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Mining Maximal Cliques through an Intuitionistic Fuzzy Graph

S. Venkatesh[1,*] *and S. Sujatha*[2]

[1] Department of Mathematics, J.J. College of Engineering and Technology, Tiruchirappalli, Tamil Nadu, India.
[2] Department of Computer Applications, University College of Engineering, BIT campus, Anna University, Tiruchirappalli, Tamil Nadu, India.

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Abstract: This paper deals with an inspiring and real-world problem of mining maximal cliques in an intuitionistic fuzzy graph G where the edges are weighted by the degrees of membership together with non-membership values. By using fuzzy cuts (α,β) such that $0 \leq \alpha\beta \leq 1$, a modified concept of $C_{\alpha\beta}$ maximal clique is proposed in an intuitionistic fuzzy graph. To find $C_{\alpha\beta}$ maximal cliques in an intuitionistic fuzzy graph, we present an effective mining algorithm based on this $C_{\alpha\beta}$ is introduced.

Keywords: Intuitionistic Fuzzy Graph, $C_{\alpha\beta}$ maximal clique, degree of non-membership, fuzzy cuts, cardinality of edges, fuzzy cut arc.

1 Introduction

The analysis of huge databases and receiving the knowledge discovery is a critical task in making a suitable decision in real time industry problems. Large data sets frequently contain information that is unreliable in nature. Identification of dense substructure is a very important task that comes from the graph. There are many applications in data processing which incorporates the cluster and community detection in social and biological networks [5]. Its additionally potential that the foremost fundamental dense sub structure in an exceedingly graph will unremarkably employed in a band a wholly connected sub graph [13]. Naturally, maximal cliques attract us due its special character that it never be contained within the other clique.

It is one of the largest issues to identify the all maximal cliques in a graph from the graph mining which has been used in several settings probably from social networks the overlapping communities are found out [5, 13], the e-mail networks are analysed and the bio-informatics issues are recognized. The relationships themselves become a probabilistic naturally in a number of the cases as we tend to face the link of one person influences another person in an exceedingly social network.

Last few years, the Fuzzy theory become faded compare to the Graph theory which dominates completely the earlier. Fuzzy set has been developed with the concept of prospective field which is recognized as an Knowledge base. In 1965, ‘fuzzy sets’ was published by Zadeh [17] as his inspirational work, in which he thoroughly explained the fuzzy set theory. The fuzzy set and fuzzy relations are put in to argumentative discussion by Kaufmann [8]. The concept of fuzzy analogues on various graph theoretic concepts is identified by Azriel [14] in 1975. Some important clarifications are introduced on fuzzy graphs by Rosenfeld Bhattacharya [2]. Moreover he obtained fuzzy graph theory outcomes as concerning center and eccentricity. The representing information and relationships between objects of Sunitha and Mathew [16] are suitably displayed by the fuzzy graph theory.

Atanassov [1] has taken much efforts in the analysis of fuzzy sets and defined it. The identical factor is more outlined by Zadeh, that he significantly extended the conception of “Intuitionistic Fuzzy Sets” and explored their basic properties. This applications of Intuitionistic fuzzy sets is originated by him that is employed in knowledgeable systems, systems theory. In addition to that a plenty of operations and relations over intuitionistic fuzzy sets are has explained by him.

* Corresponding author e-mail: venkateshjjcet@gmail.com

Atanassov's intuitionistic fuzzy graph is identified as an exceptional case and analysed the modules by Parvathy and Karunambigai [11]. Operations on Intuitionistic Fuzzy Graphs (IFGs) are also done by them [12]. Antony Shannon and Atanassov [15] deliberate a original generalization of the intuitionistic fuzzy graphs. The fuzzy graph model has fascinated plenty of responsiveness who works on data mining from the societies of data science and fuzzy logic. In this paper, we design a novel methodology on maximal clique in IFGs and its properties are studied.

