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Step-Stress Partially Accelerated Life Tests Model in Estimation of Inverse Weibull Parameters under Progressive Type-II Censoring

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Abstract: In this paper, inverse Weibull (IW) distribution with the step-stress model and progressive type-II censoring data are considered. The maximum likelihood and Bayesian estimation are discussed for the distribution parameters and the acceleration factor. The outline criteria in Bayesian approach are settled under utilized non-informative and gamma informative priors under balanced-squared error and balanced linear-exponential loss functions with the help of MCMC method. Finally, the numerical example and simulation study are constructed to assess the obtaining results.

Keywords: Step-stress partially accelerated life tests; Progressively type-II censoring; Inverse Weibull distribution; Maximum likelihood estimation; Bayesian estimation; MCMC method.

1 Introduction

In the accelerated life tests (ALT), items are tested under stress that are higher than usual stress so that more failure information can be collected within a shorter time. This suggests the failure time is a function of the alleged ”stress factor” and higher stress bring speedier failure. For example, some component have a long life at lower temperatures but it fails speedier at a higher temperature . In most ALT experiments, all test units are run at least at one of the stress conditions unless the units fail or are edited some time recently later stress conditions begin. A partially ALT is one type of the ALT schemes, and it allows part of the test units to be run at a normal condition throughout the entire testing period of time. The partially ALT can be led utilizing different stress schemes such as constant stress partially ALT (CS-PALT) and step stress partially ALT (SS-PALT). Different types of ALT are presented in Nelson [1], the commonly used are SS-PALT and CS-PALT. In the SS-PALT model, a test item is first run at normal condition and, if it does not file for a predefined time, then it is kept running at accelerated condition until failure happens or the observation is censored. More details about the partially ALT are considered by different author, Goel [2] talked about the estimation issue of the acceleration factor utilizing maximum likelihood and Bayesian methods for items having the exponential distribution and uniform distribution in the case of complete sampling, DeGroot and Goel [3] utilized the Bayesian approach, with various loss functions, to estimate the parameters of the exponential distribution and the acceleration factor in case of complete sampling. Additionally, Bhattacharyya and Soejoeti [4] estimated the parameters of the Weibull distribution and the acceleration factor using the maximum likelihood method in the case of the complete data. Bai and Chung [5] utilized the maximum likelihood method to estimate the scale parameter and the acceleration factor for exponentially distributed lifetime utilizing type-I censoring data. Bai et al. [6] examined the estimation issue of parameters for items having lognormal distribution.

Censoring is common in life tests because of time limits and other restrictions on data collection. It is noted that one can use type-II censoring scheme to save time and money. However, this sampling scheme does not allow the removal of test items from the test at any time

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point other than the final termination point. The most common censoring schemes which has several applications in reliability experiments is progressive type-II censoring, see Balakrishnan and Kundu [7]. The progressive type-II censoring has the flexibility of permitting evacuation of units at focuses other than the terminal purpose of the investigation. Another preferred standpoint of progressive censoring is that the degeneration data of the test units is acquired from those expelled units. Although the scheme is more flexible in terms of the expulsion of units see Balakrishnan and Aggarwala [8] . In this paper, we concentrate on SS-PALT under progressive type-II censoring which can be described as follows.

Suppose that n units are put on a life testing experiment and let $X_1, X_2, ..., X_n$ be their relating lifetimes. We assume that $X_i, i = 1, 2, ..., n$ are independent and identically distributed with probability density function (PDF) $f(\mathbf{x})$ and cumulative distribution function (CDF) $F(x)$. Prior to the experiment, an integer $m < n$ is resolved and the progressive type-II censoring scheme $(R_1, R_2, ..., R_m)$ with $R_i > 0$ and $n = m + \sum_{i=1}^{m} R_i$ is specified. During the experiment, the i-th failure is observed and quickly after the failure, R_i functioning items are randomly removed from the test. We mean the m totally observed lifetimes by $X_{i:m:n}^{(R_1, R_2, ..., R_m)}$, $i = 1, 2, ..., m$, which are the observed progressively type-II right censored sample. For comfort, we will smother the censoring scheme in the notation of the $X_{i:m:n}$'s. We also denote the observed values of such a progressively type-II right censored sample by $x_{1:m:n} < x_{2:m:n} < ... < x_{m:m:n}$.

In spite of the fact that censoring can help abbreviate the time and lessen the cost, with an ever increasing number of items having high caliber and long life, censoring can't meet the requests of gathering enough data about the items' lifetimes as quickly as time permits, so the ALTs are broadly utilized in reliability analysis. As of late, the mix of accelerated life test with the progressive censoring scheme has drawing in consideration of few researchers. Ismail [9,10] studied the SS-PALT model respectively, with adaptive type-II progressively hybrid censored data and type-I progressively hybrid censored data from Weibull distribution. Recently Lui et al. [11] studied reliability analysis of masked data in adaptive SS-PALT with progressive removal.

The step-stress models is examined widely in the writing. Miller and Nelson [12] discussed the optimal simple SS- ALT plans for the exponential distribution in the case of complete data. Gouno et al. [13] explored The ideal step-stress test for the exponential distribution with progressive type-I censoring. The simple SS- ALT under type-II censoring, assuming a cumulative exposure model for exponential distribution is considered by Balakrishnan et al. [14]. Srivastava and Shukla [15] derived the optimal plan for simple SS-ALT under the log-logistic distribution by minimizing the asymptotic variance of the MLE of the median life at the design stress. Srivastava and Shukla [16] cosidered the optimal test plan for simple step-stress under the log-logistic model in the case of censored data. Srivastava and Mittal [17] introduced The optimal simple SS-ALT for truncated logistic distribution with censoring. Ismail [18] inferred the MLEs of parameters of Weibull distribution in view of hybrid censored data, assuming a tampered random variable model. Ismail [19] got the MLEs of parameters of Weibull distribution and the acceleration factor under progressive hybrid censoring schemes. the simple SS-ALT under progressive first-failure censoring, assuming a tampered random variable model for Weibull distribution is considered by Mohie El-Din et al. [20]. Mohie El-Din et al. [21] discussed Bayes estimation for SS-ALT to power generalized Weibull distribution under progressive censoring, using a tampered random variable model.

A two-parameter IW distribution is presented in literature by Killer and Kamath [22] as a reasonable model to depict debasement marvels of mechanical parts of diesel motors. It is found widespread applications in reliability engineering, bioengineering and numerous different territories of biological disciplines. Also, to analyze lifetime data indicating unimodal hazard function, IW distribution might be considered as a suitable model. The numerical property and the use of IW distribution are talked about in monographs, for instance, see Reiss and Thomas [23]. Many creators have concentrated the properties of the IW distribution. Calabria and Pulcini [24] given an imperative interpretation of this distribution in the context of load-strength relationship for a mechanical component. Keller et al. [25] are called attention to that the debasement wonders of element parts of diesel motors can be all around portrayed by the IW model. Bayesian appraisals in view of record values from the IW lifetime model is talked about by Sultan [25]. Kundu and Howlader [26] depicted the Bayesian inference and prediction of future observation for censored data under the suspicion that both obscure parameters have independent gamma priors. Musleh and Helu [27] viewed as the statistical inferences about the obscure parameters of the IW distribution based on progressively type-II censoring utilizing traditional and Bayesian methods. Bayesian and maximum likelihood estimations of the IW parameters under progressive type-II censoring are portrayed by Sultan et al. [28]. All the more as of late, Xiuyun and Zaizai [29] concentrated the Bayesian estimation and prediction for the IW distribution under general progressive censoring.

This article can be described as follows. In Section 2 the proposed model is portrayed. The maximum likelihood estimators and the asymptotic variances of the parameters are gotten for SS-PALT under progressive type-II censored data in Section 3. In Section 4, Bayesian estimation for this model is portrayed. Applications are given in Section 5 to

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show the theoretical results. The conclusion of the study is talked about in Section 6.

