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Application Of Backstep Control In Diseased Prey-Predator System

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Abstract: In this paper,backstep control is employed to a three species diseased biological system having two prey viz. susceptible and infected prey and a predator. Using the back stepping control design, the controllability conditions are framed. The non-linear feedback control approach is implemented to derive the global asymptotic stability conditions for the biological system. Diverse set of parametric value are used and the corresponding chaotic behavior of the system is obtained. Finally numerical simulations by Matlab are executed to explore the effect of back stepping control in the system.

Keywords: Prey-predator system, Backstep control, Global stability.

1 Introduction

Mathematical models have been widely in use to exaggerate the ecological populations that is afflicted by various infectious diseases. These diseases play a key role in regulating the population size of the species. These models are described by differential equations among which the prey-predator model is of specific interest to ecologists and mathematicians.

In the recent years, the study of the infected prey-predator system has been the area of interest focused .eco-epidemiology is the ultimate genre researches by theoretical ecologists and epidemiologists. The influence of epidemics on predation was first studied by Anderson and May [20,21]. Arino et al [19]; Beltrami and Carroll [5] and Venturino [6], [17] explaines that the ecosystem of the disease spread population can be described by eco-epidemic models. Anderson and May [21]. Hadeler and Freedman [16] proved that the prey-predator populations are destabilized by the invading diseases. The infection in the predator species can experience a stabilizing effect as explained by Hilker and Schmitz [8]. Majority of the eco-epidemiological models that exist have an infected prey population. Chattopadhyay and Arino [13] investigated the epidemics in the predator-prey models with infected predator. Hsieh and Hsiao [24] considered a predator-prey model with infection in both populations to account for the possibility of a contagious disease crossing species barrier from prey to predator. Venturino [18] portrayed a similar idea on diseases in interacting species model. In the subsequent time, many authors Xiao et al [25], Bera et al [26] proposed and studied different prey-predator models in the presence of disease.

Though feedback controllers are powerful in controlling linear,nonlinear and several uncertain systems,it is more sophisticated. It is currently in trend to investigate ecological systems with backstep controllers. The controllability conditions for chaotic dynamics control are determined using backstep controllers with the expectation that this may improve the stability analysis of the system. In control theory, backstepping is a technique for designing stability controls for non-linear dynamical system. This approach is a recursive method for stabilizing the origin of a system. The control process terminates when the final external control is reached.

Hwang et al [2] proposed a linear suitable to control chaotic systems about the fixed points or limit cycles. Chui and Cheng [3,4,10] analyzed and ensured the possibility of applying the conventional feedback controllers to chaotic systems. John J.K. and Amritkar [14], Sorrentino et al[9] suggested an adaptive synchronization and control method. Major worksin

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nonlinear systems related to chaotic control problems by Wang et al[11]. An exclusive method have been suggested by Wang [23] to control chaos.Awad Ei-Gohary [1] proved that three species prey-predator population can be asymptotically stabilized to its trivial equilibrium point using feedback control. Pyragas [15] has discussed about how to choose the feedback coefficient and delay time for a system to be effective.

The rest of the paper is structured as follows. In section 2, the mathematical model with of the biological system has been investigated. The positivity and boundedness of the system is explained in section 3. In section 4, the Existence of equilibrium points with feasibility conditions is analyzed. Local stability analysis of all possible equilibrium points are discussed in section 5. Global stability analysis for the equilibrium points are analyzed in section 6.The backstep controlled system is explained in section 7. Numerical simulations are carried out to illustrate the chaotic behavior and justify the numerical manipulations prey -predator model in section 8.

2 Mathematical model

In this paper, a continuous time prey-predator system with two prey viz. susceptible and infected prey and a predator is taken into account. It is assumed that the susceptible prey population is developed on the basis of logistic law and only the infected prey is predated. Now to formulate the mathematical model of a prey-predator system with disease in prey population, we make the following assumptions:

i) The prey population grows logistically with intrinsic growth rate r and environmental carrying capacity k

ii) In the presence of infection, the prey population is divided into two groups namely susceptible prey denoted by $X(t)$ and infected prey denoted by $Y(t)$ at all time, the total population is $P(t) = X(t) + Y(t)$

iii) The disease is spread among the prey population alone and the disease is not genetically inherited. The infected prey populations do not recover or become immune.

iv) Assume that the disease transmission follows the simple law of mass action $aX(t)Y(t)$ where a as the transmission rate.

v) Assume that the predator population consumes only on infected prey with Beddington-De angelis functional response function is of the form

$$f(Y,Z) = \frac{bZ}{1+bhY+Z}$$

Where b the total is attack rate for predator or predation coefficient and is the handling time of predator to prey. The model becomes:

$$\begin{aligned}
\frac{dX}{dT} &= rX\left(1-\frac{X}{k}\right) - aXY - H_1X \\
\frac{dY}{dT} &= aXY - \frac{bYZ}{1+bhY+z} - H_2Y \qquad (1)\\
\frac{dZ}{dT} &= \frac{ebYZ}{1+bhY+Z} - dZ
\end{aligned}$$

Here the parameters $X(t), Y(t), Z(t)$ denote the susceptible, infected prey and predator population respectively. The parameters r, k, a, d, e, H_1, H_2 denotes the growth rate of susceptible prey population, the environmental carrying capacity, the rate of transmission from susceptible to infected prey population, death rate of predator, the conversion efficiency rate and harvesting rate of susceptible and infected prey respectively.

