

# Magnetohydrodynamic Peristaltic Flow of Jeffry Nanofluid with Heat Transfer Through a Porous Medium in a Vertical Tube

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**Abstract:** The present paper deals with the peristaltic motion of a non-Newtonian nanofluid with heat transfer through a porous medium inside a vertical tube. The system is stressed by a uniform magnetic field. The viscous dissipation, internal heat generation with radiation effects are considered. A Rung-Kutta-Merson method and a Newton iteration in a shooting and matching technique are used to find the solutions of the momentum, temperature and nanoparticles equations. The numerical formula of the axial velocity, temperature and nanoparticles are obtained as functions of the physical parameters of the problem. Numerical calculations are carried out for these formula. The effects of physical parameters of the problem on this formula are discussed numerically and illustrated graphically through a set of figures.

**Keywords:** Peristaltic flow; non-Newtonian nanofluid; porous medium; heat transfer.

## 1 Introduction

Peristaltic pumping is the transport of a fluid via traveling waves imposed on the walls of a tube or channel. The study of peristaltic flow is of special interest for several applications in industry and physiology. Especially the peristaltic transport of non-Newtonian fluids is a topic of major interest of the researches in the field of physiological world. Such interest is stimulated because of its occurrence in several physiological processes including chyme movement in the gastrointestinal tract, urine transport from kidney to bladder, transport of bile in bile duct, in roller and finger pumps, in vasomotion of small blood vessels.

Mekheimer and Arabi [1] studied the non linear peristaltic transport of MHD flow through a porous medium. Kumari et al. [2] studied peristaltic pumping of a conducting Jeffrey fluid in a vertical porous channel

with heat transfer. The effect of wall properties on the peristaltic flow of a non-Newtonian fluid is studied by Hayat et al. [3]. Eldabe et al. [4] studied effects of chemical reaction with heat and mass transfer on peristaltic motion of power-law fluid in an asymmetric channel with wall properties. Also, Eldabe and Abou-zeid [5] analyzed the problem of magnetohydrodynamic peristaltic flow with heat and mass transfer of micropolar biviscosity fluid through a porous medium between two co-axial ducts.

Effects of chemical reaction, heat, and mass transfer on non-Newtonian fluid flow through porous medium in a vertical peristaltic tube have been studied by El-Sayed et. al. [6]. Effects of heat and mass transfer on MHD peristaltic flow of a non-Newtonian fluid through a porous medium between two co-axial cylinders have been discussed by Shaaban and Abou-zeid [7]. Eldabe and Abou-zeid [8] examined the wall properties effect on

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peristaltic transport of micropolar non-Newtonian fluid with heat and mass transfer. Hall effects on peristaltic flow of Maxwell fluid in a porous medium are discussed by Hayat et. al. [9].

There has been great interest of researches about the transport of nanofluids due to its properties to enhance the thermal conductivity as compared to base fluid. Nanofluid is a liquid containing nanometer-sized particles (having diameter less than 100 nm), called nanoparticles. The nanoparticles are made up of metals, oxides and carbides or carbon nanotubes. Nanofluids are used to increase the rate of heat transfer of microchips in computers, microelectronics, transportation, fuel cells and manufacturing.

Most of liquids such as water, glycol and oil have low-thermal conductivity. To increase the thermal conductivity of such fluids suspended nanosized metallic particles (titanium, copper, gold, iron or their oxides) in the fluids. Choi [10] was the first who initiated this nanofluid technology. Peristaltic flow of a nanofluid under the effect of hall current and porous medium has been studied by Nowar [11]. The problem of mixed convection of gliding motion of bacteria on power-law nanoslime through a non-Darcy porous medium has been analyzed by Abou-zeid et al. [12], they considered non-Darcian and viscous dispersion effects in their analysis. Abou-zeid [13] studied homotopy perturbation method to MHD non-Newtonian nanofluid flow through a porous medium in eccentric annuli with peristalsis, Endoscopic effects on peristaltic flow of a nanofluid have been discussed by Akbar and Nadeem [14], Ellahi et. al. [15] studied a theoretical study of Prandtl nanofluid in a rectangular duct through peristaltic transport.

