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Mathematical Model for Problem of Stresses in Thermo-Magneto-Piezoelectric Material

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Abstract: In this paper, an analytical solution for the stresses in a homogeneous, transversely isotropic, piezo-thermo-elastic material has investigated. The generalized theories of thermo-elasticity have used to investigate the problem. The surface subjected to thermally insulated or isothermal and electrically shorted boundary conditions. Finally, in order to illustrate the analytical development, numerical solution has carried out piezo - thermo-elastic material. The corresponding simulated results of various physical quantities such as displacements and stresses have presented graphically.

Keywords: Stresses, Transversely isotropic, Magneto- material, Piezoelectricity.

1 Introduction

The interaction between the magnetic and thermal fields plays a vital role in geophysics for understanding the effect of Earth's magnetic field on seismic waves. With the development of active material systems, there is a significant interest in the coupling effects between the elastic, magnetic and temperature for their application in sensing and actuation [1–3] Propagation of waves in piezoelectric plates has been an active research area for several decades because of the application in piezoelectric transducers, resonators, filters, actuators and other devices such as microelectromechanical systems (MEMS). A number of exact solutions of the three-dimensional dynamical equations have obtained for widely used materials such as ceramics, various crystal cuts of quartz and materials of other symmetries. Yang et al. [4] discussed the detailed studies and analysis of piezoelectric vibratory gyroscopes in the recent publications. A comprehensive review of the work on piezoelectricity and related fields has done by Yang et al. [5, 6].

Sharma and Kumar [7, 9] have studied the propagation of plane harmonic waves in piezo-thermo-elastic materials. Sharma and Othman [8] investigated the effect of rotation on generalized thermo-viscoelastic Rayleigh–Lamb waves in plates. Sharma and Pal [9] investigated the propagation of Lamb waves in a transversely isotropic, charge and stress-free piezo-thermo-elastic plate in the context of conventional coupled theory of piezo-thermo-elasticity. They studied the wave characteristics, such as phase velocity and attenuation coefficient of the waves in Cadmium Selenide (CdSe) material. Sharma and Thakur [10] studied the effect of rotation on Rayleigh–Lamb waves in magneto-thermo-elastic plates. Recently, Sharma et al. [11–14] have studied the effect of rotation on Rayleigh waves in piezo-thermo-elastic half space. ("Reflection of piezothermoelastic waves from the charge and stress free boundary of a transversely isotropic half space).

A.M.Abd-Alla and Mahmoud [15, 16] studied on problem of radial vibrations in non-homogeneity isotropic cylinder under influence of initial stress and magnetic field, influence of rotation and generalized magneto-thermo-elastic on Rayleigh waves in a granular medium under effect of initial stress and gravity field. Mahmoud [17, 18] studied analytical solution of wave propagation in non-homogeneous orthotropic rotating elastic media and studied effect of rotation and magnetic field through porous medium on Peristaltic transport of a

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Jeffrey fluid in tube. Ting, [19] studied surface waves in a rotating anisotropic elastic half space. Yang [20] studied piezoelectric vibratory gyroscopes. Zhou and Jiang [21] studied effects of Coriolis force and centrifugal force on acoustic waves propagating along the surface of a piezoelectric half space. Mahmoud [24] discussed analytical solution for free vibrations of elasto-dynamic orthotropic hollow sphere under the influence of rotation,. A.M.Abd-Alla and Mahmoud [23, 25] studied magneto-thermo-elastic problem in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model. They also studied Influence of magnetic field on free vibrations in elastodynamic problem of orthotropic hollow sphere.

In the present work, the displacement components, stresses components, electric potential, the temperature in a homogeneous, transversely isotropic, piezo-thermo-elastic material has investigated. The corresponding numerical results of various physical quantities have presented graphically.