2 Related work

Now-a-days a large amount of recent work on demonstrating fuzzy on possibility of communities recognition and clustering using fuzzy logic. The problem of finding most fuzzy cliques in fuzzy graphs is generated by Bandyapadhyay. Then, the proposed problem was treated to reach the quadratic of 0-1 programming problem which should be unconstrained. A clique that can attain other vertices with greater membership degrees and combine them into cliques is defined as the maximum fuzzy clique. The problem of counting top-*k* cliques with the top-k highest likelihood of existence from unsure graph, that differs from [8], a current research work aiming on mining the maximal cliques form unsure graph [10] is explained by the Zou et al. [18]. A new concept, named α-largest clique in an unsure graph is also defined by them. Roberto De Virgilio et al. [13] for computing all largest cliques of an arbitrarily massive network in a distributing environment and theoretical results showing the correctness and completeness over the sparse graphs. Obviously, the weights on the edges of a fuzzy graph refer to the degrees of membership. Motivated by those differences this paper explores a new approach of mining (α,β)-maximal cliques from intuitionistic fuzzy graphs.

3 Main Results

Definition 3.1. *In an intuitionistic fuzzy graph G, the set of vertices $C \subseteq V$ the degree of membership of C termed as cdm of (C,G) and the degree of non-membership of C, termed as cdnm(C,G). Let $C \subseteq V$ be the set of vertices. Let α and β intuitionistic fuzzy cuts, then C is called $C_{\alpha\beta}$-clique if* $\mathrm{cdm}(C,G) \geq \alpha$ *and* $\mathrm{cdnm}(C,G) \leq \beta$.

Definition 3.2. *For an intuitionistic fuzzy graph $G = (V,E,\mu,\gamma)$ and a intuitionistic fuzzy cuts (α,β), a set $M \subseteq V$ is defined as a $C_{\alpha\beta}$-maximal clique if*

1. *M is a $C_{\alpha\beta}$-clique in G.*
2. *There is no vertex $v \in (V/M)$ such that $M \cup \{v\}$ is $C_{\alpha\beta}$-clique in G.*

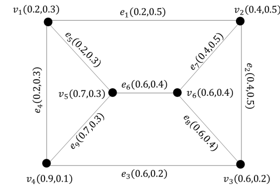

Fig. 1: Graph with fuzzy cut α,β.

Example 3.3. Consider Fig. 1:

Consider, the set of vertices and edges as $V = \{v_i;\, i = 1,2,\ldots,6\}$ and $E = \{e_j;\, j = 1,2,\ldots,9\}$ in G.

Let $\alpha = 0.1$ and $\beta = 0.7$ be the fuzzy cuts for cliques degree of membership and non membership respectively.

Let us take $\{e_4,e_5,e_9\} \in E$.
$\mathrm{cdm}\{e_4,e_5,e_9\}\,\min\{0.2,0.7,0.2\} = 0.2 \geq \alpha$.
$\mathrm{cdnm}\{e_4,e_5,e_9\} = \max\{0.3,0.3,0.3\} = 0.3 \leq \beta$.

Definition 3.4. *Let $G = (V,E)$ be an intuitionistic fuzzy graph with fuzzy cut λ, an edge $e \in E(G)$ is said to be intuitionistic fuzzy cut arc if cardinality of that edge greater than λ.*

Example 3.5. Consider Fig. 2

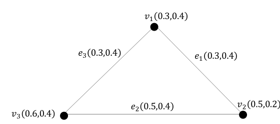

Fig. 2: Graph with fuzzy cut arc.

Here $V = \{v_i;\, i = 1,2,3\}$ and $E = \{e_j;\, j = 1,2,3\}$. Let $\lambda = 0.6$ be the intuitionistic fuzzy cut. The cardinality of three edges $\{e_1,e_2,e_3\}$ are (0.45, 0.55, 0.85) respectively. Here the edge e_3 is the fuzzy cut arc in G. Hence the cardinality of $e_3 = 0.85 > \lambda$.