2 Description of the model

Let the random variable Y representing the lifetime of a product has the IW distribution with the shape and scale parameters β and λ, respectively. The PDF of Y is

$$f_Y(y) = \lambda\beta y^{-\beta-1}e^{-\lambda y^{-\beta}},\ \ y>0,\ \ \beta,\lambda>0, \tag{1}$$

and CDF

$$F_Y(y) = e^{-\lambda y^{-\beta}}. \tag{2}$$

The survival and hazard rate functions of the IW(β, λ) distribution are

$$S_Y(y) = 1-e^{-\lambda y^{-\beta}}, \tag{3}$$

and

$$h_Y(y) = \lambda\beta y^{-\beta-1}e^{-\lambda y^{-\beta}}\left(1-e^{-\lambda y^{-\beta}}\right)^{-1}. \tag{4}$$

The PDF of Y under SS-PALT model can be given by

$$f(y) = \begin{cases} 0, & y\le 0 \\ f_1(y) = f_Y(y), & 0<y\le\tau \\ f_2(y), & y>\tau \end{cases} \tag{5}$$

Where

$$f_2(y) = \lambda\beta(\tau+\alpha(y-\tau))^{-\beta-1}e^{-\lambda(\tau+\alpha(y-\tau))^{-\beta}},$$
$$y>0, \beta>0, \lambda>0, \tag{6}$$

that is obtained by the transformation-variable technique using the density in (1) and the model proposed by DeGroot and Goel [3] which is given by

$$Y = \begin{cases} T & if\ T\le\tau \\ \tau+(T-\tau)/\alpha & if\ T>\tau \end{cases}, \tag{7}$$

where T is the lifetime of the unit under normal utilize condition, τ is the stress change time and α is the acceleration factor, $\alpha>1$.

3 Maximum likelihood estimation

This segment examines the way toward getting the maximum likelihood estimates (MLEs) of the parameters β, λ and α based on progressively type-II censored data under the SS-PALT model. Both point and interval estimations of the parameters are discussed. Let n independent units are placed on a life test with corresponding lifetimes $Y_1, Y_2, \ldots, Y_n$ being independent and identically distributed as IW distribution with PDF given in Eq. (1). We denote the m completely ordered lifetimes by

$$y_{1:m:n} < y_{2:m:n} < \ldots < y_{J:m:n} < \tau < y_{J+1:m:n} \cdots < y_{m:m:n}, \tag{8}$$

where J is the number of failed units at use condition.

3.1 Point estimation

In this subsection, the MLEs of the unknown parameters based on the observed progressive type-II censoring data from IW distribution are given. We provide the likelihood function under SS-PALT as follows:

$$\begin{aligned} L(\alpha,\beta,\lambda|\underline{y}) &\propto \prod_{i=1}^{J} f_1(y_i)[1-F_1(y_i)]^{R_i} \\ &\quad\times\prod_{i=J+1}^{m} f_2(y_i)[1-F_2(y_i)]^{R_i} \\ &\propto (\lambda\beta)^m\prod_{i=1}^{J} y_i^{-\beta-1}e^{-\lambda y_i^{-\beta}}[1-e^{-\lambda y_i^{-\beta}}]^{R_i} \\ &\quad\times\prod_{i=J+1}^{m}\alpha(\Phi_i(\alpha))^{-\beta-1}e^{-\lambda(\Phi_i(\alpha))^{-\beta}} \\ &\quad\times[1-e^{-\lambda(\Phi_i(\alpha))^{-\beta}}]^{R_i} \end{aligned} \tag{9}$$

where

$$y_{1:m:n} < y_{2:m:n} < \ldots < y_{J:m:n} < \tau < y_{J+1:m:n} \cdots < y_{m:m:n},$$

and

$$F_2(y) = e^{-\lambda(\tau+\alpha(y_i-\tau))^{-\beta}}, \Phi_i(\alpha) = \tau+\alpha(y_i-\tau).$$

The natural logarithm of the likelihood function $\ell(\alpha,\beta,\lambda|\underline{y}) = \log L(\alpha,\beta,\lambda|\underline{y})$ is given by

$$\begin{aligned} \ell(\alpha,\beta,\lambda|\underline{y}) &= m\log(\lambda\beta)+(m-J)\log\alpha \\ &\quad-\lambda\Big(\sum_{i=1}^{J} y_i^{-\beta}+\sum_{i=J+1}^{m}(\Phi_i(\alpha))^{-\beta}\Big) \\ &\quad-(\beta+1)\Big(\sum_{i=1}^{J}\log(y_i)+\sum_{i=J+1}^{m}\log(\Phi_i(\alpha))\Big) \\ &\quad+\sum_{i=1}^{J} R_i\log\left(1-e^{-\lambda y_i^{-\beta}}\right) \\ &\quad+\sum_{i=J+1}^{m} R_i\log\left(1-e^{-\lambda(\Phi_i(\alpha))^{-\beta}}\right). \end{aligned} \tag{10}$$

Therefore the likelihood equations for β, λ, and α respectively, given by

$$\begin{aligned} \frac{\partial\ell}{\partial\beta} &= \frac{m}{\beta}+\lambda\Big(\sum_{i=1}^{J} y_i^{-\beta}\log(y_i)+\sum_{i=J+1}^{m}\Phi_i(\alpha)^{-\beta}\log(\Phi_i(\alpha))\Big) \\ &\quad-\Big(\sum_{i=1}^{J}\log(y_i)+\sum_{i=J+1}^{m}\log(\Phi_i(\alpha))\Big) \\ &\quad+\lambda\Big(\sum_{i=1}^{J} R_i\log(y_i)W(y_i,\beta,\lambda) \\ &\quad+\sum_{i=J+1}^{m} R_i\log(\Phi_i(\alpha))W(\Phi_i(\alpha),\beta,\lambda)\Big) = 0. \end{aligned} \tag{11}$$

$$\frac{\partial \ell}{\partial \lambda} = \frac{m}{\lambda} - \sum_{i=1}^{J} y_i^{-\beta} - \sum_{i=J+1}^{m} \left(\Phi_i(\alpha)\right)^{-\beta} + \sum_{i=1}^{J} R_i \times W(y_i, \beta, \lambda) + \sum_{i=J+1}^{m} R_i W(\Phi_i(\alpha), \beta, \lambda) = 0.$$

$$\frac{\partial \ell}{\partial \alpha} = \frac{(m-J)}{\alpha} + Z(t_i, \beta, \lambda) = 0, \qquad (12)$$

where

$$W(t, \beta, \lambda) = \frac{t^{-\beta} e^{-\lambda t^{-\beta}}}{\left(1 - e^{-\lambda t^{-\beta}}\right)}, \; t = y \; or \Phi(\alpha),$$

$$Z(t, \beta, \lambda) = -(\beta+1) \sum_{i=J+1}^{m} \frac{(y_i - \tau)}{\left(1 - e^{-\lambda x_{2i}^{-\beta}}\right)} + \beta\lambda \sum_{i=J+1}^{m} R_i \frac{(y_i - \tau) W(\Phi_i(\alpha), \beta, \lambda)}{\Phi_i(\alpha)}. \qquad (13)$$

From (12) the MLE of α for given β and λ, given by

$$\hat{\alpha}(\beta, \lambda) = \frac{-(m-J)}{Z(t_i, \beta, \lambda)}. \qquad (14)$$

The MLEs $\hat{\beta}$ and $\hat{\lambda}$ can be obtained by solve the system of equations $\frac{\partial \ell}{\partial \beta} = \frac{\partial \ell}{\partial \lambda} = 0$ numerically with any iteration method such as Newton Rapheson metod.

