Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/110528

Modeling and Analysis of a Real Time Spherical Tank Process for Sewage Treatment Plant

D. Mercy[1,*] *and S. M. Girirajkumar*[2]

[1] Department of EEE, St. Joseph's College of Engineering & Technology, Thanjavur, India
[2] Department of ICE, Saranathan College of Engineering, Tiruchirappalli, India

Received: 5 Jul. 2017, Revised: 14 Aug. 2017, Accepted: 24 Aug. 2017
Published online: 1 Sep. 2017

Abstract: We discuss the tuning of PID controllers for a nonlinear unstable process models using Particle Swarm Optimization (PSO) algorithm. The effectiveness of this scheme is validated through a comparative study with classical controller tuning methods, internal model control method and heuristic method such as Particle Swarm Optimization (PSO). A real time implementation of the proposed method is carried on a nonlinear spherical tank process using LabVIEW module; this design can be applicable for sewage treatment plant and it is apparent that the PSO algorithm performs well on a nonlinear unstable process models considered in this work. The PSO tuned controller offers enhanced process characteristics such as better time domain specifications, smooth reference tracking, supply disturbance rejection, and error minimization compared to other controller tuning methods.

Keywords: PID Controller, PSO, Nonlinear Process, Spherical tank process.

1 Introduction

A stand alone control algorithm used to tune the linear process in a classical way, usually called as PID (Proportional Integral Derivative) controller. PID controller is one of the earliest and most popular controllers. The improved PID and classical PID have been allied in various kinds of industry's control fields, as its tuning methods are developing. The PID controller was proposed by Norm Minorsky in 1922. Now days the researchers are mainly concentrating on the adaptive and optimized controller which deals with more complicated process. Many researchers develop various evolutionary algorithms for the tuning of PID controller even though Zeigler Nichols PID tuning is the base for all tuning methods. Bhawana Singh and Neelu Joshi discussed about the various classical and optimized tuning methods [1]. Marshiana et al proposes the controller design for nonlinear systems using Fractional order PI controller (FOPIC) technique which offers victory as a result of valuable methods in differentiation and integration and they offer additional flexibly in the controller design [2].

Suji Prasad et al proposed the particle swarm optimization based PID controller tuning used for the performance analysis of two tank spherical interacting level control system [3]. S.Nithya et al proposed the model based tuning methods of PID controller for a real time systems [4]. Based on the literature review, the proposed system includes classical PID tuning and optimized PID tuning for a spherical tank process. Classical PID tuning method is applicable for the linear process and for a nonlinear process optimized PID tuning is proposed. Spherical tank system is a difficult and important criterion due to its nature of the shape which can increases the nonlinearity of the process.To overcome the nonlinearity of the spherical tank process optimized control techniques are widely used. In the proposed system mathematical modeling of a spherical tank system is derived and the final transfer function is obtained. Using the transfer function, simulation and the real time results are obtained for various tuning methods and their values. The response curves are plotted for each tuning values. Comparative study is performed to analyze the response curves based on the time domain specifications and error criteria. From the results PSO based PID tuning method provide better results.

* Corresponding author e-mail: mercyphd@rediffmail.com

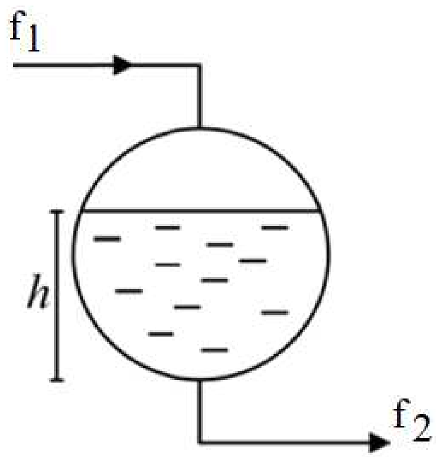

Fig. 1: Structure of spherical tank system.

2 Mathematical modeling of a spherical tank system

The proposed spherical tank system, identified as a nonlinear complex structure is shown in Fig. 1, where
h =Total height of the spherical tank,
f_1 =Input flow rate of spherical tank and
f_2 = Output flow rate of spherical tank.
Mathematical modeling of spherical tank system is derived based on the structure and the output transfer function is obtained. The output equations are well formulated and assumed as a process model structure with optimization. Optimized PID tuning is an effective tool for tuning of a controller. To derive the mathematical modeling of a spherical tank, the input of the process should be initialized and the input and output relations should be known and should be properly defined. The primary task is to understand the system and the system need to be investigated to realize the incident of nonlinearity present in the system dynamics. Now days, the utility of Particle Swarm Optimization Algorithm is extensively increasing because of its high accurate, fast and optimal responses compared with conventional techniques [5].