Theorem 3.6. *Let C be a $C_{\alpha\beta}$-clique in G. Then for every edge $e \in E_c$, $\mu(e) \geq \alpha$ and $\gamma(e) \leq \beta$ holds.*

Proof. Let G be an IFG and C be the set of vertices in V. Since (α,β)-clique satisfies a condition that the $\min\{e \in E_c/\mu(e)\}$ should be greater than α and $\max\{e \in E_c/\gamma(e)\}$ should be smaller than β. Obviously $\mu(e) \geq \alpha$ and $\gamma(e) \leq \beta$.

Theorem 3.7. *Let v_1 and v_2 be two set of vertices in Intuitionistic fuzzy graph G, if*

1. $\mathrm{cdm}(v_1,G) \leq \mathrm{cdm}(v_2,G)$, *then* $\mathrm{cdm}(v_1 \cup v_2,G) = \mathrm{cdm}(v_2,G)$
2. $\mathrm{cdnm}(v_1,G) \geq \mathrm{cdnm}(v_2,G)$, *then* $\mathrm{cdnm}(v_1 \cup v_2,G) = \mathrm{cdnm}(v_1,G)$

Proof. (i) Consider the two set of vertices $v_1,v_2 \in G$.

Let $d_\mu(v_1)$ and $d_\mu(v_2)$ represent the minimal degree of membership in G. Let α be the fuzzy cut in G.

cdm(v_1,G) represent the minimum of each edge $e \in E_{v_1}$, such that cdm$(v_1,G) \geq \alpha$. Also cdm(v_2,G) represents the minimum values of each edge $e \in E_{v_2}$ such that cdm$(v_2,G) \leq \alpha$. Suppose $d_\mu(v_1) \leq d_\mu(v_2)$ then cdm$(v_1 \cup v_2,G)$ = cdm(v_1,G), since the value of cdm(v_1,G) is less than the value of cdm(v_2,G). That is we are considering minimal degree of membership. Hence the results holds good.

(ii) Similarly $d_\gamma(v_1)$ and $d_\gamma(v_2)$ represents the degree of non-membership in G. Here cdnm$(v_1,G) \geq \beta$ and cdnm$(v_2,G) \geq \beta$. If $d_\mu(v_1) \geq d_\mu(v_2)$, then cdnm$(v_1 \cup v_2,G)$ = cdnm(v_1,G), since the value of cdnm(v_1,G) is greater than the value of cdnm(v_2,G).

Example 3.8. Consider Fig. 3.

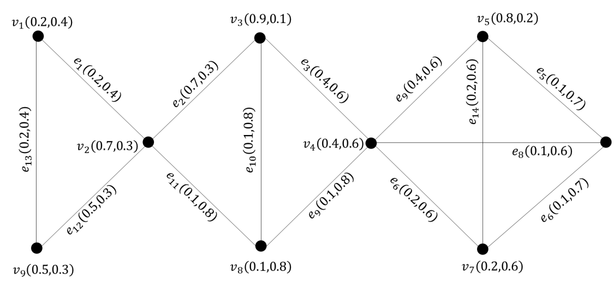

Fig. 3: Graph with clique degree of membership and non membership.

Take IFG $G = (V,E)$ with vertices $V = \{v_i; i = 1,2,\ldots,9\}$ and $E = \{e_j; j = 1,2,\ldots,14\}$.

Let $v_1 \subset V$ be the set of vertices $V_1 = \{v_5,v_6,v_7\}$. Let $\alpha = 0.1$ be the fuzzy cut in G.