3.2 *Interval estimation*

Here, the approximate confidence intervals of the parameters are inferred in light of the asymptotic distribution of the ML estimators of the elements of the vector of unknown parameters β, λ and α.
The observed Fisher information matrix is described as takes after:

$$\Delta = \begin{pmatrix} -\frac{\partial^2 \ell}{\partial \beta^2} & -\frac{\partial^2 \ell}{\partial \beta \partial \lambda} & -\frac{\partial^2 \ell}{\partial \beta \partial \alpha} \\ -\frac{\partial^2 \ell}{\partial \lambda \partial \beta} & -\frac{\partial^2 \ell}{\partial \lambda^2} & -\frac{\partial^2 \ell}{\partial \lambda \partial \alpha} \\ -\frac{\partial^2 \ell}{\partial \alpha \partial \beta} & -\frac{\partial^2 \ell}{\partial \alpha \partial \lambda} & -\frac{\partial^2 \ell}{\partial \alpha^2} \end{pmatrix}_{(\hat{\beta}, \hat{\lambda}, \hat{\alpha})}, \qquad (15)$$

whose components are given as:

$$\frac{\partial^2 \ell}{\partial \beta^2} = -\frac{m}{\beta^2} - \lambda \sum_{i=1}^{J} y_i^{-\beta} (\log(y_i))^2 + \lambda \sum_{i=J+1}^{m} \left(\Phi_i(\alpha)\right)^{-\beta} (\log(y_i))^2 + \lambda \sum_{i=1}^{J} R_i \log(y_i) W^{(\beta)}(y_i, \beta, \lambda) + \lambda \sum_{i=J+1}^{m} R_i \log(\Phi_i(\alpha)) W^{(\beta)}(\Phi_i(\alpha), \beta, \lambda). \qquad (16)$$

$$\frac{\partial^2 \ell}{\partial \lambda^2} = -\frac{m}{\lambda^2} + \sum_{i=1}^{J} R_i W^{(\lambda)}(y_i, \beta, \lambda) + \sum_{i=J+1}^{m} R_i W^{(\lambda)}(\Phi_i(\alpha), \beta, \lambda). \qquad (17)$$

$$\frac{\partial^2 \ell}{\partial \alpha^2} = -\frac{(m-J)}{\alpha^2} + \beta\lambda \sum_{i=J+1}^{m} R_i \frac{(y_i - \tau) W^{(\alpha)}(\Phi_i(\alpha), \beta, \lambda)}{\Phi_i(\alpha)} + \beta\lambda \sum_{i=J+1}^{m} R_i \frac{(y_i - \tau)^2 W(\Phi_i(\alpha), \beta, \lambda)}{\Phi_i^2(\alpha)}, \qquad (18)$$

$$\frac{\partial^2 \ell}{\partial \beta \partial \lambda} = \frac{\partial^2 \ell}{\partial \lambda \partial \beta} = \sum_{i=1}^{J} y_i^{-\beta} \log(y_i) + \sum_{i=J+1}^{m} \left(\Phi_i(\alpha)\right)^{-\beta} \log(y_i) + \sum_{i=1}^{J} R_i \log(y_i) W(y_i, \beta, \lambda) + \sum_{i=J+1}^{m} R_i \log(\Phi_i(\alpha)) W(\Phi_i(\alpha), \beta, \lambda) \qquad (19)$$

$$\frac{\partial^2 \ell}{\partial \beta \partial \alpha} = \frac{\partial^2 \ell}{\partial \alpha \partial \beta} = -\beta\lambda \sum_{i=J+1}^{m} (y_i - \tau) \left(\Phi_i(\alpha)\right)^{-\beta-1} \log(y_i) - \sum_{i=J+1}^{m} \frac{(y_i - \tau)}{\Phi_i(\alpha)} + \lambda \sum_{i=J+1}^{m} R_i \frac{(y_i - \tau)}{(\Phi_i(\alpha)} + \lambda \sum_{i=J+1}^{m} R_i \frac{W^{(\alpha)}(\Phi_i(\alpha), \beta, \lambda)}{W(\Phi_i(\alpha), \beta, \lambda)}. \qquad (20)$$

$$\frac{\partial^2 \ell}{\partial \lambda \partial \alpha} = \frac{\partial^2 \ell}{\partial \alpha \partial \lambda} = \beta \sum_{i=J+1}^{m} (y_i - \tau) \left(\Phi_i(\alpha)\right)^{-\beta-1} + \sum_{i=J+1}^{m} R_i W^{(\alpha)}(\Phi_i(\alpha), \beta, \lambda). \qquad (21)$$

where

$$W^{(\alpha)}(\Phi_i(\alpha), \beta, \lambda) = \frac{\partial W(\Phi_i(\alpha), \beta, \lambda)}{\partial q}, q = \beta, \lambda \, or \; \alpha.$$

The two-sided 100(1- γ)% normal approximation confidence interval of β , λ and α can be gotten as:

$$\left(\hat{\beta} \pm z_{\gamma/2} \widehat{se}(\hat{\beta})\right), \left(\hat{\lambda} \pm z_{\gamma/2} \widehat{se}(\hat{\lambda})\right) \; and \; \left(\hat{\alpha} \pm z_{\gamma/2} \widehat{se}(\hat{\alpha})\right), \qquad (22)$$

where $\widehat{se}(\cdot)$ is the square root of the diagonal element of Δ^{-1} comparing to every parameter, and $z_{\gamma/2}$ is the quantile $100(1-\gamma/2)$% of the standard normal distribution.

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4 Bayes estimation

In the parameters estimated by the Bayesian approach, the prior ought to mirror our prior information about the parameters. The selection of prior distribution is often based on the type of prior information available to us. When we have little or no data about the parameter, a non-informative prior ought to be utilized. In numerous practical situations, the data about the parameters are accessible in an a free way, see Basu et al. [30]. In this section, we take an informative prior distribution for the parameter λ as the gamma with the scale parameter a and shape parameter b, the hyperparameters a and b can be easily evaluated if we consider any two independent information for λ. When no any prior informations or less informations are available then non-informative prior may be suited. The above considered prior may be regarded as a non-informative prior by setting the values of hyper-parameters are to be zero. Also, we consider, the following non informative priors for β and α.

$$\pi_2^*(\beta) \propto \frac{1}{\beta},\ \pi_3^*(\alpha) \propto \frac{1}{\alpha},\ \beta > 0\ and\, \alpha > 1$$

Therefore, the joint prior of the three parameters β, λ and α can be communicated by

$$\pi^*(\beta,\lambda,\alpha) \propto \lambda^{a-1}\beta^{-1}\alpha^{-1}e^{-\lambda b},\ \beta > 0, \lambda > 0, \alpha > 1 \quad (23)$$

The joint posterior distribution, based on SS-PALT progressively type-II censored data and the set of all parameters are obtained from Equations (8) and (23), and given by

$$\begin{aligned}\pi(\alpha,\lambda,\beta|\underline{y}) \propto\ & \lambda^{m+a-1}\beta^{m-1}\alpha^{m-J} \\ & \times \exp[-\lambda(b+\sum_{i=1}^{J} y_i^{-\beta} + \sum_{i=J+1}^{m}(\Phi_i(\alpha))^{-\beta})] \\ & \times \prod_{i=1}^{J} y_i^{-\beta-1} \prod_{i=J+1}^{m}(\Phi_i(\alpha))^{-\beta-1} \\ & \times \prod_{i=1}^{J}[1-e^{-\lambda y_i^{-\beta}}]^{R_i} \times \prod_{i=J+1}^{m}[1-e^{-\lambda(\Phi_i(\alpha))^{-\beta}}]^{R_i}. \end{aligned} \quad (24)$$

Therefor, the joint posterior distribution function of parameters β , λ and α, can be written as

$$\pi(\alpha,\lambda,\beta|\underline{y}) = \frac{L(\alpha,\lambda,\beta|\underline{y})\,\pi^*(\beta,\lambda,\alpha)}{\int_0^\infty\int_0^\infty\int_0^\infty L(\alpha,\lambda,\beta|\underline{y})\,\pi^*(\beta,\lambda,\alpha)\,d\beta\, d\lambda\, d\alpha}. \quad (25)$$

By and large the Bayes estimates cannot be communicated in express structures. So, approximate Bayes estimates are gotten under noninformative prior and informative prior using Markov chain Monte Carlo (MCMC) method. We propose the accompanying MCMC method to draw samples from the posterior density function and after that to compute the Bayes estimates and the HPD credible intervals. We use the Gibbs sampling procedure to compute HPD credible, the MCMC method, Gibbs with MH algorithm (see Hastings [31]) are used to compute Bayes estimates and HPD credible intervals of the parameters, for more insight about MCMC perusers may see for example, Ahmed [32], Al-Sobhi and Soliman [33].

