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A One-Dimensional Mathematical Model for the Source Reconstruction by the Maximum Entropy Principle

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Abstract: This paper is concerned with the localization problem of a source belonging to a domain monitored by a network of detectors. A mathematical model is proposed within an inverse problem framework which is based on the maximum information entropy principle. Specifically the connection between the measurements released by the detectors and the sources is obtained by assuming that each detector has a visibility domain which is modeled by introducing a visibility function. A computational sensitivity analysis is performed on the number of detectors and on the visibility functions. The results are of great interest in the applied sciences.

Keywords: Under-determined system, Inverse problem, Shannon entropy

1 Introduction

The definition of inversion formalisms for the inverse source problem has gained much attention and various approaches have been proposed, see, among others, [1,2,3,4]. The main interest is the possibility to obtain information about the position of a source from a set of measurements. In particular the tools of the inverse problem theory have been employed [5,6,7] especially in the case of an ill-posed inverse problem where the existence or the uniqueness of the solution cannot be established. The attention has been mainly focused on the ill-posed inverse problems where the solution, if any, does not depend continuously on the initial data. In this context regularization methods have been proposed which consists in replacing an ill-posed problem with a family of neighboring well-posed problems [8,9,10].

The present paper deals with the source localization problem. Specifically the inverse problem framework proposed in [11] is employed for the derivation of a one-dimensional model. Accordingly it is assumed that in a one-dimensional domain a monitoring network is arranged. The network consists of m detectors which are able to release m measurements. The existence of a linear operator (data kernel) is conjectured; the kernel models the connection between the measurements and the $n > m$ sources (under-determined system, ill-posed problem). The kernel is assumed to depend on the visibility domain of the detectors and on a state function. The visibility domain is modeled by introducing a visibility function which is assumed to be a linear combination of straight lines. The source is thus retraced by employing the maximum Shannon entropy principle [12,13,14] which ensures the well-posedness of the inverse problem (and in particular the uniqueness of the problem). The Shannon entropy is employed because it is considered the best measure of uncertainty, and the probability distribution which maximizes it represents the best current state of knowledge. The reader is referred to the review papers [15,16,17,18] and the references cited therein. However in the pertinent literature further entropy functions have been defined, see the review paper [19].

It is worth stressing that inverse problems are usually proposed within the framework of ordinary differential equations [20,21], partial differential equations [22], kinetic theory [23,24,25,26], fluid mechanics [27]. The reader is also addressed to the books [28,29] and the review paper [30] for recent contributions.

This paper is divided into 3 more sections which follow this introduction. Specifically Section 2 reviews the recently proposed source problem framework [11], which couples the inverse problem theory [29] with the information theory [28]. Section 3 is devoted to the derivation of a specific one-dimensional model by

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defining a visibility function for each detectors; a computational analysis is performed on the number of detectors and on the visibility functions. In particular the section is divided into two subsequently subsections: the first subsection deals with the case of two detectors while the second subsection is devoted to three detectors case. Finally Section 4 concludes the paper with a critical analysis and references to future research directions.

2 The underlying inverse problem framework

This section is devoted to the fundamentals of the mathematical theory that has been recently proposed in [11]. Specifically the problem under consideration is the localization (reconstruction) of $n \in \mathbb{N}$ sources within a domain $\Omega \subset \mathbb{R}^k$, for $k \in \{1,2,3\}$. The domain is monitored by a network $\mathscr{N}$ of $m < n$ detectors which are able to release the measurements which consist of a macroscopic description (particle density) of the sample air. It is assumed that depending on the air sampled, the detectors are able to have a zone of visibility which is here modeled by assuming the existence of a visibility function $\varphi_j(x)$, with $x \in \Omega$ and for $j \in \{1,2,\dots,m\}$. Bearing all above in mind, the jth-detector releases a measure μ_j, for $j \in \{1,2,\dots,m\}$, which is related to the source s_i, $i \in \{1,2,\dots,n\}$. The mathematical framework is derived by conjecturing the linking between the measurements and the sources by means of the existence of an operator (kernel) $\mathbf{K}[\Gamma(x)](s) : \mathscr{S} \to \mathscr{M}$, where $\mathscr{S}$ is the source space and $\mathscr{M}$ the measurements space; Γ denotes one of the kernel arguments which takes into account the visibility functions of the detectors and other state variables. The kernel is assumed to be a sufficiently regular function of its arguments. Bearing all above in mind, the underlying mathematical framework consists in the resolution of the following inverse problem:

$$\mu = \mathbf{K}[\Gamma(x)](s). \tag{1}$$

where $\mu = (\mu_1, \mu_2, \dots, \mu_m) \in \mathbb{R}^{m,1}$ is the m-dimensional measurement vector, and $s = (s_1, s_2, \dots, s_n) \in \mathbb{R}^{n,1}$ is the n-dimensional source vector. In particular this paper focuses on the linear relation between sources and measurements, namely:

$$\mu = \mathbf{K}[\Gamma(x)]\, s, \tag{2}$$

where $\mathbf{K}[\Gamma(x)] = [K_{ji}[\Gamma(x)]] \in \mathbb{R}^{m,n}$. Accordingly the inverse problem reads:

$$\begin{cases} \displaystyle\sum_{i=1}^{n} s_i = 1, \\ \mu_j = \displaystyle\sum_{i=1}^{n} K_{ji}[\Gamma(x)]\, s_i, \quad j \in \{1,2,\dots,m\}. \end{cases} \tag{3}$$

As known for the well-posedness of inverse problem (3) it is sufficient that the solution $s \in \mathscr{S}$ is unique for any measurement vector $\mu \in \mathscr{M}$. Since the problem under consideration is an under-determined system ($m < n$), the uniqueness is established by searching the solution $s \in \mathbb{R}^{n,1}$ that maximizes the standard information entropy of Shannon $H : \mathbb{R}^{n,1} \to [0, \ln(n)]$ where:

$$H[s] = -\sum_{i=1}^{n} s_i \ln s_i. \tag{4}$$

Let $\lambda = (\lambda_0, \lambda_1, \lambda_2, \dots, \lambda_m)$, the related lagrangian function reads:

$$\mathscr{L}[\mathbf{K}[\Gamma]](s,\lambda) = -\sum_{i=1}^{n} s_i \ln s_i - (\lambda_0 - 1)\left(\sum_{i=1}^{n} s_i - 1\right) + \sum_{j=1}^{m} \lambda_j \left(\mu_j - \sum_{i=1}^{n} K_{ji}[\Gamma] s_i\right). \tag{5}$$

The vector solution $s^H = (s_1^H, s_2^H, \dots, s_n^H) \in \mathscr{S}$ of the inverse problem (3) is thus as follows:

$$s_H = \underset{s \in \mathscr{S}_\mu}{\operatorname{argmax}} H[s]. \tag{6}$$

As shown in [11], the maximum entropy solution of (3) reads:

$$s_i^H[\Gamma] = \frac{\exp\left(-\displaystyle\sum_{j=1}^{m} \lambda_j K_{ji}[\Gamma]\right)}{\displaystyle\sum_{i=1}^{n} \exp\left(-\sum_{j=1}^{m} \lambda_j K_{ji}[\Gamma]\right)}, \quad i \in \{1,2,\dots,n\}, \tag{7}$$

where the Lagrange multiplier λ_j, for $j \in \{1,2,\dots,m\}$, is solution of the following problem:

$$\frac{\partial}{\partial \lambda_j} \ln\left(\sum_{i=1}^{n} \exp\left(-\sum_{j=1}^{m} \lambda_j K_{ji}[\Gamma]\right)\right) = -\mu_j. \tag{8}$$

According to (7) the maximum value of the Shannon entropy depends on the measurements as follows:

$$H_{\max} = \lambda_0 + \sum_{j=1}^{m} \lambda_j \mu_j. \tag{9}$$

Remark 1. If each Lagrangian multiplier $\lambda_i = 0$, for $i \in \{1,2,\dots,m\}$, then the exponential model (7) collapses to a uniform distribution where $s_i = 1/n$ and the Shannon entropy H attains its maximum value, namely $\ln(n)$.