The nonlinear dynamics of the system is expressed by the FOPDT (first order process with delay time).

$$dV/dt = f_1 - f_2 \tag{1}$$

where, V =Volume of the spherical tank, f_1 =Input flow rate of spherical tank and f_2 =Output flow rate of spherical tank.

The volume of the spherical tank system is obtained based on the height of the tank and is given by,

$$V = \frac{4}{3}\pi h^3. \tag{2}$$

Solving the equations (1) and (2) the standard transfer function of nonlinear spherical tank system is obtained and is written as,

$$\frac{H(S)}{Q1(S)} = \frac{Re^{-\theta s}}{\tau S+1} \tag{3}$$

where R = Gain constant, θ = Time delay, τ = Time constant.

The standard transfer function is modeled with standard step input to obtained the response curve. The timing corresponds to the 35.3% and 85.3% the value of t_1 and t_2 are calculated. The parameters τ and θ are calculated as follows [9].

$$\tau = 0.67(t_2 - t_1) \tag{4}$$
$$\theta = 1.3t_1 - 0.29t_2 \tag{5}$$

The input and output flow rate of the spherical tank process is kept constant to approach the equilibrium state and the output is noted for each increment. Different readings are observed until the spherical tank process reaches the stable state. By substituting the gain constant, time delay, time constant in the transfer function (3) and it is approximated as FOPDT model [6]. The derived process model of a spherical tank process is given by,

$$G(s) = \frac{8.94e^{-4.6s}}{35.65S+1}, \tag{6}$$

where, Gain constant $R = 8.94$, Time delay $\theta = 4.6$ sec, Time constant $\tau = 35.65$.

Eq. (6) describes about the transfer function of a real time spherical tank process used in sewage treatment plant. The obtained transfer function is tuned with different tuning methods and the response curve is plotted. The plotted results are compared based on the time domain specifications and error values. The best tuning method is concluded based on the plotted results.

3 Classical PID tuning methods

"PID" is a short form for Proportional Integral Derivative controller that includes elements with three functions. The PID controller is a conventional controller combines the three error functions that is control error, integration of the error value, and derivation of the error value. The PID controller is helpful to control the output results, if three terms of controllers are constructed with inclusion of nonlinear function [7].

There are widespread papers present distinct methods to layout nonlinear PID controller. Including all nonlinear controllers some of the nonlinear controllers are mostly used in engineering applications. The reason for that is the linear controller is converted as a nonlinear based on simple specifications. Controller tuning is a process of adjusting the control parameters K_p, K_i and K_d to reach the optimum values and to obtain the desired control response. Tuning of the controller is essential for maintaining the system stability. The following tuning rules are effectively used to tune the PID controller of a spherical tank system in a classical way [8].

(1) Ziegler–Nichols (Z–N) method (1942): Controller tuning is based on the value ultimate gain and the value of ultimate period.
(2) Cohen–Coon (C–C) method (1953): Tuning of the controller is Based on Process reaction curve.
(3) Shinskey tuning method (1990)

$$K_c = \frac{0.889\tau}{K\tau_d}, \quad T_i = 0.70\tau_d, \quad T_d = 1.75\tau_d$$

(4) Maclaurin tuning method (1990)

$$K_c = \frac{T_i}{k(\lambda+\tau d)}, \quad T_i = \tau + \frac{\tau_d^2}{2(\lambda+\tau_d)},$$
$$T_d = \left(\frac{\tau^2 d}{2(\lambda+\tau d)}\right)\left(1-\frac{\tau d}{3Ti}\right)$$

(5) Connel tuning method (1987): Tuning of the controller is Based on Process reaction.

$$K_c = \frac{1.6\tau}{k\tau d}, \quad T_i = 1.6667\tau_d, \quad T_d = 0.4\tau d$$

(6) Astrom and Hagglund tuning method(1984): Integral gains of PID controller.

$$K_c = \frac{0.94\tau}{K\tau_d}, \quad T_i = 2\tau_d, \quad T_d = 0.5\tau_d.$$

4 PSO tuning of a spherical tank system

Particle Swarm Optimization is a strong random functional technique is initiated by the scattering of particles in the search space and swarm intelligence. PSO is a concept related to the problem solving based on public interaction. Particle Swarm Optimization method was invented by James Kennedy and Russell Eberhart.This method exploit a numerous representatives that compose a flock scattering around in the search space and come across for the high quality solution. Each particle in the search space is treated as a point which fine-tunes its airborne terminology according to its personal practice and the airborne information of the additional particles present in the system [9].

