Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/110325

Linear and Non-linear Trajectories of Oblique Penetration into a Granular System

Waseem Ghazi Alshanti[1,*] *and Ahmad Ghazi Alshanti*[2]

[1] Department of Mathematics, University of Hail, K S A.
[2] Department of Computer Engineering, Girne American University, Cyprus, Turkey.

Received: 2 Feb. 2017, Revised: 25 Mar. 2017, Accepted: 29 Mar. 2917
Published online: 1 May 2017

Abstract: In this paper, a two-dimensional discrete element model based on the soft particle method has been carried out to capture the dynamical behavior of a two-dimensional granular bed impacted by a projectile penetrating it obliquely. We systematically study the influence of the projectile's impact angle on the penetration profile of the projectile underneath the granular bed. The outcomes of the model suggest that the trajectory of the projectile exhibits a significant transition from linear to non-linear gradually with the decreasing of the projectile's impact angle. A function that models the profiles of the projectile's motion beneath the granular bed is also given.

Keywords: Discrete element method, Granular system, Projectile, Oblique impact.

1 Introduction

Nevertheless, the science of granular systems has an extensive research history that has deep roots in time. A general theory that governs the mechanical behavior of the granular systems has not so far been recognized. It is known that particulate systems cannot be treated as gas and fluid or solid as they have a hybrid state between these three phases. There are still no reasonable clarifications for many phenomena due to the complicated behaviors of these particulate systems. One of these challenging phenomena is the dynamical behavior of a granular bed impacted by a projectile penetrating at normal and non-normal angles.

For normal penetration of a projectile into a granular bed, a power relationship between projectile's penetration depth and projectile mass-to-area ratio has been suggested [1]. Also, the deceleration of a projectile in a granular media is characterized with an equation developed from general physical considerations [2]. Experimental and numerical simulations were employed by using discrete element method (DEM) to study the wave propagation of particulate aggregation struck by a projectile at relatively low velocities [3] and [4]. Further, it is reported that the penetration increases linearly with the momentum of the projectile [5]. The rebound velocity of a projectile depends on the lateral gap between the particles of the granular system as well as the impact position of the projectile [6]. Hou et. al [7] found that the behavior of a granular system is similar to a fluid during initial impact. Scaling laws of the projectile's penetration distance with both its impact velocity and the total drop distance were presented [8] and [9]. More and more scaling laws relevant to the normal penetration process were proposed [10,11,12]. Ciamarra et. al [13] reported that the time taken for a projectile to decelerate to a stop is independent of its impact velocity. The energy exchange between the projectile and the granular bed was investigated numerically by implementing the DEM [14]. The influence of packing on both the force chains and the velocity field of the particles within a granular system has been also a point of interest through the granular system literature [15]. Recently, analytic approach has been introduced to determine the penetration depth and stopping time for a projectile impacting a granular bed [16].

Unlike the normal impact case, the dynamics of the oblique impact possess a wide range of complex features. Therefore, studies related to oblique impact with granular materials have not been adequate for understanding the dynamics of the this impact process. So far, few attempts have been made to investigate dynamical problems

* Corresponding author e-mail: waseemalshanti@yahoo.com

© 2017 NSP
Natural Sciences Publishing Cor.

related to the oblique impact process. One of these problems is the connection between the projectile's trajectory and the magnitude of its impact angle. It is found that the trajectory profile of a projectile penetrating obliquely into a granular bed exhibits linear and non-linear traces depending on the value of the projectile's impact angle [17]. Moreover, the characteristics of the velocity field of the bed's particles through impacting are affected by the magnitude of the impact angle as well the value of the rotational (angular) velocity of the projectile [18]. The influences of the density ratio and diameter ratio on the oblique penetration depth of a spinning projectile has been considered [19]. The stopping time of the projectile during oblique penetration process is also considered [20].

The main purpose of this paper is to establish a two dimensional discrete element model in order to study the oblique penetration of a projectile into a granular system. The model investigates the traces of the projectile's trajectories for various values of impact angle.

