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On The Capacitated Step-Fixed Charge and Facility Location Problem: A Row Perturbation Heuristic

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Abstract: The classical Transportation Problem (TP) Tableau which utilizes continuous variable cost has been used to model and solve distribution problems. However, many real distribution problem decisions which require various combination of fixed and variable cost and having several mixed variables of the binary integers and continuous types make this approach limited. This challenge requires new integrated models that are also NP hard for which exact algorithms such as branch and bound, cutting plane algorithm may be inefficient to use as the problem size increases in practical business cases. We present in this paper, an integrated model of Facility Location (FL) and Step-Fixed Charge Transportation Problem (SFCTP). This problem is solved using a solution heuristic that utilizes relaxation and linearization approach to recast it to the classical TP as a starting solution. For the improved solution, a low cost and efficient perturbation heuristic that works in a row-wise manner is developed. We also propose a lower bound based on literature as a guide in achieving a solution. Lastly, a numerical example is presented to illustrate the procedures of the solution.

Keywords: Facility location, step-fixed charge, linearization and relaxation, row perturbation heuristic

1 Introduction

Decisions of different time horizons such as facility location, route selection and load consolidation are often encountered in distribution planning. While models exist in literature that supports making each of these decisions separately, there is a need to plan them in an integrated manner if global optimality is intended. The classical distribution model or transportation problem is an example of a single decision model which uses variable routing cost and is solved using the transportation tableau. However, this simplistic approximation may not be realistic in many business cases, and has prompted research into the area of transportation planning with fixed charges. These fixed charges are often incurred when siting facilities and during route selection planning for distribution. This means there is the need to plan both the location of the storage facilities like depots and warehouses in an integrated manner with the selection of route for the distribution of the materials, in which there are fixed charges along the route. There does not seem to have been many solutions provided for such problems.
The complexity of the problem may be further increased by considering the fact that in route planning today, there may be economy or dis-economy of scale in the consolidation of load. Such may be due to price break or volume discounts economies or long distance traffic dis-economy amongst others, and these are becoming more real on daily basis as supply chains become longer. This means we may be dealing with an integrated problem of facility location, load consolidation and vehicle routing with fixed charge and price breaks as a typical instance. This type of problem is not unusual in practice, and so, there is a need to develop a solution for such problem.

Integrated models and solutions such as the Fixed Charge Transportation Problem (FCTP) and the Step-Fixed Charge Transportation Problem (SFCTP) are examples of variants of the classical TP. These variants relax the entire linearity assumptions of the classical transportation cost objectives. [1] described the FCTP as one in which there is a variable cost and a fixed cost incurred for opening a transportation route with a shipment greater than zero. Also according to [2] in the analysis of another variant of the FCTP, known as the SFCTP, there can be more than one fixed cost incurred for opening a transportation route with the objective function behaving like a step function or a piece wise linear function.

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The FCTP has attracted a lot of research interests where the fixed charges in the problem statement are either at the source or along the routes. Models and solutions presented by [1,3,4,5,6] show that the fixed charges could occur either at the source or along the routes. Recently, new models and solutions have emerged with various variants of the FCTP such as in the work of [6,7,8,9]. The SFCTP variant of the FCTP is seen in the multi-item volume (or weight) transportation cost discounting models of [10] and volume discount on distribution cost of [11,12]. Research in the field of the SFCTP is growing as indicated by [2,13,14] with new problem-type models and solution techniques being continually developed. [5] gave practical applications such as increasing taxes due to high turnover or some increasing fees paid after attaining some user level. [14] extended the Fixed Charge Solid Transportation Problem (FCSTP) by presenting a lagrangian relaxation heuristic for the Step-Fixed Charge Solid Transportation Problem (SFCSTP) to solve large instances of the problem.

As indicated by [14] and [16] the FCTP has been argued to be an NP-hard problem. Similarly [14] has shown the SFCTP to be much harder to solve than the FCTP. This is quite logical due to the fact that the step function introduces more non- linearity into the objective function, making the problem much more difficult to solve. In order to reduce the expensive computations of exact methods such as the cutting plane algorithm, branch and bound for mixed integer and integer variable problems which could provide optimal solutions but inefficiently with increase in problem size, heuristics have been developed to solve the SFCTP. Although heuristics have the possibility of terminating quickly at a local optimum and giving suboptimal solutions as indicated by [2], in most instances their solutions are good and efficient [17,18,5] in solving the FCTP and SFCTP proceeded with good initial solutions through binary integer variables linearization and relaxation. Moreover, in the case of [5], their improved solution to the SFCTP is obtained through the use of a perturbation logic.

The cost of facility location is a type of fixed charge that FCTP problems and most importantly SFCTP problems have not been considered extensively. [19] presented an integrated step-fixed charge with facility location costs problem and referred to it as Capacitated Facility Location Problem (CFLP) with Piecewise Linear Transportation Cost (PLTC). While the fixed charges for both the FCTP and SFCTP are incurred as a result of the use of a route from a supply point, facility location costs are incurred due to siting or opening of facilities before any shipping are done through the routes [20]. Facility location fixed charge as described by [21] is therefore a longer horizon decision and different from the route selection fixed charge. Merging this with FCTP or SFCTP gives a problem that seeks to optimize both Facility Location Problem (FLP) and Step-Fixed Charge Transportation Problem (SFCTP) decisions together. This problem can be described as the Step-Fixed Charge Transportation and Location Problem (SFCTLP).

