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Problem of Longitudinal and Secondary Vertically Waves Reflection and Transmission during Two Media in the Context of Three Magneto-thermoelastic Theories with Varies Fields

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Abstract: The aim of this paper is study the problem of longitudinal wave (p-wave) and secondary vertically (SV-wave) reflection and transmission during two media in the context of three magneto-thermoelastic theories with varies fields effects as magnetic field and initial stress. We deal the problem of reflection and transmission of thermoelastic waves at interface between two different media in the presence of initial stress and magnetic field, in the context of CT (Classical theory), GL (Green-Lindsay) and DPL (Dual Phase Lag). The boundary conditions applied at the interface as continuity of the displacement, neglecting the tangential displacement, continuity of normal force per unit initial area, neglecting the tangential stress and continuity of the temperature. The amplitudes ratios for the incident p- and SV- waves and so the refracted are obtained. The reflection and transmission coefficients ratios for the incident waves are computed numerically and the results are represented graphically to show the physical meaning of the phenomena.

Keywords: Longitudinal (p-), secondary vertically (SV-), reflection, transmission, magnetic, thermoelasticity

1 Introduction

During the recent four decades, more attentions have been made on the generalized thermoelasticity theory because of its utilitarian aspects in diverse fields, especially in engineering, physics, structure mechanics, biology, geology, geophysics, acoustics, plasma physics and etc. The generalized thermoelasticity theories were developed to eliminate the paradox inherent in the classical theories predicting infinite speed of propagation of heat. The generalized thermoelasticity theories admit the so-called second-sound effects, predicting only finite velocity of propagation of heat. The two theories (LS and GL) ensure finite speeds of propagation for the heat wave. The theory of elasticity with non uniform heat which was in half-space subjected of thermal shock in this context which is known as the theory of uncoupled thermoelasticity and the temperature is governed by a parabolic partial differential equation in temperature term only has been discussed by Danilovskaya [1]. Widespread attention has been given to thermoelasticity theories which consider finite speed for the propagation of thermal signal. Initial stresses develop in the medium due to various reasons, such as the difference of temperature, process of quenching shot pinning and cold working, slow process of creep, differential external forces, and gravity variations.

The earth is under high initial stress and, therefore, it is of great interest to study the effect of these stresses on the propagation of elastic waves. A lot of systematic studies have been carried out on the propagation of elastic waves. Biot [2] showed that the acoustic propagation under initial stresses would be fundamentally different from that under stress free state. Lord and Shulman [3] reported a new theory based on a modified Fourier's law of heat conduction with one relaxation time. Later on, a more rigorous theory of thermoelasticity was formulated by Green and Lindsay [4] introducing two relaxation

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times. These non-classical theories are often regarded as the generalized dynamic theories of thermoelasticity. Various problems have been investigated and discussed in the light of these two theories and the studies reveal some interesting phenomena. Green and Naghdi [5,6] re-examined the basic postulates of thermo-mechanics and discussed undamped heat waves in an elastic solid. Green and Naghdi [7], Chandrasekharaiah [8] discussed different problems in thermoelasticy without energy dissipation. The representative theories in the frame of generalized thermoelasticity are presented by Hetnarski and Ignaczak [9]. Singh [10] investigated reflection and transmission of plane harmonic waves at the interface between liquid and micropolar viscoelastic solid with stretch. Kumar and Sarathi [11] studied reflection and refraction of thermoelastic plane waves at the interface between two thermoelastic media without energy dissipation. Othman and Song [12] discussed plane waves reflection from an elastic solid half-space under hydrostatic initial stress without energy dissipation. Problems on wave propagation phenomena in coupled or generalized thermoelasticity were discussed by Sinha and Elsibai [13] and Abd-alla and Al-Dawy [14]. Abd-alla et al. [15] investigated the reflection of generalized magneto-thermo-viscoelastic waves at the boundary of a semi-infinite solid adjacent to vacuum. Sinha and Elsibai [16] investigated the reflection and refraction of thermoelastic waves at the interface of two semi-infinite media with two relaxation times. Abd-Alla and Abo-Dahab [17] discussed the influence of the viscosity on the reflection and transmission of plane shear elastic waves at the interface of two magnetized semi-infinite media. The generalized magneto-thermoelasticity model with two relaxation times in an isotropic elastic medium under the effect of reference temperature on the modulus of elasticity investigated by Othman and Song [18].

Estimation of the magnetic field effect in an elastic solid half-space under thermoelastic diffusion is discussed by Abo-Dahab and Singh [19]. The impact of magnetic field, initial pressure, and hydrostatic initial stress on the reflection of P and SV waves considering a Green Lindsay theory discussed by Abo-Dahab and Mohamed [20]. Abo-Dahab et al. [21] studied the rotation and magnetic field effects on P wave reflection from a stress-free surface of an elastic half-space with voids under one thermal relaxation time. Reflection of P and SV waves from stress-free surface of an elastic half-space under the influence of magnetic field and hydrostatic initial stress without energy dissipation is investigated by Abo-Dahab [22]. Abo-Dahab et al. [23] studied relaxation times and magnetic field effects on the reflection of thermoelastic waves from isothermal and insulated boundaries of a half-space. Abo-Dahab and Asad [24] estimated Maxwell's stresses effect on the reflection and transmission of plane waves between two thermo-elastic media in the context of GN model. Deswal et al. [25] studied the reflection and refraction at an interface between two dissimilar, thermally conducting viscous liquid half-spaces. Chakraborty and Singh [26] studied the problem of reflection and refraction of thermo-elastic wave under normal initial stress at a solid-solid interface under perfect boundary condition. Abd-Alla et al. [27] studied the radial deformation and the corresponding stresses in a homogeneous annular fin of an isotropic material. Abo-Dahab and Singh [28] investigated the effects of rotation and voids on the reflection of P waves from stress-free surface of an elastic half-space under magnetic field, initial stress and without energy dissipation. Reflection and refraction of P-, SV- and thermal waves, at an initially stressed solid-liquid interface in generalized thermoelasticity has been discussed by Singh and Chakraborty [29]. Abd-Alla et al. [30] investigated SV-waves incidence at interface between solid-liquid media under electromagnetic field and initial stress in the context of three thermoelastic theories. Abo-Dahab and Salama [31] discussed plane thermoelastic waves reflection and transmission between two solid media under perfect boundary conditions and initial stress without and with influence of a magnetic field. Abd-Alla et al. [32] investigated the effect of rotation on the peristaltic flow of a micropolar fluid through a porous medium in the presence of an external magnetic field. Abd-Alla et al. [33] discussed the effects of rotation and initial stress on the peristaltic transport of a fourth grade fluid with heat transfer and induced magnetic field. Song, et al. [36] investigated the reflection and refraction of micropolar magneto-thermoviscoelastic waves at the interface between two micropolar viscoelastic media. Othman and Abbas [37] discussed a solution to a thermal-shock problem of generalized thermoelasticity for a non-homogeneous isotropic hollow cylinder.

Recently, Abo-Dahab et al. ([38]-[40]) investigated the influence of varies external effects on waves propagation between two solid-liquid media in the context of thermoelastic theories. Said and Othman [41] investigated effect of mechanical force, rotation and moving internal heat source on a two-temperature fiber-reinforced thermoelastic medium with two theories.

The aim of this paper is study the problem of longitudinal and secondary vertically waves reflection and transmission during two media in the context of three magneto-thermoelastic theories with varies fields effects as magnetic field and initial stress. We deal with the problem of reflection and transmission of thermoelastic waves at interface between two different media in the presence of initial stress and magnetic field, in the context of CT (Classical theory), GL (Green-Lindsay) and DPL (Dual Phase Lag) models. The boundary conditions is applied at the interface as continuity of the displacement, vanishing of the tangential displacement, continuity of normal force per unit initial area, vanishing of the tangential stress and continuity of temperature. The amplitudes ratios for the incident p- and SV- waves and so the refracted are obtained. The reflection and transmission coefficients for the incident waves are computed

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numerically and the results are represented graphically to show the physical meaning of the phenomena.

2 Formulation of the problem

Let us consider a plane interface between solid half-space homogeneous anisotropic elastic and liquid half-space are with a primary temperature and electromagnetic field acts on Z- direction. The solid medium placed under initial stress. A plane SV-waves is incident in medium M on the plane interface which is reflected to thermal waves (dilatational waves), p-waves (dilatational waves), and SV-waves (rotational waves) and the rest of the waves continues to travel in the other medium M' after refraction, as T- waves and p-waves, as shown in Figure 1.

