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An Adjustable Reduction Approach of Interval-valued Intuitionistic Fuzzy Soft Sets for Decision Making

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Abstract: Parameter reduction methods have been adopted by many scientific communities since it provides the basic concept for removing irrelevant features and subset of parameters that provides the same descriptive or decisionability as the entire set of parameters. Several reduction approaches have been proposed for fuzzy-soft set in making decision of datasets with fuzzy-soft values.However, existing fuzzy soft set reduction approaches suffer to handle interval-valued intuitionistic fuzzy soft datasets. To overcome this issue, we introduce an adjustable reduction approach of interval-valued intuitionistic fuzzy soft sets (AR-IIFSS) for decision making. The novelty of APR-IIFSS is that we generalize the existing approaches on reduction of fuzzy soft sets (R-FSS), interval-valued fuzzy soft sets (R-ITFSS), and intuitionistic fuzzy soft sets (R-ICFSS) for decision making. Therefore, this is the first attempt on reduction approach of interval-valued intuitionistic fuzzy soft datasets. We also introduce an adjustable reduction approach of weighted interval-valued intuitionistic fuzzy soft sets (AR-WIIFSS) and investigate its application for decision making. We make extensive analysis for AR-IIFSS and AR-WIIFSS approaches to show their feasibility in practical applications of decision making.

Keywords: Soft sets, Interval-valued intuitionistic fuzzy soft sets, Level soft sets, Reduction, Decision making

1 Introduction

Many decision making problems in economics, education, engineering, environment, social science, and medical science often involves datasets that contain various types of uncertainties and imprecise information.A wide range of classical theories such as probability theory, fuzzy set theory [1], rough set theory [2], vague set [3] and the interval mathematics are exist and often useful as mathematical tools to characterize uncertainty and imprecise data,however each of these theories has its inherent difficulties as pointed out in [4][5]. The reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theories. In 1999, Molodtsov [4] firstly introduced soft set theory as a new mathematical tool for dealing with vagueness, uncertainties and imprecise data, which is free from the difficulties affecting existing methods. Unlike previous theories mentioned above, soft set theory uses parameterization definition as its main vehicle. Molodtsov has successfully applied the soft set theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement and so on [4]. At present, work on the soft set theory is progressing rapidly and many important results have been achieved.Maji, *et al.* presented soft set theory and its algebraic properties [6]. Xiao, e*t al.* presented exclusive disjunctive soft sets [7]. Xu, *et al.* introducedvague soft sets and their properties [8].Acar, *et al.* presented soft sets and soft rings [9]. Feng, *et al.* presented soft set theory in application on semirings [10]. Aktas and Cagman presented soft sets and soft groups [11]. Qin, *et al.* presented a technique Selecting Clustering Attribute using Soft Set Approach [12]. Mamat, *et al.* [13] improved the technqie of [12] by introducing maximum

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attribute relative of soft set for clustering attribute selection.Zou and Xiao proposed data analysis approaches of soft sets under incomplete information [14]. Qin *et al.*[15] improved the technique of [14] by proposing a novel data filling approach for an incomplete soft set. Herawan and Derisproposed the idea of using soft set theory for maximal association rules mining [16].Three different types of hybrid models were presented, namely fuzzy soft set, rough soft sets, soft rough sets, and soft-rough fuzzy sets. Aygunoglu and Aygun introduced fuzzy soft groups [17]. Maji presented intuitionistic fuzzy soft sets [18]. Yang, *et al.* presented combination of interval-valued fuzzy set and soft set [19]. Majumdar and Samanta proposed the idea of generalized fuzzy soft sets [20]. Feng, *et al.* presented combination of soft sets with fuzzy sets and rough sets [21]. Maji and Roy proposed a fuzzy soft set approach for decision making problems [22]. Xiao, *et al.* proposed ahybrid fuzzy soft sets forecasting approach [23].

At the same time, there has been some progress concerning practical applications of soft set theory in decision making. The research of soft set-based decision making comes from idea on parameter reduction. Parameter reduction in soft set theory is aimed to reduce (delete) irrelevant parameter and at the same time preserving original decision choice. Maji *et al.* [24] firstly proposed the idea soft sets to solve the decision making problem with the help of rough set-based reduction [25]. Chen *et al.* [26] and Kong *et al.* [27] presented parameterization reduction and normal parameter reduction of soft sets respectively to improve the soft sets based decision making of [24]. Ma, *et al.* [28] presented a New Efficient Normal Parameter Reduction Algorithm of Soft Sets to improve the drawbacks of [24][25][26]. Roy and Maji presented a novel method to cope with fuzzy soft sets based on decision making problems [29]. The method involved construction of a comparison table from a fuzzy soft set in a parametric sense for decision-making. Kong *et al.* [30] pointed out that the Roy-Maji method [29] was incorrect and they presented a revised algorithm. They discussed the validity of the Roy-Maji method [29] and showed its limitations. Ma, *et al.* presented comprehensive parameter reduction approaches of the interval-valued fuzzy soft sets and its related algorithms [31].

The research on reduction of hybrid fuzzy soft set for decision making has also received much attention. By means of level soft sets, Feng *et al.* gave deeper insights into reduction on fuzzy soft sets (R-FSS) and presented an adjustable approach to fuzzy soft sets based decision making [32]. Yang *et al.* [33] proposed a combination of interval-valued fuzzy set and soft set. Jiang *et al.* applied the reduction interval-valued fuzzy soft sets (R-ITFSS) to analyze a decision making problem [34]. The method they used is based on fuzzy choice value. Furthermore, Feng *et al.* [35] gave another insight into decision making involving interval-valued fuzzy soft sets. They analyzed the inherent drawbacks of fuzzy choice value based method and proposed a flexible scheme by using reductive fuzzy and level soft sets. Similarly, Jiang *et al.* [36] presented an adjustable reduction approach to intuitionistic fuzzy soft sets (R-ICFSS) for decision making by using level soft sets of intuitionistic fuzzy soft sets.However, in reviewing the reduction approaches of hybrid fuzzy soft set mentioned above, they suffer to handle interval-valued intuitionistic fuzzy soft datasets.