4.1 Bayesian estimation using MCMC method

from Equations (24) the full conditional probability posterior distribution of β, λ and α are given by

$$\begin{aligned}\pi_\alpha(\alpha|\lambda,\beta,\underline{y}) \propto\ & \alpha^{m-J}\prod_{i=J+1}^{m}(\Phi_i(\alpha))^{-\beta-1} \\ & \times \prod_{i=J+1}^{m}[1-e^{-\lambda(\Phi_i(\alpha))^{-\beta}}]^{R_i}, \end{aligned} \quad (26)$$

$$\begin{aligned}\pi_\lambda(\lambda|\alpha,\beta,\underline{y}) \propto\ & \lambda^{m+a-1} \\ & \times \exp[-\lambda(b+\sum_{i=1}^{J} y_i^{-\beta} + \sum_{i=J+1}^{m}(\Phi_i(\alpha))^{-\beta})] \\ & \times \prod_{i=1}^{J}[1-e^{-\lambda y_i^{-\beta}}]^{R_i} \\ & \times \prod_{i=J+1}^{m}[1-e^{-\lambda(\Phi_i(\alpha))^{-\beta}}]^{R_i}. \end{aligned} \quad (27)$$

and

$$\begin{aligned}\pi_\beta(\beta|\alpha,\lambda,\underline{x}) \propto\ & \exp\left[-\lambda\left(\sum_{i=1}^{J} y_i^{-\beta} + \sum_{i=J+1}^{m}(\Phi_i(\alpha))^{-\beta}\right)\right] \\ & \times \beta^{m-1}\prod_{i=1}^{J} y_i^{-\beta-1}\prod_{i=J+1}^{m}(\Phi_i(\alpha))^{-\beta-1} \\ & \times \prod_{i=1}^{J}[1-e^{-\lambda y_i^{-\beta}}]^{R_i} \\ & \times \prod_{i=J+1}^{m}[1-e^{-\lambda(\Phi_i(\alpha))^{-\beta}}]^{R_i}. \end{aligned} \quad (28)$$

Therefore, the posterior distribution of α, β and λ cannot be diminished scientifically to an outstanding distribution, and in this manner it is unrealistic to test specifically by standard techniques. Therefore, we use the Metropolis-Hasting (MH) algorithm with the normal proposal distribution to generate a random sample from the posterior densities of α, β and α. We utilize the accompanying calculation to register the Bayes estimate of β, λ and α

Step 1: Start with an $(\beta^{(0)}, \lambda^{(0)}, \alpha^{(0)})$.
Step 2: Set $j = 1$.
Step 3: Using MH algorithm, with the proposal distribution $q(\lambda) = N(\lambda^{(j-1)}, \widehat{se}(\hat{\lambda}))$, generate $\lambda^{(j)}$ from $\pi_\lambda(\lambda^{(j-1)}|\alpha^{(j)}, \beta^{(j-1)}, \underline{y})$.
Step 4: Using MH, generate $\beta^{(j)}$ from $\pi_\beta(\beta^{(j-1)}|\alpha^{(j)}, \lambda^{(j)}, \underline{y})$, with the $N(\beta^{(j-1)}, \widehat{se}(\hat{\beta}))$ proposal distribution.
Step 5: Using MH, generate $\alpha^{(j)}$ from $\pi_\alpha(\alpha^{(j-1)}|\beta^{(j)}, \lambda^{(j)}, \underline{y})$, with the $N(\alpha^{(j-1)}, \widehat{se}(\hat{\alpha}))$ proposal distribution.
Step 6: Set $j = j + 1$.

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Step 7: Repeat Steps 2 to 6, N times, and obtain the posterior samples $\beta^{(j)}$, $\lambda^{(j)}$, and $\alpha^{(j)}$, $j = 1,2,3,...,N$.

Control after MCMC sampling is generally performed to dispose the burn-in, i.e., the early samples, on account of the fact that the initial iteration value is dependably arbitrarily chosen and just the steady iteration values are required. The initial iteration value in this study is MLEs rather than arbitrary estimation. Hence,we do not dispose the burn-in. The major advantage of this technique is that the Markov chain would converge fast. These samples are utilized to compute the Bayes estimates, and to construct the HPD credible intervals for β, λ and α.

4.2 Bayes estimation based on balanced loss function

In Bayesian approach, choosing a solitary esteem that speaks to the best estimate of an unknown parameter, one must indicate a loss function. The balanced loss function makes a harmony amongst classical and Bayesian methodologies, and provides an estimate that is a linear combination of ML and Bayes estimates. Ahmadi et al. [34] recommended the utilization of alleged balanced loss function, to be in the form

$$L_{\rho,\omega,\delta_0}(\theta,\delta) = \omega q(\theta)\rho(\delta_0,\delta) + (1-\omega)q(\theta)\rho(\theta,\delta) \quad (29)$$

where $\omega \in [0,1)$, $q(\theta)$ is an appropriate positive weight function and $\rho(\theta,\delta)$ is a subjective loss function when estimating θ by δ. The parameter δ_0 is a chosen prior estimator of θ, obtained for example from the criterion of ML, least squares or moment among others. A general development with regard to Bayes estimators under $L_{\rho,\omega,\delta_0}(\theta,\delta)$ is given, namely by relating such estimators to Bayes solutions to the unbalanced case, i.e., $L_{\rho,\omega,\delta_0}(\theta,\delta)$, with $\omega = 0$. $L_{\rho,\omega,\delta_0}(\theta,\delta)$ can be specialized to various choices of loss function, such as for squared error loss (SEL) and LINEX loss (LINEX) loss functions. By choosing $\rho(\theta,\delta) = (\delta-\theta)^2$ and $q(\theta) = 1$, the Eq (26) decreased to the balanced SEL (BSEL) function, used by Ahmadi et al. [35] furthermore, starting late by Soliman et al. [36] and Ahmed [37], in the form

$$\delta_{\omega,\delta_0}(\theta,\delta) = \omega(\delta-\delta_0)^2 + (1-\omega)(\delta-\theta)^2, \quad (30)$$

and the corresponding Bayes estimate of the function θ is given by

$$\hat{\theta}_{BS} = \omega\delta_0 + (1-\omega)E(\theta|\underline{x}). \quad (31)$$

By choosing $q(\theta)$=1 and $\rho(\theta,\delta) = e^{c(\delta-\theta)}$-$c(\delta-\theta)-1$, in Eq. (29) reduced to the balanced LINEX loss (BLINEXL) function, in the form:

$$\hat{\theta}_{BL} = -\frac{1}{c}log\left[\omega e^{-c\delta_0} + (1-\omega)E\left(e^{-c\theta}|\underline{x}\right)\right], \quad (32)$$

where $c \neq 0$ is the shape parameter of BLINEXL function. The balanced loss functionis gotten extensive consideration in the writing. Rodrigues and Zellner [37] connected the balanced loss function in the estimation of mean time to failure. Gruber [38] considered the observational Bayes and approximate minimum mean SE estimator under a general balanced loss function. Okasha [39] constructed the Bayesian and the E-Bayesian techniques for estimating the scale parameter, reliability and hazard functions of the Lomax distribution in view of type-II censored and by considering the BSE loss function. Under both the balanced loss function Soliman et al. [35] considered the Bayesian inference of the modified Weibull lifetime parameters when the data are progressively censored. Likewise, Ahmed [32] considered Bayesian estimation based balanced loss function under progressive type-II censoring from two-parameter bathtub-shaped lifetime model. The Bayes estimators for the entropy of the Weibull distribution in light of the symmetric and asymmetric loss functions, for instance, the squared error, LINEX and general entropy loss functions, are given by Cho et al. [40]. Using Equations $(25)-(27)$, the approximate Bayes estimates under the BSE and BLINEX loss functions for $\theta = (\beta,\lambda,\alpha)$ are provided, respectively, by:

$$\hat{\theta}_{BS} = \omega\ \hat{\theta}_{ML} + (1-\omega)\frac{\sum_{i=M+1}^{N}\theta^{(i)}}{N}, \quad (33)$$

and

$$\hat{\theta}_{BL} = -\frac{1}{c}log\left[\omega e^{-a\hat{\theta}_{ML}} + (1-\omega)\frac{\sum_{i=M+1}^{N}e^{-a\theta^{(i)}}}{N}\right]. \quad (34)$$

