subset of a Hausdorff topological vector space E and $T : X \to 2^X$ be a set-valued map. Suppose that $\alpha : X \times X \to \mathbb{R}^+$. We say that T is an α-admissible set-valued mapping if it satisfies the following condition:*

$$\alpha(x,y) \geq 1 \textit{ implies } \alpha(\mu, \nu) \geq 1, \ x, y \in X, \ \mu \in Tx, \ \nu \in Ty.$$

The purpose of this paper is to study fixed points for the KKM* family satisfying the α-admissible Meir-Keeler-type set contractions with respect to the set-measure σ_p of noncompactness in the context of Hausdorff topological vector spaces.

2 Fixed point results for the α-admissible weaker ψ-Meir-Keeler-type set contractions

Applying the weaker Meir-Keeler mapping, we introduce the following notion of weaker Meir-Keeler mapping in a bounded subset X of a Hausdorff topological vector space E with respect to the set-measure of noncompactness.

Definition 5. *Let X be a nonempty subset of a Hausdorff topological vector space E. Then $\psi : X \to \mathbb{R}^+$ is called a weaker Meir-Keeler mapping in X if there exists $P \in \mathscr{P}$ such that the following condition holds:*
($*$) for each $\eta > 0$ there exists $\gamma > 0$ such that

$$\eta \leq \sigma_p(A) < \eta + \gamma \Longrightarrow \exists n_0 \in \mathbb{N}, \psi^{n_o}(\sigma_p(A)) < \eta,$$

for each $p \in P$, and for each bounded subset A of X.

We next define the notion of an α-admissible weaker ψ-Meir-Keeler-type set contraction on a Hausdorff topological vector space E.

Definition 6. *Let X be a nonempty subset of a Hausdorff topological vector space E, let $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ be a weaker Meir-Keeler mapping with $\{\psi^n(t)\}_{n\in\mathbb{N}}$ a decreasing function for all $t \in \mathbb{R}^+$. A mapping $T : X \to 2^E$ is said to be an α-admissible weaker ψ-Meir-Keeler-type set* *contraction if, there exists $P \in \mathscr{P}$ such that the following condition holds:*

$(**)$ *for each $\eta > 0$ there exists $\gamma > 0$ such that*

$$\eta \leq \psi(\sigma_p(A)) < \eta + \gamma \Longrightarrow \alpha(x,y)\sigma_p(T(A)) < \eta,$$

for each $A \subset X$ where A and $T(A)$ are bounded, $x, y \in A$, and $p \in P$.

*Remark.*Note that if T is an α-admissible weaker ψ-Meir-Keeler-type set contraction, then from above Definition, it is easy to get the following inequality

$$\alpha(x,y)\sigma_p(T(A)) \leq \psi(\sigma_p(A)).$$

The following lemma and its consequent theorem will play important roles in this paper.

Lemma 1.*[2] Let X be a nonempty almost convex subset of a Hausdorff topological vector space E, and let Y, Z be two topological spaces. Then*

(1) *if $T \in KKM^*(X,Y)$ and $f \in \mathscr{C}(Y,Z)$, then $fT \in KKM^*(X,Z)$.*
(2) *if $T \in KKM^*(X,Y)$ and D is a nonempty almost convex subset of X, then $T|_D \in KKM^*(D,Y)$.*

Theorem 1.*[6] Let X be a nonempty almost convex subset of a locally convex space E. If $T \in KKM^*(X,X)$ is compact and closed, then T has a fixed point in X.*

We state our main theorem, as follows:

Theorem 2.*Let X be a nonempty almost convex subset of a Hausdorff topological vector space E. Suppose that $T : X \to 2^X$ is an α-admissible weaker ψ-Meir-Keeler-type set contraction with $int(T(x)) \neq \phi$ for each $x \in X$, and that there exists $x_0 \in X$ such that $\alpha(x_0,x_0) \geq 1$. Then X contains a precompact almost convex subset K of X with $T(K) \subset K$.*

Proof. Since T is an α-admissible weaker ψ-Meir-Keeler-type set contraction, there exists $P \in \mathscr{P}$ such that

$$\alpha(x,y)\sigma_p(T(A)) \leq \psi(\sigma_p(A)),$$

for each $p \in P$ and $A \subset X$, $x, y \in A$.

Take $y \in X$. Let $X_0 = X$ and

$$X_{n+1} = X \cap int(co(T(X_n) \cup \{y\}))), \text{ for all } n \in \mathbb{N}.$$

Then, we have the following conclusions:

(1) by Proposition 2, X_n is nonempty and almost convex, for each $n \in \mathbb{N}$,
(2) $X_{n+1} \subset X_n$, for each $n \in \mathbb{N}$,
(3) $T(X_n) \subset X_{n+1}$, for each $n \in \mathbb{N}$.