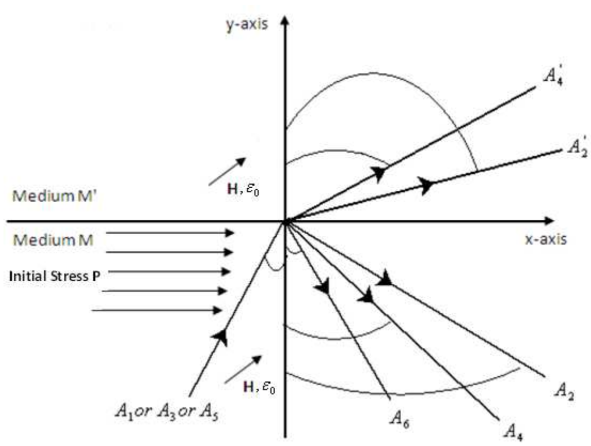

Fig. 1: Geometry of the problem.

Where, θ is the angle of incidence for a plane waves, θ_1 and θ_2 are the angle of reflected waves, θ_1' and θ_2' are the angle of transmitting waves, H and ε_0 is the electromagnetic field vector acting on Z-direction, gravity in z-axis direction, A_1, A_3 and A_5 are the amplitudes of the incident waves, A_2, A_4 and A_6 are the amplitudes of reflected waves, A_2' and A_4' are the amplitudes of the transmitted T- and SV-waves, respectively. We assume a Cartesian coordinate system with origin 'O' on the plane $y=0$. Since we consider a two-dimensional problem, we restrict our analysis to plane strain parallel to the oxy plane. Hence, all the field variables depend only on x , y and time t.

The initial stress effects on medium M only as shown in Figure 2, and $S_{11}$and S_{22} are the normal stresses in and directions, respectively.

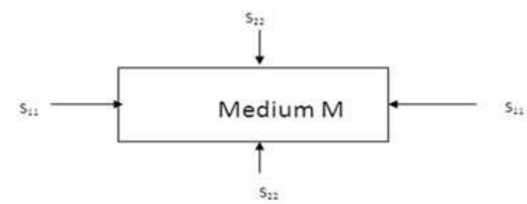

Fig. 2: Components of initial stress in the solid medium.

3 Basic equations

1) The dynamical equations of motion in rotating frame of reference for a plane strain under initial stress in the absence of a heat source and presence of Lorentz's force, given by Biot (1956) is

$$\frac{\partial S_{11}}{\partial x}+\frac{\partial S_{21}}{\partial y}-P\frac{\partial \bar{\omega}}{\partial x}-\rho g\frac{\partial v}{\partial x}+F_x=\rho\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\frac{\partial S_{21}}{\partial x}+\frac{\partial S_{22}}{\partial y}-P\frac{\partial \bar{\omega}}{\partial x}+\rho g\frac{\partial u}{\partial x}+F_y=\rho\frac{\partial^2 v}{\partial t^2} \tag{2}$$

where, $\bar{\omega}=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$, F_x and F_y are components of the electromagnetic field in x and y directions, respectively.

2) The stress-strain relations with incremental isotropy is given by Biot (1956)

$$\left.\begin{array}{l} S_{11}=(\lambda+2\mu+P)e_{xx}+(\lambda+P)e_{yy}-\gamma\left(T+\tau_1\frac{\partial}{\partial t}T\right), \\ S_{22}=\lambda e_{xx}+(\lambda+2\mu)e_{yy}-\gamma\left(T+\tau_1\frac{\partial}{\partial t}T\right), \\ S_{12}=2\mu e_{xy} \end{array}\right\} \tag{2}$$

3) The incremental strain-components was given by Biot (1956)

$$e_{xx}=\frac{\partial u}{\partial x},\quad e_{yy}=\frac{\partial v}{\partial y},\quad e_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial u}\right), \tag{3}$$

4) The modified heat conduction equation is

$$K\left(1+\tau_\Theta\frac{\partial}{\partial t}\right)\nabla^2 T=\rho C_e\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)+\gamma T_0\left[\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\tau_0\delta_{ij}\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right] \tag{4}$$

where, C_e is specific heat per unit mass, e_{ij} is strain components, K is thermal conductivity, P is initial stress, S_{11}, S_{12}, S_{22} are incremental stress components, λ and μ are Lame's constants, T_0 is natural temperature of the medium, δ_{ij} is Kronecker delta, T is absolute temperature of the medium, τ_0 and $\tau_1$are thermal relaxation times, α_t is coefficient of linear thermal expansion, u_i is components of the displacement vector, $\bar{\omega}$ is magnitude of local rotation, τ_Θ is the phase-lag of the gradient of temperature.

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5) Taking into account the absence of displacement current, the linearized Maxwell's equations governing the electromagnetic fields for a slowly moving solid medium having a perfect electrical conductivity are

$$\begin{aligned} &\vec{J}+\varepsilon_0\frac{\partial \vec{E}}{\partial t}=\operatorname{curl}\vec{h} \\ &-\mu_e\frac{\partial \vec{h}}{\partial t}=\operatorname{curl}\vec{E} \\ &\operatorname{div}\vec{h}=0 \\ &\operatorname{div}\vec{E}=0 \\ &\vec{E}=-\mu_e\left(\frac{\partial \vec{u}}{\partial t}\times\vec{H}_0\right) \\ &\vec{h}=\operatorname{curl}(\vec{u}\times\vec{H}_0) \end{aligned} \tag{5}$$

where, $\vec{H}_0=(0,0,H)$, $\vec{H}=\vec{H}_0+\vec{h}\,(x,y,t)$ in Cartesian coordinates (x,y,z), $\vec{H}(0,0,H)$.

Using Equation (5) we obtain

$$F_x=\mu_e H^2\left[\frac{\partial e}{\partial x}-\varepsilon_0\frac{\partial^2 u}{\partial t^2}\right], \tag{3}$$

$$F_y=\mu_e H^2\left[\frac{\partial e}{\partial y}-\varepsilon_0\frac{\partial^2 v}{\partial t^2}\right], \tag{4}$$

$$F_z=0 \tag{5}$$

where, $\vec{E}$ is an electric intensity vector, $\vec{F}$ is Lorentz's body force vector, $\vec{h}$ is perturbed magnetic field vector, $\vec{H}$ is magnetic field vector, $\vec{H}_0$ is primary constant magnetic field vector, $\vec{J}$ is an electric current density vector, μ_e is magnetic permeability, ε_0 electric permeability.

6) Maxwell's stress equation given in the form as:

$$\tau_{ij}=\mu_e\left[H_i h_j+H_j h_i-(\vec{H}_k.\vec{h}_k)\delta_{ij}\right], i,h=1,2,3 \tag{7a}$$

where, τ_{ij} is Maxwell's stress tensor, which reduces to

$$\tau_{11}=\tau_{22}=\mu_e H^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right),\quad \tau_{12}=0 \tag{7b}$$

3.1 *Solution of the problem*

Substituting from equations (2), (3) and (6) into equation (1), we get

$$\begin{aligned} &\left(\lambda+2\mu+P+\mu_e H^2\right)\frac{\partial^2 u}{\partial x^2}+\left(\lambda+\mu+\frac{P}{2}+\mu_e H^2\right)\frac{\partial^2 v}{\partial x\partial y}+\left(\lambda+\frac{P}{2}\right)\frac{\partial^2 u}{\partial y^2}-\rho g\frac{\partial v}{\partial x} \\ =\;&\left(\rho+\varepsilon_0\mu_e^2H^2\right)\frac{\partial^2 u}{\partial t^2}+\gamma\left(\frac{\partial T}{\partial x}+\tau_1\frac{\partial^2 T}{\partial x\partial t}\right) \end{aligned} \tag{6}$$

$$\begin{aligned} &\left(\mu-\frac{P}{2}\right)\frac{\partial^2 v}{\partial x^2}+\left(\lambda+\mu+\frac{P}{2}+\mu_e H^2\right)\frac{\partial^2 u}{\partial x\partial y}+\left(\lambda+2\mu+\mu_e H^2\right)\frac{\partial^2 v}{\partial y^2}+\rho g\frac{\partial u}{\partial x} \\ =\;&\left(\rho+\varepsilon_0\mu_e^2H^2\right)\frac{\partial^2 v}{\partial t^2}+\gamma\left(\frac{\partial T}{\partial y}+\tau_1\frac{\partial^2 T}{\partial y\partial t}\right) \end{aligned} \tag{7}$$

To separate the dilatational and rotational components of strain, we introduce displacement scalar and vector potentials Φ and Ψ defined by the following relations:

$$u=\frac{\partial \Phi}{\partial x}-\frac{\partial \Psi}{\partial y}$$

$$v=\frac{\partial \Phi}{\partial y}+\frac{\partial \Psi}{\partial x}$$

where, $\vec{\Psi}=(0,0,\Psi)$.