To minimize the number of parameters involved with the model system, it is extremely useful to write the system in non-dimensionalzed form. For this purpose introduce the variables X, Y and T as follow

$$x \to \frac{X}{k}, y \to \frac{Y}{k}, z \to \frac{Z}{bhk} \text{ and } t \to Tr \qquad (2)$$

In terms of the non-dimensionalized variables the model system (1) become

$$\begin{aligned}
\frac{dx}{dt} &= x(1-x) - \alpha xy - h_1x \equiv xp(x,y) \\
\frac{dy}{dt} &= \alpha xy - \frac{\beta yz}{c+y+z} - h_2y \equiv yq(x,y,z) \qquad (3)\\
\frac{dz}{dt} &= \frac{\delta yz}{c+y+z} - yz \equiv zr(x,y,z)
\end{aligned}$$

Where the relation between the dimensional and non-dimensional parameters are given by:

$$\alpha = \frac{ak}{r}, \beta = \frac{b}{r}, e = \frac{e}{rh}, h_1 = \frac{H_1}{r}, h_2 = \frac{H_2}{r}, \gamma = \frac{d}{r}$$

Subject to the positive initial conditions:

$$x(0) \geq 0,\ y(0) \geq 0,\ z(0) \geq 0 \qquad (4)$$

The system (3) is defined on the set

$$R^3_+ = \left\{(x,y,z) \in R^3 / x \geq 0, y \geq 0, z \geq 0\right\} \qquad (5)$$

3 Positivity and boundedness of solution:

It is important to show positivity and boundedness for the system (3) as they represent populations. Positivity implies that the population survives and boundedness may be interpreted as a natural restriction to growth as a consequence of limited resources.

In the following theorem, we show that solution of system (3) together with initial condition (4) is positive and bounded to establish that the model formulation is ecologically meaningful.

Theorem 3.1.

All solution of $(x(t), y(t), z(t))$ of the system (3) with initial condition (4) are positive for all $t \geq 0$

Proof. Equation (3) together with initial condition (4) gives

$$x(t) = x(0)exp\int_0^t p(x(s), y(s))ds > 0$$

$$y(t) = y(0)exp\int_0^t q(x(s), y(s), z(s))ds > 0$$

$$z(t) = z(0)exp\int_0^t r(x(s), y(s), z(s))ds > 0$$

Hence all solutions starting from interior of the first octant(Int R^3_+) remain in it for future time.

Theorem 3.2.

All the non-negative solutions of the model system (3) that state in R^3_+ are uniformly bounded

Proof. Let $x(t), y(t), z(t)$ be any solution of the system (3) Since from the first equation of model (3)

$$\frac{dx}{dt} \leq x(1-x)$$

we have $\lim_{t\to\infty} sup\, x(t) \leq 1$

Let $\xi = x + y + \frac{\beta}{\delta}z$

Therefore

$$\frac{d\xi}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \frac{\beta}{\delta}\frac{dz}{dt} \tag{6}$$

Substituting (3) in equation (6),we get

$$\begin{aligned}\frac{d\xi}{dt} + m\xi &= x((1+m-h_1)-x) + (m-h_2)y \\ &\frac{\beta z}{\delta}(m-\gamma) \\ &\leq x((1+m-h_1)-x)\end{aligned}$$

$$\frac{d\xi}{dt} + m\xi \leq \mu \text{ since } (1+m-h_1) = \mu \text{ (say))}$$

Applying Lemma on differential inequalities Birkoff [12], we obtain

$$0 \leq \xi(x,y,z) \leq \frac{\mu}{m}\left(1-e^{-m}\right) + \frac{\xi(x(0), y(0), z(0))}{e^m}$$

and for $t \to \infty$ we have

$$0 \leq \xi(x,y,z) \leq \frac{\mu}{m}$$

Thus all solutions of system (3) enter into the region

$$\Gamma = \left\{(x,y,z) \in R^3_+ : 0 \leq \xi \leq \frac{\mu}{m} + \varepsilon, \forall \varepsilon > 0\right\} \tag{7}$$

This completes the proof.