1. Consider a clique degree of membership:
   Clique degree of membership
   $(V_1,G) = \min(0.1,0.2,0.1) = 0.1 \geq \alpha$.
   Let $v_2 \subset V$ be the set of vertices $V_2 = \{v_4,v_5,v_7\}$ then clique degree membership,
   $\text{cdm}(V_2,G) = \min(0.2,0.5,0.2) = 0.2 \geq \alpha$.
   Also consider $V_1 \cup V_2 \subset V$ be the set of vertices.
   $V_1 \cup V_2 = \{v_4,v_5,v_6,v_7\}$ clique degree of membership
   $\text{cdm}(\{V_1 \cup V_2\},G) = \min(0.1,0.2,0.5,0.1,0.1,0.2) = 0.1 \geq \alpha$.
   This implies $\text{cdm}\{(V_1,G)\} \leq \text{cdm}\{(V_2,G)\}$.
   Hence, $\text{cdm}(V_1 \cup V_2,G) \leq \text{cdm}(V_1,G)$.
2. Consider a clique degree of non-membership.
   Let $\beta = 0.7$ be the fuzzy cut in G.
   Clique degree of non-membership

$$\begin{aligned} \text{cdnm}(V_1,G) &= \max(0.4,0.7,0.7) = 0.7 \geq \beta, \\ \text{cdnm}(V_2,G) &= \max(0.6,0.6,0.6) = 0.6 \geq \beta. \\ \text{cdnm}(\{V_1 \cup V_2\},G) &= \max(0.6,0.6,0.6,0.6,0.7,0.4) \\ &= 0.7 \geq \alpha. \end{aligned}$$

   Here $\text{cdnm}(V_1,G) < \text{cdnm}(V_2,G)$.
   Therefore, $\text{cdnm}\{(V_1 \cup V_2,G)\} = \text{cdnm}(V_1,G)$.

Theorem 3.9. *An IFG G is maximal clique graph if it contains fuzzy cut arc λ and there exist at least one clique after mining.*

Proof. Let λ be the fuzzy cut of the edge set E in G.
Case(i): If Intuitionistic fuzzy graph is complete. Then after mining the results holds good. Hence G is a maximal clique graph.
Case(ii): Suppose G is not complete. To prove this theorem we consider the edges of $E(G)$. Let n =number of edges of $E(G)$. After mining together with fuzzy cut λ.

1. If $n = 1$ obviously it is a clique, then the theorem holds good.
2. If $n = 2$ and each vertex is connected then theorem does not holds. Since by the definition of Intuitionistic fuzzy graph, the graph does not contain self loop and parallel edges.
3. If $n = 3$ and each vertex is connected with each other. Then we get a complete graph. The theorem holds good.

Therefore, In general, maximal clique graph is obtained only by choosing fuzzy cut arc λ properly in such a way that it should not violate the condition that the graph with at least one clique.

4 Modified Fuzzy formal Analysis

The Modified Fuzzy Formal Analysis (MFFA) is fully explained in this section. It executes both membership and non-membership values. Mainly the provides the definition of it and concentrate on the structure and construction procedure of Fuzzy formal Analysis lattice.

Definition 4.1(Fuzzy formal context with membership and non-membership values).
A fuzzy formal context represented as a four tuple $M = (O,A,R,S)$ where $R = \phi(o,A)$, O is the set of objects, A is a set of attributes and R is a fuzzy set on domain $O \times A$. Each relation $(o,a) \in R$, $o \in O$, $a \in A$ has a membership value $\mu(o,a)$ in $[0,1]$ under the relation R, $S = \psi(O,A)$ is a fuzzy set on a domain $O \times A$. Each relation $(o,a) \in S$ has a membership value $\gamma(o,a) \in (0,1)$ under the relation S.