3 A one-dimensional model

This section deals with the derivation of a specific model within the framework proposed in the previous section. Specifically it is assumed that $\Omega = [a,b]$, with $a < b$. The domain $[a,b]$ is monitored by m detectors and the source s

consists of $n > m$ points which cover a subdomain Σ of $[a,b]$. The visibility function of each detector, which is set casewise, is assumed to be a linear combination of linear functions (straight lines). In particular a numerical sensitivity analysis on the number of detectors and on the source position is performed. Specifically the set of numerical simulations, is performed for two different networks of detectors, and more precisely $\mathscr{N}_1 = \{C_1^1, C_2^1\}$ and $\mathscr{N}_2 = \{C_1^2, C_2^2, C_3^2\}$. The computational analysis is depicted by representing the distribution function of s which attains, according to the maximum entropy principle, its maximum value in the vicinity of the source (source localized).

3.1 The case of two detectors

The domain Ω is monitored by a network $\mathscr{N}_1$ composed by the two detectors $C_1^1 = (a,0)$ and $C_2^1 = (b,0)$. The visibility functions $\varphi_1(x)$ and $\varphi_2(x)$ of the detectors are chosen as follows:

$$\varphi_1(x) = \begin{cases} a_1 x + b_1 & a \leq x \leq \alpha_1 \\ c_1 & \text{otherwise} \end{cases}$$

and

$$\varphi_2(x) = \begin{cases} a_2 x + b_2 & \alpha_2 \leq x \leq b \\ c_2 & \text{otherwise} \end{cases}$$

with $(a_i; b_i; c_i; \alpha_i)$, for $i \in \{1,2\}$, set casewise. A sensitivity analysis on the sources position and on the visibility function is now performed. Specifically, the case of a set of sources which are allocated in the center of the domain is considered. The cases of a set of sources which are allocated near the detector C_1^1 and near the the detector C_2^1 are also taken into account. The set of numerical simulations is performed for two different choices of the visibility function of detectors, namely $\Upsilon_1 = \{C_1^1(\varphi_1^1), C_2^2(\varphi_2^1)\}$ and $\Upsilon_2 = \{C_1^2(\varphi_1^2), C_2^2(\varphi_2^2)\}$. The Figures 1 and 2 show how the network is arranged in the domain $\Omega = [0,10]$ and how the coefficients of the two visibility functions are chosen, namely Υ_1 and Υ_2. According to Figure 1, in the network $\mathscr{N}_1(\Upsilon_1)$ it is assumed that the visibility of a detector decreases as the distance between the source and the detectors increases. In this case there exist subdomains of the domain Ω which are visible to both the two detectors, and subdomains which are visible to one detector only. According to Figure 2, in the network $\mathscr{N}_1(\Upsilon_2)$ it is assumed that the visibility of a detector decreases as the distance between the source and the detectors increases but there is always a visibility after a certain distance. In this latter case every subdomain of the domain Ω is visible to both the two detectors.

As first case of source localization, we consider a set of sources placed in the center of the domain Ω and specifically in the subdomain $\Sigma_c = [4,5;5,5]$. As the Figure 3 shows, the reconstruction of the source in the network $\mathscr{N}_1(\Upsilon_1)$ presents an interval of incertitude (constant curve) dues to the choice of the visibility function. Indeed as shown in Figure 4 the interval of incertitude is reduced in the network $\mathscr{N}_1(\Upsilon_2)$.

As second case we consider the source localization problem for an asymmetric case; specifically the source

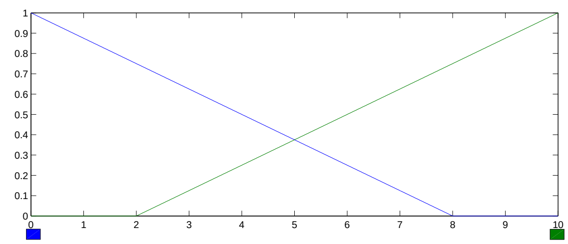

Fig. 1: The network $\mathscr{N}_1(\Upsilon_1)$ with detectors $C_1^1 = (0,0)$ and $C_2^1 = (10,0)$, visibility functions $(a_1;b_1;c_1;\alpha_1) = (-1/8;1;0;8)$, $(a_2;b_2;c_2;\alpha_2) = (1/8;-1/4;0;2)$, respectively.