The PSO algorithm has to follow three steps and it has to repeat the steps still reaching the stopping condition.

–Calculate the fitness of each particle.
–Revise individual and global best fitness and positions.
–Revise the velocity and location of each particle.

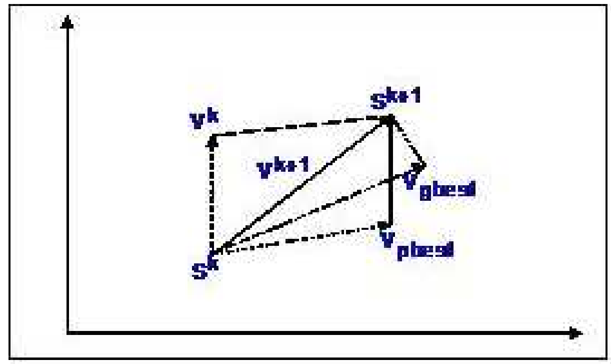

Fig. 2: Graphical Representation of PSO Values.

Fig. 2 represents the graphical analysis of PSO values in the search space, where
s^k: Present searching point,
s^{k+1}: Customized search point,
v^k: Present velocity,
v^{k+1}: Customized velocity,
v_{pBest}: Localized Velocity,
v_{gBest}: Globalized Velocity.
Every particle in the search space sustain the track of records and are related with the high efficient result that have been recognized by the particles. This is known as personal best, pBest. The obtained new finest value is tracked using the PSO technique is the finest value accomplished up to now through every particle in the locality of that particle. It is known as gBest. The fundamental idea of PSO lies on the speeding up of each particle in the pathway of its pBest and the gBest positions, by means of a subjective biased speeding up at each time step [10].

Pseudo Code is a basic code for implementing the PSO algorithm. The basic code for executing the algorithm is given below

Start the process
    For each and every particle
    Initiate each particle in the search space
Stop the process
Repeat the process
    For each and every particle
    Calculate the fitness value
    If calculated value is better than the best value
    Set the current value as new *pBest*
Stop the process
Choose the particle with the best fitness value from all
        the particles and named as the *gBest*.
    For each and every particle
    Estimate the particle velocity
    Renew the particle location
Stop the process [11]

In the proposed system optimized tuning values are identified based on iteration values. From the classical PID tuning methods best K_p, K_i, K_d values are obtained. The obtained values are used to initialize the PSO tuning

Table 1: Parameter Initialization.

Population Dimension	50
Iteration Count	100
Constant Velocity, c_1	2
Constant Velocity, c_2	2

Table 2: PID Tuning Parameters.

Tuning Methods	K_p	K_i	K_d
Astrom & Hagglund	1.64	0.258	2.48
Cohen & Coon	2.03	0.24	2.38
IMC	0.18	0.09	0.25
Shinskey	1.85	0.31	3.54
Connel	2.47	0.49	1.36
Maclaurin	1.34	0.11	1.39
PSO	0.11	0.01	0.12

method and initialization of PSO tuning includes the parameter initialization process is shown in Table 1. To introduce PSO, numerous parameters want to be described. Parameter initialization is a process of initiating the dimension of the search space, number of iterations and velocity constants. The dimension of the flock satisfies the necessity of global optimization and working out cost. Initial inputing of the parameters are as per the table. After the completion of the iteration global best and local best values are obtained.

4.1 PID Tuning Parameters

PID parameters are calculated using various tuning methods and the K_p, K_i, K_d values are tabulated in Table 2 and best tuning values are analysed.

4.2 Performance Index

The most significant method of applying the PSO algorithm is to choose the objective function which is used to estimate the fitness of each Particle. Most of the process uses performance indices as an objective function. The objective functions are Mean of the Squared Error, Integral of Time Absolute Error, Integral of Absolute Error, and Integral of the Squared Error. Based on the above objective function various error criteria were calculated for each tuning methods and the error values are compared. The PID controller is employed to reduce the error value and it will be defined more thoroughly based on the error criterion. If the performance indices values are smaller it gives the best results and for higher values it will not provide good results [12].