The aim of this paper is to investigate the peristaltic flow of an electrically conducting non-Newtonian nanofluid in a vertical symmetric tube through a porous medium. Jeffry non-Newtonian constitutive model is employed for the transport fluid. The system of non-linear partial differential equations which describe this motion subjected to the appropriate boundary conditions are solved numerically by using Rung-Kutta-Merson method with shooting and matching technique. Numerical results for the axial velocity, temperature and nanofluid phenomena are determined. The effects of different parameters on these expressions have been discussed through several graphs.

## 2 Mathematical formulation

Let us consider the peristaltic flow of a Jeffry fluid through a vertical tube, the cylindrical polar coordinates system  $(R, Z)$  are used, where  $R$  is along the radius of the tube and  $Z$  coincides with axis of the tube. The geometry of the tube wall is given by

$$h = a + b \cos\left[\frac{2\pi}{\lambda}(Z - ct)\right], \quad (1)$$

where  $a$  is the radius of the tube,  $b$  is the amplitude of the peristaltic wave,  $\lambda$  is the wave length,  $c$  is the wave propagation speed, and  $t$  is the time.

The constitutive equations for an incompressible Jeffry nanofluid are given by:

$$\underline{\tau} = -P\underline{I} + \underline{S}, \quad (2)$$

$$S = \frac{\mu}{1 + \lambda_1}(\dot{\gamma} + \lambda_2\ddot{\gamma}). \quad (3)$$

where  $\underline{S}$  is the extra stress tensor,  $P$  is the pressure,  $\underline{I}$  is the identity tensor,  $\mu$  is the dynamic viscosity,  $\lambda_1$  is the ratio of relaxation to retardation times,  $\lambda_2$  is the retardation time,  $\dot{\gamma}$  (vector quantity) is the shear rate, and dots denote the differentiation with respect to time. The fundamental equations governing the flow of an incompressible Jeffry nanofluid are given by:

$$\nabla \cdot \underline{V} = 0, \quad (4)$$

$$\rho_f \frac{d\underline{V}}{dt} = \nabla \cdot \underline{\tau} + \mu_e \underline{J} \wedge \underline{B} - \frac{\mu}{\kappa^*} \underline{V} + \underline{F}, \quad (5)$$

$$(\rho c)_f \frac{dT}{dt} = k_c \nabla^2 T + \underline{\tau} \cdot \nabla \underline{V} - (\rho c)_p \left[ D_B \nabla \phi \nabla T + \left( \frac{D_T}{T_0} \right) (\nabla T)^2 \right] + Q_0 T, \quad (6)$$

$$\frac{d\phi}{dt} = D_B \nabla^2 \phi + \left( \frac{D_T}{T_0} \right) \nabla^2 T, \quad (7)$$

$$\left. \begin{aligned} \underline{J} &= \sigma(\underline{E} + \underline{V} \wedge \underline{B}), \nabla \cdot \underline{B} = 0, \\ \nabla \wedge \underline{B} &= \mu_m \underline{J}, \nabla \wedge \underline{E} = -\frac{\partial \underline{B}}{\partial t} \end{aligned} \right\}, \quad (8)$$

where  $\underline{V}$  is the velocity,  $\kappa^*$  is the permeability,  $\underline{J}$  is the current density,  $\underline{B} = (B_0, 0, 0)$  is the magnetic field,  $\sigma$  is the electrical conductivity,  $\mu_m$  is the magnetic permeability,  $\underline{E}$  is the electric field,  $\underline{\tau}$  is Cauchy stress tensor and is defined by Eqs. (2) and (3),  $\underline{F}$  is the body forces,  $\rho_f$  is the density of the fluid,  $\rho_p$  is the density of the particle,  $(\rho c)_f$  and  $(\rho c)_p$  are heat capacity of the fluid and effective heat capacity of the nanoparticle material,  $T$

is the temperature of the fluid,  $\phi$  is the nanoparticle phenomena,  $k_c$  is the thermal conductivity,  $D_B$  is the Brownian diffusion coefficient,  $D_T$  is the thermophoretic diffusion coefficient,  $Q_0$  is the constant heat addition and absorption.