4 Existence of equilibrium points with feasibility conditions

The system (3) may have the following equilibrium points.

i) The trivial equilibrium point $E_0(0,0,0)$ always exists.

ii) The disease and predator free equilibrium point $E_1(\bar{x}, 0, 0)$ exists where $\bar{x} = 1 - h_1$ provided with the condition.

$$1 - h_1 > 0 \tag{8}$$

iii) In the absence of predator species the susceptible and infected prey species can survive. The predator free equilibrium point $E_2(\bar{x}, \bar{y}, 0)$ exists where $(\bar{x}, \bar{y})$ are given as $\bar{x} = \frac{h_2}{\alpha}, \bar{y} = \frac{\alpha(1-h_1)-h_2}{\alpha^2}$

Provided with the conditions

$$\alpha(1-h_1) > h_2 \tag{9}$$

iv) The co-existence positive equilibrium point $E_3(x^*, y^*, z^*)$ exists in the interior of the first octant if and only if there is a positive solution to the following algebraic non-linear system

$$1 - x - \alpha y - h_1 = 0 \tag{10}$$

$$\alpha x - h_2 - \frac{\beta z}{c+y+z} = 0 \tag{11}$$

$$\frac{\delta y}{c+y+z} - \gamma = 0 \tag{12}$$

Solving the above set of equation, we get

$$\begin{aligned} x^* &= 1 - \alpha y^* - h_1 \\ y^* &= \frac{\delta(c+z^*)}{\delta-\gamma} \\ z^* &= \frac{(\alpha x^* - h_2)(c+y^*)}{\beta - (\alpha x^* - h_2)} \end{aligned} \tag{13}$$

Provided with the conditions

$$\alpha x^* > h_2, \delta > \gamma \text{ and } \beta > (ax^* - h_2) \tag{14}$$

5 Local and Global stability analysis:

In this section, we shall examine the stability of the system (3) at all the possible equilibrium points by using the Jacobian matrix.

Theorem 5.1.

The trivial equilibrium point E_0 is locally asymptotically stable in the $x-y-z$ direction, if $h_1 > 1$.otherwise unstable.

Proof

The Jacobian matrix associated with the equilibrium point at $E_0(0,0,0)$ is

$$V(E_0)=\begin{bmatrix}(-1+h_1) & 0 & 0\\ 0 & -h_2 & 0\\ 0 & 0 & -\gamma\end{bmatrix}$$

The eigenvalues of E_0 are

$$\lambda_1 = 1-h_1, \lambda_2 = -h_2, \lambda_3 = -\gamma$$

If $h_1 > 1$ in this case $\lambda_1, \lambda_2, \lambda_3$ are negative, hence is locally asymptotically stable in the direction.

If $h_1 < 1$ in this case two of the eigen value is negative and one of them is positive. So E_0 is asymptotically stable in the $y-z$ direction and unstable in x direction.

This completes the proof.

Theorem 5.2.

The disease and predator free equilibrium point E_1 is locally asymptotically stable in the $x-y-z$ plane if $h_1 < 1$ and $\alpha(1-h_1) < h_2$.

But if $h_1 > 1$ and $\alpha(1-h_1) > h_2$ in this case it is stable in z direction and unstable in $x-y$ direction.

Proof:

The Jacobian matrix associated with the equilibrium point at $E_1(1-h_1, 0, 0)$ is

$$V(E_1)=\begin{bmatrix}(-1+h_1) & -\alpha(1-h_1) & 0\\ 0 & \alpha(1-h_1)-h_2 & 0\\ 0 & 0 & -\gamma\end{bmatrix}$$

The eigenvalues of E_1 are $\lambda_1 = -1+h_1$, $\lambda_2 = \alpha(1-h_1)-h_2$, $\lambda_3 = -\gamma$ If $h_1 < 1$ and $\alpha(1-h_1) < h_2$ in this case all the eigenvalues are negative.

Hence E_1 is asymptotically stable in the $x-y-z$ direction. But if $h_1 > 1$ and $\alpha(1-h_1) > h_2$ in this case one of the eigenvalue is negative and two of them is positive so it is stable in z direction and unstable in $x-y$ direction.

This completes the proof.

Theorem 5.3.

The predator free equilibrium point E_2 is asymptotically stable in the $x-y-z$ plane if $\alpha < (\alpha h_1 + h_2)$ and $(c\alpha^2+\alpha) < (\alpha h_1 + h_2)$

Proof:

The Jacobian matrix associated with the equilibrium point at E_2 is

$$V(E_2) = \begin{bmatrix}\frac{-h_2}{\alpha} & -h_2 & 0\\ \frac{\alpha-(\alpha h_1+h_2)}{\alpha} & 0 & \frac{\beta(\alpha-(\alpha h_1+h_2))}{c\alpha^2+\alpha-(\alpha h_1+h_2)}\\ 0 & 0 & \frac{\delta(\alpha-(\alpha h_1+h_2))}{c\alpha^2+\alpha-(\alpha h_1+h_2)}-\gamma\end{bmatrix}$$