Therefore, to make descriptions of the objective world more realistic, practical and accurate in some cases, an approach on parameter reduction of interval-valued intuitionistic fuzzy soft set for decision making is needed. Concretely, in this paper we further generalize the approaches introduced by Feng *et al.* [32] and Jiang *et al.* [36]. In summary, the contributions in this work are shown as below:

a) We introduce an adjustable reduction approach to interval-valued intuitionistic fuzzy soft set (AR-IIFSS). The interval-valued intuitionistic fuzzy soft set (IIFSS) is a combination of the intuitionistic fuzzy and interval-valued fuzzy soft set theories i.e. the theory is a more general fuzzy soft set model. The AR-IIFSS is used for decision-making which based on reductive intuitionistic fuzzy and level soft sets. This is the first attempt on reduction of interval-valued intuitionistic fuzzy soft set for decision making.
b) We employ the reductive intuitionistic fuzzy soft set computing technique to convert an interval-valued intuitionistic fuzzy soft set into an intuitionistic fuzzy soft set.
c) We use level soft sets of intuitionistic fuzzy soft sets to convert the intuitionistic fuzzy soft set into a standard soft set. Furthermore, we perform decision-making on the standard soft set.
d) We also introduce an adjustable reduction approach of weighted interval-valued intuitionistic fuzzy soft set (AR-WIIFSS).
e) We investigate applications of proposed AR-IIFSS andAR-WIIFSSapproaches for decision-making.

The rest of this paper is organized as follows. Section 2 briefly reviews some background on soft sets, fuzzy soft sets and several kinds of extension models of fuzzy soft sets. Section 3 firstly recalls the concept of reductive fuzzy soft sets and defines a new concept of reductive intuitionistic fuzzy soft sets. Secondly,section 3 recalls the level soft sets. Thirdly, section 3 presents reduction of interval-valued intuitionistic fuzzy soft set algorithm and illustrate some examples. Section 4 is devoted to adjustable reduction approach of weighted interval-valued intuitionistic fuzzy soft sets for decision making. Finally, the conclusions of this paper are given in Section 5.

2 Theoretical Background

In this section, we recall the basic notions of fuzzy sets, soft sets, fuzzy soft sets, intuitionistic fuzzy soft sets, interval-valued fuzzy soft sets, and interval-valued intuitionistic fuzzy soft sets.

Definition 2.1 *Let U be a non-empty set of universe. A fuzzy set is a pair (U,m), where $m : U \to [0,1]$. For each $x \in U$ the value $m(x)$ is called the grade of membership of x in (U,m).*

Let U be an initial universe of objects, E be the set of parameters in relation to objects in $U, P(U)$ denotes the power set of U.The definition of soft set is given below.

Definition 2.2*(See [4])* *A pair (F,E) is called a soft set over U, where $F$is a mapping given by $F : E \to P(U)$.*

From definition, a soft set (F,E) over the universe U is a parameterized family of the subsets of the universe U, which gives an approximate description of the objects in U. For any parameter $e \in E$, the subset $F(e) \subseteq U$ may be considered as the set of e-approximate elements in the soft set (F,E). In the following definition, we recall the notion of fuzzy soft sets as a combination of Definitions 2.1 and 2.2 which is quoted from Maji *et al.* [37]. They introduced the notion of fuzzy soft sets as a fuzzy generalization of soft sets.

Definition 2.3*(See [37])* *Let F(U) be the set of all fuzzy subsets in a universe U and E be a set of parameters. A pair (F,E) is called a fuzzy soft set over U, where F is a mapping given by $F : E \to F(U)$.*

From Definition 2.3, fuzzy subsets are used as substitutes for the crisp subsets. Hence every soft set may be considered as a fuzzy soft set. Generally speaking, for any parameter $e \in E, F(e)$ is a fuzzy subset of U, which is called the fuzzy value set of parameter e.

Before introducing the notion of the interval-valued fuzzy soft sets, let us give the concept of the interval-valued fuzzy sets [38].

Definition 2.4*(See [38]).* *An interval-valued fuzzy set Y on an universe U is a mapping $Y : U \to$ Int([0,1]), where Int([0,1]) stands for the set of all closed subintervals of [0,1]. The set of all interval-valued fuzzy sets on U is denoted by $I(U)$.*

Suppose $Y \in I(U), \forall x \in U, Y(x) = \lfloor Y^-(x), Y^+(x) \rfloor$ is called the degree of membership of element x to Y. The $Y^-(x)$ and $Y^+(x)$ are referred to as the lower and upper degrees of membership x to Y, where $0 \le Y^-(x) \le Y^+(x) \le 1$.

Definition 2.5*(See [33]).* *Let U be an initial universe and E be a set of parameters. A pair (F,E) is called an interval-valued fuzzy soft set over U, where F is a mapping given by $F : E \to I(U)$.*

An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of U. For every $e \in E$, $F(e)$ is referred as the interval fuzzy value set of parameter e. Clearly, $F(e)$ can be written as $F(e) = \{(x, F(e)^-(x), F(e)^+(x)) : x \in U\}$,where $F(e)^-(x)$ and $F(e)^+(x)$ be the lower and upper degrees of membership of x to $F(e)$, respectively. An interval-valued fuzzy soft set is also a special case of a soft set because it is still a mapping from parameters to $I(U)$.

In the following, we will introduce the notion of intuitionistic fuzzy soft sets. Firstly, let us briefly introduce the concept of intuitionistic fuzzy sets [39].

Definition 2.6*(See [39]).* *Let E be a fixed set. An intuitionistic fuzzy set in E is an object having the form $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in A\}$, where the functions $\mu_A : E \to [0,1]$ and $\gamma_A : E \to [0,1]$ define the degree of membership and the degree of non-membership respectively of the element x to the set A.*

From Definition 2.6 for every $x \in E$, clearly we have $0 \le \mu_A(x) + \gamma_A(x) \le 1$. By introducing the concept of intuitionistic fuzzy sets into the theory of soft sets, Maji *et al.* [40][41] proposed the concept of the intuitionistic fuzzy soft sets as follows.