The Bayesian credible interval for the parameter is obtained by using the generated MCMC samples. By arranging the posterior sample $\theta^{(j)}$, $j = 1,2,3,...,N$ as $\theta_{(1)} < \theta_{(2)} < ... < \theta_{\theta_{(N)}}$, using the algorithm proposed by Chen and Shao [42], the $100(1-\gamma/2)\%$ HPD credible intervals for $\theta = (\beta,\lambda,\alpha)$ is given by

$$\left(\theta_{(j)},\theta_{(j+[N(1-\gamma/2)])}\right), \quad (35)$$

where j is chosen such that

$$\theta_{(j+[N(1-\gamma/2)])} - \theta_{(j)} = \min_{1\le i\le \gamma/2N}\left(\theta_{(i+[N(1-\gamma/2)])} - \theta_{(i)}\right); j = 1,2,...,N.$$

5 Application

In this section we lead a simulation study and dissect an illustrative example are directed to investigate the performances of the MLEs and Bayes estimates .

5.1 Simulation study

In this section, simulation studies are directed to evaluate the performances of the MLEs and Bayes estimates in terms of mean square errors (MSEs). For a given n, m, τ, β, λ , α and censoring schemes (CSs) R_i, $i = 1,2,...,m$, th estimation procedure is performed according to the following algorithm.

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Step 1: Set the values of n, m, τ, β, λ , α and R_i, $i = 1, 2, ..., m$.
Step 2: For given values of the prior parameters a and b generate λ from Gamma (a, b)
Step 3: Use the model given by Eq (5) to generate progressively censored data for given n, m the set of data can be considered as

$$y^R_{1:m:n} < y^R_{2:m:n} < ... < y^R_{J:m:n} < \tau < y^R_{J+1:m:n} ... < y^R_{m:m:n}.$$

Where $R = (R_1, R_2, ..., R_m)$, $\sum_{i=1}^{m} R_i = n - m$
Step 4: Use the progressive censored data to compute the MLEs of the parameters. Th Newton Raphson method is applied for solving the nonlinear system in Eqs. (9), (10), and (11) to compute the MLEs of the parameters.
Step 5: Calculate the Bayes estimates of the model parameters relative to BSEL and BLINEXL function based on MCMC approximation with $N = 11000$.
Step 6: Calculate the approximate confience bounds and CIs with confience levels 95% for the three parameters of the model
Step 7: Replicate the Steps $3-6$, 1000 times.
Step 8: Compute the average values of the MSEs associated with the MLEs, BSE and BLINEX loss function of the parameters.
Step 9: Repeat steps $1-8$ with diffrent values of n, m and R_i, $i = 1, 2, ..., m$.

5.2 Simulation procedure

In this subsection results are presented to compute the MLEs of the unknown parameters β and λ as well as the acceleration factor α. The Newton–Raphson technique is connected for solving the nonlinear system to acquire the MLEs of the parameters and register the approximate intervals. We likewise compute the Bayes estimates of the unknown parameters in light of the MCMC sampling strategy. For diffrent values of n, m and R_i , $i = 1, 2, ..., m$, and different censoring schemes; details of the schemes are given in Table 1. Different progressive censoring schemes are considered with notation that $(5, 0^3)$ means $(5, 0, 0, 0)$. we have used $\beta = 1, \lambda = 1$, $\alpha = 1.5$ and $\tau = 1.0$. For Bayesian estimation, we are used informative prior for the parameter λ, we used the hyperparameters value as $a = 1$ and $b = 1$. We compute the average estimates (AE) and the average MSE of the estimates based on 1000 replications. Results are reported in Tables $2-4$. In all cases BSE and BLINEX loss functions, with $\omega = 0.2, 0.8$, are utilized for registering the Bayes estimates. Table 5 reports the average length (AL) of confidence intervals and Bayesian credible intervals with coverage percentages (CP) for β, λ and α. The CIs are computed in view of 10000 MCMC tests. The initial values for the parameters for running the MCMC sampler algorithm are taken to be their MLEs. According to the results from the Tables $2-4$. As sample size n increases, the MSEs of estimators of all the unknown parameters decrease. For settled n, the MSEs of estimators decrease as m increases. From Table 5 we find that the CPs of the approximate confidence intervals and Bayesian CIs are very near ostensible level. Also, in most cases, the Bayesian CIs are marginally shorter length than that of the approximate confidence intervals.Henceforth, we prescribe to utilize Bayesian CIs over approximate confidence intervals. When $\omega = 0.8$ all results of Bayes estimates under both BSE and BLINEX loss functions for the parameters are very like corresponding MLEs.

Table 1: Several CSs for the simulation study.

CS	n	m	R	CS	n	m	R
[1]	50	40	$(10, 0^{39})$	[8]	70	50	$(20, 0^{49})$
[2]			$(0^{15}, 1^{10}, 0^{15})$	[9]			$(0^{15}, 1^{20}, 0^{15})$
[3]			$(0^{39}, 10)$	[10]			$(0^{49}, 20)$
[4]	50	45	$(5, 0^{44})$	[11]	70	60	$(10, 0^{59})$
[5]			$(0^{20}, 1^5, 0^{20})$	[12]			$(0^{25}, 1^{10}, 0^{25})$
[6]			$(0^{44}, 5)$	[13]			$(0^{59}, 10)$
[7]	50	50	(0^{50})	[14]	70	70	(0^{70})

5.3 Numerical example

We simulate a set of lifetime data following IW distribution under progressive Type-II censoring in SS-ALT. The data are presented as: 0.120162, 0.285467, 0.294413, 0.310609, 0.417991, 0.454602, 0.519122, 0.525779, 0.554423, 0.613962, 0.63692, 0.754227, 0.754288, 0.77136, 0.83592, 0.874847, 0.881625, 0.916359, 0.93054, 0.951793, 0.952627, 0.963777. this simulated observations based on $n = 60$, $m = 55$, $\beta = 1$, $\lambda = 1$, $\alpha = 1.4$, and $R = (1^5, 0^{50})$. The MLEs of model parameters and acceleration factor β, λ and α are

$$\hat{\beta} = 0.9778, \qquad \hat{\lambda} = 1.0162, \qquad \hat{\alpha} = 1.30554.$$

The inverse of Fisher information matrix Δ^{-1} is given as follows:

$$\hat{\Delta}^{-1} = \begin{pmatrix} 0.0140 & -0.0099 & -0.0325 \\ -0.0099 & 0.0252 & 0.0341 \\ -0.0325 & 0.0341 & 0.2065 \end{pmatrix},$$

The estimated variances of estimates of $\hat{\beta}$; $\hat{\lambda}$ and $\hat{\alpha}$ are

$$\widehat{var}(\hat{\beta}) = 0.0140, \quad \widehat{var}(\hat{\lambda}) = 0.0252, \quad \widehat{var}(\hat{\alpha}) = 0.2065.$$

To find the standard errors of $\hat{\beta}$, $\hat{\lambda}$ and $\hat{\alpha}$, we take the square root of the diagonal elements of $\hat{\Delta}^{-1}$, 95% confidence intervals for the parameters β, λ and the acceleration factor α are$0.7455 \leq \beta \leq 1.2101$, $0.7050 \leq \lambda \leq 1.3275$ and $0.4147 \leq \alpha \leq 2.1964$. Now we compute the Bayes estimates of β, λ and α, we assume $a = b = 0$. Fig. (1), (3) and (5) shows the trace plots of

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Table 2: AE and MSEs of ML and Bayes estimates of α.