2 The DEM and numerical simulation scheme

The soft-particle discrete method which originally was introduced by Cundall and Strack [21] is adopted in the present numerical simulations. In a typical simulation, a 250×250 *mm* granular bed is generated by employing the random packing method. The bed's particles are two-dimensional discs with a uniform distribution of diameters $0.9mm \leq d_p \leq 1.4mm$ and density $\rho_b = 1.0 \times 10^3 kg/m^3$. The granular bed is subjected to the oblique impact of projectile with diameter $d_p = 0.55 \times 10^{-2} m$, density $\rho_p = 2.5 \times 10^3 kg/m^3$ and a given impact velocity $\mathbf{v} = 15m/sec$. The projectile strikes the middle of the horizontal free surface of the granular bed with different values of the impact angle θ ranging from 10° to 90°. See Figure (1). For a typical simulation,

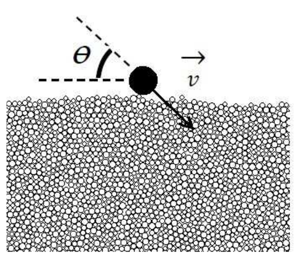


Fig. 1: Projectile with impact velocity $\vec{v}$ strikes the middle of the horizontal free surface of a granular bed at impact angle θ.

the particles bear two types of forces: contact forces and gravitational force. Unlike the gravitational body force, the particle contact forces act only when the particle is in contact with other particles and/or with a boundary wall. The normal contact force is modelled by a damped linear spring, while the tangential contact force by a linear spring in series with a frictional sliding element model as shown in Figure(2). The interaction forces developed

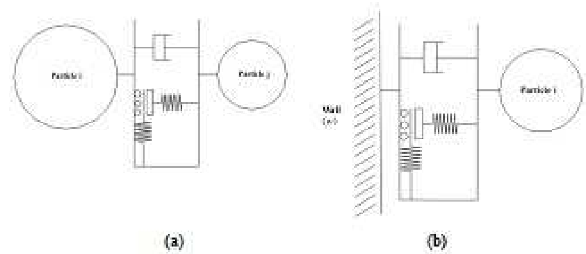


Fig. 2: (a)Contact mechanism between particle i and j (b)Particle-wall contact between particle i and wall w.

between two particles or between particle and boundary wall are calculated based on the physical properties and the relative velocities. The dynamic equations for the motion for each of the particles under these forces are

$$m_i \frac{d^2\mathbf{r}_i}{dt^2}(t) = \sum_{j=1, j\neq i}^{N_p} \mathbf{F}_{c,ij} + \sum_{k=1}^{N_w} \mathbf{F}_{c,iw_k} + \mathbf{F}_{gi}, \quad (1)$$

$$I_i \frac{d^2\theta_i}{dt^2}(t) = \sum_{j=1, j\neq i}^{N_p} (\mathbf{r}_i \times \mathbf{F}_{c,ij}) + \sum_{k=1}^{N_w} \left(\mathbf{r}_i \times \mathbf{F}_{c,iw_k}\right), \quad (2)$$

where m_i, I_i, $\mathbf{r}_i(t)$ and $\theta_i(t)$ are, respectively, the mass, rotational moment of inertia, position and rotational vectors of the center of particle i. $\mathbf{F}_{gi}$, $\mathbf{F}_{c,ij}$ and $\mathbf{F}_{c,iw_k}$ are, respectively, gravitational body force and contact forces acting on particle i due to particle j and boundary wall w_k. N_p and N_w are, respectively, the number of particles and walls in the simulation.

In the analysis of the system, any contact force is decomposed into normal and tangential components. For any typical pair of particles i and j, if they are in contact, then the contact force formula is

$$\mathbf{F}_{ij}(t) = \mathbf{F}_{ij,\mathbf{n}}(t)\mathbf{n} + \mathbf{F}_{ij,\mathbf{s}}(t)\mathbf{s}, \quad (3)$$

where $\mathbf{n}$ and $\mathbf{s}$ are unit vectors in the normal and shear directions of the contact plane, $\mathbf{F}_{ij,\mathbf{n}}\mathbf{n}$ and $\mathbf{F}_{ij,\mathbf{s}}\mathbf{s}$ are, respectively, the magnitudes of the normal contact force and shear contact force namely,

$$\mathbf{F}_{ij,\mathbf{n}}\mathbf{n}(t) = -k_{\mathbf{n}}\delta_{ij,\mathbf{n}}(t), \quad (4)$$

© 2017 NSP
Natural Sciences Publishing Cor.