5 Experimental setup

The non-linear behaviour of the spherical tank system is identified by constant input flow rate. The maximum height of the tank is 20 cm. Input to the tank is incremented step wise, the current to the system is maintained at 4–20 mA and passes all the way through the serial port RS-232 along DAQ interface unit. Through manual control method, specified transform at input value the output response of the process is documented. Using controller tuning methods the time constant and delay time of a FOPTD process is constructed using tangent method based on its point of inflection.

Fig. 3: Experimental Setup of the Automated Process & the Real Time Setup of the Proposed System

5.1 Real Time Setup

Fig. 3 shows the experimental setup of the automated process and the real time setup of the proposed system consists of spherical tank process, water reservoir, centrifugal pump, rotameter, an electro pneumatic converter and pneumatic control valve. The output signal from the process is interfaced with a computer using compact DAQ through RS-232 serial port. Thus the coding were developed using LabVIEW software and interfaced using DAQ module.

In Fig. 3 water reservoir is used as a storage tank. Centrifugal pump is used to pump the water from the reservoir and circulates the water throughout the plant. The rotameter is an industrial flowmeter used to measure the flowrate of liquids and gases. An electro pneumatic converter converts a 4 to 20 mA input signal to a directly proportional (3 to 15 psi) pneumatic output signal. Pneumatic control valve is used to control the flow rate. The pneumatic valve used here is “air to close valve” which is used to adjusts the water flow in the spherical tank system. The height of the spherical tank process is obtained through computational method and broadcasted in the form of current range between (4–20) mA.

Table 3: Technical specifications.

Parts Used	Description
Materials used in Spherical Tank	Material: Stainless Steel Diameter: 76.5 cm, Volume: 7.15 litres
Volume of Storage tank	Volume: 10 litres Stainless Steel
Type of Sensor	RF Capacitance Type
Size of the Control valve	1/4” Pneumatic actuated Type: Air to open Input (3–15) psi
Range of Rotameter	Range (0–18) lpm
Size of Air regulator	1/4” BSP Range (0–2.2) bar
I/P converter	Input level-20 psi and current range (4–20) mA Output level-(3–15)psi
Pressure gauge	Range 1(0-30) psi Range 2 (0–100) psi
Pump	Centrifugal pump 0.5 HP

Hardware and software of the system are interfaced by means of DAQ system. The input to the system is regulated and tuned using optimal tuning method. The control action is performed by executing LabVIEW coding, based on the hardware interface. The control signal controls the valve position thus controls the level of the spherical tank. The technical specifications of the spherical tank process is described in Table 3.

6 Results and discussion

The output reaction of spherical tank system using PID controller is determined and the results are recorded. The response of the controllers are estimated and evaluated in the form of rise time, overshoot and settling time with existence of measurement noise. The controller output is evaluated based on the performance index, if the error values are lesser than the controller is consider as a best controller. PSO tuning terminology provides an iteration based analysis were we can get the optimized local best and global best values. This value can be used to get quick steady state response.

6.1 Block diagram of a simulation process.

Fig. 4 represents the block diagram of a spherical tank process. The closed loop simulation diagram of the spherical tank process is shown in figure. Closed loop system of a real time process consists of input block, output block, error detector, controller, plant and a feedback loop. The main advantages of designing the closed loop system includes accurate response, sensitive to input variations, flexible operation, reliable output, dynamic response and automatic error corrections. In this proposed block diagram closed loop system is widely preferred due to its automatic error correction factor for spherical tank process used in sewage treatment plant. In this plant disturbances occurs in various parts of the plant which affects the working of the process and it leads to get the incorrect output response. Automatic correction of errors is achieved by implementation of closed loop spherical tank process and it is designed using LabVIEW software. In LabVIEW input and output of the process is initiated in the front panel and the designing of the system is performed in the block diagram panel. The block descriptions are as follows; initially the standard step input is connected to the error detector as a reference input and the output of the process is connected as feedback to the error detector for comparison purpose. The error value is fed to the controller blocks which consists of PID controllers specified with proportional value (K_p), integral value (K_i) and derivative value (K_d). The above stated values are calculated with the controller tuning formulas for the derived spherical tank transfer function. The multiple response of the system is combined and plotted in a single graph using multiplexer. The collector is used to get the response in a collective manner with respect to time, thus the display device helps to project the output response in a graphical way. The output response is analyzed and the results were compared with error analysis and time domain analysis. Based on the comparative study best tuning method is identified.