Let  $U$  and  $W$  be the respective velocity components in the radial and axial directions in the fixed frame, respectively. For the unsteady two-dimensional flow, the velocity components and the temperature may be written as follows:

$$\underline{V} = (U(R, Z), 0, W(R, Z)), \text{ and } T = T(R, Z).$$

Introducing a wave frame  $(r, z)$  moving with velocity  $c$  away from the fixed frame  $(R, Z)$  by the transformation

$$\begin{aligned} r &= R, z = Z - ct, \\ u &= U, w = W - c. \end{aligned} \tag{9}$$

The governing continuity, momentum, temperature, and nanoparticles equations become

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{10}$$

$$\begin{aligned} \rho_f \left( u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) &= -\frac{\partial P}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (r S_{rr}) \\ + \frac{\partial}{\partial z} (S_{rz}) - \frac{S_{\theta\theta}}{r} - \sigma B_0^2 u - \frac{\mu}{\kappa^*} u, \end{aligned} \tag{11}$$

$$\begin{aligned} \rho_f \left( u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) &= -\frac{\partial P}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (r S_{rz}) + \frac{\partial}{\partial z} (S_{zz}) \\ - \sigma B_0^2 w - \frac{\mu}{\kappa^*} w - \rho g \alpha (T - T_0) - \rho g \alpha (\phi - \phi_0), \end{aligned} \tag{12}$$

$$\begin{aligned} \rho c_p \left( u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) &= k_c \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) \\ + \frac{1}{1+\lambda_1} \left( 2 \left( \frac{\partial u}{\partial r} \right)^2 + 2 \left( \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right)^2 \right) \\ + (\rho c)_p [D_B \left( \frac{\partial \phi}{\partial r} \frac{\partial T}{\partial r} + \frac{\partial \phi}{\partial z} \frac{\partial T}{\partial z} \right) + \left( \frac{D_T}{T_0} \right) \left[ \left( \frac{\partial T}{\partial r} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right]] + Q_0 T, \end{aligned} \tag{13}$$

$$\begin{aligned} \left( u \frac{\partial \phi}{\partial r} + w \frac{\partial \phi}{\partial z} \right) &= D_B \left( \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} \right) + \\ \left( \frac{D_T}{T_0} \right) \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right). \end{aligned} \tag{14}$$

The boundary conditions for this system are given by

$$\left. \begin{aligned} \frac{\partial w}{\partial r} = 0, \frac{\partial T}{\partial r} = 0, \frac{\partial \phi}{\partial r} = 0 \quad \text{at } r = 0, \\ w = 0, T = 0, \phi = 0 \quad \text{at } r = h \end{aligned} \right\}. \tag{15}$$

The appropriate non-dimensional variables for the flow are defined as

$$\left. \begin{aligned} r^* &= \frac{r}{a}, z^* = \frac{z}{\lambda}, u^* = \frac{\lambda u}{ac}, w^* = \frac{w}{c}, P^* = \frac{a^2 P}{\lambda \mu c}, t^* = \frac{c}{\lambda} t, \\ t^* &= \frac{c}{\lambda} t, \theta = \frac{T - T_0}{T_1 - T_0}, f = \frac{\phi - \phi_0}{\phi_1 - \phi_0}, S^* = \frac{aS}{\mu c}, h^* = \frac{h}{a}, \delta = \frac{a}{\lambda} \\ \phi &= \frac{b}{a}, M = \sqrt{\frac{\sigma}{\mu}} B_0 a, D_a = \frac{\kappa^*}{a^2}, Re = \frac{\rho c a}{\mu}, \\ Gr &= \frac{\rho g \alpha a^2 (T_1 - T_0)}{\mu c}, Br = \frac{\rho g \alpha a^2 (\phi_1 - \phi_0)}{\mu c}, \overline{\sigma} = \frac{(\rho c)_p}{(\rho c)_f}, \\ P_r &= \frac{\nu \rho c_p}{k_c}, E_c = \frac{c^2}{c_p (T_1 - T_0)}, Nt = \frac{\overline{\sigma} D_T (T_1 - T_0)}{\nu T_0}, \\ Nb &= \frac{\overline{\sigma} D_B (\phi_1 - \phi_0)}{\nu T_0} \end{aligned} \right\}, \tag{16}$$

where  $\delta$  is the wave number,  $Re$  is Reynolds number,  $M$  is the magnetic parameter,  $D_a$  is Darcy number,  $E_c$  is Eckert number,  $P_r$  is Prandtl number,  $Gr$  is the local temperature Grashof number,  $Br$  is the local nanoparticle Grashof number,  $Nt$  is the thermophoresis parameter, and  $Nb$  is Brownian motion parameter.