Table 1: Normalized simulation parameters of Oblique penetration

Parameter	Value
Number of particles N_p	10500
Bed particle diameter $d_b/d_\circ$ (where $d_\circ = 1.0mm$)	0.9 - 1.4
Bed particle density $\rho_b/\rho_\circ$ (where $\rho_\circ = 1.0 \times 10^3 kg/m^3$)	1
Projectile diameter $d_p/d_\circ$ (where $d_\circ = 1.0mm$)	5.5
Projectile density $\rho_p/\rho_\circ$ (where $\rho_\circ = 1.0 \times 10^3 kg/m^3$)	2.5×10^3
Particle-particle normal spring stiffness $k_{\mathbf{n},pp}/\left(\rho_\circ \mathbf{g} d_\circ^2\right)$	1.266×10^5
Particle-particle normal dashpot coefficient $c_{\mathbf{n},pp}/\left(\rho_\circ \mathbf{g}^{0.5} d_\circ^{2.5}\right)$	2.524×10^2
Particle-particle tangential spring stiffness $k_{\mathbf{s},pp}/\left(\rho_\circ \mathbf{g} d_\circ^2\right)$	1.266×10^5
Particle-particle friction coefficient μ_{pp}	1.0
Particle-wall normal spring stiffness $k_{\mathbf{n},pw}$	1.566×10^5
Particle-wall normal dashpot coefficient $c_{\mathbf{n},pw}/\left(\rho_\circ \mathbf{g}^{0.5} d_\circ^{2.5}\right)$	2.01×10^2
Particle-wall tangential spring stiffness $k_{\mathbf{s},pw}/\left(\rho_\circ \mathbf{g} d_\circ^2\right)$	3.006×10^4
Particle-wall friction coefficient μ_{pw}	1.0
Gravity in x-direction $\lVert \mathbf{g}_x \rVert / \lVert \mathbf{g} \rVert$	0
Gravity in y-direction $\lVert \mathbf{g}_y \rVert / \lVert \mathbf{g} \rVert$	-9.8
Time step $\Delta t \sqrt{\lVert \mathbf{g} \rVert / d_\circ}$	5.37×10^{-6}

$$\mathbf{F}_{ij,\mathbf{s}}(t)\mathbf{s} = -sign\left[\delta(t)\right] \cdot \min\left\{k_{\mathbf{n}}\left|\delta_{ij,\mathbf{s}}(t)\right|, \mu \mathbf{F}^{e}_{ij,\mathbf{n}}(t)\right\}, \tag{5}$$

where $k_{\mathbf{n}}$ and $k_{\mathbf{s}}$ are, respectively, the particle-particle normal and tangential spring coefficients, $\mathbf{F}^{e}_{ij,\mathbf{n}}(t)$ and μ are, respectively, the elastic contribution of the contact force between the particles i and j in the normal direction (**n** direction) and the friction coefficient of the granular particles,

$$\delta_{ij,\mathbf{n}}(t) = (R_i + R_j) - \left|\mathbf{r}_i(t) - \mathbf{r}_j(t)\right| \tag{6}$$

and

$$\delta_{ij,\mathbf{s}}(t) = \int_{t_\circ}^{t} \left(\dot{\mathbf{r}}_i(\eta) - \dot{\mathbf{r}}_j(\eta)\right) \cdot \mathbf{s}\, d\eta \tag{7}$$

are, respectively, the normal compression and the tangential displacement between the particles i and j over the time step $\Delta t = t - t_\circ$, R_i and R_j are the radii of the particles i and j.