Definition 2.7*(See [40]).* *Let U be a set of universe, E be a set of parameters, and $S(U)$ denotes the set of all intuitionistic fuzzy sets of U. A pair (F,A) is called an intuitionistic fuzzy soft set over U, where F is a mapping given by $F : A \to S(U)$.*

Generally speaking, for every $e \in E$, $F(e)$ is an intuitionistic fuzzy subset of U and it is called intuitionistic fuzzy value set of parameter e. Clearly, $F(e)$ can be written as an intuitionistic fuzzy set such that $F(e) = \{(x, \mu_{F(e)(x)}, \gamma_{F(e)}(x)) \| x \in U\}$,where $\mu_{F(e)}(x)$ and $\gamma_{F(e)}(x)$ be the membership and non-membership functions respectively. Finally, in the following definition we introduce the concepts of interval-valued intuitionistic fuzzy set [42] and interval-valued intuitionistic fuzzy soft set [43].

Definition 2.8*(See [42].)* *An interval-valued intuitionistic fuzzy set on a universe Y is an object of the form $A = \{(x, \mu_A(x), \gamma_A(x)) \| x \in Y\}$, where $\mu_A(x) : Y \to Int([0,1])$ and $\gamma_A(x) : Y \to Int([0,1])$, where $Int([0,1])$ stands for the set of all closed subintervals of $[0,1]$, which satisfy the condition of $\forall x \in Y$, sup $\mu_A(x)+$ sup $\gamma_A(x) \le 1$*

By introducing the concept of interval-valued intuitionistic fuzzy set into the theory of soft sets, Jiang *et al.* [43] proposed the concept of the interval-valued intuitionistic fuzzy soft set as follows.

Definition 2.9*(See [43].* *Let U be a set of universe, E be a set of parameters, and $E(U)$ denotes the set of all interval-valued intuitionistic fuzzy sets of U. A pair (F,E) is called an interval-valued intuitionistic fuzzy soft set over U, where F is a mapping given by $F : E \to E(U)$.*

An interval-valued intuitionistic fuzzy soft set is also a special case of a soft set because it is still a mapping from parameters to $E(U)$. For every $e \in A$, $F(e)$ is an interval-valued intuitionistic fuzzy set of U. The $F(e)$ can be written as $F(e) = \{(x, \mu_{F(e)}(x), \gamma_{F(e)}(x)|x \in U\}$,where $\mu_{F(e)}(x)$ is the interval-valued fuzzy membership degree that object x holds on parameter e, $\gamma_{F(e)}(x)$ is the interval-valued fuzzy membership degree that object x does not hold on parameter e.

For illustration of an interval-valued intuitionistic fuzzy soft set, we consider the following example.

Example 2.1 Consider an interval-valued intuitionistic fuzzy soft set (F,E) which describes the "attractiveness of houses" that Mr. X is considering for purchase. Suppose there are six houses under consideration, namely the universes $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, and the parameter set $E = \{e_1, e_2, e_3, e_4, e_5\}$, where e_i stand for "beautiful", "large", "cheap", "modern" and "in green surroundings", for $i = 1,2,3,4,5$ respectively. Suppose that

$$F(e_1)=\{\langle h_1,[0.7,0.8],[0.1,0.2]\rangle \langle h_2,[0.85,0.9],[0.05,0.1]\rangle, \langle h_3,[0.5,0.7],[0.2,0.3]\rangle \langle h_4,[0.4,0.6],[0.3,0.4]\rangle, \langle h_5,[0.6,0.8],[0.15,0.2]\rangle \langle h_6,[0.3,0.5],[0.3,0.45]\rangle\};$$

$$F(e_2)=\{\langle h_1,[0.82,0.84],[0.05,0.15]\rangle \langle h_2,[0.7,0.74],[0.17,0.25]\rangle \langle h_3,[0.86,0.9],[0.04,0.1]\rangle \langle h_4,[0.52,0.64],[0.23,0.35]\rangle \langle h_5,[0.3,0.35],[0.5,0.65]\rangle \langle h_6,[0.5,0.68],[0.25,0.3]\rangle\};$$

$$F(e_3)=\{\langle h_1,[0.52,0.72],[0.18,0.25]\rangle \langle h_2,[0.7,0.75],[0.1,0.23]\rangle \langle h_3,[0.6,0.7],[0.2,0.28]\rangle \langle h_4,[0.72,0.78],[0.11,0.21]\rangle \langle h_5,[0.58,0.68],[0.18,0.3]\rangle \langle h_6,[0.33,0.43],[0.5,0.55]\rangle\};$$

$$F(e_4)=\{\langle h_1,[0.55,0.6],[0.3,0.35]\rangle \langle h_2,[0.7,0.75],[0.15,0.25]\rangle \langle h_3,[0.2,0.3],[0.5,0.6]\rangle \langle h_4,[0.3,0.5],[0.4,0.5]\rangle \langle h_5,[0.68,0.77],[0.1,0.2]\rangle \langle h_6,[0.62,0.65],[0.15,0.35]\rangle\};$$

$$F(e_5)=\{\langle h_1,[0.7,0.8],[0.1,0.2]\rangle \langle h_2,[0.75,0.9],[0.05,0.1]\rangle \langle h_3,[0.65,0.8],[0.15,0.2]\rangle \langle h_4,[0.8,0.9],[0.05,0.1]\rangle \langle h_5,[0.72,0.85],[0.1,0.15]\rangle \langle h_6,[0.84,0.93],[0.04,0.07]\rangle\};$$

The tabular representation of (F,E) is shown in Table 2.1.

From Table 2.1, obviously, we can see that the precise evaluation for each object on each parameter is unknown while the lower and upper limits of such an evaluation are given. For example, we cannot present the precise membership degree and non-membership degree of how beautiful house h_1 is, however, house h_1 is at least beautiful on the membership degree of 0.7 and it is the most beautiful on the membership degree of 0.8; house h_1 is not at least beautiful on the non-membership degree of 0.1 and it is not the most beautiful on the non-membership degree of 0.

