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Hydromagnetic Stability of a Self-gravitating Oscillating Fluid Cylinder

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Abstract: The hydro-magnetic stability of a self-gravitating oscillating medium with streams of variable velocities for fluid cylinder has been defined and investigated. The streaming is unstable, but the magnetic field has a significant stabilizing effect. Under certain conditions, the rotating forces have a stabilizing effect. Using suitable and specific conditionsto distinguish between stable and unstable domains, the stability criterion is derived and investigated numerically and analytically. The effects of inertial self-gravity, and electromagnetic forces on the stability of a fluid cylinder are studied. All basic functions and equations have been solved after defining the problem.

Keywords: Hydro magnetic, Magnetic Field, Oscillating and Self-Gravitating

1 Introduction

Using suitable and specific conditions, analytically and numerically, the stability criterion is derived and discussed for the purpose of identifying the characteristics of stable and unstable domains. See Radwan [13] Moreover, Chandrasekhar [4] demonstrated the magneto-hydro-dynamic`s stability of a complete fluid cylinder permeated by a homogeneous magnetic field. There are tests which were executed to determine the stability of an annular fluid jet. Also, Chandrasekhar [4] gives the classic example of a gas cylinder submerged in a liquid's capillary instability for axisymmetric perturbation. Drazin and Reid [7], Hassan [10], Elazab et al. [8], and Hassan [10] Cheng examined the unpredictability of a gas jet in a liquid that can't be compressed. However, we must point out that Cheng's results [6] are not to be taken lightly, where for all modes, the dispersion relation was valid. The axisymmetric magneto-hydrodynamic self-gravitating stability of a fluid cylinder is studied, as is the magneto-hydrodynamic stability of an oscillating fluid cylinder in the presence of a magnetic field. Discussed by Barakat. M [3]. Modes of Mehring C and Sirignano [12], axisymmetric capillary waves on thin annular liquid sheets are explored. The purpose of this research is to determine the self-gravitating stability for a confined liquid with a magnetic field, all symmetric and asymmetric perturbation modes of a fluid cylinder exist.

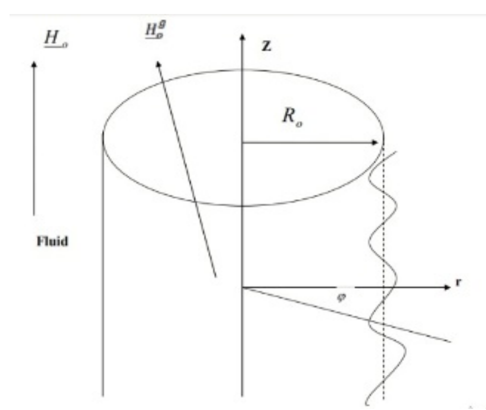

Fig. I: self-gravitation Hydromagnetic cylindrical Fluid sketch.

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2 The Problem's Formation

We take into account a fluid cylinder with a uniform cross-section of (radiusR_0), the fluid is as assumed to be incompressible, non-viscous and non-dissipative of permeality coefficient. There is a uniform axial magnetic field inside the fluid, which surrounds the fluid jet and has negligible motion.

$$H_0^{(i)} = (0,0,H_0) \quad (1)$$

While the encompassing locale outside the liquid is given by

$$H_0^{(e)} = (0,0,\alpha H_0) \quad (2)$$

where H_0 is the intensity of the magnetic field and α is a parameter, the fluid is assumed to be streaming with oscillating velocity...

$$u_0 = (0,0,U\cos\Omega t) \quad (3)$$

Where Ω is the oscillating frequency of the fluid at t=0
U is the amplitude of velocityu_0.

The components of $H_0^{(i)}$, $H_0^{(e)}$ and u_0 are taken into consideration along cylinder coordinates (r,φ, z) with the fluid cylinder's axis coincident with the z-axis. The combined force of self-gravitating, magneto dynamic, and pressure gradient forces acts on the fluid.