From equations (8) and (10), we get the following equations:

$$\nabla^2\Phi=\frac{\left(\rho+\varepsilon_0\mu_e^2H^2\right)}{\left(\lambda+2\mu+P+\mu_e H^2\right)}\left(\frac{\partial^2\Phi}{\partial t^2}\right)+\frac{\gamma}{\left(\lambda+2\mu+P+\mu_e H^2\right)}\left(T+\tau_1\frac{\partial T}{\partial t}\right)+\rho g\frac{\partial \Phi}{\partial y} \tag{11}$$

$$\nabla^2\Psi=\frac{\left(\rho+\varepsilon_0\mu_e^2H^2\right)}{\left(\lambda+\frac{P}{2}\right)}\left[\frac{\partial^2\Psi}{\partial t^2}\right]-\rho g\frac{\partial \Psi}{\partial x} \tag{12}$$

From equations (9) and (10), we get,

$$\nabla^2\Phi=\frac{\left(\rho+\varepsilon_0\mu_e^2H^2\right)}{\left(\lambda+2\mu+\mu_e H^2\right)}\left(\frac{\partial^2\Phi}{\partial t^2}\right)+\frac{\gamma}{\left(\lambda+2\mu+\mu_e H^2\right)}\left(T+\tau_1\frac{\partial T}{\partial t}\right) \tag{13}$$

$$\nabla^2\Psi=\frac{\left(\rho+\varepsilon_0\mu_e^2H^2\right)}{\left(\mu-\frac{P}{2}\right)}\left[\frac{\partial^2\Psi}{\partial t^2}\right]+\rho g\frac{\partial \Psi}{\partial y} \tag{14}$$

where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

Using equation (10) in equation (4), we get

$$K\left(1+\tau_\Theta\frac{\partial}{\partial t}\right)\nabla^2T=\rho C_e\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)+\gamma T_0\frac{\partial}{\partial t}\left(1+\tau_0\delta_{ij}\frac{\partial}{\partial t}\right)\nabla^2\Phi \tag{15}$$

We study the above basic equations for the following three different theories:

(i)Classical and dynamical coupled theory (1956) (CD) $(\delta_{ij}=0,\ \tau_0=0,\ \tau_1=0,\tau_\Theta=0)$

(ii)Green and Lindsay's theory (1972) (GL) $(\delta_{ij}=0,\ \tau_1\geq\tau_0>0,\tau_\Theta=0)$

(iii)Dual phase-lag theory (DPL) $(\delta_{ij}=1,\ \tau_0>0,\ \tau_1=0,\ 0\leq\tau_\Theta<\tau_0)$

Equations (11) and (14) rewritten as:

$$\nabla^2\Phi=\frac{\alpha}{C_1^2(1+R_H)}\frac{\partial^2\Phi}{\partial t^2}+\frac{\gamma}{\rho C_1^2(1+R_H)}\left(T+\tau_1\frac{\partial T}{\partial t}\right)+\rho g\frac{\partial \Phi}{\partial y} \tag{16}$$

$$\nabla^2\Psi=\frac{\alpha}{C_2^2}\left[\frac{\partial^2\Psi}{\partial t^2}\right]+\rho g\frac{\partial \Psi}{\partial y} \tag{17}$$

where,

$$R_H=\frac{C_A^2}{C_1^2},\quad C_1^2=\frac{\lambda+2\mu+P}{\rho},\quad C_2^2=\frac{\mu-\frac{P}{2}}{\rho},\quad C_A^2=\frac{\mu_e H^2}{\rho},\quad C^2=\frac{1}{\mu_e\varepsilon_0},\quad \alpha=1+\frac{C_A^2}{C^2}$$

Here, R_H, C_A, C_1 and C_2 represent the Alfven speed, the sensitive part of magnetic field, velocities of isothermal dilatational and rotational waves respectively, in medium M.

Eliminating T from equations (15) and (16), we obtain a fourth order differential equation in terms of Φ as

$$T=\left(1+\tau_1\frac{\partial}{\partial t}\right)^{-1}\left[\frac{\rho C_1^2(1+R_H)}{\gamma}\nabla^2\Phi-\frac{\rho\alpha}{\gamma}\frac{\partial^2\Phi}{\partial t^2}-\frac{\rho^2 g C_1^2(1+R_H)}{\gamma}\frac{\partial \Phi}{\partial y}\right] \tag{18}$$

$$\begin{aligned} &C_3^2(1+R_H)\left(1+\tau_\Theta\frac{\partial}{\partial t}\right)\nabla^4\Phi-\left[(1+R_H+\varepsilon_T)\frac{\partial}{\partial t}+\left((1+R_H)\tau_0+\varepsilon_T\tau_1+\varepsilon_T\tau_0\delta_{ij}+\frac{C_3^2}{C_4^2}\right)\frac{\partial^2}{\partial t^2}-C_3^2(1+R_H)\frac{\partial}{\partial y}\right. \\ &\left.+\left(\frac{C_3^2}{C_4^2}\tau_\Theta+\varepsilon_T\tau_0\tau_1\delta_{ij}\right)\frac{\partial^3}{\partial t^3}-C_3^2(1+R_H)\tau_\Theta\frac{\partial^2}{\partial t\partial y}+\rho g(1+R_H)\frac{\partial^2}{\partial t\partial y}\left(1+\tau_0\frac{\partial}{\partial t}\right)\right]+\frac{1}{C_4^2}\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial^3\Phi}{\partial t^3}=0 \end{aligned} \tag{19}$$

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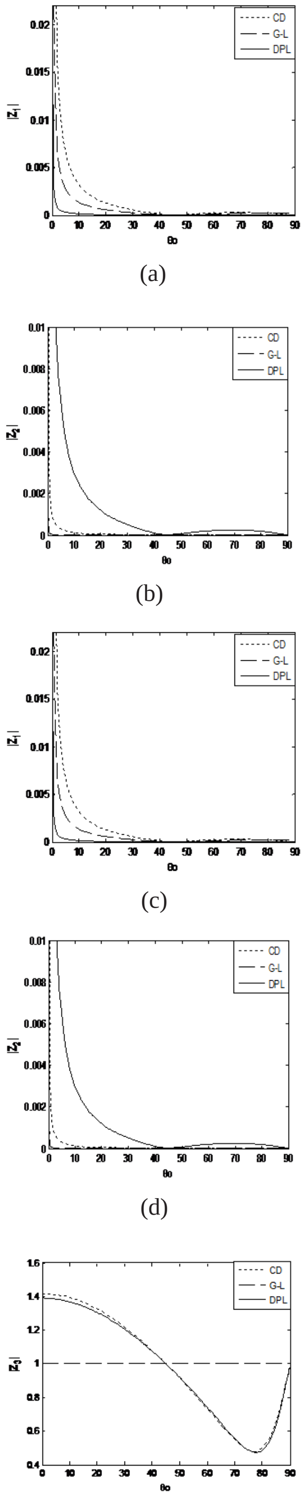

(a) (b) (c) (d) (e)

Fig. 3: Variation of the amplitudes $|Z_i|$ $(i = 1,2,...,5)$ with the angle of incidence of SV-waves for three models of:$P = (1.7)(10^{11})$, $H = 10^3$, $\varepsilon_0 = 0.5$

where, $C_3^2 = \frac{K}{\rho C_e}, C_{33}^2 = \frac{K}{C_e}$, $\varepsilon_T = \frac{T_0\gamma^2}{\rho^2 C_e C_1^2}$, $C_4^2 = \frac{C_1^2}{\alpha}$ (ε_T is thermoelastic coupling constant of the solid medium M).

We assume now the solution takes the following form:

$$\Phi = f(y)\exp[ik(x-ct)], \tag{8}$$

$$\Psi = s(y)\exp[ik(x-ct)], \tag{9}$$

$$T = h(y)\exp[ik(x-ct)] \tag{10}$$

where, $c = \frac{\omega}{k}$.