Facility location in itself is known to be NP hard, and so is vehicle routing and load consolidation, and so, it is expected that the problem considered would be NP hard. The usual approach, therefore, is to either simplify the problem through the transformation of the original problem into some more solvable approximation, or through the relaxation of some original constraints, or to solve the original problem using some heuristic, or in certain instances, have some combinations of all these approaches. These approaches are seen in integrated facility location models of [19,22,23].

This paper considers a heuristic to solve the SFCTLP, and is illustrated with a small hands-on problem size example. This is done in order to show an in-depth understanding of the workings of our solutions for the SFCTLP in a similar manner to [1,24]. The main objective in this article is about minimizing the traditional distribution problem cost of a source to destination where a minimum number of facilities with known capacities have to be chosen from amongst other competing capacitated facilities or locations with fixed location costs in order to ship an item through routes with step-fixed costs.

This SFCTLP emanates also as a variant of the SFCTP in like fashion as the SFCTP and SFCSTP. We proceed by discussing the formulation of the SFCTP and reviewing known starting initial solutions for solving it, before presenting the integrated model for the SFCTLP, followed by a solution heuristic and then a numerical example.

2 SFCTP Model Formulation

The classical TP, and its variants such as FCTP, SFCTP, SFCSTP are described as m suppliers and n demand point distribution problems, where m denotes the number of sources (factories, warehouses or distribution centers) and n refers to the number of customers or demand points. There are supply and demand requirements which often are represented as capacities S_i and demand D_j for each source i and demand point j respectively over a known time period. The m suppliers incur a unit transportation cost c_{ij} per unit distance and a fixed charge h_{ij} whenever a transportation route is opened (utilized for shipping) under constraints of supply capacity meeting a typical demand of transportation algorithm.

There are more than one set of fixed charges in the route (i, j) when step-fixed charges are considered. In the SFCSTP, the fixed charges are represented by the vehicle cost of conveying different volumes of the load. While, in SFCTP, the fixed charges may be incurred either through duties, taxes or vehicle costs of different volumes transported. The number of fixed charges depends on the number of break points in the step function desired. In this case, two steps of fixed charges, h_{ij1} and h_{ij2}, are considered without loss of generality. The fixed charge h_{ij1} is incurred when a route is opened and termed as

H_{ij1} in the objective function and the second h_{ij2} is incurred when the shipment load (or transported unit) exceeds an amount A_{ij}, and termed as H_{ij2} in the objective function also. A_{ij} is referred to as the break point and may be fixed or varying per route (i, j) depending on the model under consideration. When there is load distribution in any route i.e. $x_{ij} \geq 0$, h_{ij1} is incurred. While h_{ij2} is incurred when $x_{ij} \geq A_{ij}$.
The standard mathematical model for the SFCTP is represented below:
Min Z =

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\,x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} g_{ijk}h_{ijk} \quad (1)$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = S_i y_i \qquad \forall\ i = 1-m \quad (2)$$

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad \forall\ j = 1-n \quad (3)$$

$$\sum_{i=1}^{m} S_i y_i = \sum_{j=1}^{n} D_j \qquad \forall\ i = 1-m,\ \forall\ j = 1-n \quad (4)$$

Where $g_{ij1} = \begin{cases} 1 & x_{ij} > 0 \\ 0 & Otherwise \end{cases}$, $g_{ij2} = \begin{cases} 1 & x_{ij} > A_{ij} \\ 0 & Otherwise \end{cases}$

$y_i = 0\ or\ 1$

$x_{ij} \geq 0$

[2] noted that the solution methods of the SFCTP depend on the break point position i.e. if $A_{ij} < \min(S_i, D_j)$ or $A_{ij} \geq \min(S_i, D_j)$. If $A_{ij} \geq \min(S_i, D_j)$, the optimal solution to the SFCTP is an optimal solution to FCTP. The SFCTP solution heuristic of [5] and the SFCTLP heuristic presented in this paper work through building a relaxed cost matrix which are modifications of [18] relaxation for the FCTP. In our model the binary integer z_{ij} associated with the fixed charges h_{ij} (standard SFCTP model above) or H_{ij} (in our model below) is replaced by x_{ij}/M_{ij} where $M_{ij} = \min(S_i, D_j)$. Thus a relaxed cost matrix is formed. [5] followed in similar fashion to obtain a first relaxed cost matrix $C_{ij} = c_{ij} + \frac{h_{ij1} + h_{ij2}}{M_{ij}}$ or $C_{ij} = c_{ij} + \frac{H_{ij1} + H_{ij2}}{M_{ij}}$ and second relaxed cost matrix $C_{ij} = c_{ij} + \frac{h_{ij2}}{M_{ij}-A_{ij}}$ or $C_{ij} = c_{ij} + \frac{H_{ij2}}{M_{ij}-A_{ij}}$.
To improve their initial solution of SFCTP, [5] demonstrated that the number of basic variables for a near optimal solution of the SFCTP having two steps (or tiers) can be greater than $(m+n-1)$ that is traditionally expected for a classical TP. They considered a minimization model of the step-fixed charge problem and presented a numerical example to support their claim. They also noted that for a two tier or two step-fixed charge problem where load distribution x_{ij} is such that $x_{ij} \leq A_{ij}$ or $x_{ij} > A_{ij}$, perturbation moves would result in above or below A_{ij} distribution. This is quite logical as it expected that some optimal load values would occur at the break points. They also established that using the transportation problem would create solutions with $(m+n-1)$ or less to which a particular perturbation would be needed to redistribute the load units to take advantage of the fixed charges along the routes.
In Figure 1 and 2 below, the cost objective pattern with different fixed-cost values and the expected linearization as illustrated by [5] are shown.
[2] however showed the limitation of the second relaxed cost in the works of [5] when $M_{ij} \leq A_{ij}$ with C_{ij} (relaxed cost) not giving a positive result. They further proposed three formulas for calculating relaxed cost (C_{ij}) which are based on firstly whether $A_{ij} < M_{ij}$ or $A_{ij} \geq M_{ij}$, secondly on A_{ij} being included or not in the formula and thirdly on the number of $M_{ij} - A_{ij}$ shipments done. They used f_{ij1} and f_{ij2} as their route fixed cost in their formulas as represented below.
The first one is given as