Concerning the current model's stability, the basic equations for that are synthesis of hydrodynamic equations and Maxwell equations

$$\rho\left[\frac{\partial u}{\partial t} + (\underline{u}.\nabla)\underline{u}\right] = \rho\nabla V + \mu(\nabla\wedge\underline{H})\wedge\underline{H} - \nabla p \quad (4)$$

$$\nabla.\underline{u}=0 \quad (5)$$

$$\frac{\partial H}{\partial t} = \nabla\wedge(\underline{u}\wedge\underline{H}) \quad (6)$$

$$\nabla.\underline{H} = 0 \quad (7)$$

$$\nabla^2 V = -4\pi G\rho \quad (8)$$

$$\nabla.\underline{H}^{(e)} = 0 \quad (9)$$

$$\nabla\wedge\underline{H}^{(e)}=0 \quad (10)$$

$$\nabla^2 V^{(e)} = 0 \quad (11)$$

Along the interface of fluid

$$P_s = T(\nabla.\underline{N_s}) \quad (12)$$

Where

$$\underline{N_s} = \frac{\nabla f(r,\varphi,z;t)}{|\nabla f(r,\varphi,z;t)|} \quad (13)$$

Which u and p are the fluid velocity vector and kinematic pressure, T the coefficient of surface tension, N_s the unit vector normal to the fluid interface where

$$f(r,\varphi,z;t)=0 \quad (14)$$

3 State of equilibrium

Equation (4) can be written as

$$\rho\left[\frac{\partial u}{\partial t} + (\underline{u}.\nabla)\underline{u}\right] - \mu(\underline{H}.\nabla)\underline{H} = -\nabla\Pi \quad (15)$$

Where

$$\Pi = p + \rho V + \frac{\mu}{2}(\underline{H}.\underline{H}) \quad (16)$$

Where π total magneto hydrodynamic pressure. The basic equations (4)-(16) are solved with taking equations (1)-(3) in the unperturbed state and applying the boundary conditions at r=R_0we get

$$\Pi_0 = p_0 - \rho V_0 + \frac{\mu}{2}(\underline{H_0}.\underline{H_0}) = const. \quad (17)$$

p_{0s}=T/R_0

But the balance of the pressure

p_0=$\Pi_0 + \rho V_0 - \frac{\mu}{2}(\underline{H_0}.\underline{H_0})$

The self-gravitating potentials V_0 and $V_0^{(e)}$ in the equilibrium satisfy

$\nabla^2 V_0 = -4\pi G\rho$ (18)

$\nabla^2 V_0^{(ex)} = 0$ (19)

The solutions of equations (18), (19)

$V_0 = -\pi\rho G r^2 + c_1$ (20)

$V_0^{(ex)} = c_2 \ln r + c_3$ (21)

Wherec_1 , c_2 *and* c_3 are integration constants that must be identified in conjunction with boundary conditions.

$c_1 = 0$

c_2=-$2\pi G\rho R_0^2$ (22)

c_3=-$\pi G\rho R_0^2$+$2\pi G\rho R_0^2 \ln R_0$ (23)

therefore

$V_0 = -\pi G\rho r^2$ (24)

$V_0^{(ex)} = -\pi G\rho R_0^2 \left[1 + 2\ln\left(\frac{r}{R_0}\right)\right]$ (25)

by balancing the pressure a cross the boundary surface r=R_0

rating the fluid pressure p_0 in the equilibrium state is given by

$p_0 = \frac{T}{R_0} + \pi G\rho^2 (R_0^2 - r^2) + \frac{\mu}{2}(\alpha^2 - 1)H_0^2$ (26)

In the equilibrium state as $\alpha = 1$, we observe that there is no donating in the magnetic field ,Outside of the cylinder the magnetic field becomes active.

When$R_0 > r$, the self-gravitating force donates to p_0 in a positive manner; when r> R_0, it donates in a negative manner, and when r=R_0 , it donates nothing at all.

4 Perturbed States

Every physical quantity Q(r,φ z; t) can be developed as for minor deviations from the equilibrium state:

Q(r,$\varphi, z; t$) =$Q_0(r) + \varepsilon(t) Q_1(r, \varphi, z) + \cdots$ (27)

where

$Q_1 = \varepsilon_0 q_1(r) \exp\big(\sigma t + i(kz + m\varphi)\big)$ (28)

the modified form of the formula in the cylindrical interface is given by

r=$R_0 + R_1 + \cdots$ (29)

with

$R_1 = \varepsilon(t) \exp\big(i(kz + m\varphi)\big)$ (30)

where

$\varepsilon(t) = \varepsilon_0 \exp(\sigma t)$

The height of the surface wave measured from the unperterbuted state. From eq. (27) and (30) in the basic equations (4) - (14), the pertinent perturbation equations are given by