Let $x_1 \in T(x_0)$. Since T is α-admissible and $\alpha(x_0,x_0) \geq 1$ and using (2) and (3), we have that

$$x_1 \in T(x_0) \subset T(X_0) \subset X_1,$$

and

$$\alpha(x_1,x_1) \geq 1.$$

Continuing this process, we can construct a sequence $\{x_n\}$ such that for all $n \in \mathbb{N} \cup \{0\}$

$$x_{n+1} \in Tx_n \subset T(X_n) \subset X_{n+1}$$

and hence we have

$$\alpha(x_{n+1},x_{n+1}) \geq 1, \text{ for all } n \in \mathbb{N}.$$

From above argument and by the properties of the set measure φ_p, we deduce that for each $n \in \mathbb{N}$

$$\begin{aligned}\sigma_p(X_{n+1}) &\leq \sigma_p(int(co(T(X_n) \cup \{y\})))) \\ &\leq \sigma_p(co(T(X_n) \cup \{y\}))) \\ &= \sigma_p(T(X_n)) \\ &\leq \alpha(x_{n+1},x_{n+1})\sigma_p(T(X_n)) \\ &\leq \psi(\sigma_p(T(X_n))) \\ &\leq \psi(\sigma_p(X_n)),\end{aligned}$$

and then we get

$$\sigma_p(X_{n+1}) \leq \psi^n(\sigma_p(X_0)).$$

Since $\{\psi^n(\sigma_p(X_0))\}_{n \in \mathbb{N}}$ is decreasing, it must converge to some $\eta \geq 0$. We claim that $\eta = 0$. On the contrary, assume that $\eta > 0$. Since $\psi : X \to \mathbb{R}^+$ is a weaker Meir-Keeler mapping, there exists $\delta > 0$ such that for the bounded set X_0

$$\eta \leq \psi(\sigma_p(X_0))) < \eta + \delta \Longrightarrow \exists n_0 \in \mathbb{N}, \ \psi^{n_0}(\sigma_p(X_0))) < \eta.$$

Since $\lim_{n \to \infty} \psi^n(\sigma_p(X_0))) = \eta$, there exists $m_0 \in \mathbb{N}$ such that $\eta \leq \psi^p(\sigma_p(X_0))) < \delta + \eta$, for all $m \geq m_0$. Thus, we conclude that $\psi^{m_0+n_0}(\sigma_p(X_0)) < \eta$. So we get a contradiction. Therefore $\lim_{n \to \infty} \psi^n(\sigma_p(X_0)) = 0$, that is,

$$\lim_{n \to \infty} \sigma_p(X_{n+1}) = 0.$$

Take $X_\infty = \cap_{n \in \mathbb{N} \cup \{0\}} X_n$. Then X_∞ is a nonempty precompact almost convex subset of X, and, by (2), (3), we also have that $T(X_\infty) \subset X_\infty$. □

*Remark.*In the process of the proof of Theorem 2, we call the set X_∞ a precompact-inducing almost convex subset of X.

By applying Theorem 1 and Theorem 2, we conclude that the following fixed point theorem.

Theorem 3.*Let X be a nonempty almost convex subset of a locally convex space E. Suppose that $T \in KKM^*(X,X)$ is an α-admissible weaker ψ-Meir-Keeler-type set contraction with $int(T(x)) \neq \phi$ for each $x \in X$ and $\overline{T(X)} \subset X$, and that there exists $x_0 \in X$ such that $\alpha(x_0,x_0) \geq 1$. Then T has a fixed point in X.*

Proof. By using Theorem 2 and Remark 2, we get a precompact-inducing almost convex subset X_∞ of X, and we also conclude that

$$\lim_{n\to\infty} \sigma_p(T(X_{n+1})) = 0.$$

Hence $T(X_\infty)$ is a precompact subset of X and $\overline{T(X_\infty)}$ is a compact subset of X. The rest follows from Lemma 1 and Theorem 1. □

By Theorem 3, we can get the following fixed point theorem for convex sets.

Theorem 4.*Let X be a nonempty convex subset of a locally convex space E. Suppose that $T \in KKM(X,X)$ is an α-admissible weaker ψ-Meir-Keeler-type set contraction with $\overline{T(X)} \subset X$, and that there exists $x_0 \in X$ such that $\alpha(x_0,x_0) \geq 1$. Then T has a fixed point in X.*

3 Fixed point results for the α-admissible Meir-Keeler-type set contractions

Applying the Meir-Keeler mapping, we define the notion of an α-admissible Meir-Keeler-type set contraction on a Hausdorff topological vector space E with respect to the set-measure of noncompactness.