$$C_{ij} = \begin{cases} c_{ij} + \frac{f_{ij1}}{M_{ij}} & if\ A_{ij} \geq M_{ij} \\ c_{ij} + \frac{f_{ij1}+f_{ij2}}{M_{ij}} & if\ A_{ij} < M_{ij} \end{cases} \quad \forall(i,j)\ \ 5(a)$$

The second given as

$$C_{ij} = \begin{cases} c_{ij} + \frac{f_{ij1}}{M_{ij}} & if\ A_{ij} \geq M_{ij} \\ c_{ij} + \frac{f_{ij2}}{M_{ij}-A_{ij}} & if\ A_{ij} < M_{ij} \end{cases} \quad \forall(i,j)\ \ 5(b)$$

The third given as $C_{ij} =$

$$\begin{cases} c_{ij} + \frac{f_{ij1}}{M_{ij}} & if\ A_{ij} \geq M_{ij} \\ c_{ij} + \frac{f_{ij2}}{A_{ij}} + \frac{f_{ij1}}{M_{ij}-A_{ij}} & if\ A_{ij} < M_{ij} \end{cases} \quad \forall(i,j)\ \ 5(c)$$

From their analyses they came to the conclusion that the first formula gave the best approximation when compared to [18] and [5] they also made suggestions as to using the other formulas as better starting solutions for the SFCTP.

3 SFCTLP Problem Structure and Formulation

Two methods are utilized for the linearization and relaxation of the initial solution development of SFCLTP. The first procedure is as described by [18] , [5] and [2] which employs the transportation model variable cost structure to form a relaxed cost matrix. As discussed in earlier sections, the position of A_{ij} i.e. $A_{ij} < M_{ij}$ or $A_{ij} \geq M_{ij}$ in developing the relaxed or reduced transportation cost matrix would have an effect on the SFCTP solution found. We also note that the break point position i.e. $A_{ij} < M_{ij}$ or $A_{ij} \geq M_{ij}$ for any problem involving a two tier fixed-charge cost on a route would affect the relaxation and perturbation pattern when seeking for a solution heuristic. Therefore, we have

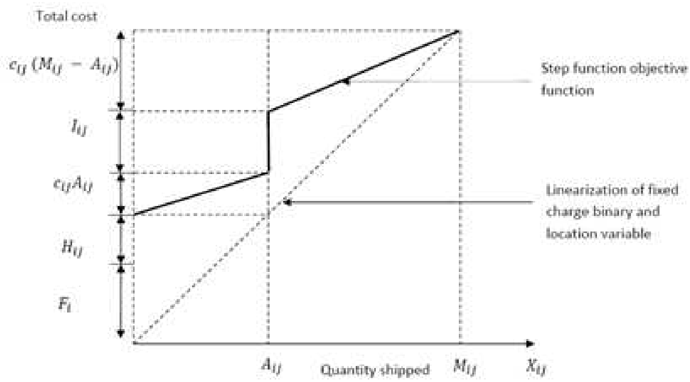

Fig. 1: Two-step linearization and relaxation Structure [5]

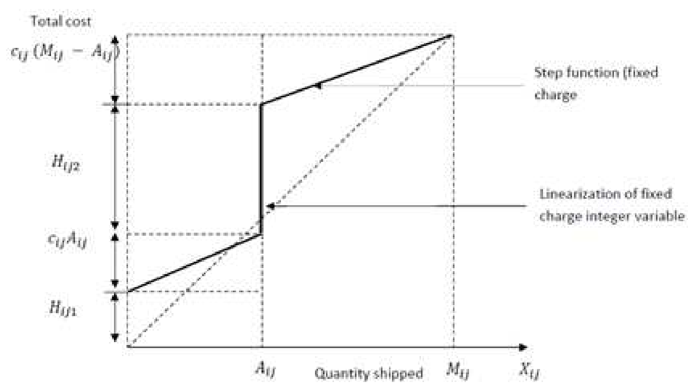

Fig. 2: Linearization and relaxation structure when $H_{ij1} < H_{ij2}$

extended the model of [18] , [5] and more importantly the second formula by Altassan, El-Sherbiny [2] by creating our starting SFCTP part of the problem using

$$C_{ij} = \begin{cases} c_{ij} + \frac{h_{ij1}}{M_{ij}} & if\ A_{ij} \geq M_{ij} \\ c_{ij} + \frac{h_{ij1} + h_{ij2}}{M_{ij} - A_{ij}} & if\ A_{ij} < M_{ij} \end{cases} \quad \forall (i,j) \ \ 5(d)$$

We have used a summation of route fixed costs $h_{ij1} + h_{ij2}$ or ($H_{ij1} + H_{ij2}$) instead of h_{ij2} (H_{ij2}) alone to account for incurring the fixed cost h_{ij1} (H_{ij1}) whenever a route is opened before h_{ij2} is incurred due to the break point A_{ij} . Also, we have used h_{ij} instead of f_{ij} used in the route fixed costs.