A neighboring cell technique is implemented in the present simulations in order to reduce the number of contact checks for each particle. Once all forces and moments acting on particle due to contacts are calculated, the updated velocities and positions of all particles can be obtained by numerical integration of Newton's equation of motion. The simulation input parameters include: physical properties of the granular particles under consideration, initial conditions and boundary conditions. The initial conditions include: the initial positions and velocities of all particles and the geometry of the boundary conditions. Using the density of particle, gravitational acceleration, and particle diameter all parameters are normalized. A description of mechanical and environmental dimensionless simulation parameters are given below in Table (1). According to Alshanti and Hu [8], three different stages of the impact process can be recognized: impact, penetration and collapse. In Figure (3), the projectile initially hits the horizontal surface of the granular bed at a non-normal impact angle and then penetrates obliquely the granular bed. A momentary crater is formed and gradually disappears due to the collapse of the bed's granules. Through impact and penetration stages, the projectile bears finite collisions and hence, several forces acting on it. These forces waste the projectile kinetic energy and therefore, the projectile impact velocity decelerates till the projectile comes to rest after achieving the maximum penetration depth underneath the bed surface.

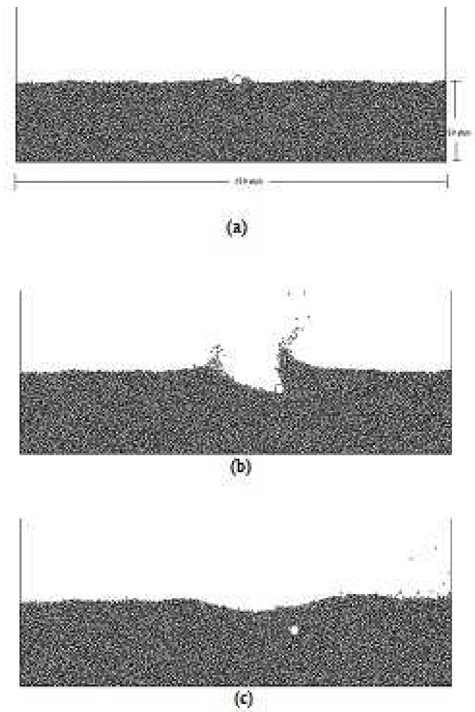


Fig. 3: Simulation sequential snapshot of oblique penetration with $\theta = 45^\circ$ (a) Impact stage $t = 0.2\,\text{sec}$ (b) Penetration stage $t = 2.2\,\text{sec}$ (c) Collapse stage$t = 19.8\,\text{sec}$.

3 Results and discussion

To analyze the relationship between the trajectory of the projectile and its impact angle, a series of simulations are conducted. For a typical simulation, a range of impact angle from $\theta = 10^\circ$ to $\theta = 90^\circ$ is considered while other simulation's parameters are kept identical as listed in Table 1 including the projectile's impact velocity $\mathbf{v} = 15m/sec$. Figure 4 shows the projectile's penetration profiles below the granular bed surface for different values of θ ranging from $\theta = 10^\circ$ to $\theta = 90^\circ$. It is

© 2017 NSP
Natural Sciences Publishing Cor.

obvious that the magnitude of the projectile's impact angle plays a significant role in forming the profile of the its trajectory under the granular bed surface. For small values of impact angle, namely, $\theta = 10^\circ$ to $\theta = 45^\circ$ the projectile's trajectories exhibit non-linear traces. Alternatively, for large values of impact angle, namely, $\theta = 55^\circ$ to $\theta = 90^\circ$, the trajectories of the projectile are linear traces.

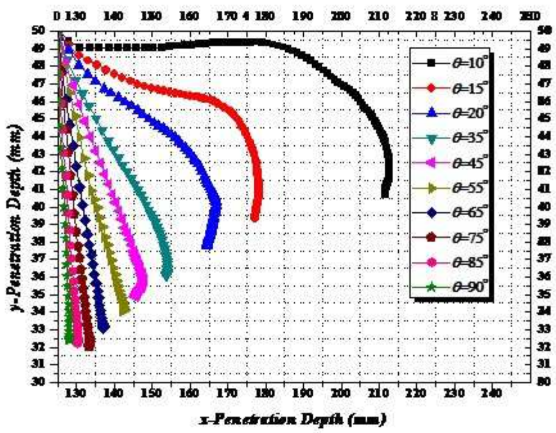


Fig. 4: Projectile's trajectories of oblique penetration at different values of impact angles with fixed impact velocity $\vec{v} = 15m/\sec$