Substituting from equations (20) into equation (19), we get:

$$\frac{d^2f}{dy^2} + C\frac{d^3f}{dy^3} + A\frac{d^2f}{dy^2} + D\frac{df}{dy} + Bf(y) = 0 \tag{21}$$

where

$$A = -2k^2 + \frac{ikc}{C_3^{*2}}(1+R_H+\varepsilon_T) + \frac{k^2c^2}{C_3^{*2}}\left((1+R_H)\tau_0 + \varepsilon_T\tau_1 + \varepsilon_T\tau_0\delta_{ij} + \frac{C_3^2}{C_1^2}\right) - \frac{ik^3c^3}{C_3^{*2}}\left(\frac{C_3^2}{C_4^2}\tau_\Theta + \varepsilon_T\tau_0\tau_1\delta_{ij}\right),$$

$$B = k^4 - \frac{k^4c^2}{C_3^{*2}}\left((1+R_H)\tau_0 + \varepsilon_T\tau_1 + \varepsilon_T\tau_0\delta_{ij} + \frac{C_3^2}{C_1^2}\right) + \frac{ik^3c^3}{C_3^{*2}}\left(\frac{C_3^2}{C_4^2}\tau_\Theta + \varepsilon_T\tau_0\tau_1\delta_{ij}\right) + \frac{ik^3c^3}{C_4^2C_3^{*2}}\left(1 - ikc\tau_0 - \frac{C_4^2}{c^2}\right)(1+R_H+\varepsilon_T),$$

$$D = \frac{(Lk^2 - iLk^3c\tau_\Theta + iMk^3c + M\tau_0k^4c^2)}{C_3^{*2}},$$

$$C = \frac{-L + iLk^3c\tau_\Theta - iMkc - M\tau_0k^2c^2)}{C_3^{*2}}.$$

and

$$C_3^{*2} = C_3^2(1+R_H)(1-i\omega\tau_\Theta), \qquad L = \frac{K}{C_e}(1+R_H), \quad M = \rho g(1+R_H)$$

Equation (21) is a fourth order differential equation in f(y), the solution gives four values of $f(y)$, the function in equation (20) takes the form

$$\Phi = [A_1\exp(ikm_1y) + A_2\exp(ikm_2y) + A_3\exp(ikm_3y) + A_4\exp(ikm_4y)]\exp[ik(x-ct)] \tag{22}$$

Substituting equation (20) into equation (17), we get

$$\frac{d^2s}{dy^2} - \rho g\frac{ds}{dy} + k^2\left(\frac{c^2\alpha}{C_2^2} - 1\right)s = 0 \tag{23}$$

Equation (23) suggests that the solution yields two values of $g(y)$, the function in equation (20) written as

$$\Psi = [A_5\exp(ikm_5y) + A_6\exp(ikm_6y)]\exp[ik(x-ct)] \tag{24}$$

The constants A_i $(i = 1,2,...,6)$ represent the amplitudes of incident SV-waves and reflected (thermal wave) T-, P- and SV-waves, respectively.

Substituting from equations (20) and (22) into equation (11), we get the value of $h(y)$, the function T in equation (20) becomes

$$T = \frac{\rho\alpha}{\gamma\tau}[F_1A_1\exp(ikm_1y) + F_2A_2\exp(ikm_2y) + F_3A_3\exp(ikm_3y) + F_4A_4\exp(ikm_4y)]\exp[ik(x-ct)] \tag{25}$$

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where

$$\tau = 1 - i\omega\tau_1, \quad F_1 = -\frac{C_1^2}{\rho}(1+R_H)(\omega^2 + k^2 m_1) + \omega^2 - \frac{\rho C_1^2 g}{\alpha}(1+R_H)(im_I), \qquad I = 1,2,3,4$$

Setting $\mu = P = 0$ in equations (1)-(4) we obtain the basic equations for a non-viscous liquid medium in the presence of body forces, we get displacement equations and temperature equation for the liquid medium M' as follows:

$$(\lambda' + \mu_e' H^2)\frac{\partial^2 u'}{\partial x^2} + (\lambda' + \mu_e' H^2)\frac{\partial^2 v'}{\partial x \partial y} = \rho' \frac{\partial^2 u'}{\partial t^2} + \gamma' \left(\frac{\partial T'}{\partial x} + \tau_1' \frac{\partial^2 T'}{\partial x \partial t}\right) - \rho' g' \frac{\partial v'}{\partial x} \tag{26}$$

$$(\lambda' + \mu_e' H^2)\frac{\partial^2 u'}{\partial x \partial y} + (\lambda' + \mu_e' H^2)\frac{\partial^2 v'}{\partial y^2} = \left(\rho' + \mu_e'^2 \varepsilon_0' H^2\right)\frac{\partial^2 v'}{\partial t^2} + \gamma' \left(\frac{\partial T'}{\partial y} + \tau_1' \frac{\partial^2 T'}{\partial y \partial t}\right) + \rho' g' \frac{\partial u'}{\partial x} \tag{27}$$

$$K' \left(1 + \tau_\Theta' \frac{\partial}{\partial t}\right)\nabla^2 T' = \rho' C_e' \left(\frac{\partial T'}{\partial t} + \tau_0' \frac{\partial^2 T'}{\partial t^2}\right) + T_0 \gamma' \frac{\partial}{\partial t}\left(1 + \tau_0' \delta_{ij} \frac{\partial}{\partial t}\right)\nabla^2 \Phi' \tag{28}$$

The primes have been used to designate the corresponding quantities in the liquid medium M' as already been defined in case of solid medium M.

Taking

$$u = \frac{\partial \Phi'}{\partial x}, \qquad v = \frac{\partial \Phi'}{\partial y} \tag{29}$$

We obtain

$$\nabla^2 \Phi' = \frac{\alpha'}{C_1'^2 (1 + R_H')}\frac{\partial^2 \Phi'}{\partial t^2} + \frac{\gamma'}{\rho' C_1'^2 (1 + R_H')}\left(1 + \tau_1' \frac{\partial}{\partial t}\right) T' + \rho' g' \frac{\partial \Phi'}{\partial y} \tag{30}$$

$$K' \left(1 + \tau_\Theta' \frac{\partial}{\partial t}\right)\nabla^2 T' = \rho' C_e' \left(\frac{\partial T'}{\partial t} + \tau_0' \frac{\partial^2 T'}{\partial t^2}\right) + T_0 \gamma' \frac{\partial}{\partial t}\left(1 + \tau_0' \delta_{ij} \frac{\partial}{\partial t}\right)\nabla^2 \Phi' \tag{31}$$

where, $C_1'^2 = \dfrac{\lambda'}{\acute{\rho}}$.

Solving equations (30) and (31) and proceeding exactly in a similar way as in solid medium M, we get the appropriate solution for Φ' and T' as

$$\Phi' = [A_2' \exp(ikm_2' y) + A_4' \exp(ikm_4' y)] \exp[ik(x - ct)] \tag{32}$$

$$T' = \frac{\rho' \alpha'}{\gamma''}[F_1' A_2' \exp(ikm_2' y) + F_2' A_4' \exp(ikm_4' y)] \exp[ik(x - ct)] \tag{33}$$

where,

$$\tau' = 1 - i\omega\tau_1'. \tag{34}$$

The constants A_2' and A_4' represent the amplitudes of refracted thermal and p-waves, respectively.

3.2 Boundary conditions

1) Normal displacement is continuous at the interface, i.e. $v = v'$,this leads to

$$\frac{\partial \Phi}{\partial y} + \frac{\partial \Psi}{\partial x} = \frac{\partial \Phi'}{\partial y} \tag{35}$$

Using Equations (22), (24) and (32) in the above continuity relation, we get,

$$\{A_1(m_1)\exp(ikm_1 y) + A_2(m_2)\exp(ikm_2 y) + A_3(m_3)\exp(ikm_3 y) + A_4(m_4)\exp(ikm_4 y)\} + \{A_5 \exp(ikm_5 y) + A_6 \exp(ikm_6 y)\} = \left\{A_2'\left(m_2'\right)\exp\left(ikm_2' y\right) + A_4'\left(m_4'\right)\exp\left(ikm_4' y\right)\right\}$$

at y=0

$$m_1 A_1 + m_2 A_2 + m_3 A_3 + m_4 A_4 + A_5 + A_6 - m_2' A_2' - m_4' A_4' = 0 \tag{36}$$

2) Tangential displacement must vanish at the interface i.e. $u = 0$.

This leads to $\frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial y} = 0$

Using equations (22) and (24) in the above boundary condition, we get

$$\{A_1 \exp(ikm_1 y) + A_2 \exp(ikm_2 y) + A_3 \exp(ikm_3 y) + A_4 \exp(ikm_4 y)\} - \{A_5(m_5)\exp(ikm_5 y) + A_6(m_6)\exp(ikm_6 y)\} = 0$$

at y=0

$$A_1 + A_2 + A_3 + A_4 - m_5 A_5 - m_6 A_6 = 0 \tag{37}$$

3) Normal force per unit initial area must be continuous at the interface i.e. $\nabla f_y = \nabla f_y'$

This leads to $s_{22} + \tau_{22} = s_{22}' + \tau_{22}'$

where, $\tau_{ij} = \mu_e$ (2), (3) and (7) for medium M and their corresponding equations for medium M' and using equations (10) and (32), we obtain

$$\left(\lambda + \mu_e H^2\right)\left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}\right) + 2\mu\left(\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial \Psi}{\partial x \partial y}\right) - \gamma\left(T + \tau_1 \frac{\partial T}{\partial t}\right) = \left(\lambda' + \mu_e' H'^2\right)\left(\frac{\partial^2 \Phi'}{\partial x^2} + \frac{\partial^2 \Phi'}{\partial y^2}\right) - \gamma'\left(T' + \tau_1 \frac{\partial T'}{\partial t}\right) \tag{38}$$