2 Formulation of the problem

In this work, One assume a transversely isotropic homogeneous, piezo-thermo-elastic material. The material's thickness is $2d$ at uniform temperature T_0 in undisturbed state. The origin of the coordinate system (x,y,z)on the middle surface of the material used in this work. One have chosen the xy-plane so that it coincides with the middle surface and z-axis normal to it along the thickness. The x-axis chosen in the wave propagation direction to ensure that all the particles on a line parallel to y-axis are equally spaced. Accordingly, not all the field quantities depend on yz-coordinates. Material surfaces are represented by $x=\pm d$ which are governed by isothermal, electrically shorted (closed circuit), stress free and thermally insulated boundary conditions. Let $u(x,t)=(u,0,0)$ represents the displacement vector, $\phi(x,t)$ represents the electric potential, $\psi(x,t)$ represents the magnetic potential and $T(x,t)$refers to temperature change in the material. They considered this case in the non-existence of heat sources, charge density, and body forces in dimensionless form linear generalized theories of piezo-thermo-elasticity. The hexagonal crystal symmetry electric displacement along with the constitutive relations take the following form:

$$\sigma_{xx}=c_{11}\varepsilon_{xx}-\beta_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T, \tag{1}$$

$$\sigma_{yy}=c_{12}\varepsilon_{xx}-\beta_1\left(1+t_1\delta_{2k}\frac{\partial}{\partial t}\right)T, \tag{2}$$

$$D_x=\varsigma_{11}E_x+m_{11}H_x, \tag{4}$$

$$B_x=m_{11}E_x+\mu_{11}H_x, \tag{5}$$

where, $\beta_1=(c_{11}+c_{12})\,\gamma_1+c_{13}\gamma_3, \qquad \beta_3=2c_{13}\gamma_1$

Where c_{ij} are isothermal elastic tensor, σ_{ij} is stress vector, $\varepsilon_{ij}$is strain tensor, ς_{ij} is electric permittivity, e_{ij} is piezoelectric parameter, t_1 is thermal relaxation time, β_1, β_3 are the isothermal thermo-elastic parameters, γ_1, γ_3 , k_1, is the coefficient of linear thermal expansion and thermal conductivities along and perpendicular to the axis of symmetry, respectively. The relation between the electric field vector E_i and the electric potential ϕ , and Similarly, the magnetic field H_i is related to the magnetic potential ψ as: is given by:

$$E_x=-\frac{\partial}{\partial x}\phi, \qquad H_x=-\frac{\partial\psi}{\partial x}, \tag{6}$$

$$\frac{\partial}{\partial x}\sigma_{xx}=\rho\,\frac{\partial^2}{\partial t^2}u, \tag{7}$$

$$\frac{\partial}{\partial x}D_x=0, \tag{8}$$

$$\frac{\partial}{\partial x}B_x=0, \tag{9}$$

$$K_{11}\frac{\partial^2}{\partial x^2}T-\rho C_e\left(\frac{\partial}{\partial t}+t_0\frac{\partial^2}{\partial t^2}\right)T \;=\beta_1T_0\left(1+t_0\delta_{1k}\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial x\partial t}u\right). \tag{10}$$

Using Eqs. (7-10) and Eqs. (1-6), One get

$$c_{11}\frac{\partial^2}{\partial x^2}u-\beta_1\left(\frac{\partial}{\partial x}T+\delta_{2k}t_1\frac{\partial^2}{\partial x\partial t}T\right)\;=\;\rho\,\frac{\partial^2}{\partial t^2}u, \tag{11}$$

$$\varsigma_{11}\frac{\partial^2}{\partial x^2}\phi+m_{11}\frac{\partial^2}{\partial x^2}\psi\;=\;0, \tag{12}$$

$$m_{11}\frac{\partial^2}{\partial x^2}\phi+\mu_{11}\frac{\partial^2}{\partial x^2}\psi\;=\;0, \tag{13}$$

$$\frac{K_{11}}{\rho C_e}\frac{\partial^2}{\partial x^2}T-\left(\frac{\partial}{\partial t}+t_0\frac{\partial^2}{\partial t^2}\right)T\;=\;\frac{\beta_1T_0}{\rho C_e}\left(1+\delta_{1k}t_0\frac{\partial}{\partial t}\right)\frac{\partial^2}{\partial x\partial t}u \tag{14}$$

In order to simplify, One will implement the following dimensionless variables.