Definition 4.2. *Suppose $M = (O,A,R,S)$ is a modified fuzzy formal concept and α,β is a confidence threshold for $X \subseteq O$ and $Y \subseteq A$ are defined in the following operations:*
For membership values,
$X^* = \{a \in A \backslash \forall o \in X : \mu(o,a) \geq \alpha\}$
and $Y^* = \{o \in O \backslash \forall a \in Y : \mu(o,a) \geq \alpha\}$
For Non-membership values
$X^{**} = \{a \in A \backslash \forall o \in X : \gamma(o,a) \leq \beta\}$
and $Y^{**} = \{o \in O \backslash \forall a \in Y : \gamma(o,a) \leq \beta\}$

Definition 4.3. *A Fuzzy concept of a fuzzy formal context M with a fuzzy cut α,β in a pair $\{X_j = \phi(x),Y\}$ where $X \subseteq O, Y \subseteq O, Y \subseteq A, X^* = Y, Y^* = X$ and $X^{**} = Y, Y^{**} = X$ for both membership and non-membership values. Each* *object $O \in \phi(x)$ is defined as $\mu_0 = \min_{a\in y}\mu(o,a)$. Similarly $\gamma_0 = \min_{a\in y}\gamma(o,a)$. Particularly, if $Y = \{\ \}$, then $M_0 = 1$ for every O.*

Definition 4.4.
A collection of all $C(M)$ of a fuzzy formal context $L = (C(M), \leq)$ with the partial order $\leq$.

Definition 4.5.
We define intuitionistic fuzzy matrix in G, by assuming the fuzzy cuts α, β in such a way that the accurate result of any mining maximal clique has been obtained.
Let $\alpha_{ij} = \max_i \mu_{ij}$ and $\sigma_{ij} = \min_i \gamma_{ij}$.
Let us define

$$\gamma_{ij} = \begin{cases} \max(\alpha_i, \alpha_j), & \text{if } \mu_{ij} \neq 0 \\ 0, & \text{if } i = j \end{cases}$$

and

$$S_{ij} = \begin{cases} \min(\alpha_i, \alpha_j), & \text{if } \gamma_{ij} \neq 0 \\ 0, & \text{if } i = j. \end{cases}$$

Example 4.1. Consider Fig. 4.

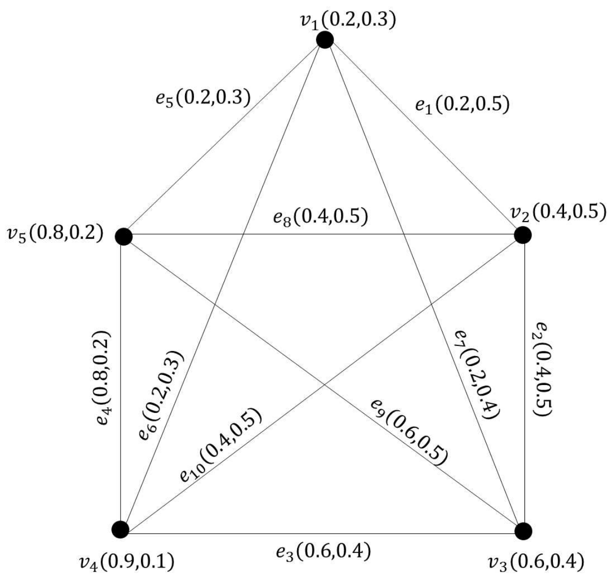

Fig. 4: Graph contains matrix values γ_{ij} and S_{ij}.

Therefore, $\gamma_{ij} = \begin{pmatrix} 0 & 0.4 & 0.5 & 0.5 \\ 0.4 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0 \end{pmatrix}$ and

$$S_{ij} = \begin{pmatrix} 0 & 0.4 & 0.3 & 0.3 \\ 0.4 & 0 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0 \end{pmatrix}.$$

Intuitionistic fuzzy formal context reconstruction. Let $\alpha = 0.5$ and $\beta = 0.3$ be the fuzzy cuts in G. Here the membership values that are less than α are sieved out from above matrix and the non-membership values that are greater than β are strained out from above matrix. Hence the refined fuzzy formal context is shown below:

$$\gamma_{ij} = \begin{pmatrix} 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0 \end{pmatrix} \text{ and } S_{ij} = \begin{pmatrix} 0 & 0 & 0.3 & 0.3 \\ 0 & 0 & 0.3 & 0.3 \\ 0.3 & 0.3 & 0 & 0.3 \\ 0.3 & 0.3 & 0.3 & 0 \end{pmatrix}.$$

We extend this concept of mining in huge level by considering $n \times n$ matrix (see Fig. 5).