CS	MLE	MCMC								
		BSEL			BLINEX					
		$\omega=0$	$\omega=0.2$	$\omega=0.8$	$\omega=0$		$\omega=0.2$		$\omega=0.8$	
					$c=1$	$c=5$	$c=1$	$c=5$	$c=1$	$c=5$
[1]	1.4100 (0.1232)	1.2420 (0.0937)	1.2756 (0.0886)	1.3764 (0.1062)	1.1766 (0.1317)	1.2613 (0.3112)	1.2164 (0.1165)	1.2973 (0.2755)	1.3551 (0.1072)	1.1892 (0.1384)
[2]	1.3188 (0.1661)	1.2002 (0.1259)	1.2239 (0.1255)	1.2951 (0.1496)	1.4906 (0.0201)	1.2347 (0.3468)	1.4820 (0.0288)	1.2679 (0.3133)	1.3993 (0.0876)	1.1395 (0.1872)
[3]	1.3132 (0.1745)	1.1911 (0.1334)	1.2155 (0.1321)	1.2887 (0.1568)	1.4864 (0.0278)	1.2140 (0.3700)	1.4788 (0.0369)	1.2474 (0.3348)	1.3959 (0.0960)	1.2215 (0.1993)
[4]	1.3933 (0.1308)	1.2460 (0.0930)	1.2754 (0.0908)	1.3638 (0.113435)	1.1837 (0.1289)	1.2774 (0.2962)	1.2196 (0.1173)	1.2123 (0.2633)	1.3442 (0.1148)	1.2950 (0.1407)
[5]	1.2357 (0.2135)	1.1391 (0.1711)	1.1584 (0.1713)	1.2164 (0.1968)	1.2727 (0.2226)	1.2013 (0.3913)	1.2003 (0.2114)	1.2405 (0.3503)	1.1971 (0.2029)	1.2650 (0.2496)
[6]	1.3936 (0.1288)	1.2774 (0.0832)	1.3007 (0.0850)	1.3704 (0.1124)	1.2217 (0.1113)	1.2314 (0.2470)	1.2516 (0.1056)	1.0643 (0.2202)	1.3540 (0.1138)	1.2324 (0.1277)
[7]	1.2999 (0.1639)	1.2067 (0.1232)	1.2253 (0.1245)	1.2812 (0.1489)	1.1467 (0.16131)	1.2844 (0.2965)	1.1731 (0.1536)	1.2232 (0.2624)	1.2643 (0.1529)	1.1451 (0.1829)
[8]	1.5003 (0.0996)	1.3074 (0.0601)	1.3460 (0.0567)	1.4617 (0.0804)	1.2496 (0.0863)	1.2483 (0.2239)	1.2928 (0.0747)	1.2851 (0.1938)	1.4419 (0.0786)	1.2820 (0.0870)
[9]	1.1390 (0.2851)	1.0785 (0.2306)	1.0906 (0.2349)	1.1269 (0.2676)	1.0140 (0.2863)	1.2209 (0.4613)	1.0350 (0.2790)	1.2848 (0.4230)	1.1093 (0.2760)	1.2947 (0.3279)
[10]	1.0878 (0.3327)	1.0350 (0.2738)	1.0455 (0.2784)	1.0772 (0.3138)	1.0698 (0.3351)	1.1806 (0.5547)	1.0893 (0.3274)	1.1084 (0.5196)	1.0592 (0.3233)	1.1465 (0.3812)
[11]	1.4660 (0.0920)	1.3192 (0.0544)	1.3486 (0.0538)	1.4367 (0.0763)	1.2651 (0.0775)	1.0751 (0.1993)	1.3002 (0.0699)	1.1099 (0.1728)	1.4199 (0.0758)	1.2896 (0.0829)
[12]	1.2218 (0.2173)	1.1463 (0.1730)	1.1614 (0.1756)	1.2067 (0.2021)	1.0863 (0.2173)	1.0288 (0.3633)	1.1096 (0.2100)	1.0654 (0.3282)	1.1902 (0.2078)	1.0791 (0.2447)
[13]	1.1427 (0.2720)	1.1148 (0.2171)	1.1204 (0.2244)	1.1372 (0.2573)	1.0620 (0.2574)	1.0961 (0.4126)	1.0760 (0.2562)	1.0222 (0.3866)	1.1241 (0.2640)	1.0453 (0.2944)
[14]	1.3831 (0.1302)	1.2861 (0.0810)	1.3055 (0.0846)	1.3637 (0.1141)	1.235 (0.1055)	1.0593 (0.2232)	1.2609 (0.1025)	1.0904 (0.2001)	1.3491 (0.1155)	1.2450 (0.1247)

With each scheme the first row represents the average relative estimate and the second row MSE is reported with in bracket immediately below.

10000 MCMC samples for posterior distribution of β, λ and α. It show that the MCMC procedure converges very well. Fig. (2), (4) and (6) provide the histogram plots of generated β, λ and α. It is observed that the histograms of the generated posteriors match quite well with the theoretical posterior density functions. Therefore, MCMC samples can be used for estimate the unknown parameters and constructing the approximate CIs, under squared error loss function ($\omega=0$), we compute the approximate Bayes estimates of β, λ and α using MCMC method and they are

$$\hat{\beta}_{BS}=0.9490, \qquad \hat{\lambda}_{BS}=1.0546, \qquad \hat{\alpha}_{BS}=1.4976,$$

and the associated 95% symmetric Cls are given by

$$0.7256 \leq \beta \leq 1.1873,\ 0.7613 \leq \lambda \leq 1.3899 \quad and \quad 0.7085 \leq \alpha \leq 2.7042,$$

Under LINEX loss function ($\omega=0$), we compute the approximate Bayes estimates of β, λ and α, and they are

$$\hat{\beta}_{BL}=0.9548, \qquad \hat{\lambda}_{BL}=1.0469, \qquad \hat{\alpha}_{BL}=1.4592,$$

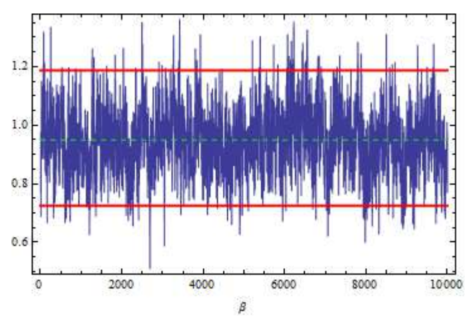

Fig. 1 Simulation number of β obtained by MCMC method.

We also compute the approximate Bayes estimates of β, λ and α under both BSE and BLINEX loss function with $\omega=0.2,0,8$ and they are in Table 6.

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Table 3: AE and MSEs of ML and Bayes estimates of λ .