The corresponding characteristic equation for E_2 is

$$\lambda^3 + p_1\lambda^2 + p_2\lambda + p_3 = 0 \tag{15}$$

where

$$p_1 = \frac{h_1}{\alpha} - \frac{\delta(\alpha-(\alpha h_1+h_2))}{c\alpha^2+\alpha-(\alpha h_1+h_2)} + \gamma$$

$$p_2 = \left[(h_2)\left(\frac{\alpha-(\alpha h_1+h_2)}{\alpha}\right)\right] + \left[\left(\frac{-h_2}{\alpha}\right)\left(\frac{\delta(\alpha-(\alpha h_1+h_2))}{c\alpha^2+\alpha-(\alpha h_1+h_2)}-\gamma\right)\right]$$

$$p_3 = h_2\left(1-\left(h_1+\frac{h_2}{\alpha}\right)\right)\left(\frac{\delta((\alpha h\_1+h_2)-\alpha)}{c\alpha^2+\alpha-(\alpha h_1+h_2)}+\gamma\right)$$

By using Routh-Hurwitz criteria If $p_1 > 0, p_3 > 0$ and $p_1p_2 - p_3 > 0$ then E_2 is locally asymptotically stable. Now the straight forward calculation gives $\alpha < (\alpha h_1 + h_2)$ and $(\alpha^2+\alpha) < (\alpha h_1+h_2)$, implies E_2 is locally asymptotically stable.

Theorem 5.4.

The co-existence equilibrium point of the system (3) exists, if is locally asymptotically stable if following conditions hold

$$(ax^* - h_2) < \frac{\beta(cz^*+z^{*2}}{(c+y^*+z^*)^2},$$

$$\delta((cy+y^{*2}) < \gamma(c+y^*+z^*)^2$$

Proof:

The Jacobian matrix at the interior point $E_3(x^*, y^*, z^*)$ is given below:

$$V(E_3) = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix} \text{ where}$$

$$a_{11} = 1-2x^*-\alpha y^*-h_1;\ a_{12} = -\alpha x^*;$$

$$a_{21} = \alpha y^*;\ a_{22} = ax^*-h_2-\frac{\beta(cz^*+z^*)}{(c+y^*+z^*)^2};$$

$$a_{23} = \frac{\beta(cy^*+y^{*^2})}{(c+y^*+z^*)^2};$$

$$a_{31} = 0;\ a_{32} = \frac{\delta(cz^*+z^{*2})}{(c+y^*+z^*)^2};\ a_{33} = \frac{\delta(cy^*+y^{*2})}{(c+y^*+z^*)^2}-\gamma$$

Then corresponding characteristic equation becomes

$$\lambda^3 + S_1\lambda^2 + S_2\lambda + S_3 = 0 \tag{16}$$

where

$$\begin{aligned}S_1 &= -(a_{11}+a_{22}+a_{33})\\ S_2 &= a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33}+a_{12}a_{21}a_{23}a_{32}\\ S_3 &= -[(a_{11}a_{22}a_{33}-a_{11}a_{23}a_{32})-a_{12}(a_{21}a_{32})]\\ &= a_{11}a_{23}a_{32}+a_{12}a_{21}a_{32}-a_{11}a_{22}a_{33}\end{aligned}$$

$$\begin{aligned}S_1S_2-S_3 = {} & [a_{12}a_{21}(a_{11}+a_{22}+a_{33})+a_{23}a_{32}(a_{22}+a_{33})]\\ & -\left[a_{11}^2(a_{22}+a_{33})+a_{22}^2(a_{11}+a_{33})\right.\\ & \left.+a_{33}^2(a_{11}+a_{22})-2a_{11}a_{22}a_{33}-a_{12}a_{21}a_{32}\right]\end{aligned}$$

The sufficient condition for $S_1 > 0, S_3 > 0, S_1S_2 - S_3 > 0$ are as follows $a_{11} \le 0, a_{22} \le 0, a_{33} \le 0$ Which implies the conditions

$$(ax^*-h_2) < \frac{\beta(cz^*+z^{*2})}{(c+y^*+z^*)^2}$$

$$\delta(cy+y^{*2}) < \gamma(c+y^*+z^*)^2$$

Thus if the condition stated in the theorem holds, then all the Rouh-Hurwitz criteria

(i) $S_1 > 0$

(ii)$S_2 > 0$

(iii)$S_1S_2 - S_3 > 0$

are satisfied and the system is locally asymptotically stable.

6 Global Stability Analysis :

In this section, we shall study the global dynamics of the system (3) around the predator free equilibrium point and co-existence equilibrium point $E_3(x^*,y^*,z^*)$.

Theorem 6.1

The predator free equilibrium point E_2 is globally asymptotically stable in the interior of the quadrant of $x-y$ plane.

Proof

$$\text{Let} \quad H(x,y) = \frac{1}{xy} \tag{17}$$

Clearly $H(x,y)$ is positive in the interior of the positive quadrant of $x-y$ plane.

$$h'(x,y) = x(1-x) - \alpha xy - h_1 x$$
$$h''(x,y) = \alpha xy - h_2 y$$

$$\begin{aligned} \text{Then} \quad \Delta(x,y) &= \frac{\partial}{\partial x}(h'H) + \frac{\partial}{\partial y}(h''H) \\ &= -\frac{1}{y} < 0 \end{aligned} \tag{18}$$

By using Bendixson-Dulac criteria, we note that $\Delta(x,y)$ remains the same sign and is not identically zero in the interior of the positive quadrant of the $x-y$ plane. This completes the proof.