The second procedure develops an average relaxation method, as indicated by equation(14) below. This second method relaxes the location variable y_i by creating an average location variable value ${y_i}^a$ for all the competing locations. Through some perturbation techniques developed on the initial solution, better solutions are obtained.

Our problem is stated as the capacitated facility location problem with step-fixed Charges along the transportation routes i.e. Step-Fixed Charge Transportation and Location Problem (SFCTLP).

3.1 Model assumptions

We make the following assumptions in our model:

1.Deterministic input.
2.One stage or two echelon problem.
3.Two step-fixed charge cost.
4.Single period and single item distribution problem.

3.2 Model Parameters

i : Index for sources (plants, locations or rows).
m : Number of sources (plants , warehouses etc.).

n : Number of destinations (or demand point).
j : Index for demands (destinations or columns).
k : Index for levels or (number of steps).
c_{ij} : Unit cost of shipment on route (i, j).
S_i : Capacity for each location i.
h_{ij1} : First level fixed cost on route(i, j).
h_{ij2} : Second level fixed cost on route(i, j).
H_{ij1} : First level step-fixed cost based on load distribution.
H_{ij2} : Second level step-fixed cost based on load distribution.
x_{ij}: Allocation variable (or load distributions) along route (i, j).
y_i: Location variable for plant or source (0 or 1).
g_{ij1}: Step-fixed charge variable (determining first or second level of fixed cost).
z_{ij}: Fixed charge variable in the objective function (0 or 1).
A_{ij}: Break point for the fixed costs along the route (i, j).

3.3 Mathematical Model (Objective function and Constraints)

(Objective function) Minimize $Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m} F_i\, y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk}z_{ij}$ (6a)

Where:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij1} = \sum_{i=1}^{m}\sum_{j=1}^{n} g_{ij1}h_{ij1}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij2} = \sum_{i=1}^{m}\sum_{j=1}^{n} g_{ij2}h_{ij2}$$

$$\therefore \sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij1} + \sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij2} = \sum_{i=1}^{m}\sum_{j=1}^{n} g_{ij1}h_{ij1} + \sum_{i=1}^{m}\sum_{j=1}^{n} g_{ij2}h_{ij2}$$

Where:

$$g_{ij1} = \begin{cases} 1 & x_{ij} > 0 \\ 0 & Otherwise \end{cases}, g_{ij2} = \begin{cases} 1 & x_{ij} > A_{ij} \\ 0 & Otherwise \end{cases}$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk}z_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij1}\ z_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij2}\ z_{ij}$$

subject to (constraints):
$\sum_{j=1}^{n} x_{ij} \leq S_i y_i$
$\forall\ i = 1-m$ (7)
$\sum_{i=1}^{m} x_{ij} = D_j \quad \forall\ j = 1-n$ (8)
$\sum_{i=1}^{m} S_i y_i \geq \sum_{j=1}^{n} D_j \quad \forall\ i = 1-m,\ \forall\ j = 1-n$(9)
$x_{ij} \geq 0$ (10a)
$y_i = 0\ or\ 1\ z_{ij} = 0\ or\ 1$ (10b)
Equation 6(a) is the objective function. The first term is a variable cost, the second term is the facility location cost and third term is the route step-fixed charge cost. Equation (7) is the supply capacity constraint of each location or sources. Equation(8) is the demand constraint to be met. Equation (9) is the aggregate constraint for supply and demand balance. Equation (10a) refers to the non-negativity constraint while (10b) refer to the binary integer constraints.

4 Solution Method

Our solution method iterates through the steps and rules below in seeking for an improved solution:
Step 1: We develop an initial solution by linearization and relaxation of the binary variables (y_i *and* z_{ij}) in the model problem
Step 2: We calculate a lower bound for SFCTLP.
Step 3: Improve our initial solution through a structured perturbation procedure which we refer to as Row Perturbation Heuristic (RPH)
The RPH works through improving the initial solution method by iterating through the following well established procedures of moving to a good low cost solution in an efficient manner. [19] discussed how the use of some of the rules below can drive towards a reduced cost solution.

1. Least cost rule.
2. Utilization rule.
3. Fixed-cost elimination rule (Location fixed cost and route selection fixed cost).
4. Feasibility rule.

The heuristic uses the least cost rule to determine which sources to open and where to allocate capacities. Moreover, it allocates load units to reduce the number of fixed costs incurred i.e. facility location cost and route fixed costs by pushing load units to already open sources, closing unneeded locations in the process and also moving away from the higher tier fixed cost. Feasibility rule has been used to ensure capacity and demand constraints are satisfied during the load redistribution.

4.1 Initial Solution

This is achieved through the linearization and relaxation of integer (binary) variables i.e. the facility location y_i and fixed-charge selection z_{ij} variables. A Relaxed Transportation Problem (RTP) is thus formed as result.
Using the relaxation of integer variables described earlier:
Where: $z_{ij} = x_{ij}/M_{ij}$ and $M_{ij} = \min(S_i\ , D_j\ )$
Using equation (9), the minimum supply requirement implies that:

$$\sum_{i=1}^{m} S_i y_i \geq \sum_{j=1}^{n} D_j \qquad \forall\ i = 1-m,\ \forall\ j = 1-n \quad (11)$$

We develop a new location variable$y_i{}^a$, which is the average of $\sum_{i=1}^{m} y_i$ to help relax the location variable y_i. Thus equation (11) is restated as

$$\sum_{i=1}^{m} S_i\, y_i{}^a = \sum_{j=1}^{n} D_j \quad (12)$$

$$\therefore \sum_{i=1}^{m} S_i\, y_i{}^a = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \quad (13)\ \therefore$$

$$y_i{}^a = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}}{\sum_{i=1}^{m} S_i} \quad (14)$$