Substituting from equations (22), (24), (25), (32) and (33) in the above equation, we get

$$\begin{aligned}
&\left(C_5\left(-k^2 - k^2 m_1\right) - C_6 k^2 m_1 - \alpha F_1\right) A_1 \exp(ikm_1 y) + \left(C_5\left(-k^2 - k^2 m_3\right) - C_6 k^2 m_3 - \alpha F_3\right) A_3 \exp(ikm_3 y) + \\
&\left(C_5\left(-k^2 - k^2 m_2\right) - C_6 k^2 m_2 - \alpha F_2\right) A_2 \exp(ikm_2 y) + \left(C_5'\left(k^2 + k^2 m_2'\right) + \rho^* \alpha' F_2'\right) A_2' \exp\left(ikm_2' y\right) \\
&\left(C_5\left(-k^2 - k^2 m_4\right) - C_6 k^2 m_4 - \alpha F_4\right) A_4 \exp(ikm_4 y) + \left(C_5'\left(k^2 + k^2 m_4'\right) + \rho^* \alpha' F_4'\right) A_4' \exp\left(ikm_4' y\right) \\
&- C_6 k^2 m_5 A_5 \exp(ikm_5 y) - C_6 k^2 m_6 A_6 \exp(ikm_6 y) = 0
\end{aligned}$$

at y=0

$$\begin{aligned}
&\left(C_5\left(-k^2 - k^2 m_1\right) - C_6 k^2 m_1 - \alpha F_1\right) A_1 + \left(C_5\left(-k^2 - k^2 m_3\right) - C_6 k^2 m_3 - \alpha F_3\right) A_3 + \\
&\left(C_5\left(-k^2 - k^2 m_2\right) - C_6 k^2 m_2 - \alpha F_2\right) A_2 + \left(C_5'\left(k^2 + k^2 m_2'\right) + \rho^* \alpha' F_2'\right) A_2' \\
&\left(C_5\left(-k^2 - k^2 m_4\right) - C_6 k^2 m_4 - \alpha F_4\right) A_4 + \left(C_5'\left(k^2 + k^2 m_4'\right) + \rho^* \alpha' F_4'\right) A_4' \\
&- C_6 k^2 m_5 A_5 - C_6 k^2 m_6 A_6 = 0
\end{aligned} \tag{39}$$

where, $C_5 = \frac{\lambda + \mu_e H^2}{\rho}$, $\quad C_6 = \frac{2\mu}{\rho}$, $\quad C_5' = \frac{\lambda' + \mu_e' H'^2}{\rho'}$, $\quad C_6' = \frac{2\mu'}{\rho'}$, $\rho^* = \frac{\rho'}{\rho}$

4) Tangential force per unit initial area must vanish at the interface i.e, $\nabla f_x = 0$

This leads to $s_{12} + Pe_{xy} + \tau_{12} = 0$

Using equations (2), (3), (7), (10), (22) and (24), we obtain

$$A_1 m_1 \exp(ikm_1 y) + A_2 m_2 \exp(ikm_2 y) + A_3 m_3 \exp(ikm_3 y) + A_4 m_4 \exp(ikm_4 y) - \frac{1}{2}\left(\left(A_5 m_5^2 \exp(ikm_5 y) + A_6 m_6^2 \exp(ikm_6 y)\right) + \frac{1}{2}\left(A_5 \exp(ikm_5 y) + A_6 \exp(ikm_6 y)\right)\right) = 0$$

at y = 0

$$m_1 A_1 + m_2 A_2 + m_3 A_3 + m_4 A_4 - \frac{1}{2}\left(m_5^2 - 1\right) A_5 - \frac{1}{2}\left(m_6^2 - 1\right) A_6 = 0 \tag{40}$$

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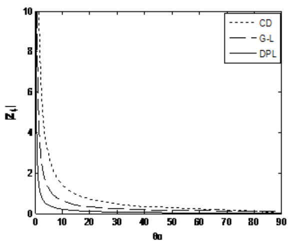

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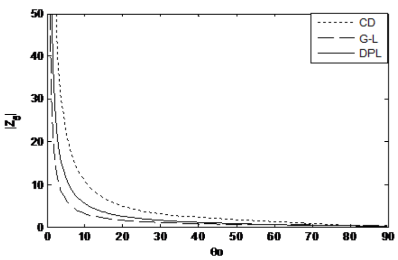
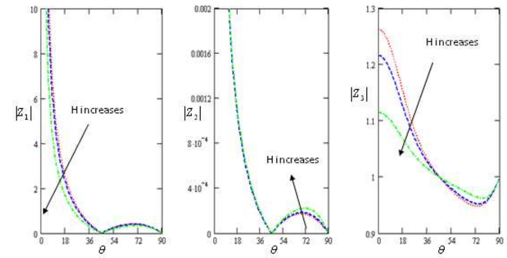


(b)

Fig. 4: Variation of the amplitudes $|Z_i|$ $(i=1,2,...,5)$ with the angle of incidence of SV-waves for variation of magnetic field: $P=(4,5,9)x(10^5)$, $H=10^3$, $\varepsilon_0=0.3$



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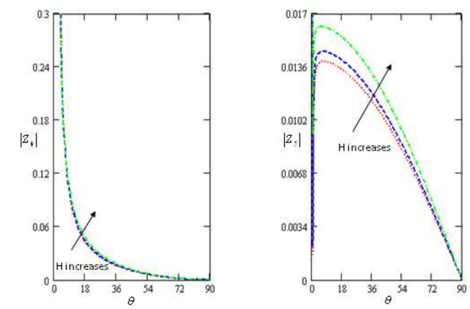


(b)

Fig. 5: Variation of the amplitudes $|Z_i|$ $(i=1,2,...,5)$ with the angle of incidence of SV-waves for variation of initial stress: $P=(1.1,1.3,1.5,1.7)x(10^{11})$, $H=10^3$, $\varepsilon_0=0.3$

5) Temperature must be continuous at the interface i.e, $T=T'$.
Using equations (25)(**??**) and (33) with making some algebraic simplifying, we get

$$\frac{\rho\alpha}{\gamma\tau}\left[F_1A_1\exp(ikm_1y)+F_2A_2\exp(ikm_2y)+F_3A_3\exp(ikm_3y)+F_4A_4\exp(ikm_4y)\right]$$
$$=\frac{\rho'\alpha'}{\gamma'\tau'}\left[F_2'A_2'\exp\left(ikm_2'y\right)+F_{4'}'A_4'\exp\left(ikm_4'y\right)\right]$$

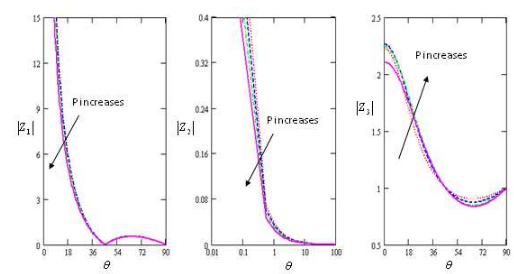
at y=0

$$F_1A_1+F_2A_2+F_3A_3+F_4A_4-\frac{\rho^*\alpha^*}{\gamma^*\tau^*}\left[F_2'A_2'+F_{4'}'A_4'\right]=0 \tag{41}$$

where, $\gamma^*=\frac{\gamma'}{\gamma}$ and $\tau^*=\frac{\tau'}{\tau}$.

3.3 *Equations for the reflection and refraction coefficients*

To consider the reflection and refraction of thermoelastic plane waves are incident at the solid-liquid interface at $y=0$ making an angle θ with the y-axis.
Case 1: For p-wave incidence, we put $c=p^{-1}\cos ec\theta$and $A_1=A_5=0$
Case 2: For SV wave incidence, we put $c=C_2\cos ec\theta$and $A_1=A_3=0$



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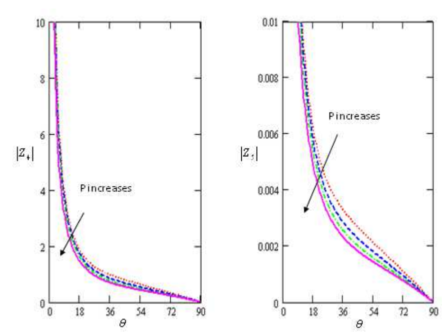

(b)

Fig. 6: Variation of the amplitudes $|Z_i|$ $(i=1,2,...,5)$with the angle of incidence of p-waves for variation of magnetic field: $P=(4,5,9)x(10^4)$, $H=10^11$, $\varepsilon_0=0.3$