Theorem 6.2

The co-existence equilibrium point $E_3(x^*,y^*,z^*)$ is globally asymptotically stable with respect to all solutions initiating in the interior of satisfy the following conditions $x < x^*$ and $zy^* > z^*y$

Proof:

The proof can be reached by using Lypunov stability theorem which gives sufficient condition. Now let us define

$$\begin{aligned} L &= S\left[x - x^* - x^* ln\left(\frac{x}{x^*}\right)\right] + T\left[y - y^* - y^* ln\left(\frac{y}{y^*}\right)\right] \\ &\quad + U\left[z - z^* - z^* ln\left(\frac{z}{z^*}\right)\right] \end{aligned} \tag{19}$$

Where S,T,U are positive constant to be chosen later. Differentiate (19) with respect to t and substitute (3) in (19). We get the following

$$\begin{aligned} \frac{dL}{dt} &= S[(1-x) - \alpha y - h_1](x-x^*) \\ &\quad + T\left[\frac{-\beta z}{c+y+z} - h_2 + \alpha x\right](y-y^*) \\ &\quad + U\left[\frac{\delta y}{c+y+z} - \gamma\right](z-z^*) \\ &= S\left[-(x-x^*) - \alpha[y-y^*]\right](x-x^*) \\ &\quad + T\left[\alpha(x-x^*) - \left[\frac{\beta z}{c+y+z} - \frac{\beta z^*}{c+y^*+z^*}\right]\right](y-y^*) \\ &\quad + U(z-z^*)\left[\frac{\delta y}{c+y+z} - \frac{\delta y^*}{c+y^*+z^*}\right] \\ &= S[-(x-x^*)](x-x^*) \\ &\quad + T\left[-\left[\frac{\beta(c(z-z^*) + (zy^* - z^*y))}{(c+y+z)(c+y^*+z^*)}\right]\right](y-y^*) \\ &\quad + U(z-z^*)\left[\frac{\delta(c(y-y^*) + (yz^* - y^*z))}{(c+y+z)(c+y^*+z^*)}\right] \end{aligned} \tag{20}$$

We choose the parameters $S=1, \delta=1, T=\frac{U}{\beta}$ then we get

$$\frac{dL}{dt} = -(x-x^*)^2 - \frac{(y^*z - yz^*)}{(c+y+z)(c+y^*+z^*)}(y-y^*)(z-z^*)$$

Then using the given condition, we see that dL/dt is negative definite. L is a Lyapunov function with respect to all solutions in the interior of the positive octant which proves the theorem

7 Introduction of Backstep control in Prey-predator system

In this section, the system with two preys viz. susceptible prey, infected prey and a predator population controlled by back stepping using nonlinear feedback control approach is studied. we initiate the study by assuming that the system (3) can be written in the suitable for

$$\begin{aligned} \frac{dx}{dt} &= x(1-x) - \alpha xy - h_1 x \\ \frac{dy}{dt} &= \alpha xy - \frac{\beta zy}{c+y+z} - h_2 y + u_1 \\ \frac{dz}{dt} &= \frac{\delta zy}{c+y+z} - \gamma z + u_2 \end{aligned} \tag{21}$$

Where u_1, u_2 are back stepping nonlinear feedback controllers which is the function of state variables and which will be suitable choice to make the trajectory of the whole system (21). As long as these feedback stabilize the system (21) converge to zero as the time t goes to infinity.(i.e) The system (21) gives $\lim_{t\to\infty}\|x(t)\| = 0$

Theorem 7.1
Using the backstep control in the three species system,

$$u_1 = -\left(1+\alpha e_1 - x - \frac{h_1}{\alpha} + \frac{h_2}{\alpha} - e_1\right)x + \frac{h_2}{\alpha} + \alpha x^2 \quad (22)$$

$$u_2 = \frac{-\delta e_2(1+\alpha e_1 - x)}{c+\alpha e_1 - x + \alpha e_2} \quad (23)$$

And with error dynamics

$$e_1 = y - \eta_1;$$
$$e_2 = z - \eta_2 \quad (24)$$

The system (21) will be asymptotically stable in the Lyapunov sense about its equilibrium state.

Proof:
The Lyapunov function of is taken as

$$F_1(x) = \frac{1}{2}x^2 \quad (25)$$

Differentiate (25) with respect to t is

$$\dot{F_1} = x\dot{x}$$
$$= x(x - x^2 - \alpha xy - h_1 x) \quad (26)$$

By defining the virtual controller
$y = \eta_1(x)$ where

$$\eta_1 = \frac{1-x}{\alpha} \quad (27)$$

By using virtual controller (27) in the above equation (26), we get

$$\dot{F_1} = -h_1 x^2 \quad (28)$$

This is a negative definite function. Now consider the Lyapunov function of (x, e_1)

$$F_2(x,e_1) = \frac{1}{2}x^2 + \frac{1}{2}e_1^2 \quad (29)$$

Consider the error dynamics

$$e_1 = y - \eta_1(x) \quad (30)$$

which implies

$$y = e_1 + \eta_1(x)$$
$$= \frac{1}{\alpha}(1+\alpha e_1 - x) \quad (31)$$

The derivative of (30) on applying (31) becomes

$$\dot{e_1} = \dot{y} + \frac{\dot{x}}{\alpha} \quad (32)$$
$$= \alpha xy - \frac{\beta yz}{c+y+z} - h_2 y + u_1$$
$$\frac{1}{\alpha}[x(1-x) - \alpha xy - h_1 x] \quad (33)$$