$$\sigma'_{ij}=\frac{\sigma_{ij}}{\beta_1T_0},\;\; \bar{\beta}=\frac{\beta_1}{c_{11}},\;\; K=\frac{K_{11}}{\rho C_e},\;\; \bar{\rho}=\frac{\rho}{c_{11}},\;\; \in=\frac{\beta_1T_0}{\rho C_e},\;\; \varepsilon=\frac{T_0\beta_1^2}{\rho C_ec_{11}},\;\; D'_i=\frac{D_i}{\beta_1T_0},$$
$$\beta_1=(c_{11}+c_{12})\,\gamma_1+c_{13}\gamma_3,\;\; \beta_3=2c_{13}\gamma_1,\;\; m_1=\frac{m_{11}}{\varsigma_{11}},\;\; \mu_1=\frac{\mu_{11}}{m_{11}}. \tag{15}$$

Eqs. (11)- (14) in the non-dimensional forms (after suppressing the primes) reduce to

$$\frac{\partial^2}{\partial x^2}u-\bar{\beta}\left(\frac{\partial}{\partial\ x}+\delta_{2k}t_1\frac{\partial^2}{\partial x\partial t}\right)T\;=\bar{\rho}\,\left(\frac{\partial^2}{\partial t^2}\right)u, \tag{16}$$

$$\frac{\partial^2}{\partial x^2}\phi+m_1\frac{\partial^2}{\partial x^2}\psi=0, \tag{17}$$

$$\frac{\partial^2}{\partial x^2}\phi+\mu_1\frac{\partial^2}{\partial x^2}\psi=0, \tag{18}$$

$$K\frac{\partial^2}{\partial x^2}T-\left(\frac{\partial}{\partial t}+t_0\frac{\partial^2}{\partial t^2}\right)T\;=\left(1+\delta_{1k}t_0\frac{\partial}{\partial t}\right)\left(\in\frac{\partial^2}{\partial x\partial t}u\right). \tag{19}$$

where $\rho$is density, C_e is specific heat at constant strain, t_0,t_1, are thermal relaxation times. The superimposed dots indicate time differentiation and comma notation has been utilized for spatial derivatives. The symbol$\delta_{jk}, j=1,2$, is the Kronecker's delta in which $k=1$corresponds to Lord and Shulman and $k=2$ refers to Green and Lindsay theory of thermo-elasticity. The aforementioned theories are abbreviated as GL and SL respectively. The prime was repressed for simplification. φ,ψ D_i and B_i, are respectively, the electric potential, magnetic potential, electrostatic displacement and magnetic induction, ε and μ_{jk} are, respectively, the dielectric and magnetic permeability coefficients, e_{kj},d_{kj} and m_{jk} are, respectively, the piezoelectric, piezo-magnetic and magneto-electric material coefficients.

3 Solution of the problem

One consider the solutions of the form:

$$u = \bar{u}(x)\, e^{i\omega t}, \tag{20}$$

$$\phi = \bar{\phi}(x)\, e^{i\omega t}, \tag{21}$$

$$\psi = \bar{\psi}(x)\, e^{i\omega t}, \tag{22}$$

$$T = \bar{T}(x)\, e^{i\omega t}. \tag{23}$$

where ω is the angular frequency, and prime was repressed for simplification. $\bar{u}(x)$, $\bar{\varphi}(x)$, $\bar{\psi}(x)$ and $\bar{T}(x)$ are the displacement component u, electric potential, magnetic potential and temperature change respectively. Using solutions (20-23) into Eqs. (16-19) to get a system of four coupled equations of components $\left(\bar{u}(x), \bar{\varphi}(x), \bar{\psi}(x), \bar{T}(x)\right)^T$,

$$(\nabla_1^2 - \rho\omega^2)\,\bar{u} - \beta_1(1 + i\omega t_1)\,\frac{d\bar{T}}{dx} = 0, \tag{24}$$

$$\nabla_1^2\bar{\phi} + m_1\nabla_1^2\bar{\psi} = 0, \tag{25}$$

$$\nabla_1^2\bar{\phi} + \mu_1\nabla_1^2\bar{\psi} = 0, \tag{26}$$

$$\varepsilon_1\,(i\omega - t_0\delta_{1k}c^2)\,\frac{d}{dx}\bar{u} - (\nabla_1^2 - (i\omega - t_0\omega^2))\,\bar{T} = 0, \tag{27}$$