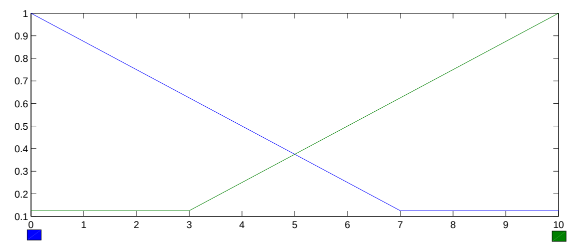

Fig. 2: The network $\mathscr{N}_1(\Upsilon_2)$ with detectors $C_2^2 = (0,0)$ and $C_2^2 = (10,0)$, visibility functions $(a_1;b_1;c_1;\alpha_1) = (-1/8;1;0,13;7)$, $(a_2;b_2;c_2;\alpha_2) = (1/8;-1/4;0,13;3)$, respectively.

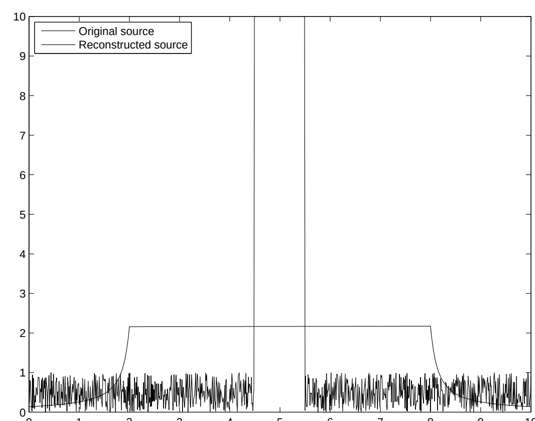

Fig. 3: The network $\mathscr{N}_1(\Upsilon_1)$ and the source subdomain $\Sigma_c = [4,5;5,5]$ (rectangle). The source reconstruction is obtained by the maximum entropy curve.

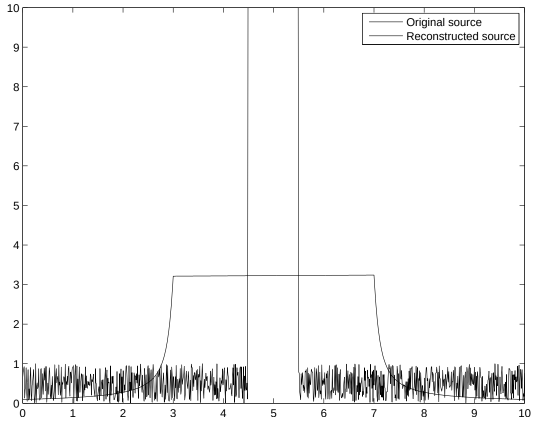

Fig. 4: The network $\mathscr{N}_1(\Upsilon_2)$ and the source subdomain $\Sigma_c = [4,5;5,5]$ (rectangle). The source reconstruction is obtained by the maximum entropy curve.

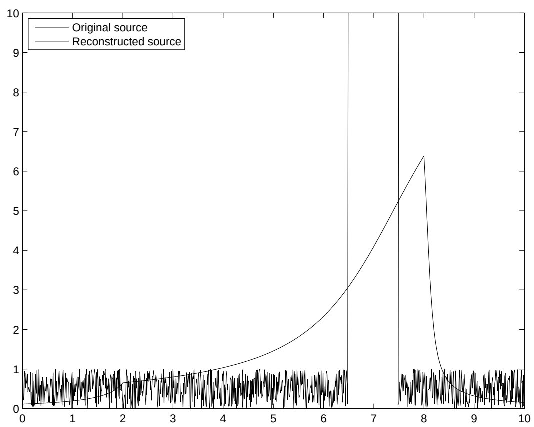

Fig. 5: The network $\mathscr{N}_1(\Upsilon_1)$ and the source subdomain $\Sigma_c = [6,5;7,5]$ (rectangle). The source reconstruction is obtained by the maximum entropy curve.