Again defining the virtual controller

$$z = \eta_2(x,y) \quad (34)$$

where $\eta_2 = 0$
By applying (31) and (32) in the equation (33) gives

$$\dot{e_1} = \left(1+\alpha e_1 - x - \frac{h_1}{\alpha} + \frac{h_2}{\alpha} - e_1\right)x - \frac{h_2}{\alpha}$$
$$-h_2 e_1 + u_1 \quad (35)$$

Now, we have

$$\frac{dx}{dt} = x(1-x) - \alpha xy - h_1 x \quad (36)$$

By using (31), the above equation (36) gives

$$\frac{dx}{dt} = -\alpha e_1 x - h_1 x \quad (37)$$

Differentiate (29) and applying (37) in (29) we get,

$$\dot{F_2} = x(-\alpha e_1 x - h_1 x)$$
$$+e_1\Big[\left(1+\alpha e_1 - x - \frac{h_1}{\alpha} + \frac{h_2}{\alpha} - e_1\right)x - \frac{h_2}{\alpha}$$
$$-h_2 e_1 + u_1\Big] \quad (38)$$

Choosing the backstepping controller (22) in equation (38) becomes

$$\dot{F_2} = -h_1 x^2 - h_2 e_1^2 \quad (39)$$

Which is negative definite function. Now consider the Lyapunov function for (x, e_1, e_2)

$$F_3(x,e_1,e_2) = \frac{1}{2}x^2 + \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \quad (40)$$

The derivative of the above function along the derivative of (40) is

$$\dot{F_3} = \dot{F_2} + e_2\dot{e_2} \quad (41)$$

Now consider the error dynamics

$$e_2 = z - \eta_2(y,x) \quad (42)$$

Let $\eta_2 = 0 \Rightarrow e_2 = z$
The derivative of the error dynamics is

$$\dot{e_2} = \dot{z} \quad (43)$$

Hence the derivative (41) along (31),(39),(43) becomes

$$\dot{F_3} = -h_1 x^2 - h_2 e_1^2$$
$$+e_2\left[\frac{\delta e_2(1+\alpha e_2 - x)}{(c+\alpha e_1 - x) + \alpha e_2} - \gamma e_2 + u_2\right] \quad (44)$$

Choosing the back step controller (23),the above derivative (44) becomes

$$\dot{F_3} = h_1 x^2 - h_2 e_1^2 - \gamma e_2^2 \quad (45)$$

This is a negative definite function. Hence the theorem proves that the system is globally asymptotically stable.

8 Numerical simulation

Numerical simulation plays the key role in the qualitative analysis of the system. The main objective of the numerical simulation is to explore the possibility of variation and the effect of back step control to this behavior. For the various choices of the parameter of the model, we have performed the simulation using MATLAB. It is observed that they show good agreement with our analytical findings. First we start by studying the density behavior of the two preys and one predator with time of the uncontrolled system.

Let R_1 be the parameter set taken as $\beta = 0.9, \delta = 0.5, \gamma = 0.2, c = 0.35, \alpha = 2.35$ With the above parameter set the system (3) has varying harvesting rate within the range $0.01 < h_1 < 0.2$ and $0.01 < h_2 < 0.2$. If $h_1 = 0.01, h_2 = 0.01$, the time series of the system (3) is as shown in figure (1) if $h_1 = 0.1, h_2 = 0.1$ the time series of the system (3) is as shown in figure (2). If $h_1 = 0.2, h_2 = 0.2$, the time series and the phase portrait is as shown in figure(3) and (4). From the figures (1-4), we observe that increase in the harvest rate, reduces the population density of the infected prey , predator with inflation in the susceptible prey.

Let R_2 be the parameter set taken as $\beta = 0.9, \delta = 0.5, \gamma = 0.2, c = 0.35, h_1 = h_2 = 0.1$. With the above parameter set, the system (3) has varying diseases transmission rate. If $\alpha = 2.35$, the corresponding time series and phase portrait are as shown in figure (5) and (6). . If $\alpha = 1.85$, the corresponding time series and phase portrait are as shown in figure (7) and (8). It is observed that the decrease in the transmission rate decreases the density of infected prey and predator population and increases the susceptible prey population.

In the controlled system (21), population density reaches the point $(0,0,0)$ as shown in figure (9), (10), (11) when the harvesting rates are set as $0.01 < h_1 < 0.2$ and $0.01 < h_2 < 0.2$ with the disease transmission rate to be in the range $1.85 < \alpha < 2.35$ (i.e) the system (21) converges to zero as $t \to \infty$. The corresponding phase portrait is shown in Figure (12). The results of numerical simulation conclude that the three species prey-predator model is globally asymptotically stable.