After dropping the star mark for simplicity, Eqs. (10)-(14) under the assumptions of long wavelength and low-Reynolds number approximations take the form

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{17}$$

$$\frac{\partial P}{\partial r} = 0, \tag{18}$$

$$\frac{\partial P}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (r S_{rz}) - \left( M^2 + \frac{1}{D_a} \right) w - Gr \theta - Br f, \tag{19}$$

$$\begin{aligned} \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} &= -\frac{P_r E_c}{1+\lambda_1} \left( \frac{\partial w}{\partial r} \right)^2 - P_r Nt \left( \frac{\partial \theta}{\partial r} \right)^2 \\ - P_r Nb \left( \frac{\partial f}{\partial r} \frac{\partial \theta}{\partial r} \right) + P_r Q_0 \theta, \end{aligned} \tag{20}$$

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} = - \left( \frac{Nt}{Nb} \right) \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right), \tag{21}$$

where

$$S_{rr} = S_{\theta\theta} = S_{zz} = 0, \quad S_{rz} = \frac{1}{1+\lambda_1} \left( \frac{\partial w}{\partial r} \right), \tag{22}$$

Thus, the boundary conditions in their dimensionless form read

$$\left. \begin{aligned} \frac{\partial w}{\partial r} = 0, \frac{\partial \theta}{\partial r} = 0, \frac{\partial f}{\partial r} = 0 \quad \text{at } r = 0, \\ w = 0, \theta = 0, f = 0 \quad \text{at } r = h \end{aligned} \right\}. \tag{23}$$

where

$$h = 1 + \phi \sin(2\pi z). \tag{24}$$

### 3 Method of solution

Let  $w = Y_1$ ,  $\theta = Y_3$ , and  $f = Y_5$ . Hence equations (19)-(21) can be written as follows:

$$\left. \begin{aligned} Y_1' &= Y_2, Y_2' = -\frac{1}{r}Y_2 \\ &+ (1 + \lambda_1) \left( \frac{\partial P}{\partial z} + \left( M^2 + \frac{1}{D_a} \right) Y_1 + GrY_3 + BrY_5 \right), \\ Y_3' &= Y_4, Y_4' = -\frac{1}{r}Y_4 \\ &- Pr \left( \frac{Ec}{(1+\lambda_1)} Y_2^2 + NtY_4^2 + NbY_4Y_6 + Q_0Y_3 \right), \\ Y_5' &= Y_6, Y_6' = -\frac{1}{r}Y_6 - \frac{Nt}{Nb} \left( Y_4 + \frac{1}{r}Y_4 \right), \end{aligned} \right\} \quad (25)$$

where prime denotes to differentiation with respect to  $r$  and the system (25) is subjected to the boundary conditions

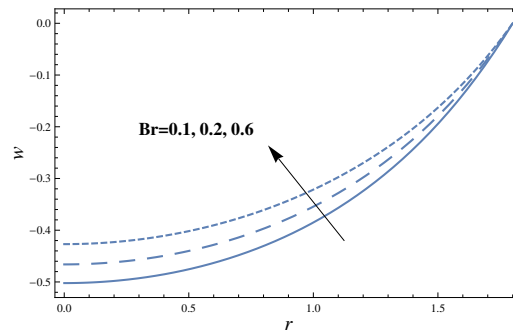
$$\left. \begin{aligned} Y_2 &= 0, Y_4 = 0, Y_6 = 0, \quad \text{at } r = 0 \\ Y_1 &= 0, Y_3 = 0, Y_5 = 0, \quad \text{at } r = h \end{aligned} \right\} \quad (26)$$

To apply shooting method, we use the subroutine D02HAF from the NAG Fortran library, which requires the supply of starting values of the missing initial and terminal conditions. The subroutine uses Rung-Kutta-Merson method with variable step size in order to control the local truncation error, then it applies modified Newton-Raphson technique mentioned before to make successive corrections to the estimated boundary values. In this problem, we start our solutions in the neighborhood of  $r = 0$ , namely,  $r = 10^{-4}$  to avoid the singularity at  $r = 0$ . The process is repeated iteratively until convergence is obtained i.e. until the absolute values of the difference between every two successive approximations of the missing conditions is less than  $\epsilon$ . (in our case  $\epsilon$  is taken =  $10^{-7}$ ).