Table 1: A tabular representation of (F,E)

U/E	e_1	e_2	e_3	e_4	e_5
h_1	[0.70,0.80], [0.10,0.20]	[0.82,0.84], [0.05,0.15]	[0.52,0.72], [0.18,0.25]	[0.55,0.60], [0.30,0.35]	[0.70,0.80], [0.10,0.20]
h_2	[0.85,0.90], [0.05,0.10]	[0.70,0.74], [0.17,0.25]	[0.70,0.75], [0.10,0.23]	[0.70,0.75], [0.15,0.25]	[0.75,0.90], [0.05,0.10]
h_3	[0.50,0.70], [0.20,0.30]	[0.86,0.90], [0.04,0.10]	[0.60,0.70], [0.20,0.28]	[0.20,0.30], [050.,0.60]	[0.65,0.80], [0.15,0.20]
h_4	[0.40,0.60], [0.30,0.40]	[0.52,0.64], [0.23,0.35]	[0.72,0.78], [0.11,0.21]	[0.30,0.50], [0.40,0.50]	[0.80,0.90], [0.05,0.10]
h_5	[0.60,0.80], [0.15,0.20]	[0.30,0.35], [0.50,0.65]	[0.58,0.68], [0.18,0.30]	[0.68,0.77], [0.10,0.20]	[0.72,0.85], [0.10,0.15]
h_6	[0.30,0.50], [0.30,0.45]	[0.50,0.68], [0.25,0.30]	[0.33,0.43], [0.50,0.55]	[0.62,0.65], [0.15,0.35]	[0.84,0.93], [0.04,0.07]

3 Proposed Parameter Reduction Approach of Interval-valued Intuitionistic Fuzzy Soft Sets and Its Algorithm

In this section we present an adjustable approach to interval-valued intuitionistic fuzzy soft set based decision making problems by combining the reductive intuitionistic fuzzy level soft sets of intuitionistic fuzzy soft sets. Firstly, we briefly recall the concept of reductive fuzzy soft sets and define a new concept of reductive intuitionistic fuzzy soft sets, and then recall the level soft sets. Finally, we present our proposed algorithm and illustrate by examples.

3.1 Reductive of Fuzzy Soft Sets and Intuitionistic Fuzzy Soft Sets

Let U be a universe set, E be a set of parameters and $A \subseteq E$. Let (F,A) be an interval-valued fuzzy soft set over U such that for every $\varepsilon \in A$, $F(\varepsilon)$ is an interval-valued fuzzy set with $F(\varepsilon)(x) = (F(\varepsilon)^-(x), F(\varepsilon)^+(x)) \forall x \in U$. The concept of reductive fuzzy soft set is proposed in [32] is shown as follows.

Definition 3.1 ***(See [32])*** *Let $\alpha\beta \in [0,1]$ and $\alpha + \beta = 1$. The vector $W = (\alpha\beta)$ is called an opinion weighting vector. The fuzzy soft set (F_W, A) over U such that $F_W(\varepsilon) = \{(x\alpha F(\varepsilon)^-(x) + \beta F(\varepsilon)^+(x)) : x \in U\} \forall \varepsilon \in A$ is called the weighted reductive fuzzy soft set of the interval-valued fuzzy soft set (F,A) with respect to the opinion weighting vector W.*

By adjusting the value of α and β , an interval-valued fuzzy soft set can be converted into a reductive fuzzy soft set decision maker. Specifically, let $\alpha = 1$ and $\beta = 0$ and $\beta = 1$, $\alpha = \beta = 0.5$ respectively. We will have three reductive fuzzy soft sets of (F,A), i.e., pessimistic reductive fuzzy soft set (PRFS) (F_-,A), optimistic reductive fuzzy soft set (ORFS) (F_+,A) and neutral reductive fuzzy soft set (NRFS) (F_N,A) . They are defined respectively as follows:

$$F_-(\varepsilon) = \{xF(\varepsilon)^-(x)) : x \in U\}, \forall \varepsilon \in A$$
$$F_+(\varepsilon) = \{xF(\varepsilon)^+(x)) : x \in U\}, \forall \varepsilon \in A$$
$$F_N(\varepsilon) = \{(x, (F(\varepsilon)^-(x)/2)|x \in U\} \forall \varepsilon \in A.$$

An interval-valued fuzzy soft set is changed to a fuzzy soft set by computing the reductive fuzzy soft set, which makes the making decision based on interval-valued fuzzy soft set much easier. Similarly, we can introduce the idea to make decision based on interval-valued intuitionistic fuzzy soft set, that is, convert both interval-valued membership degree and interval-valued non-membership degree into one fuzzy value. As a result, an interval-valued intuitionistic fuzzy soft set will be transformed to an intuitionistic fuzzy soft set, which will facilitate the making decision based on interval-valued intuitionistic fuzzy soft set. We define the notion of reductive intuitionistic fuzzy soft set as follows to illustrate the idea.

Let U be a universe set, E be a set of parameters and $A \subseteq E$. Let (F,A) be an interval-valued intuitionistic fuzzy soft set over U such that $\forall \varepsilon \in A, F(\varepsilon)$ is an interval-valued intuitionistic fuzzy set with $F(\varepsilon) = \{\langle x, \mu_{F(\varepsilon)}(x), \gamma_{F(\varepsilon)}(x)\rangle : x \in U\}, \forall x \in U$.

Definition 3.2 *Let* $\alpha, \beta, \phi, \varphi \in [0,1]$ *and* $\alpha + \beta = 1$, $\phi + \varphi = 1$. *The vector* $W = (\alpha, \beta, \phi, \varphi)$ *is called an opinion weighting vector. The intuitionistic fuzzy soft set* (F_W, A) *over* U *such that*
$F_W(\varepsilon) = \{(x, \alpha\mu^-_{F(\varepsilon)}(x) + \beta\mu^+_{F(\varepsilon)}(x), \phi\gamma^-_{F(\varepsilon)}(x) + \varphi\gamma^+_{F(\varepsilon)}(x))|x \in U\}, \forall \varepsilon \in A$
is called the weighted reductive intuitionistic fuzzy soft set of the interval-valued intuitionistic fuzzy soft set (F,A) *with respect to the opinion weighting vector* W.