The non-trivial solutions of Eq. (24)-(27) are represented by:

$$\bar{u}(x) = \frac{1}{4\lambda}\left(A_1\left(C_1e^{-A_2x} - C_2e^{A_2x}\right) + A_3(C_3e^{-A_4x} - A_3C_4e^{A_4x})\right), \tag{28}$$

$$\bar{T}(x) = C_1e^{-A_5x} + C_2e^{A_5x} + C_3e^{-A_6x} + C_4e^{A_6x}, \tag{29}$$

$$\bar{\phi}(x) = C_5x + C_6, \tag{30}$$

$$\bar{\psi}(x) = C_7x + C_8. \tag{31}$$

And substituting Eqs. (28-31) into Eqs (20-23), One obtain

$$u(x,t) = \frac{1}{4\lambda}\left(A_1\left(C_1e^{-A_2x} - C_2e^{A_2x}\right) + A_3(C_3e^{-A_4x} - C_4e^{A_4x})\right)e^{i\omega t}, \tag{32}$$

$$T(x,t) = \left(C_1e^{-A_5x} + C_2e^{A_5x} + C_3e^{-A_6x} + C_4e^{A_6x}\right)e^{i\omega t}, \tag{33}$$

$$\phi(x,t) = (C_5x + C_6)\,e^{i\omega t}, \tag{34}$$

$$\psi(x,t) = (C_7x + C_8)\,e^{i\omega t}. \tag{35}$$

And substituting Eqs. (32-35) along Eqs. (1-5), One obtain the stresses

$$\sigma_{xx} = \frac{-c_{11}}{4\lambda}\left(A_1A_2\left(C_1e^{-A_2x} + C_2e^{A_2x}\right) + A_3A_4(C_3e^{-A_4x} + C_4e^{A_4x})\right)e^{i\omega t} - (\beta_1 + i\omega t_1)\left(C_1e^{-A_5x} + C_2e^{A_5x} + C_3e^{-A_6x} + C_4e^{A_6x}\right)e^{i\omega t}, \tag{36}$$

$$\sigma_{yy} = \frac{-c_{12}}{4\lambda}\left(A_1A_2\left(C_1e^{-A_2x} + C_2e^{A_2x}\right) + A_3A_4(C_3e^{-A_4x} + C_4e^{A_4x})\right)e^{i\omega t} - (\beta_1 + i\omega t_1)\left(C_1e^{-A_5x} + C_2e^{A_5x} + C_3e^{-A_6x} + C_4e^{A_6x}\right)e^{i\omega t}, \tag{37}$$

$$\sigma_{zz} = \frac{-c_{13}}{4\lambda}\left(A_1A_2\left(C_1e^{-A_2x} + C_2e^{A_2x}\right) + A_3A_4(C_3e^{-A_4x} + C_4e^{A_4x})\right)e^{i\omega t} - (\beta_3 + i\omega t_1)\left(C_1e^{-A_5x} + C_2e^{A_5x} + C_3e^{-A_6x} + C_4e^{A_6x}\right)e^{i\omega t}, \tag{38}$$

And electrical displacement and magnetic induction as:

$$D_x = -(\varsigma_{11}C_5 + m_{11}C_7)\,e^{i\omega t}, \tag{39}$$

$$B_x = -(m_{11}C_5 + \mu_{11}C_7)\,e^{i\omega t}. \tag{40}$$

Where E_i, D_i H_i, B_i are the electric field, electrical displacement, magnetic field and magnetic induction respectively, A_i, $i = 1,2,3,4.$ and are given as following:

$$A_1 = B_2\sqrt{-2B_1}, \qquad A_2 = \frac{1}{\sqrt{2}}\frac{\sqrt{-B_1}}{c_{11}},$$
$$A_3 = B_4\sqrt{-B_3}, \qquad A_4 = \frac{1}{\sqrt{2}}\frac{\sqrt{-B_3}}{c_{11}}.$$

$$B_1 = (\sqrt{\alpha_1} + \alpha_1)\,c_{11}, \qquad B_2 = -\sqrt{\alpha_1} + \alpha_3,$$
$$B_3 = (-\sqrt{\alpha_1} + \alpha_1)\,c_{11}, \qquad B_4 = \sqrt{\alpha_1} + \alpha_3,$$

$$\lambda = \frac{c\,\varepsilon\rho\,(i + t_0\,c\,\delta_{1k})\,(c^2)\,c_{11}}{\sqrt{2}},$$