CS	MLE	MCMC									
		BSEL			BLINEX						
		$\omega=0$	$\omega=0.2$	$\omega=0.8$	$\omega=0$		$\omega=0.2$		$\omega=0.8$		
					$c=1$	$c=5$	$c=1$	$c=5$	$c=1$	$c=5$	
[1]	1.1371	1.1304	1.1317	1.1358	1.1142	1.0548	1.1186	1.0684	1.1324	1.1171	
	(0.0468)	(0.0419)	(0.0427)	(0.0457)	(0.0368)	(0.0230)	(0.0385)	(0.0257)	(0.0445)	(0.0392)	
[2]	1.0463	1.0450	1.0452	1.0460	1.1219	0.9787	1.1099	0.9899	1.0655	1.0298	
	(0.0322)	(0.0290)	(0.0294)	(0.0312)	(0.0496)	(0.0225)	(0.0458)	(0.0230)	(0.0351)	(0.0283)	
[3]	0.9789	0.9802	0.9799	0.9792	1.0560	0.9169	1.0431	0.9274	0.9975	0.9643	
	(0.0277)	(0.0238)	(0.0244)	(0.0267)	(0.0325)	(0.0262)	(0.0308)	(0.0256)	(0.0276)	(0.0262)	
[4]	1.0488	1.0424	1.0437	1.0475	1.0288	0.9787	1.0326	0.9903	1.0446	1.0317	
	(0.0289)	(0.0256)	(0.0259)	(0.0279)	(0.0237)	(0.0201)	(0.0243)	(0.0205)	(0.0275)	(0.0253)	
[5]	1.0343	1.0335	1.0336	1.0342	1.0197	0.9686	1.0224	0.9795	1.0312	1.0184	
	(0.0283)	(0.0257)	(0.0260)	(0.0275)	(0.0241)	(0.0212)	(0.0246)	(0.0214)	(0.0271)	(0.0252)	
[6]	0.8247	0.8267	0.8263	0.8251	0.8178	0.7774	0.8178	0.7855	0.8229	0.8136	
	(0.0557)	(0.0534)	(0.0536)	(0.0549)	(0.0563)	(0.0692)	(0.0559)	(0.0663)	(0.0555)	(0.0580)	
[7]	0.9788	0.9797	0.9795	0.9789	0.9675	0.9222	0.9696	0.9317	0.9764	0.9652	
	(0.0279)	(0.0255)	(0.0258)	(0.0272)	(0.0252)	(0.0269)	(0.0254)	(0.0263)	(0.0271)	(0.0266)	
[8]	1.2377	1.2213	1.2246	1.2344	1.2085	1.1604	1.2141	1.1731	1.2316	1.2187	
	(0.0799)	(0.0690)	(0.0709)	(0.0773)	(0.0628)	(0.0426)	(0.0657)	(0.0475)	(0.0759)	(0.0689)	
[9]	1.0789	1.0808	1.0805	1.0793	1.0695	1.0267	1.0712	1.0355	1.0769	1.0665	
	(0.0292)	(0.0264)	(0.0267)	(0.0283)	(0.0242)	(0.0184)	(0.0249)	(0.0196)	(0.0279)	(0.0257)	
[10]	1.0949	1.1014	1.1001	1.0962	1.0899	1.0470	1.0908	1.0639	1.0938	1.0836	
	(0.0698)	(0.0681)	(0.0682)	(0.0692)	(0.0644)	(0.0532)	(0.0652)	(0.0579)	(0.0685)	(0.0650)	
[11]	1.0971	1.0883	1.0900	1.0953	1.0778	1.0381	1.0815	1.0481	1.0931	1.0831	
	(0.0310)	(0.0270)	(0.0277)	(0.0300)	(0.0247)	(0.0181)	(0.0257)	(0.0195)	(0.0295)	(0.0269)	
[12]	1.0594	1.0554	1.0562	1.0586	1.0452	1.0065	1.0479	1.0156	1.0564	1.0470	
	(0.0256)	(0.0231)	(0.0233)	(0.0248)	(0.0215)	(0.0174)	(0.0220)	(0.0181)	(0.0245)	(0.0227)	
[13]	1.1123	1.1170	1.1161	1.1132	1.1064	1.0663	1.1075	1.0742	1.1110	1.1016	
	(0.0231)	(0.0243)	(0.0239)	(0.0231)	(0.0217)	(0.0136)	(0.0217)	(0.0148)	(0.0225)	(0.0202)	
[14]	0.9802	0.9784	0.9788	0.9798	0.9694	0.9353	0.9715	0.9431	0.9779	0.9697	
	(0.0201)	(0.0180)	(0.0182)	(0.0195)	(0.0179)	(0.0194)	(0.0182)	(0.0190)	(0.0195)	(0.0193)	

With each scheme the first row represents the average relative estimate and the second row MSE is reported with in bracket immediately below.

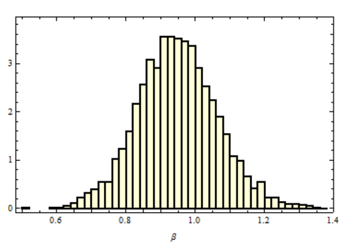

Fig. 2 Histogram of β obtained by MCMC method.

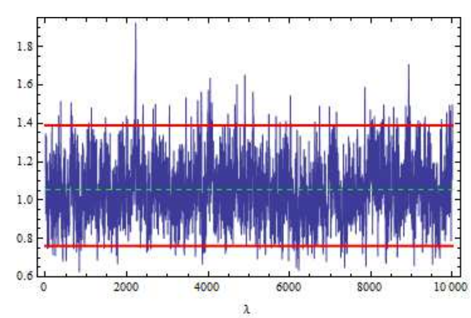

Fig. 3 Simulation number of λ obtained by MCMC method.

6 Conclusion

In this paper, we provided SS-PALT models under progressive type-II censoring when the observed data come from IW distribution. We determined the MLEs and asymptotic confidence intervals of the obscure parameters. We inferred Bayes estimators of the parameters and the acceleration parameter using non-informative and gamma informative priors under both BSE and BLINEX loss functions. We additionally proposed a few distinct strategies for constructing CIs for the parameters and the acceleration parameter. We did a recreation study to think about the execution of every one of these systems. From the simulation study, we watch that the Bayes estimates are superior to MLEs as far as

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Table 4: AE and MSEs of ML and Bayes estimates of β

CS	MLE	MCMC								
		BSEL			BLINEXL					
		$\omega=0$	$\omega=0.2$	$\omega=0.8$	$\omega=0$		$\omega=0.2$		$\omega=0.8$	
					$c=1$	$c=5$	$c=1$	$c=5$	$c=1$	$c=5$
[1]	1.0274	1.0538	1.0485	1.0327	1.0428	1.0028	1.0396	1.0068	1.0304	1.0214
	(0.0245)	(0.0230)	(0.0230)	(0.0239)	(0.0209)	(0.0158)	(0.0214)	(0.0169)	(0.0235)	(0.0218)
[2]	1.0610	1.0779	1.0745	1.0644	1.1452	1.0218	1.1314	1.0280	1.0817	1.0511
	(0.0316)	(0.0281)	(0.0285)	(0.0305)	(0.0517)	(0.0172)	(0.0474)	(0.0188)	(0.0351)	(0.0269)
[3]	1.0626	1.0780	1.0749	1.0657	1.1479	1.0204	1.1339	1.0272	1.0835	1.0520
	(0.0364)	(0.0318)	(0.0324)	(0.0352)	(0.0569)	(0.0202)	(0.0525)	(0.0220)	(0.0400)	(0.0310)
[4]	1.0455	1.0693	1.0646	1.0503	1.0588	1.0201	1.0560	1.0242	1.0480	1.0393
	(0.0230)	(0.0224)	(0.0222)	(0.0226)	(0.0202)	(0.0144)	(0.0205)	(0.0154)	(0.0221)	(0.0204)
[5]	0.9809	1.0001	0.9963	0.9847	0.9893	0.9499	0.9875	0.9550	0.9824	0.9733
	(0.0280)	(0.0232)	(0.0239)	(0.0268)	(0.0223)	(0.0214)	(0.0232)	(0.0220)	(0.0266)	(0.0257)
[6]	1.2710	1.2823	1.2801	1.2733	1.2681	1.2158	1.2685	1.2246	1.2702	1.2570
	(0.1067)	(0.1068)	(0.1064)	(0.1063)	(0.0974)	(0.0672)	(0.0988)	(0.0724)	(0.1043)	(0.0949)
[7]	1.0921	1.1071	1.1041	1.0951	1.0958	1.0544	1.0949	1.0605	1.0927	1.0828
	(0.0364)	(0.0340)	(0.0342)	(0.0356)	(0.0304)	(0.0201)	(0.0313)	(0.0221)	(0.0348)	(0.0314)
[8]	0.9663	0.9951	0.9893	0.9720	0.9877	0.9599	0.9833	0.9608	0.9704	0.9645
	(0.0149)	(0.0121)	(0.0124)	(0.0140)	(0.0118)	(0.0118)	(0.0122)	(0.0122)	(0.0140)	(0.0140)
[9]	0.9067	0.9184	0.9161	0.9091	0.9099	0.8784	0.9092	0.8831	0.9073	0.8999
	(0.0283)	(0.0218)	(0.0229)	(0.0268)	(0.0227)	(0.0275)	(0.0236)	(0.0273)	(0.0270)	(0.0276)
[10]	0.9260	0.9318	0.9306	0.9272	0.9230	0.8905	0.9235	0.8963	0.9253	0.9174
	(0.0741)	(0.0647)	(0.0664)	(0.0720)	(0.0646)	(0.0654)	(0.0662)	(0.0664)	(0.0719)	(0.0713)
[11]	1.0149	1.0382	1.0335	1.0195	1.0308	1.0034	1.0276	1.0052	1.0180	1.0121
	(0.0148)	(0.0138)	(0.0137)	(0.0144)	(0.0128)	(0.0104)	(0.0130)	(0.0110)	(0.0142)	(0.0136)
[12]	0.9250	0.9407	0.9376	0.9282	0.9330	0.9044	0.9313	0.9077	0.9265	0.9200
	(0.0253)	(0.0193)	(0.0203)	(0.0239)	(0.0197)	(0.0223)	(0.0206)	(0.0226)	(0.0240)	(0.0242)
[13]	0.9504	0.9567	0.9554	0.9517	0.9492	0.9212	0.9494	0.9263	0.9501	0.9437
	(0.0150)	(0.0129)	(0.0132)	(0.0144)	(0.0131)	(0.0153)	(0.0134)	(0.0149)	(0.0145)	(0.0147)
[14]	1.0621	1.0770	1.0740	1.0650	1.0694	1.0408	1.0678	1.0443	1.0634	1.0569
	(0.0208)	(0.0188)	(0.0190)	(0.0202)	(0.0173)	(0.0125)	(0.0177)	(0.0136)	(0.0199)	(0.0185)