By adjusting the value of α,β,ϕ and φ, an interval-valued intuitionistic fuzzy soft set can be converted into any reduction of intuitionistic fuzzy soft set decision maker desired. Specially, let $\alpha = 1$, $\beta = 0$, $\phi = 0$ and $\varphi = 1$, we have the pessimistic-pessimistic reduct intuitionistic fuzzy soft set (PPRIFS), denoted by (F_{-+}, A) and defined by

$F_{-+}(\varepsilon) = \{(x, \mu^-_{F(\varepsilon)}(x), \gamma^+_{F(\varepsilon)}(x))|x \in U\}, \forall \varepsilon \in A.$

Let $\alpha = 0$, $\beta = 1$, $\phi = 1$ and $\varphi = 0$, we have the optimistic-optimistic reductive intuitionistic fuzzy soft set (OORIFS), denoted by (F_{+-}, A) and defined by

$F_{+-}(\varepsilon) = \{(x, \mu^+_{F(\varepsilon)}(x), \gamma^-_{F(\varepsilon)}(x))|x \in U\}, \forall \varepsilon \in A.$

Let $\alpha = 0.5$, $\beta = 0.5$, $\phi = 0.5$ and $\varphi = 0.5$, we have the neutral-neutral reductive intuitionistic fuzzy soft set (NNRIFS), denoted by (F_{NN}, A) and defined by

$F_{NN}(\varepsilon) = \{(x, (\mu^-_{F(\varepsilon)}(x) + \mu^+_{F(\varepsilon)}(x))/2, (\gamma^-_{F(\varepsilon)}(x) + \gamma^+_{F(\varepsilon)}(x))/2|x \in U\}, \forall \varepsilon \in A.$

Example 3.1 Compute the PPRIFS (F_{-+}, A) , OORIFS (F_{-+}, A) and NNRIFS (F_{NN}, A) of the interval-valued intuitionistic fuzzy soft set (F,A) shown in Table 3.1. For the parameter $e_1 \in A$, we have the following intuitionistic fuzzy sets:

Table 2: Pessimistic-pessimistic reductive intuitionistic fuzzy soft set of (F,A)

U/E	e_1	e_2	e_3	e_4	e_5
h_1	[0.70, 0.20]	[0.82, 0.15]	[0.52, 0.25]	[0.55, 0.35]	[0.70,0.20]
h_2	[0.85, 0.10]	[0.70, 0.25]	[0.70, 0.23]	[0.70, 0.25]	[0.75, 0.10]
h_3	[0.50, 0.30]	[0.86, 0.10]	[0.60, 0.28]	[0.20, 0.60]	[0.65, 0.20]
h_4	[0.40, 0.40]	[0.52, 0.35]	[0.72, 0.21]	[0.30, 0.50]	[0.80, 0.10]
h_5	[0.60, 0.20]	[0.30, 0.65]	[0.58, 0.30]	[0.68, 0.20]	[0.72, 0.15]
h_6	[0.30, 0.45]	[0.50, 0.30]	[0.33, 0.55]	[0.62, 0.35]	[0.84, 0.07]

Table 3: Optimistic-optimistic reductive intuitionistic fuzzy soft set of (F,A)

U/E	e_1	e_2	e_3	e_4	e_5
h_1	[0.80, 0.10]	[0.84, 0.05]	[0.72, 0.18]	[0.60, 0.30]	[0.80,0.10]
h_2	[0.90, 0.05]	[0.74, 0.17]	[0.75, 0.10]	[0.75, 0.15]	[0.90, 0.05]
h_3	[0.70, 0.20]	[0.90, 0.04]	[0.70, 0.20]	[0.30, 0.50]	[0.80, 0.15]
h_4	[0.60, 0.30]	[0.64,0.23]	[0.78,0.11]	[0.50, 0.40]	[0.90, 0.05]
h_5	[0.80, 0.15]	[0.35, 0.50]	[0.68, 0.18]	[0.77, 0.10]	[0.85, 0.10]
h_6	[0.50, 0.30]	[0.68, 0.25]	[0.43, 0.50]	[0.65, 0.15]	[0.93, 0.04]

Table 4: Neutral-neutral reductive intuitionistic fuzzy soft set of (F,A)

U/E	e_1	e_2	e_3	e_4	e_5
h_1	[0.75, 0.15]	[0.83, 0.10]	[0.62, 0.22]	[0.58, 0.33]	[0.75,0.15]
h_2	[0.88, 0.08]	[0.72, 0.21]	[0.73, 0.17]	[0.73, 0.20]	[0.83, 0.08]
h_3	[0.60, 0.25]	[0.88, 0.07]	[0.65, 0.24]	[0.25, 0.55]	[0.73, 0.18]
h_4	[0.50, 0.35]	[0.58,0.29]	[0.75,0.16]	[0.40, 0.45]	[0.85, 0.08]
h_5	[0.70, 0.18]	[0.33, 0.58]	[0.63, 0.24]	[0.73, 0.15]	[0.79, 0.13]
h_6	[0.40, 0.38]	[0.59, 0.28]	[0.38, 0.53]	[0.64, 0.25]	[0.89, 0.06]

$$F_{+-}(e_1) = \left(\frac{k_1}{[0.7,0.2]} \; \frac{k_2}{[0.85,0.1]} \; \frac{k_3}{[0.5,0.3]} \; \frac{k_4}{[0.4,0.4]} \; \frac{k_5}{[0.6,0.2]} \; \frac{k_6}{[0.3,0.45]}\right)$$
$$F_{+-}(e_1) = \left(\frac{k_1}{[0.8,0.1]} \; \frac{k_2}{[0.9,0.05]} \; \frac{k_3}{[0.7,0.2]} \; \frac{k_4}{[0.6,0.3]} \; \frac{k_5}{[0.8,0.15]} \; \frac{k_6}{[0.5,0.3]}\right)$$
$$F_{NN}(e_1) = \left(\frac{k_1}{[0.75,0.15]} \; \frac{k_2}{[0.88,0.08]} \; \frac{k_3}{[0.6,0.25]} \; \frac{k_4}{[0.5,0.35]} \; \frac{k_5}{[0.7,0.18]} \; \frac{k_6}{[0.4,0.38]}\right)$$

Similarly we can compute other intuitionistic fuzzy sets with respect to other parameters in A. The results are shown in Tables 3.1, 3.2, and 3.3, respectively.