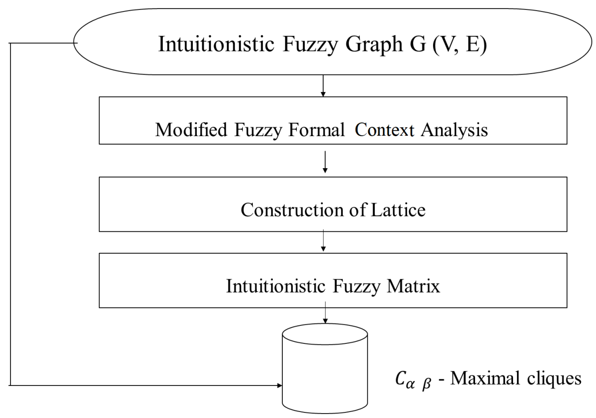

Fig. 5: Chart to get proposed solution of $C_{\alpha\beta}$-maximal cliques.

5 Algorithm

We derive an algorithm based on the above definitions and theorems. Aim of the algorithm is to mining $C_{\alpha\beta}$-maximal clique from an intuitionistic fuzzy graph.

Modified fuzzy concept analyses based on $C_{\alpha\beta}$-maximal cliques mining algorithm

Input:
IFG $G = (V, E, \mu, \gamma)$ A fuzzy cut α, β
Output:
$C_{\alpha\beta}$-maximal cliques.

1. Set $\tau = \phi$
2. Start
3. Structure a modified formal context MFFCA (G) via intuitionistic fuzzy matrix
4. Upgrade the MFFC (G) in by straining out the membership and non membership values.
5. Construct fuzzy lattice $L = (C(M), \leq)$
6. End
7. For $i = 1$ to N do
   begin
   max $= \arg\max(X_i)$
   end

8. For $i = 1$ to N do
   begin
   if $(X_i, Y_i) \in C(\text{MFFC}(G))$
   && $(X_i = Y_i)$ && $(i = \max)$
   $\tau \leftarrow (X_i, Y_i)$
   end
9. End.

6 Application

For the discovery and development of drugs between six multi-national companies and their Scientists located all over the world (see Fig. 6).

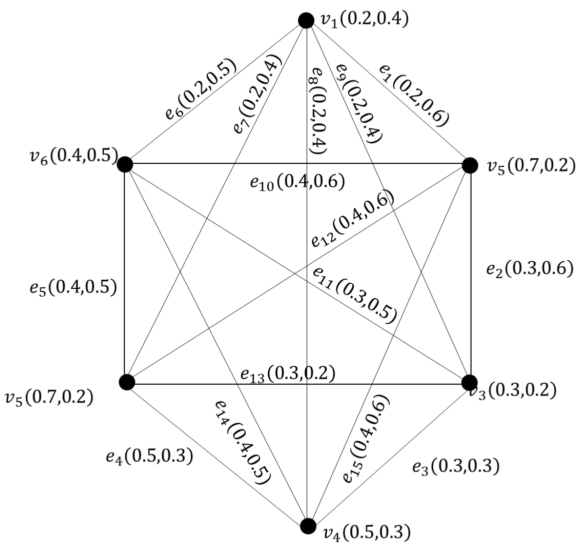

Fig. 6: Graph contains with maximal clique.

Let us consider the vertices $V = \{v_i; i = 1,2,\ldots 6\}$ be the leading pharmaceutical companies in the world and $E = \{e_j; j = 1,2,\ldots,15\}$ be the concerted strength between companies. We use maximal clique to find strong collaborative strength between the companies. Let us assume $\alpha = 0.1$ and $\beta = 0.7$ be the intuitionistic fuzzy cuts.