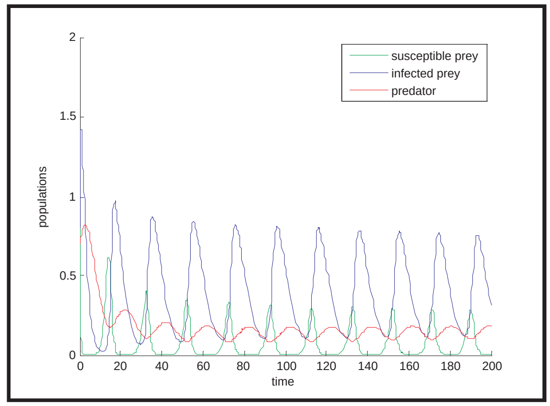

Fig 1. Time series of the system (3)

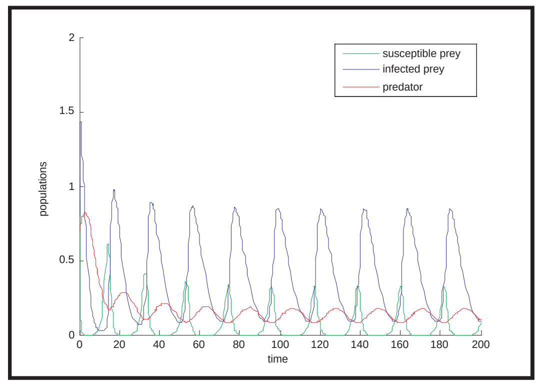

Fig 2. Time series of the system (3)

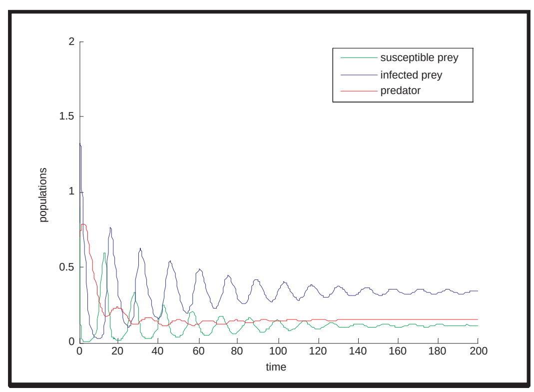

Fig 3. Time series of the system (3)

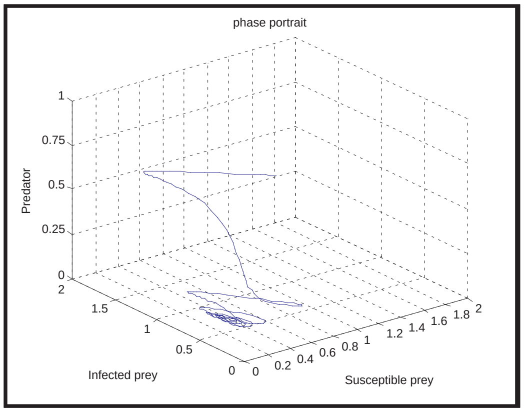

Fig 4. Phase portrait of the system (3)

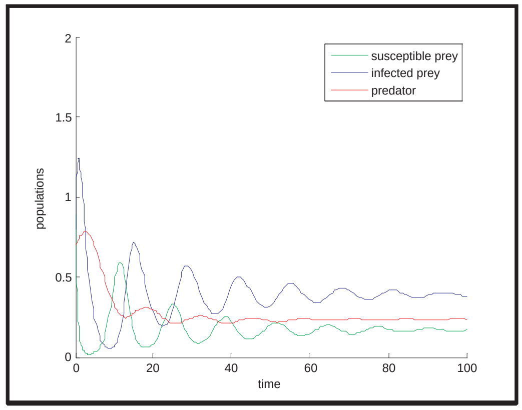

Fig 7. Time series of the system (3)

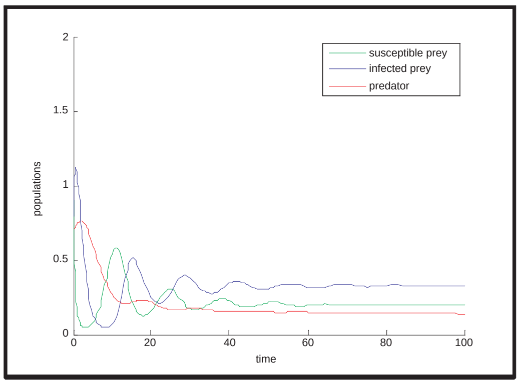

Fig 5. Time series of the system (3)