Table 4: Error analysis.

Methods	IAE	ITAE	MSE	ISE
Astrom& Hagglund	63.3	176.5	78.3	124.5
Connel	67.5	167.3	76.3	111.3
Shinskey	67.8	187.3	81.5	134.5
Maclaurin	74.8	171.4	72.9	129.5
Ziegler & Nichols	78.4	198.5	87.5	134.2
IMC	32.3	115.6	54.2	98.6
PSO	23.5	102.5	45.6	88.7

6.2 Error Analysis

Table 4 describes the different types of error values for various tuning methods. Robustness of the PID controller is analysed based on its performance index and is fixed as an objective function for optimized problems. The PID controllers tuned by the PSO based methods compared with their time domain responses and also with its objective function from the four major error criterion techniques of Integral Time of Absolute Error (ITAE), Integral of Absolute Error (IAE), Integral Square of Error (ISE) and Mean Square Error (MSE).

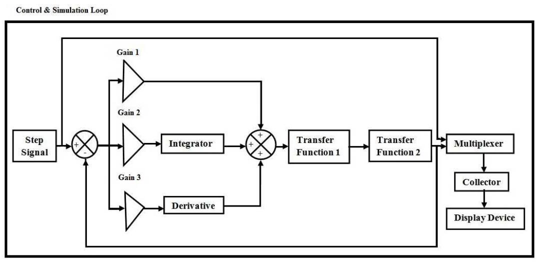

Fig. 4: Block diagram of a spherical tank closed loop simulation process.

Table 5: Time domain response values.

Methods	Rise Time	Over-shoot	Settling Time
Astrom & Hagglund	6	25	34
Cohen & Coon	4	40	19
IMC	11	0	12
Shinskey	8	30	35
Ziegler & Nichols	4	60	23
Connel	2	45	15
Maclaurin	10	0	13
PSO	12	0	10

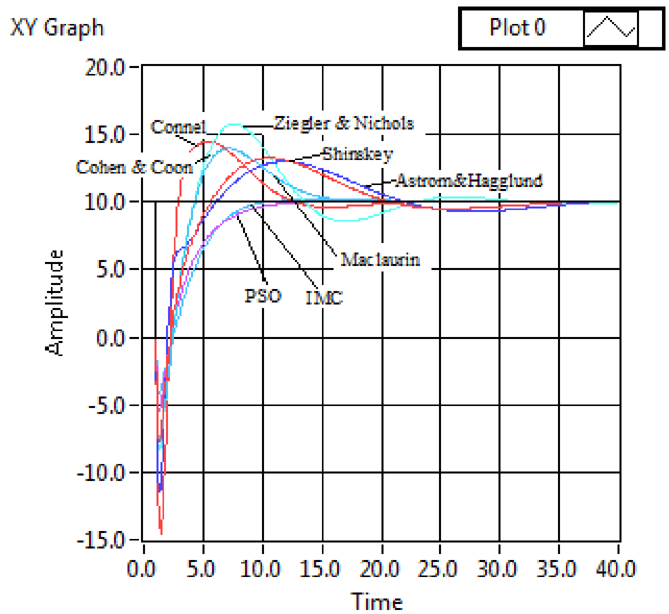

Fig. 5: Comparative graph for PID tuning methods.

Investigating the robustness of the proposed method the process uncertainties are reduced to minimum level using the error values. This error analysis can be done for the identification of best tuning method. From the Table 4 the error values of PSO tuning method is very much reduced. This shows that PSO tuning is the best tuning method.

6.3 Time domain Analysis

Table 5 includes the time domain specifications of the various tuning methods. Time domain analysis is necessary to investigate the spherical tank process in a better way. It explains clearly about the process delay time, rise time, peak overshoot and settling time. Rise time is the time to reach the 100% of the output for a given input at a very first time and peak overshoot describes about the time of output response when it reaches the peak value. Settling time is the time to reach the steady state with constant output. From the time domain analysis, PSO tuning method provides better performance based on increased rise time, smooth response without overshoot and fast settling time compared with other tuning methods.