covers the subdomain $\Sigma_c = [6,5;7,5]$. As the Figures 5 and 6 show, the source is well constructed in the two networks $\mathscr{N}_1(\Upsilon_1)$ and $\mathscr{N}_1(\Upsilon_2)$. The maximum entropy curve well detects the source. The dependence on the choice of the visibility functions is an important criterium to be considered. Indeed the error in the localization of the source subdomain $\Sigma_c = [6,5;7,5]$ within the network $\mathscr{N}_1(\Upsilon_1)$ is less than the error within the network within the network $\mathscr{N}_1(\Upsilon_2)$. Accordingly the number of networks does not appear an important criterium in this case; on the contrary the choice of the visibility function appears an important step. It is worth stressing that the symmetrical case, namely $\Sigma_c = [2,5;3,5]$, presents the same good reconstruction of the source (symmetric case).

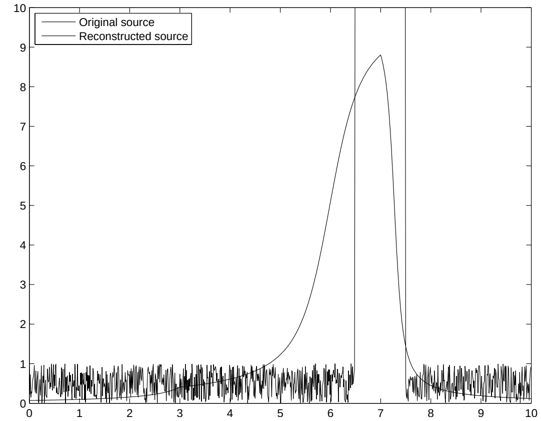

Fig. 6: The network $\mathscr{N}_1(\Upsilon_2)$ and the source subdomain $\Sigma_c = [6,5;7,5]$ (rectangle). The source reconstruction is obtained by the maximum entropy curve.

3.2 The case of three detectors

The domain Ω is assumed to be monitored by a network $\mathscr{N}_2$ composed by the following three detectors:

$$C_1^2 = (a,0), \quad C_2^2 = (\frac{a+b}{2},0), \quad C_3^2 = (b,0).$$

The visibility functions $\varphi_1(x)$, $\varphi_2(x)$ and $\varphi_3(x)$ of the detectors are chosen as follows:

$$\varphi_1(x) = \begin{cases} a_1x+b_1 & a \le x \le \alpha_1 \\ c_1 & \text{otherwise} \end{cases}$$

$$\varphi_2(x) = \begin{cases} a_2^1x+b_2^1 & a \le x \le \frac{a+b}{2} \\ a_2^2x+b_2^2 & \frac{a+b}{2} \le x \le b \end{cases}$$

and

$$\varphi_3(x) = \begin{cases} a_3x+b_3 & \alpha_3 \le x \le b \\ c_3 & \text{otherwise} \end{cases}$$

with $(a_i;b_i;c_i;\alpha_i)$ and $(a_2^j;b_2^j)$, for $i \in \{1,3\}$ and $j \in \{1,2\}$ set casewise.
Differently from the previous subsection, a sensitivity analysis on the sources position only is performed. Specifically, the case of a set of sources which are allocated in the center of the domain is considered. The cases of a set of sources which are allocated near the detector C_1^2, near the the detector C_3^2 are taken into account. Moreover the case of two source subdomains is also considered. The Figure 7 shows how the network $\mathscr{N}_2$ is arranged in the domain $\Omega = [0,10]$ and how the coefficients of the three visibility functions are chosen. According to Figure 7, in the network $\mathscr{N}_2$ it is assumed that the visibility of a detector decreases as the distance between the source and the detectors increases. The detector placed in the center of the domain allows to increase the visibility zone.