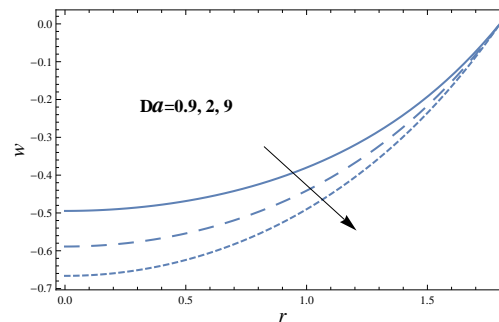
### 4 Discussion and results

To observe the quantitative effects of the parameters of the non-Newtonian fluid, nanofluid and heat transfer on the solutions obtained, a numerical computations are carried out for the formula of the axial velocity, temperature and nanoparticles.

Figs. (1) and (2) illustrate the change of the axial velocity  $w$  versus the radial coordinate  $r$  with several values of the local nanoparticle Grashof number  $Br$ , and Darcy number  $D_a$ , respectively. It is seen, from Figs. (1) and (2), that the axial velocity increases with the increase of  $Br$ , whereas it decreases as  $D_a$  increases, respectively. It is also noted that the difference of the axial velocity for different values of  $Br$  and  $D_a$  becomes greater near the axis of tube and all curves intersect at  $r = h$  which the



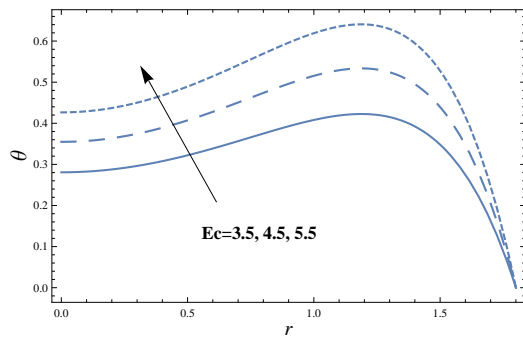
**Fig. 1:** the axial velocity profiles are plotted versus  $r$  for different values of  $Br$  for a system have the particulars  $M = 1, D_a = 0.9, Gr = 0.1, Br = 0.1, h = 1.8, \frac{\partial p}{\partial z} = 1.5, Pr = 2.5, Ec = 5.5, Nt = 3.5, Nb = 1.5, \lambda_1 = 0.01$  and  $Q_0 = 1$



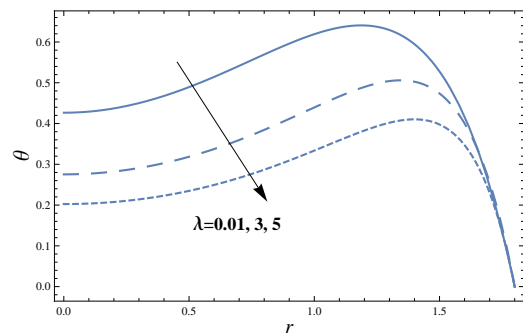
**Fig. 2:** the axial velocity profiles are plotted versus  $r$  for different values of for a system have the particulars  $M = 1, Gr = 0.1, Br = 0.1, h = 1.8, \frac{\partial p}{\partial z} = 1.5, Pr = 2.5, Ec = 5.5, Nt = 3.5, Nb = 1.5, \lambda_1 = 0.01$  and  $Q_0 = 1$

axial velocity reaches to maximum value. The result in Fig. (2) qualitatively agrees with expectations; since it is well-known that in fluid dynamics through porous media, Darcy number represents the relative effect of the permeability of the medium versus its cross-sectional area, commonly the diameter squared. The effects of both  $Gr$  and  $Nb$  on the axial velocity are found to be similar to the effect of  $D_a$  given in Fig. (2), while the effects of other parameters are found to be similar to the effect of  $Br$  given in Fig. (1), but figures are excluded to avoid any kind of repetition.