Substituting $y_i{}^a$ for y_i and $z_{ij} = x_{ij}/M_{ij}$ we transform equation (6a) as:
Minimize Z_{R1}=

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} + \sum_{i=1}^{m} F_i\, y_i{}^a + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk}\frac{x_{ij}}{M_{ij}} \quad (6b)$$

Substituting equation (14) in (6b) gives:
Minimize Z_{R1} =

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} + \sum_{i=1}^{m} F_i \left[\frac{\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}}{\sum_{i=1}^{m} S_i}\right] + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk}\frac{x_{ij}}{M_{ij}} \quad (6c)$$

$$Z_{R1} = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{\sum_{k=1}^{2} H_{ijk}}{M_{ij}}\right] x_{ij} \quad (6d)$$

Therefore

$$Z_{R1} = \sum_{i=1}^{m}\sum_{j=1}^{n} [C_{ij}]\ x_{ij} \quad (6e)$$

Where;

$$C_{ij} = c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{\sum_{k=1}^{2} H_{ijk}}{M_{ij}} \quad \forall (i,j) \quad (15)$$

However, considering the break point analyses we have made in section 3 earlier, equation (15) would be limited. Therefore using equation 5(d), equation 6(d) can further be stated as

$$Z_{R2} =$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1}}{M_{ij}}\right] x_{ij} \quad if\ A_{ij} \geq M_{ij} \quad \forall (i,j)$$

or

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1} + H_{ij2}}{M_{ij} - A_{ij}}\right] x_{ij} \quad if\ A_{ij} < M_{ij} \quad \forall (i,j) \quad (6f)$$

From equation (6f) above, the cost matrix from which the transportation tableau is constructed is given as:

$$C_{ij} = c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1}}{M_{ij}} \quad \forall (i,j) \quad if\ A_{ij} \geq M_{ij} \quad (16a)$$

or

$$C_{ij} = c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1} + H_{ij2}}{M_{ij} - A_{ij}} \quad \forall (i,j) \quad if\ A_{ij} < M_{ij} \quad (16b)$$

The linear Equation (6f) above can be solved using any optimal solution technique for transportation model (e.g. method of modified u-v distribution). This is present in optimization transportation software such as Tora.
The load distribution obtained from the relaxed cost Z_{R2} in equation (6f) is used in calculating Z in equation (6a) and would be termed the current best solution(Z^{CB}). After this, necessary perturbations following our rules are employed to arrive at another Z which is compared to the initial Z^{CB}. Comparing the values of Z^{CB} and Z , If $Z^{CB} \leq Z$ we keep (Z^{CB}) as the current best, otherwise i.e. $Z^{CB} > Z$, we term Z as the current best.

4.2 Lower bound calculations

Using equation 5(a) and our average location variable $y_i{}^a$ in equation(14) we have also extended[2] best starting solution for SFCTP. Our SFCLTP lower bound is thus calculated below.

$$Z_{LB} = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1}}{M_{ij}}\right] x_{ij} \quad if\ A_{ij} \geq M_{ij} \quad \forall (i,j)$$

or

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left[c_{ij} + \frac{\sum_{i=1}^{m} F_i}{\sum_{i=1}^{m} S_i} + \frac{H_{ij1} + H_{ij2}}{M_{ij}}\right] x_{ij} \quad if \quad A_{ij} < M_{ij}\ \forall (i,j)\ (6g)$$

4.3 Solution Improvement (using The RPH proposed):

The x_{ij} allocations obtained from the optimal solution of equation 6(f) is further perturbed using structured combinations of the least cost preference, high utilization of open locations and systematic elimination of fixed cost either by closing an open location or by preventing use of high fixed charge along the routes. The perturbation technique aims at getting a better solution while using the rules stated in section 4 as a guide. The Perturbation moves are a top-down load re-distribution along a column of each row to ensure that feasibility in demand is attained as described in the (4 × 4) transportation tableau in Figure 3 below.
S_i (location capacity) , D_i (Demand capacity). The last column represents relaxed cost summation along a row.
From equation 6(a) above, we observe that there are three cost terms in the objective function namely;

1.Variable cost $(Vc) = c_{ij}x_{ij}$.
2.Location or source fixed cost $(Lc) = (F_i y_i)$.
3.Step-fixed cost $(SFc) = (\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk}z_{ij})$.

The degree of the values obtained for each of the terms would determine the solution procedure and perturbation technique to be used. We therefore note the following scenarios out of several possible ones for our structured perturbation logic:
(a) $Vc \gg Lc\ and\ SFc$ (Variable cost having the largest value).

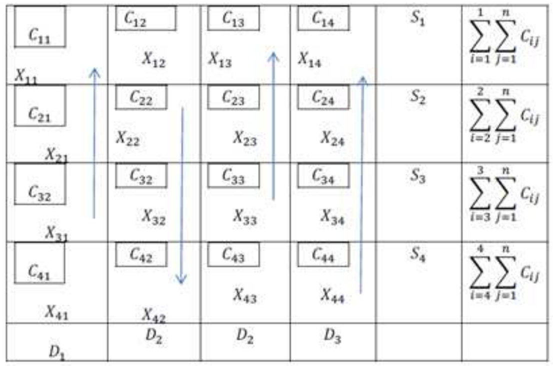

Fig. 3: Sample perturbation moves

(b) $Lc \gg Vc\ and\ SFc$ (Location cost having the largest value).
(c) $Sfc \gg Vc\ and\ Lc$ (Step-fixed cost having the largest value).
(d) $Vc \cong Lc \cong SFc$ (The three terms being approximately equal).
The summarized perturbation procedure is given below.