$$\begin{aligned}\alpha_1 = {} & -\beta_1t_1^2c^6\varepsilon^2t_0^2\,\delta_{1k}^2 + 2it_0\beta_1\delta_{1k}\varepsilon t_1(\beta_1(t_1 + t_0\delta_{1k})\,\varepsilon \\ & + t_0c_{11}\delta_{2k} + \rho)c^5 + (\beta_1^2(t_1^2 + \delta_{1k}^2t_0^2 + 4t_0\delta_{1k}t_1)\varepsilon^2 \\ & + 2(t_0^2\delta_{1k}\delta_{2k}c_{11} + (t_1(\delta_{1k} + \delta_{2k})c_{11} + \rho\delta_{1k})t_0 + t_1\rho)\beta_1\varepsilon \\ & + (t_0c_{11}\delta_{2k} - \rho)^2c^4 + (-2i\beta_1^2(t_1 + t_0\delta_{1k})\varepsilon^2 \\ & + 2i\beta_1(((-\delta_{2k} - \delta_{1k})c_{11})t_0 - \rho - t_1c_{11})\varepsilon \\ & - 2i(t_0c_{11}\delta_{2k} - \rho)c_{11})c^3 + (-\beta_1^2\varepsilon^2 - 2\beta_1(\\ & c_{11})\varepsilon - c_{11}^2)c^2\end{aligned}$$

$$\begin{aligned}\alpha_2 = {} & i\beta_1t_1c^3\varepsilon t_0\delta_{1k} + (\beta_1(t_1 + t_0\delta_{1k})\varepsilon + t_0c_{11}\delta_{2k} + \rho)c^2 \\ & - i(\beta_1\varepsilon + c_{11})c,\end{aligned}$$

$$\begin{aligned}\alpha_3 = {} & i\beta_1t_1c^3\varepsilon t_0\delta_{1k} + (\beta_1(t_1 + t_0\delta_{1k})\varepsilon + t_0c_{11}\delta_{2k} - \rho)c^2 \\ & - i(\beta_1\varepsilon + c_{11})c,\end{aligned}$$

4 Boundary conditions

The surfaces of the material proposed to be electrically shorted and thermally insulated/isothermal. Consequently, the following boundary conditions have to be satisfied at the surface of material$x = \pm d$.

(a) Mechanical condition

$$\sigma_{xx} = p_0e^{i\omega t}, \tag{41}$$

where $p_0e^{i\omega t}$ is the periodic load.

(b) Electrical conditions

$$\phi = 0, \tag{42}$$

(b) Magnetic conditions

$$\psi = 0, \tag{43}$$

(c) Thermal boundary conditions: Isothermal surfaces

$$T = 0, \tag{44}$$

5 Numerical results and discussion

In the light of explaining the analytical results, which One have got in the previous sections, One are currently showing set of numerical results for the following, two Stress free, thermally insulated piezo-thermo-elastic

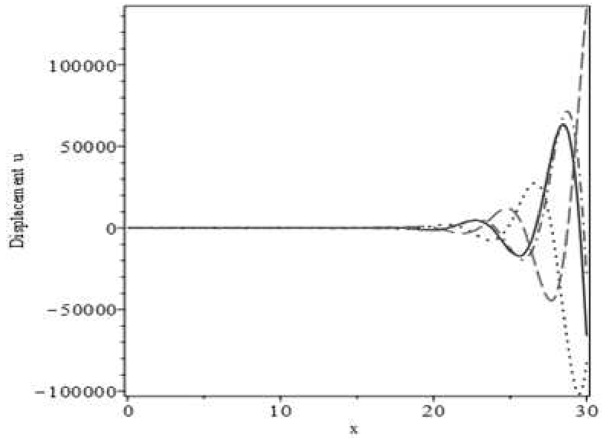

Fig. 1: Dispersion curves for displacement u, versus x with different values for time$(U2015t = 2, \ldots\ldots t = 4, --t = 6, -t = 8)$