The effects of both Eckert number  $Ec$  and the ratio of relaxation to retardation times  $\lambda_1$  on the temperature  $\theta$  are shown in Figs. (3) and (4), respectively, and it is clear that the temperature is always positive, and it increases by increasing  $Ec$ , while it decreases with the increase of  $\lambda_1$ .



**Fig. 3:** the temperature profiles are plotted versus  $r$  for different values of  $E_c$  for a system have the particulars  $M = 1, D_a = 0.9, Gr = 0.1, Br = 0.1, h = 1.8, \frac{\partial p}{\partial z} = 1.5, Pr = 2.5, Nt = 3.5, Nb = 1.5, \lambda_1 = 0.01$  and  $Q_0 = 1$

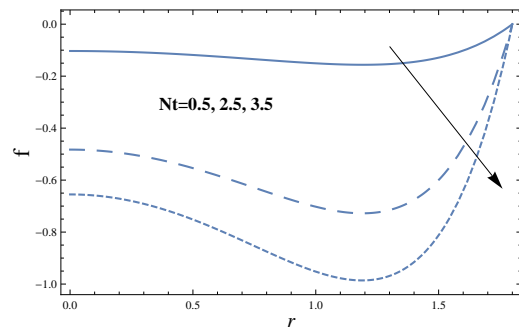


**Fig. 4:** the temperature profiles are plotted versus  $r$  for different values of  $\lambda_1$  for a system have the particulars  $M = 1, D_a = 0.9, Gr = 0.1, Br = 0.1, h = 1.8, \frac{\partial p}{\partial z} = 1.5, Pr = 2.5, Ec = 5.5, Nt = 3.5, Nb = 1.5$  and  $Q_0 = 1$

Also, it is noted that  $\theta$  increases as  $r$  increases till a maximum value, after which  $\theta$  decreases. It is also observed from Fig. (3), that there is a rise in the temperature due to the heat created by the viscous dissipation and it is in conformity with the fact that energy is stored in the fluid region due to fractional heating as a consequence of dissipation due to viscosity, and hence the temperature increases as  $E_c$  increases. The following explains the nonlinear variation of heat transfer. It is known that the effect of viscous dissipation produces heat due to drag between the fluid particles, this extra heat causes an increase of the initial fluid temperature. This increase of temperature causes an increase of the buoyant force. (This kind of influence on the buoyant force has been ignored by Gebhart [16]). The increase of the buoyant force causes an increase of the fluid velocity, the

bigger fluid velocities cause bigger drag between the fluid particles and consequently bigger viscous heating of the fluid. The new increase of fluid temperature influences the buoyant force and this procedure goes on. There is a continuous interaction between the viscous heating and the buoyant force. This mechanism produces different results in the upward and downward flow. In the upward flow where the fluid is warmer than the ambient, the extra viscous heat is added to the initial heat (the warm fluid becomes warmer) and the fluid velocity increases. In the downward flow the fluid is cooler than the ambient and the viscous heating causes an increase in the initial fluid temperature (the cold fluid becomes warmer) [17]. The results in Fig. (3) are consistent with those obtained by Abouzeid [13].

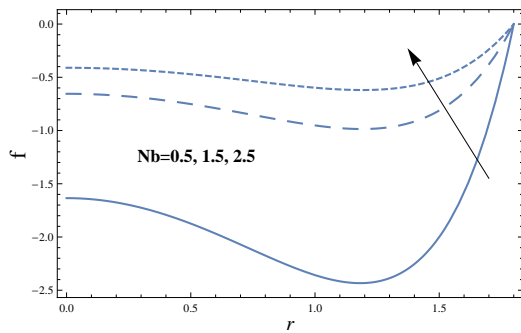
If we draw the variation of  $\theta$  with  $r$  for different values of  $D_a$  and  $Br$ , we will obtain a figure in which the behavior of the curves are the same as that obtained in Fig. (4), except that the obtained curves are very close to those obtained in Fig. (4). The effects of the other parameters are found to be similar to the effect of  $E_c$  on  $\theta$ ; these figures are excluded here to avoid any kind of repetition.