With each scheme the first row represents the average relative estimate and the second row MSE is reported with in bracket immediately below.

Table 5:AL and CPs of the 95% CIs for α , λ and β .

CS	MLE			MCMC		
	α	λ	β	α	λ	β
[1]	0.9640 (1.6153)	0.9580 (0.6593)	0.9500 (0.7742)	0.9840 (1.315)	0.9740 (0.5753)	0.9700 (0.7063)
[2]	0.9180 (1.3705)	0.9600 (0.6926)	0.9460 (0.7280)	0.9840 (1.3119)	0.9760 (0.6016)	0.9700 (0.6541)
[3]	0.9100 (1.4747)	0.9460 (0.7017)	0.9380 (0.7014)	0.9620 (1.3303)	0.9700 (0.6112)	0.9480 (0.6457)
[4]	0.9560 (1.4665)	0.9620 (0.6503)	0.9700 (0.7198)	0.9860 (1.2968)	0.9780 (0.5648)	0.9780 (0.6423)
[5]	0.9460 (1.3934)	0.9420 (0.6601)	0.9380 (0.7142)	0.9780 (1.3550)	0.9620 (0.5716)	0.9620 (0.6450)
[6]	0.9380 (1.2006)	0.9420 (0.7544)	0.9380 (0.6304)	0.9700 (1.2306)	0.9480 (0.6555)	0.9480 (0.5634)
[7]	0.9620 (1.2244)	0.9700 (0.6702)	0.9780 (0.6782)	0.9860 (1.2834)	0.9840 (0.5821)	0.9780 (0.6067)
[8]	0.9740 (1.4693)	0.9620 (0.7064)	0.9720 (0.5402)	0.9960 (1.243)	0.9760 (0.6260)	0.9760 (0.4736)
[9]	0.9760 (1.1900)	0.9480 (0.6444)	0.9700 (0.5783)	0.9780 (1.3487)	0.9620 (0.5874)	0.9760 (0.5076)
[10]	0.9620 (1.3504)	0.9380 (0.6414)	0.9620 (0.5759)	0.9700 (1.1738)	0.9480 (0.5849)	0.9620 (0.5081)
[11]	0.9820 (1.2329)	0.9340 (0.6316)	0.968 (0.5386)	0.9940 (1.2081)	0.9660 (0.5629)	0.9800 (0.4718)
[12]	0.9760 (1.2952)	0.9160 (0.6168)	0.9620 (0.5476)	0.9800 (1.1136)	0.9620 (0.5566)	0.9780 (0.4820)
[13]	0.9620 (1.7834)	0.9100 (0.6194)	0.9560 (0.5328)	0.9780 (1.2256)	0.9460 (0.5694)	0.9760 (0.4743)
[14]	0.9800 (1.2843)	0.9520 (0.5725)	0.972 (0.5451)	0.9960 (1.1864)	0.9560 (0.5243)	0.9960 (0.4804)

The number out side the bracket is the coverage probability and the number in the bracket is the length .

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Table 6:Bayes estimates under BSEL and BLINEX for a simulated data.

	ω	BSEL	BLINEX	
			$c=1$	$c=5$
α	0.2	1.4592	1.3638	1.1305
	0.8	1.4208	1.3489	1.1620
λ	0.2	1.0469	1.0367	0.9996
	0.8	1.0393	1.0316	1.0036
β	0.2	0.9548	0.9494	0.9276
	0.8	0.9605	0.9564	0.9390

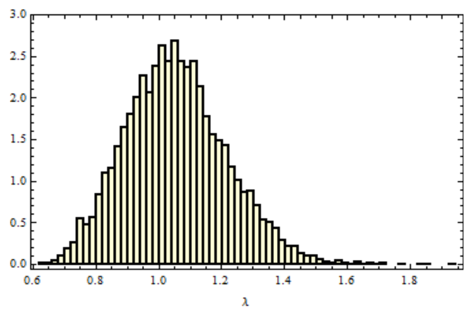

Fig. 4 Histogram of λ obtained by MCMC method.

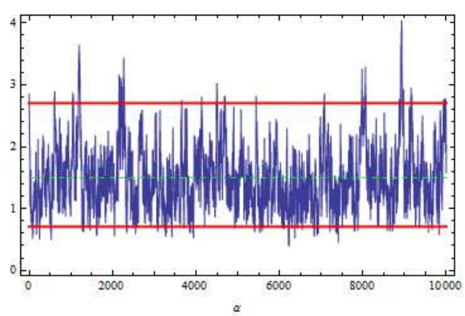

Fig. 5 Simulation number of α obtained by MCMC method.

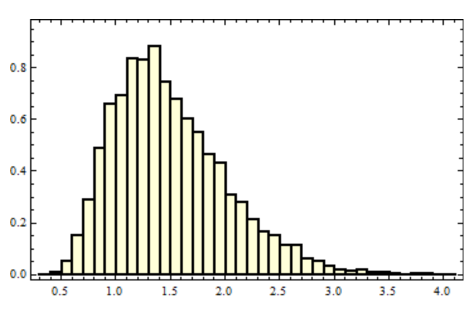

Fig. 6 Histogram of $alpha$ obtained by MCMC method.

MSEs. Likewise, the HPD credible intervals based on MH algorithm are are superior to asymptotic confidence intervals in respect of AL and CP. Besides, the length of the confidence interval is likewise diminishes as the sample size increases and CP is close to the nominal value in all sets of parameters considered here. We introduced reenacted case to represent every one of the techniques for derivation examined here and additionally to bolster the conclusions drawn.

1.For fixed values of the sample size, by increasing the observed failure times the MSEs decrease.
2.For fixed values of the sample size, the scheme II in which the censoring occurs after the first observed failure gives more accurate results through the MSEs than the other schemes.
3.Results in the censoring schemes III and IV are closed to other.
4.The approximate CIs and bootstrap-*t* CIs give more accurate results than the bootstrap-p CIs since the lengths of the former are less than the lengths of latter, for different sample sizes, and different schemes.
5.For fixed sample sizes and observed failures, the second scheme II , in which censoring occurs after the first observed failure, gives smallest lengths of the CIs for all methods.

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