This interesting phenomenon can be interpreted as follows: unlike the case for large values of the impact angle, the projectile's initial motion is close to the free surface of the granular bed when the projectile holds small values of impact angles. In this case, the projectile bears, relatively, small motion resistance due to the countable collisions with the, fairly, free bed's particles near surface. Hence, it remains moving close to the bed surface before it starts penetrating deeply. Therefore, for fixed projectile's impact velocity, the profile of its oblique penetration exhibits a considerable change from linear to a non-linear trace with the decreasing of its impact angle. Consequently, there exists a transmission value θ_t such that, $45^\circ \leq \theta_t \leq 55^\circ$ at which the projectile's trajectory turns from linear to non-linear trace.

For the current proposed oblique penetration process, we define $\forall x \in [125, 250]$, the function $T_{\mathrm{Pr}}(x)$ to describe the trajectory trace of the projectile beneath the proposed granular bed at each value of θ as follows

$$T_{\mathrm{Pr}}(x) = \begin{cases} Ae^{\frac{-x}{\alpha}} + B, & 10^\circ \leq \theta < \theta_t \\ Cx + D, & \theta_t < \theta \leq 90^\circ \end{cases},$$

where α, A, B, C and D are constants depend on the magnitude of the impact angle as well as the mechanical and environmental simulation's parameters. Table (2) shows the values of the above constants for the obtained ten projectile's trajectories given in Figure (4).

Table 2: The values of α, A, B, C and D for the obtained ten projectile's trajectories

Impact angle value (θ)	α	A	B	C	D
10°	-15.11	-5.89E-6	50.02	-	-
15°	-23.61	-0.01	51.06	-	-
20°	-28.83	-0.04	52.1	-	-
35°	119.71	180.81	-13.63	-	-
45°	17.86	21.63E3	30.23	-	-
55°	-	-	-	-0.86	156.06
65°	-	-	-	-1.31	213.28
75°	-	-	-	-2.11	312.03
85°	-	-	-	-3.02	426.36
90°	-	-	-	-5.53	739.5

4 Conclusion

In the present work, we numerically study the trajectories of a projectile obliquely penetrating a granular system. A two dimensional discrete element model based on the soft particle method is established to investigate the trace of the projectile at various impact angles with fixed impact velocity. The numerical simulations' results reveal that the model is capable to simulate the process. Moreover, it is found that the profile of the projectile's trajectory underneath the granular bed is affected the magnitude of the impact angle. For relatively small impact angles, the projectile's trajectory exhibits a non-linear trace while for large impact angle it shows a linear like behavior. A function that describes the profile of the projectile's trajectories at different impact angles is also proposed.

Acknowledgement

The authors wish to thank Prof. M. Mossa Al-sawalha and Prof. Abdallah A. Tallafha for many enlightening discussions during the preparation of this paper. Thanks are also due to anonymous reviewers for their valuable comments.

References

[1] T. Teresa, R. Fragaszy and C. Ho, Projectile penetration in granular soils, Journal of Geotechnical Engineering, vol. 117.4, pp. 658-672, (1991).

[2] Y. Boguslavskii, S. Drabkin, I. Juran and A. Salman, Theory and practice of projectile's penetration in soils, Journal of geotechnical engineering, vol. 122(10), pp. 806-812, (1996).

[3] K. Tanaka, M. Nishida, T. Kunimochi and T. Takagi, Discrete element simulation and experiment for dynamic response of two-dimensional granular matter to the impact of a spherical projectile, Powder technology, vol. 124(1), pp. 160-173, (2002).

[4] M. Nishida and Y. Tanaka, DEM simulations and experiments for projectile impacting two-dimensional particle packings including dissimilar material layers, Granular Matter, vol. 12.4, pp. 357-368, (2010).

© 2017 NSP
Natural Sciences Publishing Cor.

[5] J. de Bruyn and A, Walsh, Penetration of spheres into loose granular media, Canadian Journal of Physics, vol. 82.6, pp. 439-446, (2004).

[6] M. Nishida, K. Tanaka and Y. Matsumoto, Discrete element method simulation of the restitutive characteristics of a steel spherical projectile from a particulate aggregation, International Journal Series A Solid Mechanics and Material Engineering, vol. 47.3, pp. 438-447 (2004).