**Fig. 5:** the nanoparticles profiles are plotted versus  $r$  for different values of  $Nt$  for a system have the particulars  $M = 1, D_a = 0.9, Gr = 0.1, Br = 0.1, h = 1.8, \frac{\partial p}{\partial z} = 1.5, Pr = 2.5, Ec = 5.5, Nb = 1.5, \lambda_1 = 0.01$  and  $Q_0 = 1$

Brownian motion is the random motion of particles suspended in a fluid resulting from their collision with the quick atoms or molecules in the fluid. A physical explanation of Brownian motion was given by Einstein, who analyzed Brownian motion as the cumulative effect of innumerable collisions of the suspended particle with the molecules of the fluid. In Figs. (5) and (6), the behaviors of nanoparticles phenomena  $f$  versus the radial coordinate  $r$ , for various values of  $Nt$  and  $Nb$  are





**Fig. 6:** the nanoparticles profiles are plotted versus  $r$  for different values of  $Nb$  for a system have the particulars  $M = 1, D_a = 0.9, Gr = 0.1, Br = 0.1, h = 1.8, \frac{\partial p}{\partial z} = 1.5, Pr = 2.5, Ec = 5.5, Nt = 3.5, \lambda_1 = 0.01$  and  $Q_0 = 1$

presented respectively. It is observed from Figs. (5) and (6) that the nanoparticles phenomena increases with the increase of  $Nt$ , whereas it decreases as  $Nb$  increases, respectively. It is also noted that the difference of the nanoparticles phenomena  $f$  for different values of  $Nt$  and  $Nb$  becomes lower with increasing the radial coordinate  $r$  and reaches minimum value, after which it increases. The effects of both  $Ec$  and  $Pr$  on  $f$  are found to be similar to the effect of  $Nb$  on  $f$  shown in Fig. (6), except that the obtained curves are very close to those obtained in Fig. (6).

## 5 Conclusion

In this paper, the main aim is to obtain numerical solutions of the problem of peristaltic flow and heat transfer of an incompressible, electrically conducting Jeffery nanofluid in a vertical symmetric tube. The flow is through porous medium. Relevant equations are modulated and solved numerically for the axial velocity, temperature and nanoparticles phenomena by using Rung-Kutta-Merson method in a shooting and matching technique. Numerical calculations are presented for physical quantities of interest and their dependence on the material parameters of the fluid. The effects of these parameters are discussed by a set of graphs. The main findings from the current study can be summarized as follows:

1. By increasing  $Nt$ , the nanoparticles phenomena  $f$  increases, while it decreases by increasing values of  $Nb, Pr$  and  $Ec$ .
2. The temperature  $\theta$  is always positive.

3. The temperature  $\theta$  has an opposite behavior compared to nanoparticles behavior except that it increases with the increase of  $Nt$ .
4. The temperature  $\theta$  for different values of  $M, Nt, Nb, Pr$  and  $Ec$  increases and it decreases with the increase of  $D_a, Gr$  and  $Br$ .
5. The axial velocity  $w$  increases with the increase each of  $M$ , and  $Br$ , while it decreases as  $D_a$  and  $Gr$  increase.

## 6 Applications

1. Peristaltic pumps use flexible tubing to run through rollers in the pump head. Also, they are suitable for dispensing, metering and general transfer applications.
2. Peristaltic pumps confine the media to the tubing, so that the pump cannot contaminate the fluid and the fluid cannot contaminate the pump. So, a peristaltic pump offers easy maintenance and reduced downtime compared with other pumping technologies.
3. Peristaltic pumps are available in various configurations, from low-cost fixed-speed pumps to advanced controlled models suitable for critical metering and dispensing applications.
4. Peristaltic pumps are useful in the following
  - Juice production
  - Pizza sauce dispensing
  - Vitamin A & D injection
  - Ultra-filtration
  - Harvesting cell media
  - Acid/base dispensing
  - Adhesives for cement
  - Circuit board manufacturing
  - Dispensing glue emulsions
  - Transfer of fuels and lubricants

Information science is an interdisciplinary field concerned with applied sciences, analysis, collection, classification, manipulation, storage, retrieval, movement, dissemination, and protection of information. Practitioners within this field study the applications and usages of knowledge in organizations, along with the interaction. Every day, people use computers in new ways, scientific progress in fields like nanotechnology, nanofluid and biotechnology is almost entirely dependent on the use of computers software such as Matlab, Maple and Mathematica package. Using supercomputers, natural expectation by using a combination of observations of physical results from many sources such as mathematical representation of the behavior of the temperature and concentration.

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