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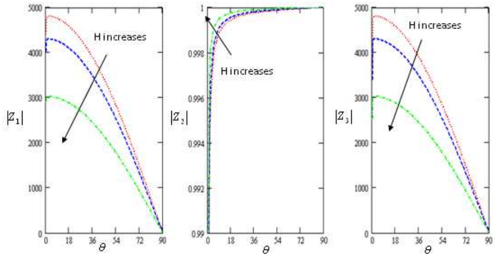

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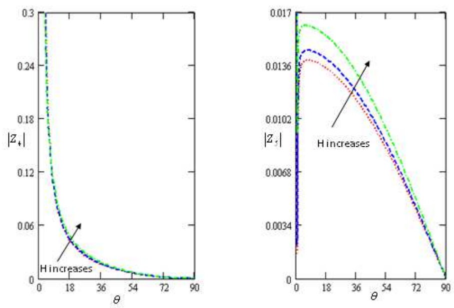
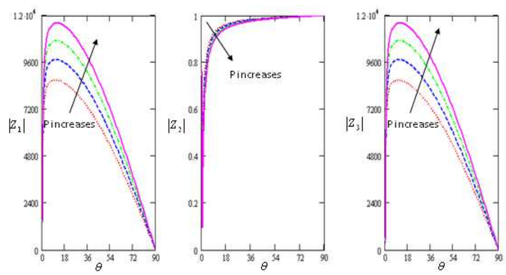
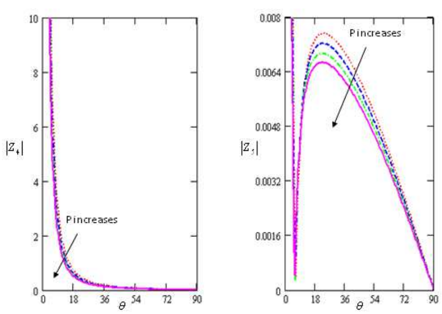


(b)

Fig. 7: Variation of the amplitudes $|Z_i|$ $(i=1,2,...,5)$ with the angle of incidence of p-waves for variation of initial stress: $P=(1.1,1.3,1.5,1.7)x(10^{11})$, $H=10^3$, $\varepsilon_0=0.3$



(a)



(b)

Fig. 8: Variation of the amplitudes $|Z_i|$ $(i=1,2,...,5)$ with the angle of incidence of p-waves for variation of electric field: $\varepsilon_0=0.1,0.2,0.4,0.5$, $P=(1.1,1.3,1.5,1.7)x(10^{11})$, $H=10^3$

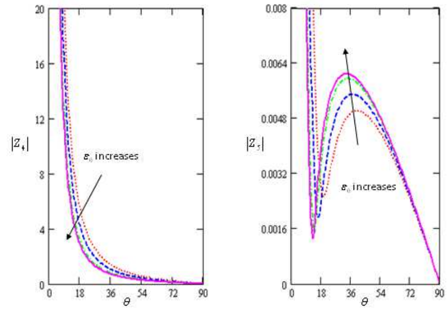


Fig. 9: Variation of the amplitudes $|Z_i|$ $(i=1,2,...,5)$ with the angle of incidence of SV-waves under dual-phase-lag theory (DPL): $P=1.1x(10^{12})$, $H=10^2$, $\varepsilon_0=0.1$

By generalization, we get a system of five non-homogeneous equations for a thermoelastic plane wave incident,

$$\sum_{i=1}^{5} a_{ij}Z_j = y_j, \quad (j=1,2,...,5) \tag{42}$$

where

$$a_{11}=m_2,\ a_{12}=m_4,\ a_{13}=1,\ a_{14}=-m_2^{'},\ a_{15}=-m_4^{'},\ a_{21}=a_{22}=1,\ a_{23}=-m_6,$$
$$a_{24}=a_{25}=0,\quad a_{31}=\left(C_5\left(-k^2-k^2m_2\right)-C_6k^2m_2-\alpha F_2\right),\quad a_{32}=\left(C_5\left(-k^2-k^2m_4\right)-C_6k^2m_4-\alpha F_4\right),$$
$$a_{33}=-C_6k^2m_6, a_{34}=\left(C_5^{'}\left(k^2+k^2m_2^{'}\right)+\rho^*\alpha^{'}F_2^{'}\right)=a_{35}=\left(C_5^{'}\left(k^2+k^2m_4^{'}\right)+\rho^*\alpha^{'}F_4^{'}\right),$$
$$a_{41}=m_2, a_{42}=m_4, a_{43}=-0.5\left(m_6^2-1\right), a_{44}=a_{45}=0,$$
$$a_{51}=F_2,\ a_{52}=F_4,\ a_{53}=0,\ a_{54}=-\frac{\rho^*\alpha^*}{\gamma^*\tau^*}F_2^{'},\quad a_{55}=-\frac{\rho^*\alpha^*}{\gamma^*\tau^*}F_4^{'}.$$

where, $(j=1,2,...,5)$ are the ratios of amplitudes of reflected thermal, P-, SV-waves and refracted thermal, P-waves for the incident waves, respectively, and

(i) For incident p-wave

$$y_1=a_{12}, y_2=-a_{22}, y_3=-a_{32}, y_4=a_{42}, y_5=-a_{52}$$

$$Z_1=\frac{A_2}{A_3}, Z_2=\frac{A_4}{A_3}, Z_3=\frac{A_6}{A_3}, Z_4=\frac{A_2'}{A_3}, Z_5=\frac{A_4'}{A_3}$$

(ii) For incident SV-wave

$$y_1=-a_{13},\ y_2=m_5,\ y_3=C_6k^2m_5,\ y_4=\tfrac{1}{2}\left(m_5^2-1\right),\ y_5=a_{53},$$
$$Z_1=\tfrac{A_2}{A_5},\ Z_2=\tfrac{A_4}{A_5},\ Z_3=\tfrac{A_6}{A_5},\ Z_4=\tfrac{A_2^{'}}{A_5},\ Z_5=\tfrac{A_4^{'}}{A_5}$$

Special case

If the gravity is neglected, equation (42) takes the form

$$\sum_{i=1}^{5} a_{ij}Z_j = y_j, \quad (j=1,2,...,5) \tag{43}$$

Where

$$a_{11}=-m_1,\ a_{12}=-m_2,\ a_{13}=1,\ a_{14}=-m_1^{`},\ a_{15}=-m_2^{`},\ a_{21}=a_{22}=1,\ a_{23}=m_3$$
$$a_{24}=a_{25}=0,\quad a_{31}=\left[-(2+\beta)+c^2\left(\frac{\alpha}{C_2^2}-\beta q^2\right)\right],\quad a_{32}=\left[-(2+\beta)+c^2\left(\frac{\alpha}{C_2^2}-\beta p^2\right)\right]$$
$$a_{33}=-(2+\beta)m_3, a_{34}=a_{35}=-\rho^*\left(1+m_3^2\right), a_{41}=-m_1, a_{42}=-m_2, a_{43}=-0.5\left(m_3^2-1\right),$$

$$a_{44}=a_{45}=0,\quad a_{51}=\left(1-\frac{(1+R_H)}{\alpha}q^2C_1^2\right),\qquad a_{52}=\left(1-\frac{(1+R_H)}{\alpha}p^2C_1^2\right),$$
$$a_{53}=0,\qquad a_{54}=-\frac{\rho^*\alpha^*}{\gamma^*\tau^*}\left(1-\frac{\left(1+R_H^{'}\right)}{\alpha^{'}}q^{'2}C_1^{'2}\right),\quad a_{55}=-\frac{\rho^*\alpha^*}{\gamma^*\tau^*}\left(1-\frac{\left(1+R_H^{'}\right)}{\alpha^{'}}p^{'2}C_1^{'2}\right)$$

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where, $(j = 1, 2, ..., 5)$ are the ratios of amplitudes of reflected thermal, P-, SV-waves and refracted thermal and p-waves for the incident waves, respectively.
where, $m_1 = m_2 = \sqrt{q^2c^2 - 1}, \quad m_3 = m_4 = \sqrt{p^2c^2 - 1}$

$$p^2 = \frac{1}{2C_4^2 C_3^{*2}} \left[C_4^2 \left(\tau_0 \left(1 + R_H \right) + \varepsilon_T \tau_1 + \varepsilon_T \tau_0 \delta_{ij} \right) + c_3^2 + \frac{i(1+R_H+\varepsilon_T)C_4^2}{\omega} - i\omega \left(c_3^2 \tau_\Theta + C_4^2 \varepsilon_T \tau_0 \tau_1 \delta_{ij} \right) + \sqrt{N} \right], \tag{44}$$

$$q^2 = \frac{1}{2C_4^2 C_3^{*2}} \left[C_4^2 \left(\tau_0 \left(1 + R_H \right) + \varepsilon_T \tau_1 + \varepsilon_T \tau_0 \delta_{ij} \right) + c_3^2 + \frac{i(1+R_H+\varepsilon_T)C_4^2}{\omega} - i\omega \left(c_3^2 \tau_\Theta + C_4^2 \varepsilon_T \tau_0 \tau_1 \delta_{ij} \right) - \sqrt{N} \right], \tag{11}$$

$$N = \left[C_4^2 \left(\tau_0 \left(1 + R_H \right) + \varepsilon_T \tau_1 + \varepsilon_T \tau_0 \delta_{ij} \right) + c_3^2 + \frac{i(1+R_H+\varepsilon_T)C_4^2}{\omega} - i\omega \left(c_3^2 \tau_\Theta + C_4^2 \varepsilon_T \tau_0 \tau_1 \delta_{ij} \right) \right]^2 - \frac{4i\left(1 - i\omega\tau_0\right) C_4^2 C_3^{*2}}{\omega}. \tag{46}$$

$$m_5 = m_6 = \sqrt{\frac{\alpha c^2}{C_2^2} - 1}. \tag{47}$$

Numerical results and discussion
In the view of ilustrate the numerical calculations, we use the physical quantities of the following media
For solid medium (M Crust)