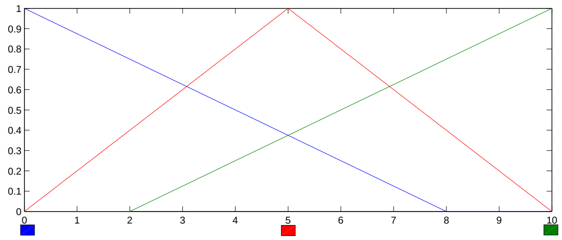

Fig. 7: The network $\mathcal{N}_2$ with detectors $C_1^2 = (0,0)$, $C_2^2 = (5,0)$ and $C_3^2 = (10,0)$, visibility functions $(a_1;b_1;c_1;\alpha_1) = (-1/8;1;0;8)$, $(a_2;b_2;c_2;\alpha_2) = (1/8;-1/4;0;2)$, $(a_2^1;b_2^1) = (1/5;0)$, and $(a_2^2;b_2^2) = (-1/5,2)$.

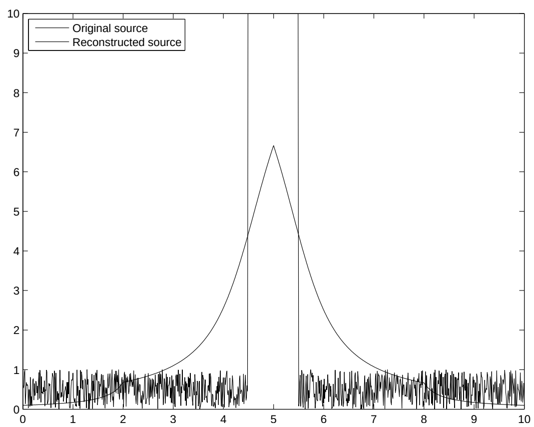

Fig. 8: The network $\mathcal{N}_2$ and the source subdomain $\Sigma_c = [4,5;5,5]$ (rectangle). The source reconstruction is obtained by the maximum entropy curve.

In the first case, we consider a set of sources placed in the center of the domain Ω and specifically in the subdomain $\Sigma_c = [4,5;5,5]$. Differently from the same set source domain analyzed in the previous subsection (see Figures 3 and 4), the source in the network $\mathcal{N}_2$ is well localized and reconstructed. Indeed as shown in Figure 8, there is not an interval of incertitude (constant curve) thanks to the presence of the detector in the center of the domain.
The Figures 9 and 10 are devoted to the reconstruction of two set source domains, the first domain near to the detector C_1^2 and the second domain near the detector C_3^2, respectively. As in the previous subsection, the source here is again well reconstructed.
Finally, the Figure 11 shows the reconstruction of two source subdomains, namely $\Sigma_c^1 = [1;2]$ and $\Sigma_c^2 = [8;9]$. In this case the entropy curve presents two maximum points localized near the sources. The numerical simulations show again how it is not important the number of detectors but their disposition and consequently their visibility function.

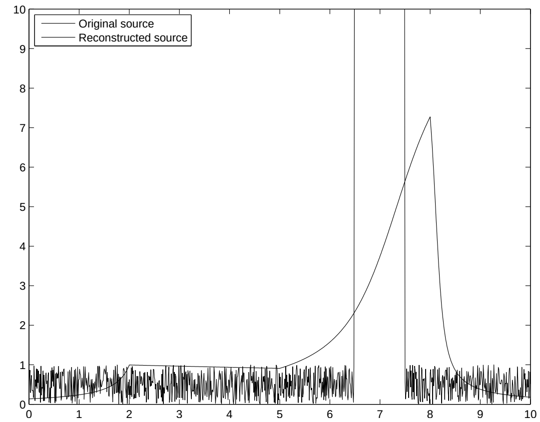

Fig. 9: The network $\mathcal{N}_2$ and the source subdomain $\Sigma_c = [6,5;7,5]$ (rectangle). The source reconstruction is obtained by the maximum entropy curve.