Definition 7.*Let X be a nonempty subset of a Hausdorff topological vector space E. A mapping $T : X \to 2^E$ is said to be an α-admissible Meir-Keeler-type set contraction if, there exists $P \in \mathscr{P}$ such that the following condition holds:*
$(***)$ for each $\eta > 0$ there exists $\gamma > 0$ such that

$$\eta \leq \sigma_p(A) < \eta + \gamma \Longrightarrow \alpha(x,y)\sigma_p(T(A)) < \eta,$$

for each $A \subset X$ where A and $T(A)$ are bounded, $x,y \in A$, and $p \in P$.

*Remark.*Note that if T is an α-admissible Meir-Keeler-type set contraction, then from above Definition, we get the following inequality

$$\alpha(x,y)\sigma_p(T(A)) \leq \sigma_p(A).$$

Theorem 5.*Let X be a nonempty almost convex subset of a Hausdorff topological vector space E. Suppose that $T : X \to 2^X$ is an α-admissible Meir-Keeler-type set contraction with $int(T(x)) \neq \phi$ for each $x \in X$, and that there exists $x_0 \in X$ such that $\alpha(x_0,x_0) \geq 1$. Then X contains a precompact almost convex subset K of X with $T(K) \subset K$.*

Proof. Since T is an α-admissible Meir-Keeler-type set contraction, there exists $P \in \mathscr{P}$ such that

$$\alpha(x,y)\sigma_p(T(A)) \leq \sigma_p(A),$$

for each $p \in P$ and $A \subset X$, $x,y \in A$.
Take $y \in X$. Let $X_0 = X$ and

$$X_{n+1} = X \cap int(co(T(X_n) \cup \{y\})), \text{ for all } n \in \mathbb{N}.$$

Then, we have the following conclusions:

(1) X_n is nonempty and almost convex, for each $n \in \mathbb{N}$,
(2) $X_{n+1} \subset X_n$, for each $n \in \mathbb{N}$,
(3) $T(X_n) \subset X_{n+1}$, for each $n \in \mathbb{N}$.

Let $x_1 \in T(x_0)$. Since T is α-admissible and $\alpha(x_0,x_0) \geq 1$ and using (2) and (3), we have that

$$x_1 \in T(x_0) \subset T(X_0) \subset X_1,$$

and

$$\alpha(x_1,x_1) \geq 1.$$

Continuing this process, we can construct a sequence $\{x_n\}$ such that for all $n \in \mathbb{N} \cup \{0\}$

$$x_{n+1} \in Tx_n \subset T(X_n) \subset X_{n+1}$$

and hence we have

$$\alpha(x_{n+1},x_{n+1}) \geq 1, \text{ for all } n \in \mathbb{N}.$$

From above argument, we deduce that for each $n \in \mathbb{N}$

$$\begin{aligned}
\sigma_p(X_{n+1}) &\leq \sigma_p(int(co(T(X_n) \cup \{y\})))) \\
&\leq \sigma_p(co(T(X_n) \cup \{y\}))) \\
&= \sigma_p(T(X_n)) \\
&\leq \alpha(x_{n+1},x_{n+1})\sigma_p(T(X_n)) \\
&\leq \sigma_p(X_n).
\end{aligned}$$

Thus the sequence $\{\sigma_p(X_n)\}$ is decreasing, it must converge to some γ.

Notice that $\eta = \inf\{\sigma_p(X_n) : n \in \mathbb{N} \cup \{0\}\}$. We claim that $\eta = 0$. Suppose, on the the contrary, that $\eta > 0$. Since T is an α-admissible Meir-Keeler-type set contraction, corresponding to η, there exist a $\delta > 0$ and a natural number k such that

$$\eta \leq \sigma_p(X_k) < \eta + \delta \Longrightarrow \sigma_p(X_{k+1}) \leq \alpha(x_k,x_k)\sigma_p(T(X_k)) < \eta.$$

This is a contradiction since $\eta = \inf\{\sigma_p(X_n) : n \in \mathbb{N} \cup \{0\}\}$. Thus, we obtain that

$$\lim_{n\to\infty} \sigma_p(X_n) = 0.$$

Let us take $X_\infty = \cap_{n\in\mathbb{N}\cup\{0\}} X_n$. Then X_∞ is a nonempty precompact subadmissible subset of X, and, by (2), (3), we also have that $T(X_\infty) \subset X_\infty$. □

From Theorem 1 and Theorem 5, we get the following fixed point theorem.

Theorem 6.*Let X be a nonempty almost convex subset of a locally convex space E. Suppose that $T \in KKM^*(X,X)$ is an α-admissible Meir-Keeler-type set contraction with $int(T(x)) \neq \phi$ for each $x \in X$ and $\overline{T(X)} \subset X$, and that there exists $x_0 \in X$ such that $\alpha(x_0,x_0) \geq 1$. Then T has a fixed point in X.*

Authors contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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