3.2 Level Soft Sets

Feng *et al.* [32] initiated the concept of level soft sets to solve fuzzy soft set based decision making problem. Subsequently, the same author applied level soft set to solve interval-valued fuzzy soft sets based decision making problem [35]. Jiang *et al.* [34] further generalized the approach introduced in [32] by applying level soft set to solve intuitionistic fuzzy soft sets based decision making. Level soft set of intuitionistic fuzzy soft set is defined as follows.

Definition 3.3*(See [34])*. *Let* $\varpi = (F,A)$ *be an intuitionistic fuzzy soft set over* U, *where* $A \subseteq E$ *and* E *is a set of parameters. For* $s,t \in [0,1]$, *the (s,t) -level soft set of* ϖ *is a crisp soft set* $L(\varpi; s,t) = (F_{(s,t)}, A)$ *defined by*

$F_{(s,t)}(\varepsilon) = L(F(\varepsilon);s,t) = \{x \in U|\mu_{F(e)}(x) \geq s$ and $\gamma_{F(\varepsilon)}(x) \leq t\}, \forall \varepsilon \in A$.

where $s \in [0,1]$ can be regarded as a given least threshold on membership values and $t \in [0,1]$ as a given greatest threshold on non-membership values.

The thresholds s and t are usually in advance chosen by decision makers according to their requirements. In Definition 3.3, the threshold pair assigned to each parameter is always the constant value pair$(s,t) \in [0,1] \times [0,1]$. However, in some decision making problems, it may happen that decision makers would like to use different threshold pairs on different parameters. To cope with such problems, a function instead of a constant value pair is used as the thresholds on membership values and non-membership values, respectively.

Definition 3.4*(See [34]). Let $\varpi = (F,A)$ be an intuitionistic fuzzy soft set over U, where $A \subseteq E$ and E is a set of parameters. Let $\lambda : A \to [0,1] \times [0,1]$ be an intuitionistic fuzzy set in A, which is called a threshold intuitionistic fuzzy set. The level soft set of ϖ with respect to λ is a crisp soft set $L(\varpi;\lambda) = (F_\lambda, A)$ defined by*

$$F_\lambda(\varepsilon) = L(F(\varepsilon);\lambda(\varepsilon)) = \{x \in U|\mu_{F(\varepsilon)}(x) \geq \mu_\lambda(\varepsilon) \text{ and } \gamma_{F(\varepsilon)}(x) \leq \gamma_\lambda(\varepsilon)\}\forall \varepsilon \in A.$$

According to the definition, four types of special level soft set are also defined in [34]. They are Mid-level soft set $L(\varpi, mid_\varpi)$, Top-Bottom-level soft set $L(\varpi, topbottom_\varpi)$, Top-Top-level soft set $L(\varpi, toptop_\varpi)$ and Bottom-bottom-level soft set $L(\varpi, bottombottom_\varpi)$. The intuitionistic fuzzy set $mid_\varpi : A \to (0,1) \times [0,1]$is called the mid-threshold of the intuitionistic fuzzy soft set ϖ and defined by

$$\mu_{mid\varpi}(\varepsilon) = \frac{1}{|U|}\sum_{x\in U}\mu_{F(\varepsilon)}(x) \text{ and}$$
$$\gamma_{mid\varpi}(\varepsilon) = \frac{1}{|U|}\sum_{X\in U}\gamma_{F(\varepsilon)}(x), \forall \varepsilon \in A.$$

The intuitionistic fuzzy set $topbottom_\varpi : A \to [0,1] \times [0,1]$ is called the top-bottom-threshold of the intuitionistic fuzzy soft set ϖ and defined by

$$\mu_{topbottom}(\varepsilon) = \max_{x\in U}\mu_{F(\varepsilon)}(x) \text{ and}$$
$$\gamma_{topbottom}(\varepsilon) = \min_{x\in U}\gamma_{F(\varepsilon)}(x), \forall \varepsilon \in A.$$

The intuitionistic fuzzy set $toptop_\varpi : A \to [0,1] \times [0,1]$ is called the top-top-threshold of the intuitionistic fuzzy soft set ϖ and defined by

$$\mu_{toptop_\varpi}(\varepsilon) = \max_{x\in U}\mu_{F(\varepsilon)}(x) \text{ and}$$
$$\gamma_{toptop_\varpi}(\varepsilon) = \max_{x\in U}\gamma_{F(\varepsilon)}(x), \forall \varepsilon \in A.$$

The intuitionistic fuzzy set $bottombottom_\varpi : A \to [0,1] \times [0,1]$ is called the bottom-bottom-threshold of the intuitionistic fuzzy soft set ϖ and defined by

$$\mu_{bottombottom_\varpi}(\varepsilon) = \min_{x\in U}\mu_{F(\varepsilon)}(x) \text{ and}$$
$$\lambda_{bottombottom_\varpi}(\varepsilon) = \min_{x\in U}\lambda_{F_\varepsilon}(x), \forall \varepsilon \in A.$$

3.3 Proposed AR-IIFSS and Its Algorithm

In this section we present our algorithm for decision making based on interval-valued intuitionistic fuzzy soft sets. From Figure 1, APR-IIFSS Algorithm comprises 6 steps, where the input is Interval-valued intuitionistic fuzzy soft set and the output is optimal decision. Step 1, the algorithm considers appropriate reductive intuitionistic fuzzy and level soft sets of intuitionistic fuzzy soft sets (step 2). In step 3, interval-valued intuitionistic fuzzy soft sets based decision making can be converted into only crisp soft sets based decision making. Firstly, by computing the reductive intuitionistic fuzzy soft set, an interval-valued intuitionistic fuzzy soft set is converted into an intuitionistic fuzzy soft set, and then the intuitionistic fuzzy soft set is further converted into a crisp soft set by using level soft sets of intuitionistic fuzzy soft sets (step 4). Finally, decision making is performed on the crisp soft set (steps 5 and 6).