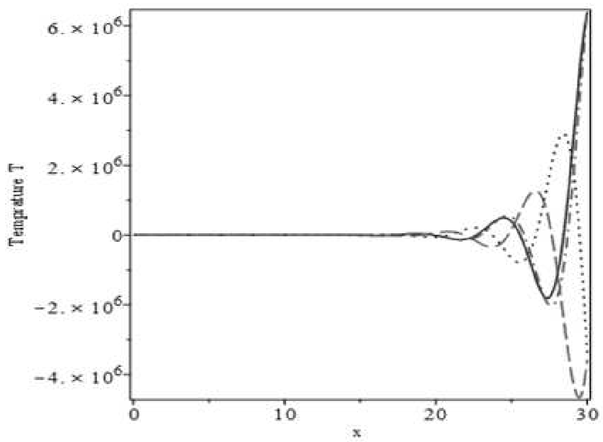

Fig. 2: Dispersion curves for temperature T, versus x with different values for time $(U2015t = 2, \ldots\ldots t = 4, --t = 6, -t = 8)$

material, electrically shorted. give the physical data for material

$c_{11} = 2.11, \quad c_{12} = 0.94, \quad c_{13} = 1.02, \quad m_{11} = 0.0074, \quad \varsigma_{11} = 10, \quad \mu_{11} = -344.66, \quad e_{15} = 106.6, \quad C_e = 260,$

$k_{11} = 1.5, \quad t_1 = 1, \quad \omega = 9 \times 10^{-1}, \quad \rho = 1.858, \quad \delta_{2k} = 1, \quad \delta_{1k} = 1, \quad C_v = 420, \quad T_0 = 298, \quad \beta_1 = 1.52.$

A dimensionless thermal relaxation time $t_0 = 0.5$ and t_1 is selected as multiple of t_0. The thermomechanical coupling factor, specific loss factor and relative frequency shifts numerically analyzed. The computed results in respected dispersion curves, Figure 1 shows the comparison between the displacement component u. The

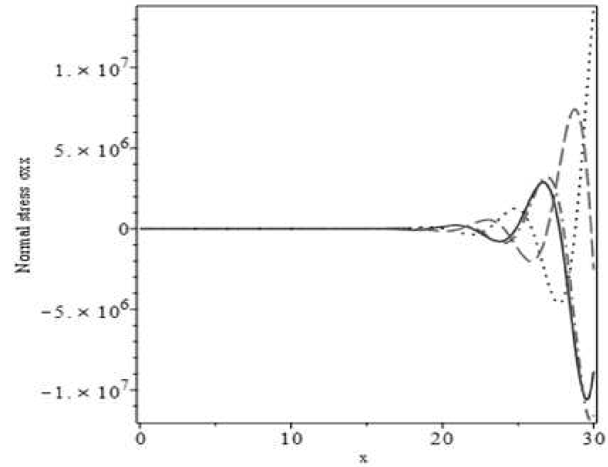

Fig. 3: Dispersion curves for normal stress $U3c3_{xx}$, versus x with different values for time $(U2015t = 2, \ldots\ldots t = 4, --t = 6, -t = 8)$

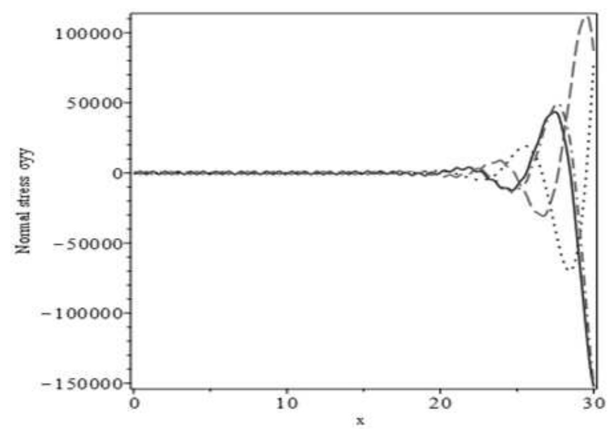

Fig. 4: Fig. 4 Dispersion curves for normal stress $U3c3yy$,versus x with different values for time $(U2015t = 2, \ldots\ldots t = 4, --t = 6, -t = 8)$

computations are carried out for the time $t = 2,\ 4,\ 6, 8$, on the surface plane. Figures 2 shows the comparison between the temperature$T$, the computations are carried out for the time $t = 2,\ 4, 6, 8$, on the surface plane. Figures 3-5, shows the comparison between the normal stresses componentsσ_{xx}, σ_{yy} andσ_{zz}, the computations carried out for the time $t = 2,\ 4, 6, 8$. Figures 6-7 shows the comparison between the electric displacement D_x and magnetic inductionB_x, the computations are carried out for the time $t = 2, 4, 6, 8$, on the surface medium. According to the above numerical results, one can observe that All the physical quantities agree with the boundary conditions. The significant effect of thermal