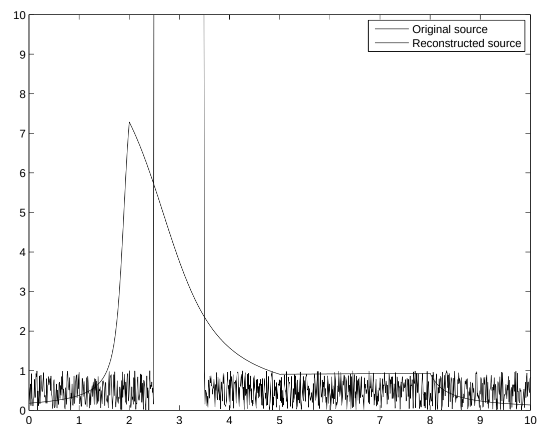

Fig. 10: The network $\mathcal{N}_2$ and the source subdomain $\Sigma_c = [2,5;3,5]$ (rectangle). The source reconstruction is obtained by the maximum entropy curve.

4 Critical analysis and research perspectives

The mathematical model proposed in this paper has been derived according to a new inverse problem framework. In particular the model is based on the definition of the visibility function of a detector. As known a detector performs the measurements of the particles that have been traveling separately before gathering inside the detector (air sampled). A sampling function describing where and when the samples are taken can be thus defined (detector function). The visibility function considered in the present paper can be related to the sampling function considering that the air sampled is made of particles that have an history (retroplume).
The performed simulations have shown that the number of detectors is not an important issue with respect to the

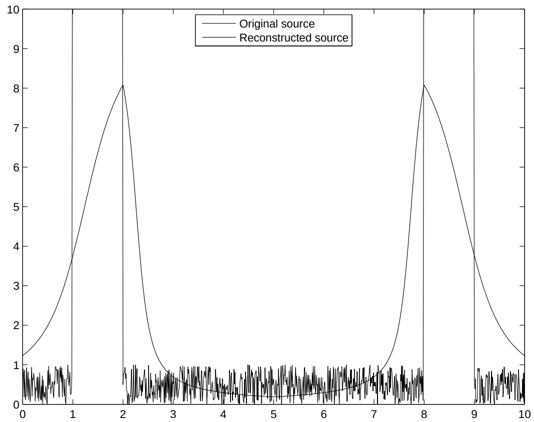

Fig. 11: The network $\mathscr{N}_2$ and the source subdomains $\Sigma_c^1 = [1;2]$ and $\Sigma_c^2 = [8;9]$ (rectangles). The source reconstruction is obtained by the maximum entropy curve.

visibility function. Indeed some of the sources considered in the previous section have been well localized in the case of two detectors thus showing that the choice of the visibility function (and then of the detector) is an important task. Accordingly the position of the detectors is the first issue to be discussed. Consequently a research perspective is the development of a strategy for the optimization process of the detectors position.
In the context of the visibility function selection an interesting choice can be pursued considering the function associated to a detector in the paper [31]. The function, called adjoint, is derived according to the transport properties of the particles. This is a work in progress and the results will be presented due course.
It is worth stressing that the solution of the model proposed in the present paper is based on the maximum Shannon entropy principle. The solution can be improved by replacing the Shannon entropy with the relative entropy [16]. However the concept of relative entropy requires an information *a priori* on the source position. This is another research perspective that can quantitative improve the source localization.