$\lambda = \mu = 3 \times 10^{10} N.m^{-2}, \quad \alpha_t = 1.0667 \times 10^{-5} K^{-1}, \quad \omega = 7.5 \times 10^{13} S^{-1}, \ T_0 = 300 K,$
$C_e = 1100 J.Kg^{-1}.K^{-1}, \quad \rho = 2900 Kg.m^{-3}, \quad K = 3 W.m^{-1}K^{-1}.$

For liquid medium (M/ Water)

$\lambda' = \mu' = 20.4 \times 10^9 N.m^{-2}, \quad \alpha_t' = 69 \times 10^{-6} K^{-1}, \ K' = 0.6 W.m^{-1}.K^{-1},$
$C_e' = 4187 J Kg^{-1}.K^{-1}, \quad \rho' = 1000 Kg.m^{-3}.$

Considering $\tau_0 = \tau_0' = 0.9, \quad \tau_1 = \tau_1' = 0.9, \quad \tau_\Theta = \tau_\Theta' = 0.8$.
Fig. 3 appears the variation of the amplitudes ratios with the angle of incidence of SV-waves for three models in thermoelasticity the couple theory (CD) and Green-Lindsay (G-L) theory, as well as the dual-phase-lag theory (DPL).$|Z_1|$,$|Z_2|$,$|Z_4|$and $|Z_5|$start from maximum values and interrupted to zero at $\theta = 90°$but $|Z_3|$arrives to unity when the angle of incidence $\theta = 90°$. It is seen that dual-phase-lag theory in$|Z_1|$and$|Z_4|$takes the smallest values comparing with the existing of two relaxation times (G-L) while if there isn't relaxation times (CD), $|Z_1|$ and$|Z_4|$take the largest value. It is clear that Green-Lindsay (G-L) theory $|Z_2|$ takes the smallest values comparing with couple theory (CD) while if there is relaxation times to dual-phase-lag theory (DPL) $|Z_2|$ takes the largest value. It is display that $|Z_3|$ makes a slight change for couple theory (CD) and dual-phase-lag theory (DPL) theory but Green-Lindsay (G-L) takes the line from the unity to unity. It is show that Green-Lindsay (G-L) theory$|Z_5|$ takes the smallest values comparing with dual-phase-lag theory (DPL) but the couple (CD) theory$|Z_5|$ takes the largest value (please see, Ref. [30]).
Fig. 4 displays the variation of the amplitudes with the angle of incidence of SV-waves for variation of initial stress under effect of magnetic field in the context of dual-phase-lag theory, it appears that $|Z_1|$,$|Z_2|$,$|Z_4|$ and $|Z_5|$ start from maximum values and interrupted to zero at $\theta = 90°$ but $|Z_3|$ arrives to unity also at $\theta = 90°$. It is seen that with the influence increasingly the magnetic field, the amplitude ratio $|Z_1|$decreases but $|Z_5|$increases with an increasing of magnetic field, but the amplitude ratios $|Z_2|$ and $|Z_3|$ are affected slightly with an increasing of H. Also, it is obvious that $|Z_3|$ decreases with the increased values of H in the interval $0^o < \theta < 45^o$ and increase with increasing of H if $45^o < \theta < 90^o$. From **Fig. 5**, it is clear that the $|Z_1|$,$|Z_2|$,$|Z_4|$ and $|Z_5|$ start from maximum values and interrupted to zero at $\theta = 90°$but $|Z_3|$ arrives to unity also at $\theta = 90°$. It is seen that $|Z_1|$,$|Z_2|$,$|Z_4|$ and $|Z_5|$ decrease with an increasing of magnetic field H but the amplitude ratio $|Z_3|$ increases in the interval $0^o < \theta < 45^o$ but decrease with increase of H if $45^o < \theta < 90^o$.
Fig. 6 depicts the variation of the reflection and refraction coefficients with the angle of incidence of P-waves under dual-phase-lag theory (DPL) in the presence of the magnetic field and constant parameters. It is obvious that $|Z_1|$ and $|Z_3|$ decrease with an increasing of H but $|Z_2|$,$|Z_4|$ and $|Z_5|$ are affected positively with the increased values of the magnetic field H. Also, it is concluded that $|Z_1|$,$|Z_3|$,$|Z_4|$ and $|Z_5|$ decrease tending to zero with an increasing of angle of incidence but $|Z_2|$ increases with an increasing of angle of incidence tends to unity. Variation of amplitudes $|Z_i|\,(i = 1, 2, ..., 5)$ with the angle of incidence of P-waves with variation of initial stress, it is seen that $|Z_1|$ and $|Z_3|$ increase with an increasing of initial stress P, but $|Z_2|$, $|Z_4|$ and $|Z_5|$ decrease. In Fig. 8, we obvious that the amplitudes $|Z_i|\,(i = 1, 2, ..., 5)$ are affected strongly by the influence of electric field, it is noted that $|Z_1|$, $|Z_2|$, $|Z_3|$ and $|Z_5|$ have strong increasing effect with an increasing of electric field, but $|Z_4|$ decreases, physically this indicate to the effect of electric field in the reflection and transmission coefficients and refers to practical results on the phenomena.
Finally, Fig. 9 depicts comparison between the amplitudes of reflection and transmission coefficients $|Z_i|\,(i = 1, 2, ..., 5)$ which indicate that $|Z_1|$, $|Z_2|$, $|Z_4|$ and $|Z_5|$ start from maximum values and interrupted to zero at $\theta = 90°$but $|Z_3|$ arrives to unity also at $\theta = 90°$. The effect of amplitude differences with changing the angle of incidence to waves $|Z_1| < |Z_4| < |Z_5| < |Z_2| < |Z_3|$ the smallest affects $|Z_1|$, but the largest affects $|Z_3|$.

4 Perspective

From the results calculated analytically and presented graphically in the context of (CD), (G-L) and (DPL) models and effect of gravity, initial stress, electromagnetic field on the reflection and transmission at the interface between solid-liquid media under perfect boundary conditions are discussed and conclude the following remarks:

1.The reflected amplitudes depend on the angle of incidence, gravity, initial stress, electromagnetic field and thermal relaxation times.

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2.The initial stress and electromagnetic field play a significant role that has the inverse trend for the reflected and transmitted waves.

3.The three thermoelastic theories have aspects effects on the reflection and refraction phenomena.

4.Analytically, we see that the influence of gravity is too clear in the results obtained in the secular equations roots as well as in the amplitudes ratios.

5.In DPL model, it appears that $|Z_1| < |Z_4| < |Z_5| < |Z_2| < |Z_3|$; the smallest affects $|Z_1|$, but the largest affects $|Z_3|$.

6.It is observed that the electric field has strong effect on the reflection coefficient that has a lot of applications, especially, in seismic waves, engineering, earthquakes, volcanoes, and acoustics that are helpful for researchers concerned with material science, designers of new materials, low-temperature physicists, as well as for those working on the development of a theory of hyperbolic propagation. Study of the phenomenon of rotation, magnetic field and diffusion are also used to improve the conditions of oil extractions [30].

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References

[1] V. Danilovskaya, Thermal stresses in an elastic half-space due to sudden heating of its boundary, Prikl Mat. Mekh. Vol. 14, pp. 316-324 (1950).

[2] M.A. Biot, Mechanics of Incremental Deformations, John Wiley and Sons, Inc., New York, 1965.

[3] H.W. Lord, Y. Shulman, A generalized dynamical theory of thermoelasticity, Journal of Mechanics Physics Solids, Vol. 15, pp. 299-309 (1967).

[4] A.E. Green and K.A. Lindsay, Thermoelasticity, J. Elasticity 2 (1972) 1-7.

[5] A.E. Green and P.M. Naghdi, A re-examination of the basic postulates of thermo-mechanics, P. Roy. Soc. Lond. A, Vol. 432, pp. 171–194 (1991).