6.4 Output Response Analysis

The output response is shown in Fig. 5. From the results it is infer that using error analysis and time domain analysis the best tuning method is identified. The output response is obtained by applying various controller tuning method for a spherical tank process. The output response is plotted against time with different colors. Error analysis describes the error values for different tuning methods and from the graphical analysis it is infer that by using PSO tuning the error values IAE, ITAE, ISE & MSE are reduced to minimum value. Time domain analysis discussed about the rise time, peak overshoot and settling time. The output response explains that using PSO tuning rise time is increased, overshoot is reduced and settles fast. In Fig. 5 violet color response is the PSO response. Comparing the error values and time domain specifications PSO tuning is identified as the best tuning for a spherical tank process used in sewage treatment plant.

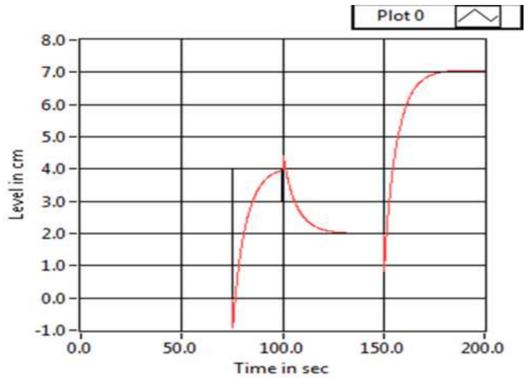

Fig. 6: Servo Response of PSO for setpoint changes at 7 cm.

6.5 Servo Response Analysis

In a closed loop feedback system there are two operating modes they are servo and regulatory modes. Design and implementation of servo and regulatory control loops are used to maximize the process efficiency. Servo response is the response of the system to setpoint changes. Fig 6 shows the servo response of a spherical tank process used in sewage treat plant. In sewage treatment plant various inputs are given at different levels and by using the servo analysis the efficiency of the plant is improved.

6.6 Regulatory Response Analysis

Regulatory response is the response of the system to load disturbance changes and is shown in Fig. 7. In a real time process there is a possibility of disturbances due to various parameters like load variations, environmental conditions, misuse of instruments, loading effects, calibration errors, etc. Regulatory response used in sewage treatment plant rejects the disturbances and provides smooth response curve.

7 Conclusion

The PSO controller tuning results are evaluated and analysed with the conventional PID tuning methods. The stated optimized tuning method provides more efficient results in terms of improved step response, reduced error, fast response time and rapid settling time over traditional PID tuning methods in the application of spherical tank based sewage water treatment plant. Results shows that the overshoot of PSO controller is reduced to zero and in the error criteria IAE value is reduced to an extent. Concluding that the PSO tuning is preferred as the best tuning technique and the entire concept is configured to implement in the sewage treatment plant for complete maintenance free operation and storage applications.

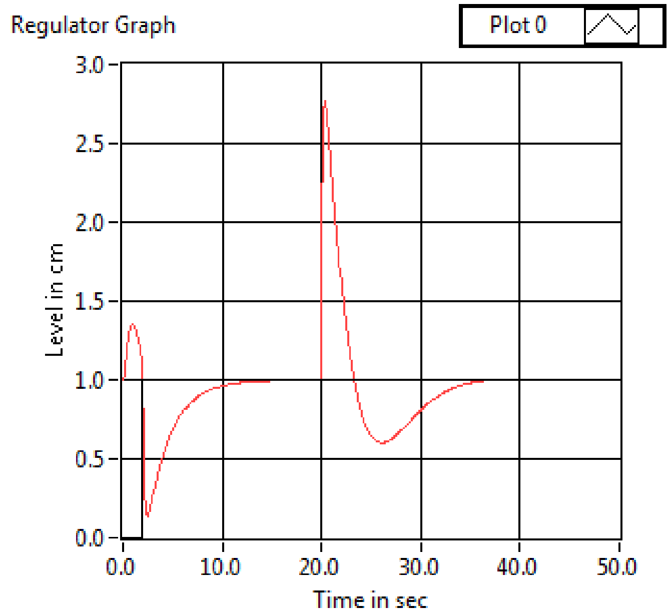

Fig. 7: Regulatory Response of PSO tuning for load disturbance at 20 seconds.