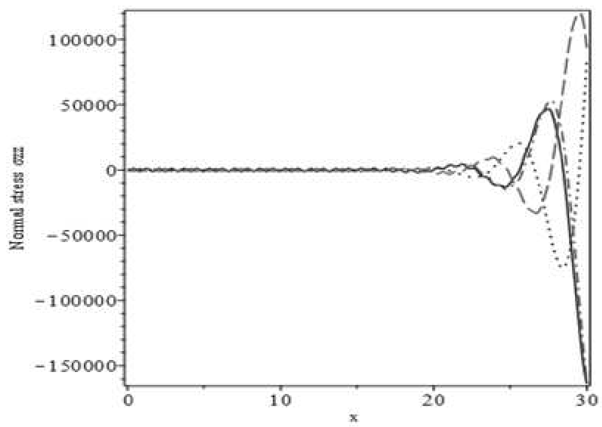

Fig. 5: Dispersion curves for normal stress $U3c3_{zz}$,versus x with different values for time ($U2015t = 2, \ldots\ldots t = 4, - - t = 6, -t = 8$)

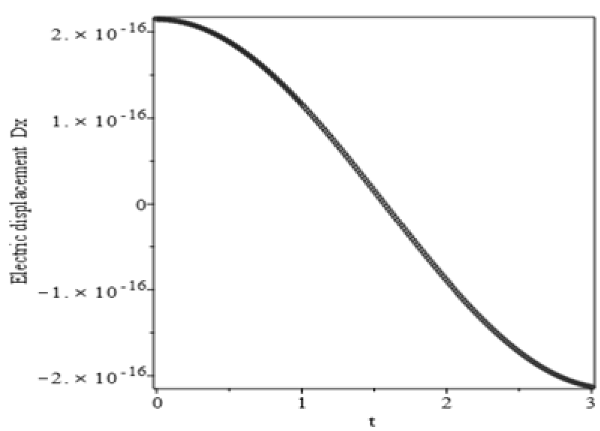

Fig. 6: Dispersion curves for electrical displacement D_x, versus x with different values for time ($U2015t = 2, \ldots\ldots t = 4, - - t = 6, -t = 8$)

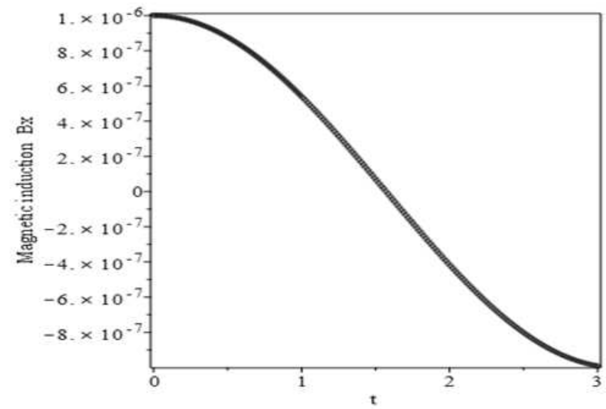

Fig. 7: Dispersion curves for magnetic induction B_x, versus x with different values for time ($U2015t = 2, \ldots\ldots t = 4, - - t = 6, -t = 8$)

time relaxation has observed in all the various physical quantities of the material, since all the profiles of considered functions are quite distinguishable.

Finally, one can observe that, the analytical solutions based upon normal mode analysis for piezoelectric thermo-elastic medium have been developed and utilized and all the functions are continuous.

6 Conclusion

The mathematical model of the mechanical stresses in a homogeneous, transversely isotropic, piezo-thermo-elastic material have investigated. The generalized theories of thermo-elasticity have used to investigate the problem. It was then subjecting the conditions, electrical and thermally insulated thermally. Results in the forms of graphs and so each variables such stresses and displacements displayed.

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