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A Novel Algorithm based on Entanglement Measurement for Improving Speed of Quantum Algorithms

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Abstract: In this paper, we draw attention to consider that the quantum entanglement measurement should be implemented as a key part during manufacturing the quantum processors and quantum micro-controllers. This paper aims to harness the complete power of quantum mechanics by the crossover between entanglement measurements and quantum gates to propose a novel quantum computer algorithms and protocols. One of these measurements that we apply is concurrence, used to measure entanglement in a two-qubit system to solve the problem under scrutiny in n-dimensional vector space. The general algorithm to reshape many of the existing quantum algorithms, and to propose a novel quantum algorithms and protocols based on entanglement measurement is proposed in this paper.

Keywords: Entanglement, concurrence, quantum algorithms, quantum processor manufacturing

1 Introduction

Quantum computations are done using quantum algorithms. They are faster comparing with the classical algorithms, e.g. and Grover algorithm [1,2], Shor algorithm [3,4], Deutsch-Jozsa algorithm [5,6] and etc[7, 8]. This high speed owing back to astonishing phenomena in quantum mechanics which are superposition and entanglement. Quantum entanglement is a unique microscopic physical phenomenon that occurs when two qubits or more are interact in ways such that the quantum state of each qubit cannot be described independently of the others, even when the qubits are separated by a vast distance. This phenomenon is called ”spooky action at a distance” by A. Einstein, B. Podolsky and N. Rosen[9]. Entanglement is a pivotal issue in quantum information and quantum computation theory and it is under continuous research [10,11,12,13,14,15,16,17]. This amazing phenomenon was controversial between A. Einstein and N. Bohr what was known as the EPR paradox. Finally quantum mechanics verified itself experimentally when set of experiments of quantum entanglement were done successfully [18,19,20]. Entanglement is an area of extremely hot research by the communities of atomic physics and quantum information processing [21], with crucial utilization in many applications, for instance quantum teleportation [22,23], satellite-based quantum key distribution [24,25], and quantum Internet [26].

In this paper, we propose a novel generalized algorithm to crossover between the quantum evolutions using quantum gates and entanglement measurement to harness the complete power of quantum parallelism and entanglement to enhance the efficiency and the speed of quantum computations.

2 Two Entangled Qubits and Entanglement Measurements

2.1 Two Entangled Qubits

The maximally entangled states of two qubits is called Bell states $\frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ or $\frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ [21,27].

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Bell states can be generated by quantum circuit depicted in Fig. 1, and used in many applications such as an unknown qubit teleportation and quantum key distribution [29,30].

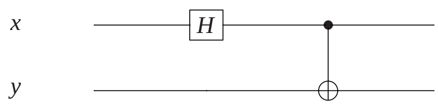

Fig. 1: Quantum circuit which produces the Bell states [21].

The quantum circuit shown in Fig. 2 produces two-qubit entangled normalized system $(\alpha|01\rangle \pm \beta|10\rangle)$ if the control qubit $|x\rangle$ is in a normalized superposition state and produces separable qubits if it is state in $|0\rangle$ or $|1\rangle$.

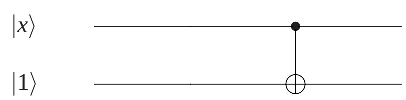

Fig. 2: A quantum circuit which produces entangled states if the control qubit $|x\rangle$ is in a normalized superposition state and produces separable states if the control qubit $|x\rangle$ in a deterministic state $|0\rangle$ or $|1\rangle$.

2.2 Two qubits Entanglement Measurement

Entanglement measurements [28,31] are used to reveal if there is an entanglement into quantum systems which are governed by n-dimension Hilbert space such that $n > 1$. There are plenty of entanglement measurements defined based on different considerations such as concurrence, negativity, quantum discord, witness and so on [33,32]. Concurrence measurement [31,32] is considered one of the most popular measurements of entanglement quantification of bipartite system, and can be defined as follows:

$$C = |\langle\phi|\sigma_y \otimes \sigma_y|\phi^\dagger\rangle|,$$

where $\sigma_y = -i|1\rangle\langle 0| + i|0\rangle\langle 1|$, and $i = \sqrt{-1}$.

Also, the concurrence of the states $\frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ is calculated theoretically as follows [34]:

$$C = 2|\alpha\beta|, \tag{1}$$

where $0 \leq C \leq 1$.