1. Using the linearization in 6(f) to obtain the starting solution and initial load distribution. The Z obtained is termed the current best (Z^{CB}).
2. (a) Calculate the values of the major terms of the objective function i.e. $Vc,\ Lc\ and\ SFc$
(b.1) If $Lc \gg Vc\ and\ SFc$ go to step (3),
(b.2) Else If $Sfc \gg Vc\ and\ Lc$ go to step (4).
(b.3) Else go to Step 1 and exit Procedure.
3. (a.1) For location cost reduction (Lc) if dummy rows are obtained from step (1)
(a.2) Yes: ignore row and capacity in calculation. Else go to Step (3b.1)
(b.1) Check if $\sum_{i=1}^{m} S_i y_i - \sum_{j=1}^{n} D_j \geq \min(S_{i=1\ \cdots m})$, excluding $i =\ dummy\ row$.
(b.2) If true proceed to 3c.1,
(b.3) Else if $\sum_{i=1}^{m} S_i y_i - \sum_{j=1}^{n} D_j = 0$ Stop and exit procedure. Return Z^{CB}
(b.4) Else go to Step (3h).
(c.1) Identify whether rows or locations with partially utilized capacities are available.
(c.2) Arrange in the order of decreasing $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}\quad for\ i = 1, 2...m$ where $C_{ij} \to$ relaxed cost matrix (Break ties arbitrarily and select largest.)
(d.1) Identify rows or locations with fully utilized capacities and arrange in the order of decreasing $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}\quad for\ i = 1, 2...m$ where $C_{ij} \to$ relaxed cost matrix (Break ties).If yes go to (3e.1). If none go to (3g).
(e.1) Is there an X_{ij} with maximum C_{ij} position according to the row identified in Step (3d.1) ?
(e.2) If Yes: Remove allocations starting with maximum C_{ij} position from open and allocated X_{ij} positions of the fully utilized rows as identified in 3(d.1) or (as per partially utilized row as in step (3g)) and add into position (i, j) of the partially utilized rows in a decreasing order of $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}\quad for\ i = 1, 2...m$ to balance the row capacity (break ties as step3c).
(e.3) If No: maximum position has no load then move to next in rank of C_{ij}, (break ties arbitrarily)
(f) Repeat step 3(e) until allocations have been completely removed in the fully or partially utilized row identified as per step (3e). Go to Step(3h).
(g) Arrange the partially utilized location or row capacity in an order of decreasing $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}\quad for\ i = 1, 2...m$, (break ties as in 3c) and select Maximum. Repeat Step (3e) to (3f).
(h.1) Use the current load distribution to calculate $Z(new)$. Compare the values of Z^{CB} and $Z(new)$
(h.2) If $Z(new) < Z^{CB}$ we term Z as the current best and go to Step (1).
(h.3) If otherwise i.e. $Z\ (new) > Z^{CB}$, Stop and exit procedure.
4. (a.1) For the Step-fixed charge cost reduction, check if any dummy rows?

(a.2) Delete any dummy rows or un-utilized locations obtained in step 1.
(b.1) Identify if rows or locations with partially utilized capacities are available.
(b.2) Arrange in the order of increasing $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}\quad for\ i = 1, 2...m$ where $C_{ij} \to$ relaxed cost matrix (Break ties arbitrarily).
(c) Identify rows and locations with fully utilized capacity and arrange in the order of increasing $\sum_{i=m}^{m}\sum_{j=1}^{n} C_{ij}\quad for\ i = 1, 2...m$. (Break ties as in 4b). If none go to (3b).
(d.1) Check If there are open and allocated x_{ij} positions greater than A_{ij} within the largest row as identified by step (4c) at maximum C_{ij} position?
(d.2) No: If maximum position has no load move to next in rank of C_{ij}

(d.3) Yes: Check if un-allocated positions x_{ij} of the row as identified by step (4b.1) can accommodate move.
(d.3.1) No : If current capacity cannot accommodate the reallocation, move to the next ranked partially utilized capacity row according to (4b.1) Proceed till the identified x_{ij} position in step (4d.1) or (4g) has been redistributed in a single step of A_{ij}. If no partially utilized row with availability go to Step (3b).
(d.3.2) Yes: Redistribute (A_{ij}) identified at (4.d.1) starting with the x_{ij} with at maximum C_{ij} position
(e) Repeat Step (4b) to (4d) until moves already taken are about to be repeated or till a position $x_{ij} - A_{ij}$ after using step (4d or 4g) becomes x_{ij} . Use the current load distribution to calculate Z . go to Step (3b).

4.4 RPH Flow Chart description

A flow chart showing the perturbation steps described above and how they iterate to improve the starting solution is presented in Figure 4 below. The flow chart symbols utilized have the same meaning as standard flow chart symbols.
RPH iterative procedure as shown in the flow chart uses the initial solution to determine quickly whether location fixed-cost elimination or upper tier route fixed-cost elimination would be appropriate to achieve an overall cost reduction. The load redistribution using the order of decreasing $\sum_{i=m}^{m}\sum_{j=1}^{n}C_{ij}$ from the fully-allocated routes during Location fixed-cost elimination, aims to reduce cost from high cost arcs or routes. Furthermore, reducing cost by value A_{ij} from maximum C_{ij} position at Step 4 prevents incurring upper tier route fixed cost. Also, load redistribution into locations with increasing order of $\sum_{i=m}^{m}\sum_{j=1}^{n}C_{ij}$ in step 4 ensures lower cost routes are utilized before higher ones. The flow chart also has the capacity to quickly arrive at a current best solution depending on the problem structure encountered while checking the condition $\sum_{i=1}^{m}S_i y_i - \sum_{j=1}^{n}D_j = 0$ and using the exit procedure of Step (2b.3).