[6] A.E. Green and P.M. Naghdi, On undamped heat waves in an elastic solid, Journal of Therm. Stresses, Vol. 15, pp. 253–264 (1992).

[7] A.E. Green and P.M. Naghdi, Thermoelasticity without energy dissipation, Journal of Elasticity, Vol. 31, pp. 189–208 (1993).

[8] D.S. Chandrasekharaiah, Hyperbolic thermoelasticity: a review of recent literature, Applied Mech. Rev., Vol. 51, No. 12, pp. 705–729 (1998).

[9] R.B. Hetnarski and J. Ignaczak, Generalized thermoelasticity, Journal of Thermal Stresses, Vol. 22, pp. 451-476 (1999).

[10] B. Singh, Reflection and transmission of plane harmonic waves at an interface between liquid and micropolar viscoelastic solid with stretch, Sadhana 25(6) (2000) 589-600.

[11] R. Kumar and P. Sarathi, Reflection and refraction of thermoelastic plane waves at an interface between two thermoelastic media without energy dissipation, Arch. Mech. 58 (1) (2006) 155–185.

[12] M.I.A. Othman and Y. Song, Reflection of plane waves from an elastic solid half-space under hydrostatic initial stress without energy dissipation, International Journal of Solids Struct., Vol. 44, pp. 5651–5664 (2007).

[13] S.B. Sinha and K.A. Elsibai, Reflection of thermoelastic waves at a solid half-space with two relaxation times, Journal of Thermal Stresses, Vol. 19, pp. 763-777 (1996).

[14] A.N. Abd-alla, A.A.S. Al-Dawy, The Reflection phenomena of SV waves in a generalized thermoelastic medium, International Journal of Math. Math. Sci., Vol. 23, No. 8, pp. 529-540 (2000).

[15] A.N. Abd-Alla, A.A. Yahia and S.M. Abo-Dahab, On the reflection of the generalized magneto-thermo-viscoelastic plane waves, Chaos, Solitons & Fractals, 16(2), pp. 211-231.

[16] S.B. Sinha, K.A. Elsibai, Reflection and refraction of thermoelastic waves at an interface of two semi-Infinite media with two thermal relaxation times, Journal of Thermal Stresses, Vol. 20, No. 2, pp.129-146 (1997).

[17] A.N. Abd-Alla and S.M. Abo-Dahab, The influence of the viscosity and the magnetic field on reflection and transmission of waves at interface between magneto-viscoelastic materials, Meccanica, Vol. 43, pp. 437-448 (2008).

[18] M.I.A. Othman and Y. Song, Reflection of magneto-thermoelastic waves with two relaxation times and temperature dependent elastic moduli, Applied Mathematical Modeling, Vol. 32, pp. 483-500 (2008).

[19] S.M. Abo-Dahab and B. Singh, Influences of magnetic field on wave propagation in generalized thermoelastic solid with diffusion, Arch. Mech., Vol. 61, pp. 121-136 (2009).

[20] S.M. Abo-Dahab and R.A. Mohamed, Influence of magnetic field and hydrostatic initial stress on reflection phenomena of P and SV waves from a generalized thermoelastic solid half-space, Journal of Vibration and Control, Vol. 16, pp. 685-699 (2010).

[21] S.M. Abo-Dahab, R.A. Mohamed and Baljeet Singh, Rotation and magnetic field effects on P wave reflection from stress-free surface elastic half-space with voids under one thermal relaxation time, Journal of Vibration and Control, Vol. 17, No 12, pp. 1827-1839 (2011).

[22] S.M. Abo-Dahab, Reflection of P and SV waves from stress-free surface elastic half-space under influence of magnetic field and hydrostatic initial stress without energy dissipation, Journal of Vibration and Control, Vol. 17, No 14, pp. 2213-2221 (2011).

[23] S.M. Abo-Dahab, R.A. Mohamed and A.M. Abd-Alla, Effect of relaxation times and magnetic field on

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the reflection of thermoelastic waves from isothermal and insulated boundaries of a half space, International Review of Physics, Vol. 5, No. 5, pp. 247-258 (2011).

[24] S.M. Abo-Dahab and A.J. Asad, Maxwell's stresses effect on reflection and transmission of plane waves between two thermo-elastic media under GN Model, International Review of Physics, Vol. 5, No. 5, pp. 286-299 (2011).

[25] S. Deswal, L. Singh and B. Singh, Reflection and refraction at an interface between two dissimilar thermally conducting viscous liquid half-spaces, International Journal of Pure and Applied Mathematics, Vol. 70, No. 6, pp. 807-824 (2011).

[26] N. Chakraborty and M.C. Singh, Reflection and refraction of a plane thermoelastic wave at a solid-solid interface under perfect boundary condition, in presence of normal initial stress, Applied Mathematical Modeling, Vol. 35, No. 11, pp. 5286-5301 (2011).

[27] A.M. Abd-Alla, S.R. Mahmoud and S.M. Abo-Dahab, On problem of transient coupled thermoelasticity of an annular fin, Meccanica, Vol. 47, pp. 1295-1306 (2012).

[28] S.M. Abo-Dahab and B. Singh, Rotational and voids effects on the reflection of P waves from stress-free surface of an elastic half-space under magnetic field, initial Stress and without energy dissipation, Applied Mathematical Modeling, Vol. 37, pp. 8999-9011 (2013).

[29] M.C. Singh and N. Chakraborty, Reflection and refraction of P-, SV- and thermal waves, at an initially stressed solid-liquid interface in generalized thermoelasticity, Applied Mathematical Modeling, Vol. 37, pp. 463-475 (2013).

[30] A.M. Abd-Alla, S.M. Abo-Dahab and A.A. Kilany, SV-waves incidence at interface between solid-liquid media under electromagnetic field and initial stress in the context of three thermoelastic theories, Journal of Thermal Stresses, Vol. 39, No. 8, pp. 960-976 (2016).

[31] S.M. Abo-Dahab and Moustafa M. Salama, A plane magnetothermoelastic waves reflection and transmission between two solid media with external heat sources and initial stress, Journal of Thermal Stresses, Vol. 37, pp. 1124-1151 (2014).

[32] A.M. Abd-Alla, S.M. Abo-Dahab and R.D. El-Semiry, Effect of rotation on peristaltic flow of a micropolar fluid through a porous medium with an external magnetic field, Journal of Magnetism and Magnetic Materials, Vol. 348, pp. 33- 43 (2013).

[33] A.M. Abd-Alla, S.M. Abo-Dahab and H.D. El-Shahrany, Effects of rotation and initial stress on peristaltic transport of fourth grade fluid with heat transfer and induced magnetic field, Journal of Magnetism and Magnetic Materials, Vol. 349, pp. 268–280 (2014).

[34] E. Choi, M. Gurnis, Thermally induced brittle deformation in oceanic lithosphere and the spacing of fracture zones, Earth Planet. Sci. Lett., Vol. 269, No. (1-2), pp. 259-270 (2008).

[35] A. Nayfeh and S.N. Nasser, Thermoelastic waves in solids with thermal relaxation, Acta Mech., Vol. 12, pp. 53-69 (1971).

[36] Y. Song, H. Xu and Y. Zhang, Reflection and refraction of micropolar magneto-thermoviscoelastic waves at the interface between two micropolar viscoelastic media, International Journal of Thermophysics, Vol. 27, No. 3, pp. 970-993 (2006) .

[37] Mohamed I. Othman and Ibrahim A. Abbas, Generalized thermoelasticity of thermal-shock problem in a non-homogeneous isotropic hollow cylinder with energy dissipation, International Journal of Thermophysics, Vol. 33, No. 5, pp. 913-923 (2012).

[38] S.M. Abo-Dahab, A.M. Abd-Alla and A. Kilicman, Propagation of p - and T-waves at interface between magnetized solid-liquid media with initial stress in the context of CT theory, Journal of Mechanical Science and Technology, Vol. 29, No. 2, pp. 579˜591 (2015).

[39] S.M. Abo-Dahab and A. Kilicman, On reflection and transmission of p- and SV-waves phenomena at the interface between solid-liquid media with magnetic field and two thermal relaxation times, Journal of Thermal Stresses, Vol. 38, pp. 447-467 (2015).

[40] S.M. Abo-Dahab, Propagation of p-, T- and SV-waves at the interface between two solid-liquid media with magnetic field and initial stress in the context of two thermoelastic theory, Canadian Journal of Physics, Vol. 93, pp. 1–17 (2015).

[41] Samia M. Said and M.I.A. Othman, Effect of mechanical force, rotation and moving internal heat source on a two-temperature fiber-reinforced thermoelastic medium with two theories, Mechanics of Time-Dependent Materials, Vol. 21, pp. 245–261 (2017).

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