2.3 The Proposed Operator

Definition 3.1. Consider two arbitrary indexed qubits i and j in k-qubit quantum register, such that the $i-indexed$ qubit is called the test qubit and $j-indexed$ qubit is called the detection qubit.
Definition 3.2. A measuring device of entanglement $D_{i,j}$ measures the concurrence between the test and detection qubits.
Definition 3.3. For arbitrary two-qubit system, an operator M_z is the operator which is applying $CNOT$ gate on the test qubit and the detection qubit, then the device $D_{i,j}$ measures the entanglement in between.

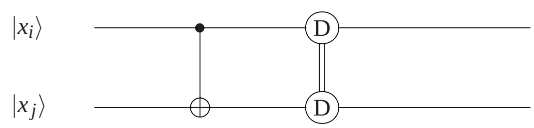

Fig. 3: Quantum circuit for the proposed M_z operator.

The circuit of the proposed operator M_z is depicted in Fig. 3. The aim of the proposed operator M_z is to check whether a test qubit is in a superposition state or not. In other words, the operator M_z, firstly, applies $CNOT$ gate on the test qubit as the control qubit and the detection qubit as a target qubit, and then measures the entanglement in between. It is worth noting that the entanglement will happen if and only if the test qubit is in a superposition. For further elaboration, let's examine a system $|\kappa\rangle$ of two-qubits one of which is in a superposition. The system can be described as follows:

$$|\kappa\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |1\rangle, \tag{2}$$

where α and β are complex numbers called the probability amplitudes and relation $|\alpha|^2 + |\beta|^2 = 1$.

So, when CNOT is applied on the system described in Eq. (2), the effect can be illustrated as follows:

$$|\kappa^*\rangle = CNOT|\kappa\rangle = \alpha|01\rangle + \beta|10\rangle, \tag{3}$$

where the second qubit is entangled with the first qubit. So, for the given bipartite quantum state $|\kappa^*\rangle$, the amount of entanglement is determined according to the concurrence relation given by Eq.(1). Some suggestions to implement the device $D_{i,j}$ are depicted in [34,35,36].

3 The proposed Algorithm

In this section, we propose a quantum algorithm that uses concurrence measurement as an essential step in quantum algorithms and protocols. In other words, the proposed algorithm shows how to use the proposed operator M_z to create an entanglement between two separable qubits, and then use concurrence measurement to solve the problem in hand.

Suppose a $n-qubit$ quantum system $|\phi\rangle$ has $N \leq 2^n$ eigen-states is given or is initialized to have the complete superposition in the first step for the algorithm in hand. If there is a specific function f, as a black box, required to be tested if it is satisfied in $|\phi\rangle$ or not. Suppose that there is a given oracle U_f of size $2^{2n+1}x2^{2n+1}$, as a black box, can test this function on the quantum system $|\phi\rangle$. The abstract problem is:

Quantum system of N states: is given or initialized in the quantum algorithm or protocol.
Given: A function f:[N] $\rightarrow \{0,1\}$.
Given: The oracle U_f (any existing or a novel algorithm or protocol).
Goal: Use U_f, M_z and the Concurrence equation, Eq. (1) or Eq. (5), to solve the problem being studied.

In order to achieve this goal, extra two qubits are added to the whole system. The first qubit is defined as the test qubit and the other is defined as the detection qubit. Such that, if the system $|\phi\rangle$ satisfies the function f then the test qubit is translated to the normalized state $\alpha|0\rangle+\beta|1\rangle$ and the entanglement is revealed between the test and the detection qubit when M_z is applied, and the concurrence value is $0 < C \leq 1$ depending on the number of states in the system $|\phi\rangle$ are satisfying the given function f. But if there are no states in the system $|\phi\rangle$ satisfied, then the state of the test qubit is unchanged and the entanglement is missed between the test and detection qubits, $C=0$, when the operator M_z is applied between the test qubit and the detection qubit. The algorithm is proposed in the following steps:

1. Register Preparation: Concatenate the given, or initialize, quantum system $|\phi\rangle$ of n qubits with the extra two qubits, the test qubit and the detection qubit, which are initialized in the state $|1\rangle$ as

$$|\psi_0\rangle = |\phi\rangle \otimes |11\rangle.$$

2. Apply the oracle U_f on $|\phi\rangle$ and the test qubit: Applying the oracle U_f will mark the solutions satisfy the given condition f.

$$|\psi_1\rangle = U_f^{\otimes n+1}|\phi,1\rangle \otimes I|1\rangle,$$

where I is 2x2 Identity operator.