5 Numerical Example

Given the supply and demand capacities, unit costs and fixed charges as in the Table1 and 2 below (adapted from [5]),we illustrate the workings of RPH.
The break point $A_{ij} = 5$ $(constant)$ through route i, j.
From equation 6(a) to 10(b) we note that;
If $x_{ij} > 0$ and ≤ 5 , $g_{ij1} = 1$, $g_{ij2} = 0$, and $z_{ij} = 1$, $therefore$;

$$H_{ij1}\ z_{ij} + H_{ij2}\ z_{ij} = g_{ij1}h_{ij1}z_{ij} + g_{ij2}h_{ij2}z_{ij} = (1) \times h_{ij1} \times (1) + (0) \times h_{ij2} \times (1) = H_{ij1}\ z_{ij}$$

If $x_{ij} > 0$ $and > 5$, $g_{ij1} = 1$ and $g_{ij2} = 1$, $z_{ij} = 1$, $therefore$;

$$[H_{ij1}\ z_{ij} + H_{ij2}\ z_{ij} = g_{ij1}h_{ij1} + g_{ij2}h_{ij2} = (1) \times h_{ij} \times (1) + (1) \times h_{ij2} \times (1) = H_{ij1}\ z_{ij} + H_{ij2}\ z_{ij}$$

If if $x_{ij} = 0$, $z_{ij} = 0$, $therefore$ $H_{ij1}\ z_{ij} + H_{ij2}\ z_{ij} = 0$

For $(i,\ j)$ position $(1,1)$, $M_{11} = 10$, and $A_{11} = 5$ thus $A_{11} < M_{11}$
From equation (16a and 16b) above, equation 16b is selected.

$$C_{11} = \left[c_{11} + \frac{F_1+F_2+F_3+F_4}{S_1+S_2+S_3+S_4} + \frac{h_{111+}\ h_{112}}{M_{11}-A_{11}}\right]$$

$$C_{11} = \left[1 + \frac{100+200+250+150}{25+25+25+25} + \frac{10+20}{10-5}\right] = 14$$

For all $(i,\ j)$ position, $A_{ij} < M_{ij}$
Equation 16b is selected for $C_{11},\ C_{12}\ldots C_{mn}$
C_{ij} relaxed cost matrix for $C_{11},\ C_{12}\ldots C_{mn}$ is given Table 3 below.

5.1 Initial solution

Tora optimization software which uses the modified u-v distribution method of solving linear transportation models is used to solve the cost matrix above (as a balanced problem) optimally to give the initial solution of the SFCLTP (Z_{R2}) represented in Table 4 below.
For our lower bound value for SFCLTP (Z_{LB}), the cost matrix below is obtained from the relaxed unit cost in equation (6g) like the relaxed costs of (16a and 16b). The load distributions after solving optimally with Tora software are presented in Table 5 below.
Using equation (6f) above;

$$Z_{R2} = (14 \times 5) + (14 \times 5) + (10.5 \times 5) + (10 \times 25) + (10.67 \times 20) + (12 \times 15) = 835.9$$

Using equation (6e) for our lower bound calculation,

$$Z_{LB} = (11 \times 10) + (10.2 \times 5) + (9.6 \times 25) + (10 \times 5) + (10 \times 15) + (11 \times 15) = 766$$

Using the load distribution for both Z_{R2} and Z_{LB} and equation (6a) for calculating Z for Z_{R2} and Z_{LB} which is represented as Z (Z_{R2}) and Z (Z_{LB}) respectively,

$$\text{Z}\ (Z_{R2}) = (14 \times 5) + (14 \times 5) + (10.5 \times 5) + (10 \times 25) + (10.67 \times 20) + (12 \times 15) + (100+200+250+150) + (10+10+10+40+40+30) = 935$$

$$\text{Z}\ (Z_{LB}) = (1 \times 10) + (2 \times 5) + (1 \times 25) + (1 \times 15) + (1 \times 5) + (2 \times 15) + (100+200+250+150) + (10+30+10+40+40+30) = 955$$

Therefore our current best solution (Z^{CB}) for the SFCLTP Z $(Z_{R2}) = 935$ with a lower bound $Z_{LB} = 766$

5.2 Improved solution (Using RPH)

In order to apply our RPH solution heuristic, the initial solution Z_{R2} matrix is labelled row and column wise as in Table 6 below.
From section 4.2 and using our initial solution Z (Z_{R2}) obtained in 5.1 we note that optimum objective function cost and current best ($\text{Z}^{CB}) = 935$.
Step (1) Current best ($\text{Z}^{CB}) = 935$.
Step (2a) Variable cost (Vc) = $c_{ij}x_{ij}$ = 95
Location or source fixed cost (Lc) = ($F_i y_i$) = 700
Step fixed cost (SFc) = ($\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2}H_{ijk}z_{ij}$) = 140