Let us consider the following sub graphs of G as $\{g_1, g_2, g_3, g_4, g_5\}$.
Let $g_1 = \{v_1, v_2, v_6\}$. Here
$\text{cdm}(e_1, e_6, e_{10}) = \min(0.2, 0.2, 0.4) = 0.2 > \alpha$.
$\text{cdnm}(e_1, e_6, e_{10}) = \max(0.6, 0.6, 0.5) = 0.6 < \beta$.
Total cardinality of the edges of $g_1 = 1.05$

Let $g_2 = \{v_1, v_2, v_3\}$. Here
$\text{cdm}(e_1, e_2, e_9) = \min(0.2, 0.3, 0.2) = 0.2 > \alpha$.
$\text{cdnm}(e_1, e_2, e_9) = \max(0.6, 0.6, 0.4) = 0.6 < \beta$.
Total cardinality of the edges of $g_2 = 1.05$

Let $g_3 = \{v_1, v_2, v_3, v_6\}$. Here
$\text{cdm}(e_1, e_2, e_{11}, e_6) = \min(0.2, 0.3, 0.3, 0.2) = 0.2 > \alpha$.
$\text{cdnm}(e_1, e_2, e_{11}, e_6) = \max(0.6, 0.6, 0.5, 0.5) = 0.6 < \beta$.
Total cardinality of the edges of $g_1 = 1.4$

Let $g_4 = \{v_1, v_2, v_5, v_6\}$. Here
$\text{cdm}(e_1, e_{12}, e_5, e_6) = \min(0.2, 0.4, 0.4, 0.2) = 0.2 > \alpha$.
$\text{cdnm}(e_1, e_{12}, e_5, e_6) = \max(0.6, 0.6, 0.5, 0.5) = 0.6 < \beta$.
Total cardinality of the edges of $g_1 = 1.5$

Let $g_5 = \{v_1, v_2, v_3, v_4\}$. Here
$\text{cdm}(e_1, e_2, e_3, e_8) = \min(0.2, 0.3, 0.3, 0.2) = 0.2 > \alpha$.
$\text{cdnm}(e_1, e_2, e_3, e_8) = \max(0.6, 0.6, 0.3, 0.4) = 0.6 < \beta$.
Total cardinality of the edges of $g_1 = 1.55$

Let us assume fuzzy arc $\lambda = 1.52$, since the cardinality of $g_5 > \lambda$. Therefore, the maximal clique is sub graph $g_5 = 1.55$. Hence the companies $\{v_i; i = 1,2,3,4\}$ has the resilient collaboration in making drugs.

7 Conclusion

In this paper, to examine the $C_{\alpha\beta}$-maximal cliques from an Intuitionistic Fuzzy Graph, the modified fuzzy formal context analysis algorithm is proposed. The main features of this algorithm are to analyses the theoretical study over the $C_{\alpha\beta}$-maximal cliques from computational fuzzy based approach. It is also anticipated to do these perceptions on the other extension of mining maximal cliques.

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S. Venkatesh received the Master degree in Mathematics from St. Joseph's College, Tiruchirappalli, Tamilnadu, India affiliated to Bharathidasan University in the year 1999. He also completed his Master of Computer application from TUK Arts College, Thanjavur, affiliated to Bharathidasan University. Currently he is working as Assistant Professor in Mathematics in J.J. college of Engineering Technology, Tiruchirappalli. Presently he is a research scholar in Anna university Chennai. He is a keen researcher in Optimization in Data mining techniques.

S. Sujatha is a Doctorate in computer applications from Periyar University, Salem and having 18 years of teaching and research experience in the field of Distributed computing, web services, Data mining, Cloud computing and its applications. Currently working as Associate Professor in the Department of computer application, Bharathidasan Institute of Technology, University College of Engineering, Anna University, Tiruchirappalli. She has published more than 40 National and International research paper in reputed journals. She has conducted sponsored seminars and workshop in the Department.