[7] M. Hou, Z. Peng, R. Liu, K. Lu and C. Chan, Dynamics of a projectile penetrating in granular systems, Physical review E, vol. 72(6), pp. 1-4, (2005).

[8] W. Alshanti and Y. Wu, Numerical Simulation of Vertical Penetration into Granular Beds, Australian Journal of Basic and Applied Sciences, vol. 5.2, pp. 182-188, (2011).

[9] M. Ambroso, R. Kamien and D. Durian, Dynamics of shallow impact cratering, Physical Review E, vol. 72.4, pp. 1-4, (2005).

[10] D. Goldman and P. Umbanhowar, Scaling and dynamics of sphere and disk impact into granular media, Physical Review E, vol. 77.2, pp. 1-4, (2008).

[11] M. Ambroso, C. Santore, A. Abat and D. Durian, Penetration depth for shallow impact cratering, Physical Review E, vol. 71(5), pp. 1-4, (2005).

[12] K. Newhall and D. Durian, Projectile-shape dependence of impact craters in loose granular media, Physical Review E, vol. 68.6, pp. 1-4, (2003).

[13] M. Ciamarra, A. Lara, A. Lee, D. Goldman, I. Vishik and H. Swinney, Dynamics of drag and force distributions for projectile impact in a granular medium, Physical review letters, vol. 92(19), pp. 1-4 (2008)

[14] F. Bourrier, F. Nicot and F. Darve, Physical processes within a 2D granular layer during an impact, Granular Matter, vol. 10.6, pp. 415-437, (2008).

[15] J. Addiss, A. Collins, F. Bobaru, K. Promratana and W. Proud, Dynamic behaviour of granular materials at impact, DYMAT, vol. 2009, pp. 59-65, (2009).

[16] J. Hinch, Particles impacting on a granular bed, Journal of Engineering Mathematics, vol. 84.1, pp. 41-48, (2014).

[17] D. Wang and X. Zheng, The scaling and dynamics of a projectile obliquely impacting a granular medium, The European Physical Journal E, vol. 35.1, pp. 1-12, (2012).

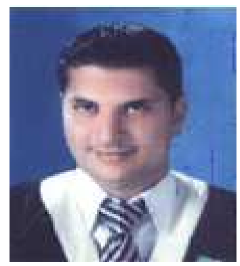
[18] D. Wang and X. Zheng, Influence of particle rotation on the oblique penetration in granular media, Physical Review E, vol. 86.6, pp. 1-4, (2012).

[19] D. Wang and X. Zheng, Effects of density ratio and diameter ratio on penetration of rotation projectile obliquely impacting a granular medium, Engineering Computations, vol. 32.4, pp. 1025-1040, (2015).

[20] D. Marghitu, L. Seunghun and M. Daniela, Experimental and Simulation Results for the Impact of a Rotating Flexible Link with a Granular Material, International Journal of Advanced Robotic Systems, vol. 11, pp. 1-9, (2014).

[21] P. Cundall and O. Strack, A discrete numerical model for granular assemblies, Geotechnique, vol. 29.1, pp. 47-65, (1979).

Waseem Ghazi Alshanti received the PhD degree in numerical analysis from the Department of Mathematics and Statistics, Curtin University, Australia, in 2010 where he worked as a lecturer from 2007 to 2010. Based on his excellent results he achieved in his PhD studies, he was awarded Curtin University completion scholarship as well as the mathematics honours and postgrad prize. Currently, he is working as Assistant Professor of Mathematics at the University of Hail. His research interests and areas of expertise include: Applied Mathematical Modelling (Fluid Dynamics, Granular Flow, Geomechanics Theory, Continmuum Mechanics Theory), Image Processing and Integral Equations.



Ahmad Ghazi Alshanti received the PhD degree in Computer Engineering from the Department of Computer Engineering, Girne American University, Cyprus, in 2017, where he worked as a lecturer from 2014 to 2016. His research interests and areas of expertise include: Wireless Sensor Network (WSN), Intrusion Detection Systems (IDS), Trusted Third Party Based Security Protocols and Mobile Agent Technology.

© 2017 NSP
Natural Sciences Publishing Cor.