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References

[1] R. Abida, M. Bocquet, N. Vercauteren, O. Isnard Design of a monitoring network over France in case of a radiological accidental release, Atmos. Environ. 42, 5205-5219 (2008).
[2] K.S. Rao, Source estimation methods for atmospheric dispersion, Atmospheric Environment 41, 6964-6973 (2007).
[3] J.-P. Issartel, Rebuilding sources of linear tracers after atmospheric concentration measurements, Atmos. Chem. Phys. 3, 2111-2125, (2003).
[4] J.-P. Issartel, M. Sharan, M. Modani, An inversion technique to retrieve the source of a tracer with an application to synthetic satellite measurements, Proc. R. Soc. A 463, 2863-2886 (2007).
[5] A. Asanov, Regularization, Uniqueness and Existence of Solutions of Volterra Equations of the First Kind, VSP, Utrecht, 1998.
[6] A.L. Bughgeim, Volterra Equations and Inverse Problems, VSP, Utrecht, 1999.
[7] A.V. Kryazhimskii, Y.S. Osipov, Inverse Problems for Ordinary Differential Equations: Dynamical Solutions, Gordon and Breach, London, 1995.
[8] R.G. Airapetyan, A.G. Ramm, A.B. Smirnova, Continuous methods for solving nonlinear ill-posed problems, Fields Institute Communications 25, 111-137 (2000).
[9] S. Crepey, Calibration of the local volatility in a trinomial tree using Tikhonov regularization, Inverse Problems 19, 91-127 (2003).
[10] S.Crepey, Calibration of the local volatility in a generalized Black-Scholes model using Tikhonov regularization, SIAM J. Math. Anal. 354, 1183-1206 (2003).
[11] C. Bianca, A new approach for the source problem based on the maximum entropy principle, Nonlinear Studies 24, 715-723 (2017).
[12] C.E. Shannon, A mathematical theory of communication, Bell Syst. Tech. J. 27, 379-423 (1948).
[13] C.E. Shannon, A mathematical theory of communication, Bell Syst. Tech. J. 27, 623-656 (1948).
[14] E.T. Jaynes, Information theory and statistical mechanics, Phys. Rev. 106, 620-630 (1957).
[15] A. Mohammad-Djafari, Entropy, information theory, information geometry and bayesian inference in data, signal and image processing and inverse problems, Entropy 17, 3989-4027 (2015).
[16] R. Kleeman, Information theory and dynamical system predictability, Entropy 13, 612-649 (2011).
[17] Y. Liu, C. Liu and D. Wang, Understanding atmospheric behaviour in terms of entropy: a review of applications of the second law of thermodynamics to meteorology, Entropy 13, 211-240 (2011).
[18] M. Batty, Space, scale, and scaling in entropy maximizing, Geographical Analysis 42, 395-421 (2010).
[19] C. Bianca, On the mathematical transport theory in microporous media: The billiard approach, Nonlinear Analysis: Hybrid Systems 4, 699-735 (2010).
[20] J. Llibre, R. Ramírez, Inverse Problems in Ordinary Differential Equations and Applications, Birkhäuser, 2016.
[21] F. Santosa, B. Weitz, An inverse problem in reaction kinetics, Journal of Mathematical Chemistry 49, 1507-1520 (2011).
[22] V. Isakov, Inverse Problems for Partial Differential Equations, Springer, 1998.
[23] Yu.E. Anikonov, Inverse Problems for Kinetic and other Evolution Equations, Inverse and ill-posed problems series, Utrecht, 2001.

[24] M.V. Neshchadim, Inverse problems for the kinetic equation of plasma physics and a uniqueness theorem, Selçuk J. Appl. Math. 4, 87-94 (2003).
[25] G Bal, Inverse problems in random media: a kinetic approach, J. Phys.: Conf. Ser. 124, 012001 (2008).
[26] C. Bianca, A. Kombargi, On the modeling of the stock market evolution by means of the information-thermostatted kinetic theory, Nonlinear Studies 24, 935-944 (2017).
[27] M. Dashti, J.C. Robinson, A.M. Stuart, Bayesian inverse problems for functions and applications to fluid mechanics, Inverse Problems 25, 115008 (2009).
[28] C.R. Smith, W.T. Grandy, eds., Maximum-Entropy and Bayesian Methods in Inverse Problems, Fundamental Theories of Physics. Reidel, Dordrecht, 1985.
[29] Kirsch, An Introduction to the Mathematical Theory of Inverse Problems, Springer, New York, Berlin, Heidelberg, 1996.
[30] F. Yaman, V.G. Yakhno and R. Potthast, Recent theory and applications on inverse problems, Mathematical Problems in Engineering 2013, 303154 (2013).
[31] M. Sharan, J.-P. Issartel, S. Kumar Singh, P. Kumar, An inversion technique for the retrieval of single-point emissions from atmospheric measurements, Proc. R. Soc. A 465, 2069-2088 (2009).

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