$\rho\left[\frac{\partial \underline{u}}{\partial t} + (\underline{u}_0.\nabla)\underline{u}_1\right] - \mu(\underline{H}_0.\nabla)\underline{H}_1 = -\nabla\Pi_1$ (31)

Where

$\Pi_1 = p_1 - \rho V_1 + \mu(H_0.H_1)$ (32)

$\nabla.\underline{u}_1$=0 (33)

$\frac{\partial H_1}{\partial t} = (\underline{H}_0.\nabla)\underline{u}_1 - (\underline{u}_0.\nabla)\underline{H}_1$ (34)

$\nabla.H_1 = 0$ (35)

$\nabla^2 V_1 = 0$ (36)

$P_{1S} = -\frac{T}{R_0^2}\left(R_1 + \frac{\partial^2 R_1}{\partial \varphi^2} + R^2 \frac{\partial^2 R_1}{\partial z}\right)$ (37)

$\nabla.\underline{H}_1^{(ex)} = 0$ (38)

$\nabla\wedge\underline{H}_1^{(ex)}$=0 (39)

$\nabla^2 V_1^{ex} = 0$ (40)

every perturbed Q(r,$\varphi, z; t$) may be expressed as

Q(r,$\varphi, z; t$) =$q_1(r) \exp\big(\sigma t + i(kz + m\varphi)\big)$ (41)

by using (28), (36) and (40) given the second-order differential equation.

From Laplace equation in cylinder coordinate eq. (36) and (40) become in the form

$V_1 = A\varepsilon_0 I_M(x) \exp\big(\sigma t + i(kz + m\varphi)\big)$ (42)

$V_1^{(ex)} = B\varepsilon_0 k_m(x)\exp\big(\sigma t + i(kz + m\varphi)\big)$ (43)

From equations (38), (34) we get

$\underline{H}_1 = \frac{ikH_0}{(\sigma + ikU\cos\Omega t)}\underline{u}_1$ (44)

by take the divergence to eq. (31) we get

$\nabla^2 \Pi_1 = 0$ (45)

Here equation (39) means the magnetic field $H_1^{(ex)}$could be a scalar function $\Psi_1^{(ex)}$

$\underline{H}_1^{(ex)} = \nabla\Psi_1^{(ex)}$ (46)

And equation (38) we get

$\nabla^2\Psi_1^{(ex)} = 0$ (47)

the fluid is incompressible, in viscid and irrational

$u_1 = \nabla\Phi$ (48)

combining equations (48), (33)

$\nabla^2\Phi_1 = 0$ (49)

From eq. (28), the variable Φ_1 ,$\pi_1 and\ \Psi_1^{(ex)}$ then nonsingular solutions of equations (45), (47) and (49)

$\Phi_1 = c_4\varepsilon_0 I_m(kr)\exp(\sigma t + i(kz + m\varphi))$ (50)

$\Pi_1 = c_5\varepsilon_0 I_m(x)\exp\ (\sigma t + i(kz + m\varphi))$ (51)

$\Phi_1^{(ex)} = c_6\varepsilon_0 k_m(x)\exp\big(\sigma t + i(kz + m\varphi)\big)$ (52)

Where $c_4, c_5\ and\ c_6$ are constant of integration which $I_m(kr) and\ k_m(kr)$ are the Bessel functions which m is the first and second type of order.

The perturbed state caused by the capillary force is the surface pressure along the cylindrical fluid interface fromequation(53)

$p_{1s} = -\frac{T}{R_0^2}(1 - m^2 - x^2)R_1$ (53)

where (x=kR_0)

5 Boundary Conditions

The boundary conditions of the problem must be satisfied by the sol. Of basic equations (4-14) in the unperterbuted state by eqs. (1-3), (17) and (23-26) while in perturbed state given by (44) and (53)

5.1Magnetic condition

It stipulates that the normal magnetic field component must remain continuous across the fluid interface.
(29) At r= R_0

$\underline{N}_0.\underline{H}_1 + \underline{N}_1.\underline{H}_0 = \underline{N}_0.\underline{H}_1^{(ex)} + \underline{N}_1.\underline{H}_0^{(ex)}$ (54)

where

$N_0 = (1,0,0) \quad , \quad N_1 = \left(0, \frac{-im}{R_0}, -ik\right)$

then,

$c_6 = \frac{i\alpha H_0}{k_m^{\backslash}(x)}$ where (x=kr) (56)