References

[1] Bhawana Singh and Neelu Joshi, "Tuning Techniques of PID controller: A review," International Journal on Emerging Technologies, **8(1)**, 2017, 4481–485.
[2] D. Marshiana and P. Thirusakthimurugan, "Comparison of Fuzzy PI Controller with Particle Swarm Optimization for a Nonlinear System", International Journal of Control Theory & Application, **9(34)**, 2016, 333–341.
[3] S.J. Suji Prasad, B. Venkatesan and I. Thirunavukkarasu, "Performance analysis of two tank spherical interacting level control system with particle swarm optimization based PID controller", International Journal of Advanced Engineering Technology, **7(2)**, 2016, 922-925.
[4] S. Nithya, N. Sivakumaran, T. Balasubramanian and N. Anantharaman "Model Based Controller design for a spherical tank process in real time", IJSSST, **9 (4)**, 2008, 25–31.
[5] S. Morkos, H. Kamal, "Optimal Tuning of PID Controller using Adaptive Hybrid Particle Swarm Optimization Algorithm", Proceeding of the Int. J. of Computers, Communications & Control, **7(1)**, 2012, 101–114.
[6] S. Nithya, N. Sivakumaran, T. Balasubramanian and N.Anantharaman, "Design of controller for nonlinear process using soft computing", Instrumentation Science and Technology, **36(4)**, 2008, 437–450.
[7] Abhishek Sharma and Nithya Venkatesan, "Comparing PI controller Performance for Non Linear Process Model", International Journal of Engineering Trends and Technology, **4(3)**, 2013, 242–245.
[8] M. Vijayakarthick and P.K. Bhaba, "Optimized Tuning of PI Controller for a Spherical Tank Level System Using New Modified Repetitive Control Strategy", International Journal of Engineering Research and Development, **3(6)**, 2013, 74–82.

[9] K.K. Avinashe and Merin Mathews, “Internal Model Control Design for Nonlinear Spherical Tank Level Process”, IJETSR, **2(8)**, 2015, 12–18.
[10] A. Ganesh Ram, S. Abraham Lincoln, “Real Time Implementation of Fuzzy Based Adaptive PI Controller for a Spherical Tank System”, IJSSST, **14(6)**, 2013, 1–8.
[11] D. Dinesh Kumar, C. Dinesh and S. Gautham, “Design and Implementation of Skogested PID Controller for Interacting Spherical Tank System”, IJAEEE, **2(4)**, 2013, 117–120.
[12] K. Hari Krishnaa, J Satheesh Kumar and Mahaboob Shaik, “Design and Development of Model Based Controller for a Spherical Tank”, International Journal of Current Engineering and Technology, **2(4)**, 2012, 374–376.

D. Mercy received her Bachelors degree in Electronics and Instrumentation Engineering from Jayaram College of Engineering and Technology, Trichy and her Post Graduate degree in Control Systems & Instrumentation Engineering from SASTRA University, Thanjavur. She is currently doing her PhD under the Faculty of Electrical Engineering in the area of Process Control in Anna University Chennai. She has around 11 years of teaching experience in various engineering colleges. Presently she is serving as an Assistant Professor and Head in the Department of Electrical and Electronics Engineering at St. Joseph’s College of Engineering and Technology, Elupatti, Thanjavur. She has published more than 15 research papers in various International Journals and Conference Proceedings. Her area of interest includes Control systems, Process Modeling, Fuzzy Logic, Genetic Algorithm and optimization techniques etc.

S. M. Girirajkumar received his Bachelors degree in Electronics and Instrumentation Engineering from Annamalai University and his M.Tech + Integrated Ph.D (M.Tech-Control System Based, Ph.D-Electrical Engineering) from SASTRA University, Thanjavur. He has around 18 years of teaching experience in various engineering colleges and around 3 years of Industrial experience. Presently he is serving as a Professor and Head in the Department of Instrumentation and Control Engineering at Saranathan College of Engineering, Trichy. He has published more than 80 research papers in various International, National Journals and Conference Proceedings. He has obtained various grants from Government of Tamilnadu and completed the funded projects. He is a recognized research supervisor for Anna University of Technology, Tiruchirappalli, and Chennai associated with EEE department. He is Guiding 4 scholars for their PhD work, through Anna University of Technology, Tiruchirappalli and Guiding 6 scholars for their PhD work, through Anna University of Technology, Chennai. His area of interest includes Control systems, Process Control, Industrial Automations, Programmable Logic Controller.