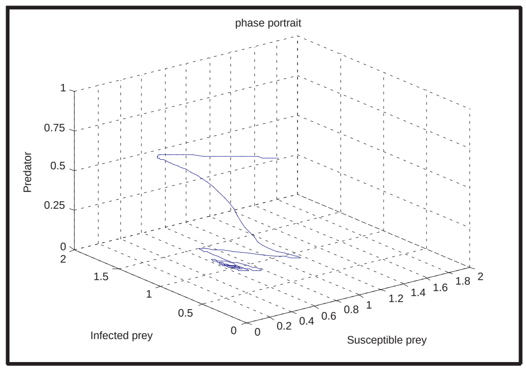

Fig 8.Phase portrait of the system (3)

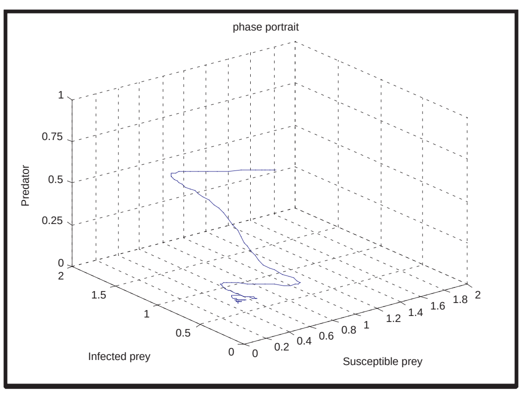

Fig 6.Phase portrait of the system (3)

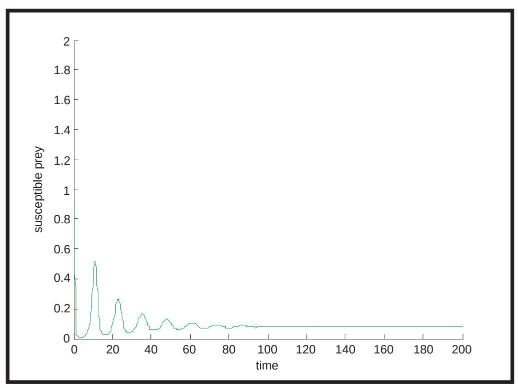

Fig 9. The variation of susceptible prey populationapproaches stability of the system(21) with backstep control

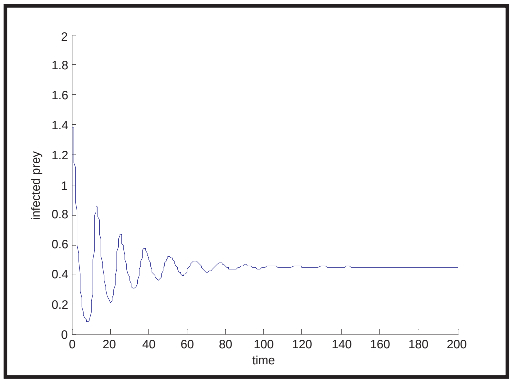

Fig 10. The variation of infected prey population approaches stability of the system(21) with backstep control

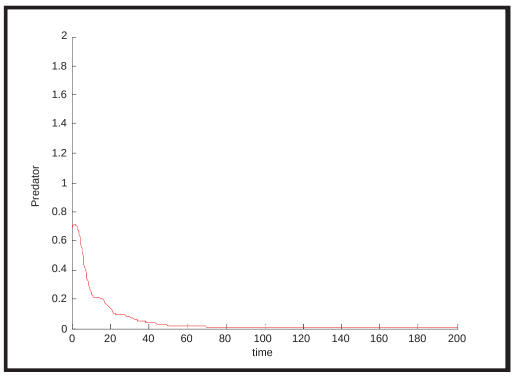

Fig 11. The variation of predator population approaches stability of the system(21) with backstep control

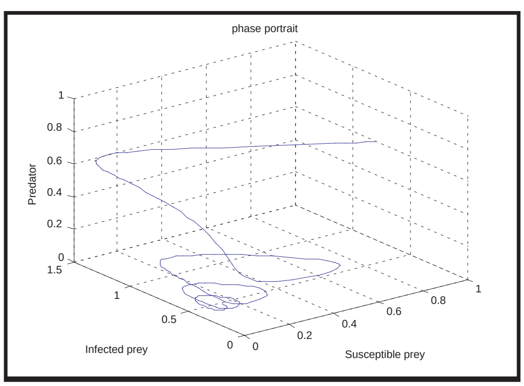

Fig 12.Phase portrait of the system approaches stability of the system (21) with backstep control

9 Conclusion

The three species biological model with infected and susceptible prey has been analyzed for its interactions. The boundedness and positivity of the system seem to hold good which indicates that the system considered is well-behaved. Existence of possible equilibrium points were obtained and the stability was analyzed at those points. The controllability conditions and the conditions for global asymptotic stability have been obtained by using the backstep control. By using the backstep control technique. It is proved that the three species prey-predator model is asymptotically stable to its trivial equilibrium point. It is observed that the decrease in the transmission rate decreases the density of infected prey and predator population and increases the susceptible prey population.Different parameter values give varying responses to the system. The chaotic behavior of this prey-predator system has been visualized by these varying values. The numerical results add to the novelty and effectiveness of the proposed work.

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