5.2 Kinematic condition

The velocity of the perturbed boundary fluid interface and the normal component of the fluid's velocity u must match.
(29)
At r=R_0

$u_{1r} = (\sigma + ikU\cos\Omega t)\varepsilon_0\exp\big(\sigma t + i(kz + m\varphi)\big)$ (57)

combining eq. (57)

$u_{1r} = \frac{\partial \phi_1}{\partial r}$

We get

$c_4 = \frac{(\sigma + ikU\cos\Omega t)}{k\ I_m^{\backslash}(x)}$ (58)

from eq. (31), (44) we get

$$\rho\left[\frac{\partial u_{1r}}{\partial t}+U\cos\Omega t\,\frac{\partial u_{1r}}{\partial z}\right]-\frac{ik\mu H_0^2}{(\sigma+ikU\cos\Omega t)}\frac{\partial u_{1r}}{\partial z}=-\frac{\partial\Pi}{\partial r} \tag{59}$$

from which we get

$$c_5=\frac{-\rho}{kI_m^{\backslash}(x)}\left[\sigma^2+2ik\sigma U\cos\Omega t-ikU\Omega\sin\Omega t-k^2U^2cos^2\Omega t\right]-\frac{\mu kH_0^2}{I_m^{\backslash}(x)} \tag{60}$$

5.3 *Self-gravitating Conditions*

(A) The self-gravitating potential must be continuous across the equilibrium surface. At r=R_0

$$V_1+R_1\frac{\partial V_0}{\partial r}=V_1^{(ex)}+R_1\frac{\partial V_0^{(ex)}}{\partial r} \tag{61}$$

(B) The derivative of the self-gravitating potential must be continuous over the initial equilibrium's surface at r=R_0

$$\frac{\partial V_1}{\partial r}+R_1\frac{\partial^2 V_0}{\partial r^2}=\frac{\partial V_1^{(ex)}}{\partial r}+R_1\frac{\partial V_0^{(ex)}}{\partial r} \tag{62}$$

sub. From eqs. (23), (24), (29), (42) and (43) we get

$$A=4\pi G\rho R_0 k_m(x) \tag{63}$$

$$B=4\pi G\rho R_0 I_m(x) \tag{64}$$

Finally, we have to apply some compatibility condition of the leap of the total stress in the fluid and framing p_{1s} across the fluid cylindrical interface (29) at r=R_0

$$p_1+R_1\frac{\partial p_0}{\partial r}+\mu(H_0.H_1)-\mu(H_0.H_1)^{(ex)}=p_{1s} \tag{65}$$

The condition can be written

$$[\Pi_1+\rho V_1]=p_{1s}-R_1\frac{\partial p_0}{\partial r}+\mu(H_0.H_1)^{(ex)} \tag{66}$$

By sub. From equations (26),(30),(48),(42),(51),(52),(53),(63),(60),(56) into condition (66) we get

$$\sigma^2+2ik\sigma U\cos\Omega t-ikU\Omega\sin\Omega t-k^2U^2cos^2\Omega t=\frac{T}{\rho R_0^3}(1-m^2-x^2)\frac{xI_m^{\backslash}(x)}{I_m(x)}+4\pi G\rho\frac{xI_m^{\backslash}(x)}{I_m(x)}\left[k_m(x)I_m(x)-\frac{1}{2}\right]+\frac{\mu H_0^2}{\rho R_0^2}\left[-x^2+\alpha^2\frac{x^2k_m(x)I_m^{\backslash}(x)}{k_m^{\backslash}(x)I_m(x)}\right] \tag{67}$$

6 General Discussions

Equation (67) is the dispersion relation of self-gravitating fluid cylinder (acted by mutual affected the electromagnetic and capillary forces)
implanted into a negligibly moving weak self-gravitating center.
IF we put Ω=0 ,eq. (67) become

$$(\sigma+ikU)^2=\frac{T}{\rho R_0^3}\left(\frac{xI_m^{\backslash}(x)}{I_m(x)}\right)(1-m^2-x^2)+4\pi G\rho\frac{xI_m^{\backslash}(x)}{I_m(x)}\left[k_m(x)I_m(x)-\frac{1}{2}\right]+\frac{\mu H_0^2}{\rho R_0^2}\left[-x^2+\alpha^2\frac{x^2K_m(x)I_m^{\backslash}(x)}{k_m^{\backslash}(x)I_m(x)}\right] \tag{68}$$

the debate of the argument in this equation, uniform fluid streaming has a destabilizing effect, and this effect exists not only in the axisymmetric mode of perturbation (m=0), but also in the non-axisymmetric mode (m≥ 1).