APR-IIFSS Algorithm
Input: Interval-valued intuitionistic fuzzy soft set (F,A)
Output: Optimal decision

1. Input an opinion weighting vector $W = (\alpha, \beta, \phi, \varphi)$ and compute the weighted reductive intuitionistic fuzzy soft set $\varpi = (F_W, A)$ of the interval-valued intuitionistic fuzzy soft set $(F,A)$with respect to the opinion weighting vector W (or choose $\varpi = PPRIFS(F_{-+}, A)$, $OORIFS(F_{-+}, A)$ or $NNRIFS(F_{NN}, A)$ of (F,A)).
2. Input a threshold intuitionistic fuzzy set $\lambda : A \to [0,1] \times [0,1]$ (or give a threshold value pair);or choose the mid-level decision rule; or choose the top-bottom-level decision rule; or choose the top-top-level decision rule; or choose the bottom-bottom-level decision rule) for decision making.
3. Compute the level soft set $L(\varpi;\lambda)$with regard to the threshold intuitionistic fuzzy set λ (or the$_{(s,t)}$-level soft set $L(\varpi;s,t)$; or the mid-level soft set $L(\varpi;mid_\varpi)$;or the top-bottom-level soft set $L(\varpi;topbottom_\varpi)$;or the top-top-level soft set $L(\varpi;toptop_\varpi)$; or the bottom-bottom-level soft set $L(\varpi;bottombottom_\varpi)$).
4. Present the level soft set $L(\varpi;\lambda)$ (or $L(\varpi;s,t)$; or $L(\varpi;mid_\varpi)$; or $L(\varpi;topbottom_\varpi)$; or $L(\varpi;toptop_\varpi)$; or $L(\varpi;bottombottom_\varpi)$)in tabular form and compute the choice value of , .
5. The optimal decision is to select o_k if $c_k = max_i c_i$.
6. If $k$has more than one value, then any one of o_k may be chosen.

Fig. 1: AR-IIFSS Algorithm

From Figure 1, there are three remarks regarding the AR-IIFSS Algorithm:

Table 5: Parts of typical schemes for interval-valued intuitionistic fuzzy soft set based decision making

Scheme name	Reductive intuitionistic fuzzy soft set	Level soft set
Pes-Topbot	PPRIFS $\varpi = (F_{-+}, A)$	$L(\varpi, topbottom_{\varpi})$
Pes-Toptop	PPRIFS $\varpi = (F_{-+}, A)$	$L(\varpi, toptop_{\varpi})$
Pes-Mid	PPRIFS $\varpi = (F_{-+}, A)$	$L(\varpi, mid_{\varpi})$
Pes-Botbot	PPRIFS $\varpi = (F_{-+}, A)$	$L(\varpi, bottombottom_{\varpi})$

a. The above algorithm is an interval-valued fuzzy extension of the algorithm of intuitionistic fuzzy soft sets based decision making (see Algorithm 1 in [14]) and an intuitionistic fuzzy extension of the algorithm of interval-valued fuzzy soft sets based decision making(see algorithm 3 in [35]);
b. The reader is referred to [24] for more details regarding the method of computing the choice value in the fifth step of the above algorithm;
c. The last step of Algorithm 1, one may go back to step 2 or step 3 to modify opinion weighting vector or the threshold so as to adjust the final optimal decision when there are too many "optimal choices" to be chosen.

From Figure 1, the advantages of AR-IIFSS Algorithm are mainly in two folds:

a. We need not treat interval-valued intuitionistic fuzzy soft sets directly in decision making but only deal with the related reductive intuitionistic soft sets and finally the crisp level soft sets after choosing certain opinion weighting vectors and thresholds. This makes our algorithm simpler and easier for application in practical problems.
b. There are a large variety of opinion weighting vectors and thresholds that can be used to find the optimal choices, hence our algorithm has great flexibility and adjustable capability. Table 3.4 gives some typical schemes that arise from AR-IIFSS algorithm by combining the reductive intuitionistic soft set PPRIFS (F_{-+}) and several typical level soft sets. As pointed out in [32], many decision making problems are essentially humanistic and subjective in nature; hence there actually does not exist a unique or uniform criterion for decision making in an imprecise environment. This adjustable feature makes AR-IIFSS algorithm not only efficient, but also more appropriate for many practical applications.

To illustrate the basic idea of AR-IIFSS algorithm, let us consider the following example.

Example 3.2 Let us reconsider the decision making problem based on the interval-valued intuitionistic fuzzy soft sets (F,A) as in Table 3.1. If we select the first scheme "Pes-Mid" in Table 3.4 to solve the problem, at first we compute the reductive intuitionistic fuzzy soft set PPRIFS $\varpi = (F_{-+}, A)$ as in Table 3.1 and then use the mid-level decision rule on $\varpi = (F_{-+}, A)$ and obtain the mid-level soft set $L(\varpi, mid_{\varpi})$, with choice values given in Table 3.5.From Table 3.5, it is clear that the maximum choice value is $c_2 = 5$. Therefore, the optimal decision is to select h_2.

Table 6: Tabular representation of the mid-level soft set $L(\varpi, mid\varpi)$ with choice values

U/E	e_1	e_2	e_3	e_4	e_5	Choice value (c_i)
h_1	1	1	0	1	0	$c_1 = 3$
h_2	1	1	1	1	1	$c_2 = 5$
h_3	0	1	1	0	0	$c_3 = 2$
h_4	0	0	1	0	1	$c_4 = 2$
h_5	1	0	1	1	0	$c_5 = 3$
h_6	0	0	0	1	1	$c_6 = 2$

In the following section, we present an adjustable approach to weighted interval-valued intuitionistic fuzzy soft sets based decision making problems by extending the approach to weighted intuitionistic fuzzy soft sets based decision making [34] or weighted interval-valued fuzzy soft sets based decision making [35].