3. Apply the Operator M_z on the test qubit and the detection qubit. The effect of M_z on $|\psi_1\rangle$ can be viewed as:

(i) Apply the CNOT gate between the test qubit and the detection qubit.

$$|\psi_2\rangle = C^{not}_{\psi_{n+1}\psi_{n+2}}|\psi_1\rangle$$

(ii) Measure the entanglement between the test qubit and the detection qubit. If there is entanglement measured, $0 < C \leq 1$, then the test qubit ψ_{n+1} is considered in a superposition state and the function f is satisfied, otherwise the test qubit ψ_{n+1} is not in a superposition state and the function f is not satisfied.

4. Find the solution of the problem under scrutiny using Eq. (1), (4) and/or (5) according to the problem in hand.

4 Analysis of the Proposed Algorithm

In this section, we discuss the proposed algorithm with the suggested operator introduced in section 2.3. We analyze the proposed algorithm assuming that the given oracle is a black box U_f, which is trying to test if a given function f is satisfied on a given quantum system $|\phi\rangle$ has $N \leq 2^n$. Suppose that the number of states in $|\phi\rangle$ which satisfy the condition f are m_0 and the number of those which are not m_1, then

$$N = m_0 + m_1 \tag{4}$$

Then according to 2^{nd} step of the proposed algorithm, after applying the oracle U_f, the state of the test qubit can be described as follows:

$$|\psi_1^{n+1}\rangle = \alpha|0\rangle + \beta|1\rangle,$$

where $m_0 = N|\alpha|^2$, $m_1 = N|\beta|^2$, iff there are $m_0 > 0$ states in the quantum system $|\phi\rangle$ satisfy the function f. But on the other hand, the state of the test qubit is:

$$|\psi_1^{n+1}\rangle = |1\rangle,$$

iff there are no states, $m_0 = 0$, satisfy the function f.

The 3^{rd} step of the proposed algorithm pertains applying M_z, there are a two nested sub-steps are executed:
(i) If there is a $m_0 > 0$ states in the quantum system $|\phi\rangle$ satisfy the function f, the CNOT gate is applied between the test qubit $|\psi_1^{n+1}\rangle$ and the detection qubit $|\psi_1^{n+2}\rangle$, there state will be:

$$\begin{aligned} |\psi_2^{n+1,n+2}\rangle &= CNOT(\alpha_{n+1}|01\rangle + \beta_{n+1}|11\rangle) \\ &= \alpha_{n+1}|0,1\oplus 0\rangle \\ &+ \beta_{n+1}|1,1\oplus 1\rangle \\ &= \alpha_{n+1}|01\rangle + \beta_{n+1}|10\rangle. \end{aligned}$$

This will produce a measurable entanglement between the test and the detection qubits in the sub-step (ii), and the concurrence between those qubits according to Eq.(1) and Eq.(4) is:

$$C = 2\frac{\sqrt{m_0(N-m_0)}}{N}. \tag{5}$$

Or:
(i) If there is no states, $m_0 = 0$, in the quantum system $|\phi\rangle$

satisfy the function f, the effect of the CNOT gate when applied on the test qubit and the detection qubit is as follows:

$$\begin{aligned}|\psi_2^{n+1,n+2}\rangle &= CNOT(|11\rangle) \\ &= |1,1\oplus 1\rangle = |10\rangle.\end{aligned}$$

This will not produce a measurable entanglement between the test and detection qubits in the sub-step (ii), and the concurrence between those qubits is $C = 0$ because the state $|10\rangle$ is a separable state.

5 Perspective

In this paper, we have tried to promote a future vision for the establishment of the novel fastest quantum algorithms. The proposed algorithm in this paper will give the potential to reshape the existing quantum algorithms and to propose novel quantum algorithms based on entanglement measurement. The proposed algorithm makes the computations of quantum algorithms fastest than its competitors which are only using unitary evolutions. This algorithm can be used to solve applications such as testing junta variables and learning Boolean functions and plenty of other algorithms in quantum computation and machine learning.

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