If we put U=0, Ω=0 and m≥ 0 ... eq. (67) become

$$\sigma^2=\frac{T}{\rho R_0^2}\left(\frac{xI_m^{\backslash}(x)}{I_m(x)}\right)(1-m^2-x^2)+4\pi G\rho\frac{xI_m^{\backslash}(x)}{I_m(x)}\left[k_m(x)I_m(x)-\frac{1}{2}\right]+\frac{\mu H_0^2}{\rho R_0^2}\left[-x^2+\alpha^2\frac{x^2k_m(x)I_m(x)}{k_m(x)I_m(x)}\right] \tag{69}$$

If we put G=0, $H_0=0$ and m=0 eq. (67) become

$$\sigma^2=\frac{T}{\rho R_0^3}\left(\frac{xI_1(x)}{I_0(x)}\right)(1-x^2)\qquad ,I_0^{\backslash}(x)=I_1(x) \tag{70}$$

this is the standard capillary instability dispersion relation. If we put G=0, $H_0=0$, m≥ 0

$$\sigma^2=\frac{T}{\rho R_0^3}\left(\frac{xI_m^{\backslash}(x)}{I_m(x)}\right)\left[I_m(x)k_{(x)}-\frac{1}{2}\right] \tag{71}$$

this relation has been derived by Chandrasekhar (6) discussing the capillary instability of fluid cylinder.
If we put T=0, H_0 *and* $m=0$,the relation (67) become

$$\sigma^2=4\pi G\rho\left(\frac{xI_1(x)}{I_0(x)}\right)\left[I_0(x)k_0(x)-\frac{1}{2}\right]\qquad ,I_0^{\backslash}(x)=I_1(x) \tag{72}$$

this relation (72) has been proven for the first time by Chandrasekhar and Fermi (12).

$$\sigma^2 = 4\pi G\rho \frac{x I_m^{\backslash}(x)}{I_m(x)}\left[I_m(x)k_m(x) - \frac{1}{2}\right] \tag{73}$$

7 Numerical Discussions

In this instance of magneto hydro gravitodynamic stability caused by the interaction of capillary, self-gravitating, and electromagnetic forces, the fluid jet model is utilized. Using numbers to discuss the relation (67)...

$$\sigma^* = \gamma + \beta + w(1 - m^2 - x^2)\frac{x I_m^{\backslash}(x)}{I_m(x)} + \frac{x I_m^{\backslash}(x)}{I_m(x)}\left[k_m(x)I_m(x) - \frac{1}{2}\right] + Nx^2\left[-1 + \alpha^2 \frac{I_m^{\backslash}(x)k_m(x)}{I_m(x)k_m^{\backslash}(x)}\right] \tag{74}$$

where $\gamma = \frac{-ikU\cos\Omega t}{(4\pi G\rho)^{\frac{1}{2}}}$, $\beta = \frac{ikU\Omega \sin\Omega t}{4\pi G\rho}$, $\sigma^* = \frac{\sigma}{(4\pi G\rho)^{\frac{1}{2}}}$, w=$\frac{T}{4\pi G\rho^2 R_0^3}$

N=$\left(\frac{H_0}{H_s}\right)^2$ which $H_s = 2\rho R_0\sqrt{\frac{\pi G}{\mu}}$,

(I) For w=0.2 conformable with N=0.1,0.4,0.7,0.9 and 1.2 it is found that gravitational magneto hydrodynamic unstable domain is

0< $x < 1.422$,

the contiguous stable domain are

1.422≤ $x < \infty$, 0< $x < \infty$. 0< $x < \infty$.

0< $x < \infty$. 0< $x < \infty$.