4 Weighted Interval-valued Intuitionistic Fuzzy Soft Sets based Decision Making

The following notion of weighted interval-valued intuitionistic fuzzy soft sets could provide a mathematical framework for modeling the interval-valued intuitionistic fuzzy soft sets based decision making problems in which all the parameters may not be equally important.

Definition 4.1 *Let U be a set of universe and E be a set of parameters. A Weighted interval-valued intuitionistic fuzzy soft set is a triple $\zeta = (F, A, f_w)$, where $(F,A)$is an interval-valued intuitionistic fuzzy soft set over U, and $f_W : A \to [0,1]$ is a weight function specifying the weight $w_j = f_w(e_j)$ for each parameter $e_j \in A$.*

AR-IIFSS algorithm (Figure 1) in the preceding section can be easily extended to deal with weighted interval-valued intuitionistic fuzzy soft set based decision making problems. The detail of AR-WIIFSS algorithm is presented in Figure 2.

The following example clearly describes how to obtain optimal decision using AR-WIIFSS Algorithm.

Example 4.1 Suppose that each parameter is associated with a weight w_i indicating its importance considered by Mr. X: for the parameter "beautiful", $w_1 = 0.8$; for the parameter "large", $w_2 = 0.9$; for the parameter "cheap", $w_3 = 0.9$; for the parameter "modern", $w_4 = 0.5$; for the parameter "in green surroundings", $w_5 = 0.7$. Thus we

AR-WIIFSS Algorithm
Input: Weighted interval-valued intuitionistic fuzzy soft set $\zeta = (F, A, f_w)$
Output: Optimal decision

1. Input an opinion weighting vector $W = (\alpha, \beta, \phi, \varphi)$ and compute the weighted reductive intuitionistic fuzzy soft set $\varpi = (F_W, A)$ of the interval-valued intuitionistic fuzzy soft set (F, A) with respect to the opinion weighting vector W (or choose $\varpi = PPRIFS(F_{-+}, A)$, $OORIFS(F_{-+}, A)$ or $NNRIFS(F_{NN}, A)$ of (F, A)).
2. Input a threshold intuitionistic fuzzy set $\lambda : A \to [0,1] \times [0,1]$ (or give a threshold value pair $(s,t) \in [0,1] \times [0,1]$);or choose the mid-level decision rule; or choose the top-bottom-level decision rule; or choose the top-top-level decision rule; or choose the bottom-bottom-level decision rule) for decision making.
3. Compute the level soft set $L(\varpi; \lambda)$ with regard to the threshold intuitionistic fuzzy set λ (or the (s,t)-level soft set $L(\varpi; s,t)$; or the mid-level soft set $L(\varpi, mid_w)$;or the top-bottom-level soft set $L(\varpi, topbottom_w)$;or the top-top-level soft set $L(\varpi, toptop_w)$; or the bottom-bottom-level soft set $L(\varpi, bottombottom_w)$).
4. Present the level soft set $L(\varpi; \lambda)$(or $L(\varpi; s,t$; or $L(\varpi; mid_w)$; or $L(\varpi; topbottom_w)$; or $L(\varpi; toptop_w)$; or $L(\varpi; bottombottom_w)$)in tabular form and compute the choice value $c_i^{'}$ of o_i , $\forall i$.
5. The optimal decision is to select o_k if $c_k^{'} = max_i c_i^{'}$.
6. If $k$has more than one value, then any one of o_k may be chosen.

Fig. 2: AR-WIIFSS Algorithm

Table 7: Tabular representation of the mid-level soft set $L(\varpi, mid_{\varpi})$ with weighted choice values (WCV)

U/E	$e_1, w_1 = 0.8$	$e_2, w_2 = 0.9$	$e_3, w_3 = 0.9$	$e_4, w_4 = 0.5$	$e_5, w_5 = 0.7$	WCV ($c_i^{'}$)
h_1	1	1	0	1	0	$c_1^{'} = 2.2$
h_2	1	1	1	1	1	$c_2^{'} = 3.8$
h_3	0	1	1	0	0	$c_3^{'} = 1.8$
h_4	0	0	1	0	1	$c_4^{'} = 1.6$
h_5	1	0	1	1	0	$c_5^{'} = 2.2$
h_6	0	0	0	1	1	$c_6^{'} = 1.2$

have a weight function $f_w : A \to [0,1]$ and the interval-valued intuitionistic fuzzy soft set (F, E) in example 1 is changed into a weighted interval-valued intuitionistic fuzzy soft set $\zeta = (F, E, f_w)$. If we still select the first scheme "Pes-Mid" in Table 3.4 to solve the problem, we shall obtain the mid-level soft set $L(\varpi, mid_{\varpi})$ with weighted choice values with tabular representation as in Table 4.1, where $\varpi = (F_{-+}, E)$.

From Table 4.1, it is clear that the maximum weighted choice value is $c_2^{'} = 3.8$. Therefore, the optimal decision is to select h_2 , namely Mr. X should buy h_2 as the best house.

5 Conclusion

In this paper, we have introduced an adjustable reduction approach to interval-valued intuitionistic fuzzy soft set (AR-IIFSS)for decision making. The proposed approach used reductive intuitionistic fuzzy soft sets and level soft sets of intuitionistic fuzzy soft sets. Unlike previous approaches, AR-IIFSS generalize the reduction of fuzzy soft sets (R-FSS), interval-valued fuzzy soft sets (R-ITFSS), and intuitionistic fuzzy soft sets (R-ICFSS) for decision making. An interval-valued intuitionistic fuzzy soft set is converted into a crisp soft after choosing certain opinion weighting vectors and thresholds. This makes the proposed algorithm simpler and easier for application in practical problems. In addition, a large variety of opinions weighting vectors and thresholds that can be used to find the optimal alternatives make our algorithm more flexible and adjustable. Moreover, we have also introduced an adjustable reduction approach of weighted interval-valued intuitionistic fuzzy soft set (AR-WIIFSS) and investigate its application for decision making.We have made extensive analysis for AR-IIFSS and AR-WIIFSS approaches to show their feasibility in practical applications of decision making.

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