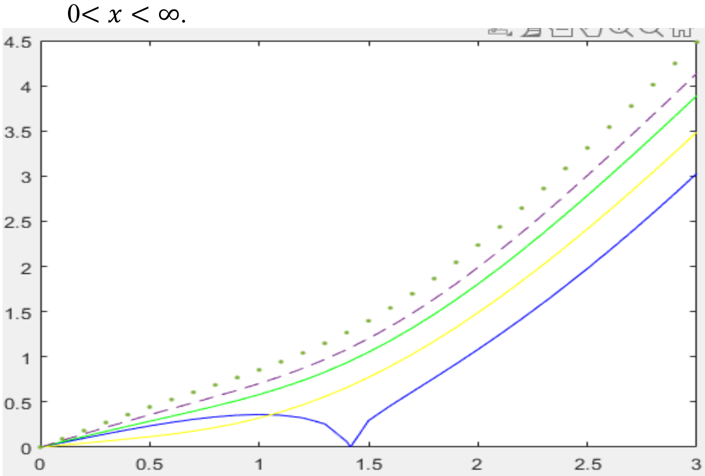

Fig. 1: For w=0.2with N=0.1,0.4,0.7,0.9 and 1.2.

(II) For w=0.4 conformable with N=0.1, 0.4, 0.7, 0.9 and 1.2 it is found that gravitational magneto hydrodynamic unstable domain is

0< $x < 1.331$, 0< $x < 0.6277$

The contiguous stable domain are

1.745≤ $x < \infty$, 0< $x < \infty$. 0< $x < \infty$

0< $x < \infty$ 0< $x < \infty$

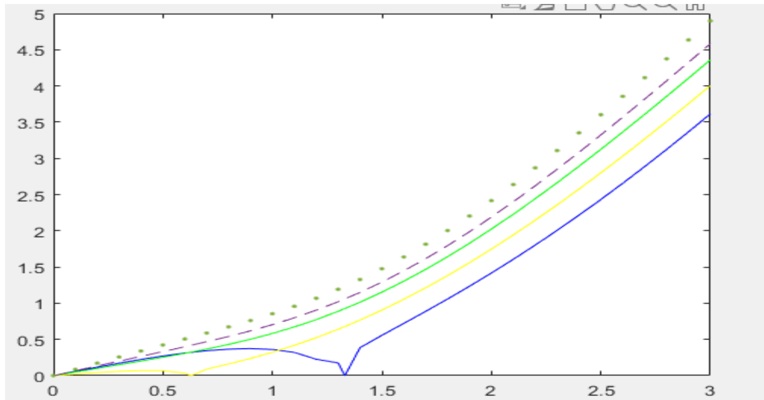

Fig. 2: For w=0.4with N=0.1,0.4,0.7,0.9 and 1.2

(III) For w=0.4,$\gamma = 0.1, \beta = 0.1$ and N=0.1, 0.4, 0.7, 0.9 and 1.2
The gravitational magneto hydrodynamic unstable domains are
0< $x < 1.436$, 0< $x < 1.030$, 0< $x < 0.637$
0< $x < 0.536$,
While the contiguous stable domain are
1.436< $x < \infty$, 1.030< $x < \infty$, 0< $x < \infty$
0.536< $x < \infty$, 0< $x < \infty$

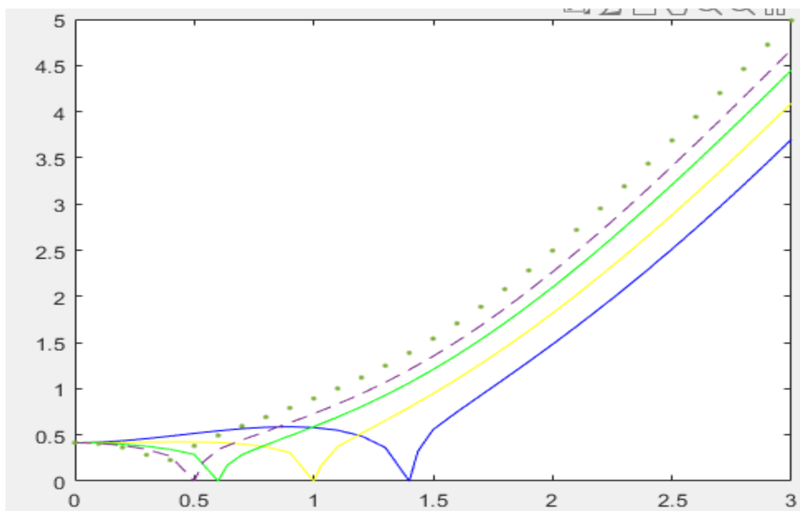

Fig. 3: For w=0.4,$\gamma = 0.1, \beta = 0.1$ and N=0.1, 0.4, 0.7, 0.9 and 1.2

(IV)For w=0.4, $\gamma = 0.7, \beta = 0.9$ and N=0.1, 0.4, 0.7, 0.9 and 1.2
The gravitational magneto hydrodynamic unstable domains are
0< $x < 1.743$, 0< $x < 1.544$, 0< $x < 1.344$
0< $x < 1.148$, 0< $x < 1.044$

while the contiguous stable domain are

1.743< $x < \infty$, 1.544< $x < \infty$, 1.344< $x < \infty$
1.148< $x < \infty$, 1.044< $x < \infty$

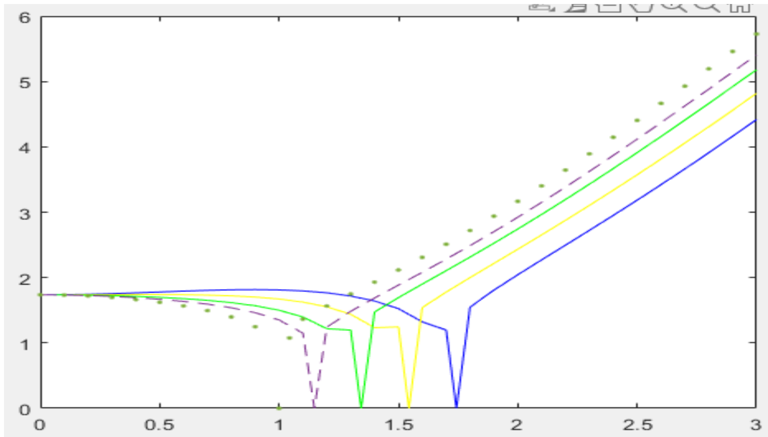

Fig. 4: For w=0.4, $\gamma = 0.7, \beta = 0.9$ and N=0.1, 0.4, 0.7, 0.9 and 1.2

(VI) For w=0.4,$\gamma = 0.9, \beta = 1.2$ and N=0.1, 0.4, 0.7, 0.9 and 1.2

The gravitational magneto hydrodynamic unstable domains are

0$< x < 1.744$, 0$< x < 1.547$, 0$< x < 1.349$

0$< x < 1.248$, 0$< x < 1.143$

while the contiguous stable domains are

1.744$< x < \infty$, 1.547$< x < \infty$, 1.349$< x < \infty$

1.248$< x < \infty$, 1.143$< x < \infty$

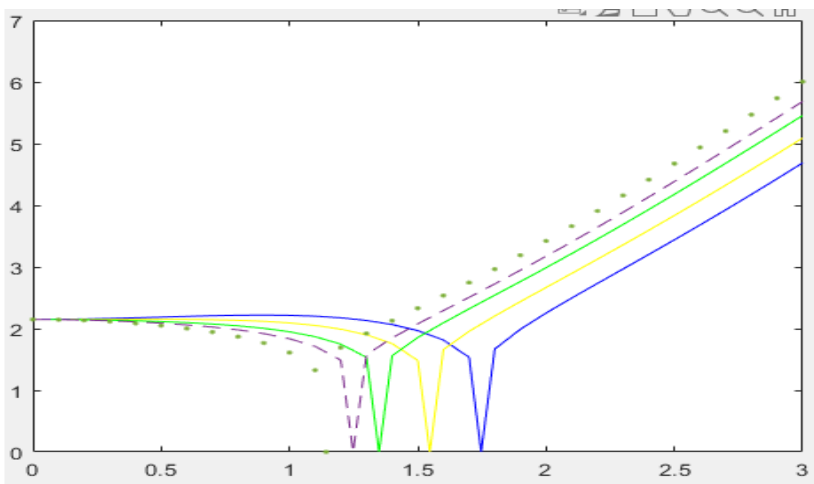

Fig. 5: For w=0.4,$\gamma = 0.9, \beta = 1.2$ and N=0.1, 0.4, 0.7, 0.9 and 1.2

8 Conclusions

From numerical analysis we get:

As N rises while velocity remains constant, the number of unstable domains decreases. This suggests that there is a stabilizing influence of the magnetic field.

The stable domains rise while the unstable domains shrink as N is increased with constant capillary force (w).

The capillary force has a strong stabilizing effect on the model.

It is found that when velocity values rise, unstable domains rise for the same values of N.This explains why the streaming effect destabilizes for both short- and long-wavelength waves.

9 Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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