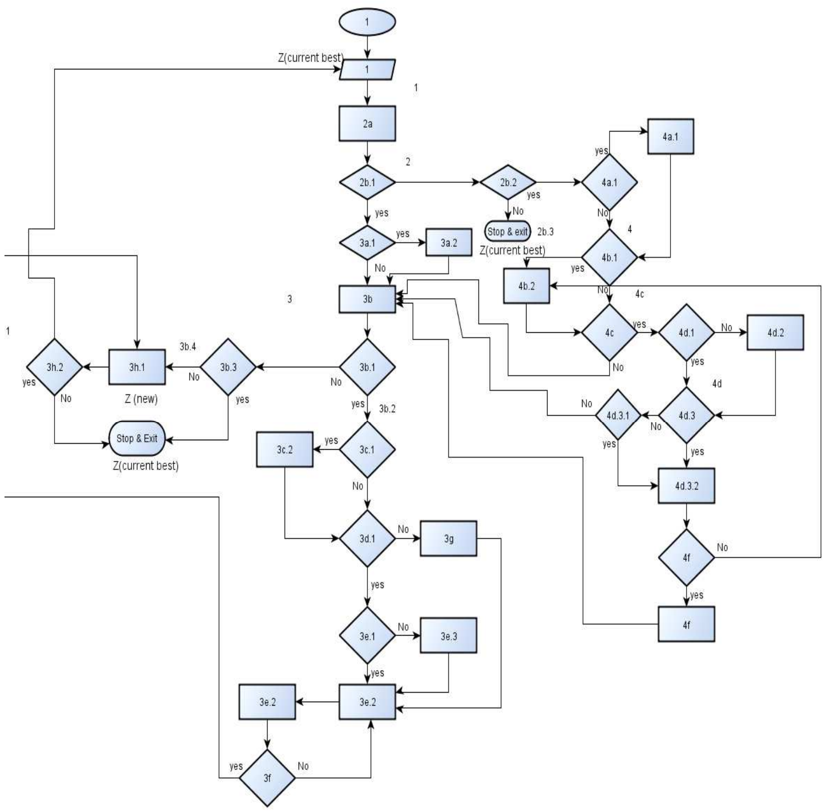

Fig. 4: Flow chart on row perturbation heuristic improving initial solution

Table 1: Supply, demand, location (set up) costs and unit cost parameters

i	S_i	F_i	j = 1	2	3	4
			c_{ij}			
1	**25**	**100**	1	3	1	3
2	**25**	**200**	2	2	3	2
3	**25**	**250**	2	1	2	1
4	**25**	**150**	1	3	1	3
D_j			**10**	**30**	**20**	**15**

Table 2: Two tier fixed charges on route i, j

i	h_{ij1} , h_{ij2}	h_{ij1} , h_{ij2}	h_{ij1} , h_{ij2}	h_{ij1} , h_{ij2}
1	10 ; 20	10 ; 10	10 ; 30	10 ; 10
2	10 ; 30	10 ; 20	10 ; 20	10 ; 20
3	10 ; 20	10 ; 30	10 ; 10	10 ; 30
4	10 ; 20	10 ; 10	10 ; 30	10 ; 10
	***j* =1**	**2**	**3**	**4**

Table 3: C_{ij} Relaxed cost matrix

14	11	10.67	12
17	10.5	12	12
15	10	10.33	12
14	11	10.67	12

Table 4: Optimal load distribution using the relaxed cost matrix

14 **5**	11	10.67	12	0 **20**	25
17	10.5 **5**	12	12 **15**	0 **5**	25
15	10 **25**	10.33	12	0	25
14 **5**	11	10.67 **20**	12	0	25
10	30	20	15	25	

Step(2b.1) Therefore since $Lc \gg Vc$ *and* $SFc \rightarrow$ Step 3

Step(3a.1) No dummy rows $\rightarrow$ Step (3b.1)

Step(3b.1) Check $\sum_{i=1}^{m} S_i y_i - \sum_{j=1}^{n} D_j \geq \min(S_{i=1\ \dots m})$ *i.e* $100 - 75 = 25$ $\rightarrow$ Step (3c.1)

Step(3c.1) Row 1 and Row 2 are partially utilized$\rightarrow$ Step (3c.2).

Step(3c.2) Arranging in decreasing $\sum_{i=m}^{m} \sum_{j=1}^{n} C_{ij}$ $for\ i = 1,2 \dots m$. In decreasing order we have(Row 2, Row1). Row 2 selected as having largest $\sum_{i=m}^{m} \sum_{j=1}^{n} C_{ij}$

Step(3d.1) Row 3 and Row 4 are fully utilized.
In order of decreasing $\sum_{i=m}^{m} \sum_{j=1}^{n} C_{ij}$ $for\ i = 1,2 \dots m$ (Row4, Row3).
Row 4 is selected $\rightarrow$Step (3e.1).
The Row selection is shown in Table 7.

Step (3e.1) Row 4, has $X_{41} = 5$ at largest $C_{ij} = C_{43}$ and also load at $X_{43} = 20$ $\rightarrow$Step (3e.2).

Step (3e.2) Remove allocation at $X_{41} = 5$ and add to position X_{21}. Remove allocation at $X_{43} = 20$ also, but no capacity to accommodate move at position X_{21}. Row 1 is selected next in the decreasing order to receive $X_{43} = 20$ $\rightarrow$Step (3f).

Step (3f) Allocations have been fully removed go to $\rightarrow$Step (3h).

Step (3h.1) While current best $Z^{CB} = 935$ New load distribution is given in Table 8 below.

$$Z\,(new) = (1 \times 5) + (2 \times 5) + (2 \times 5) + (1 \times 25) + (1 \times 20) + (2 \times 15) + (100 + 200 + 250) + (10 + 10 + 10 + 40 + 40 + 30) = 790$$

Step (3h.2) $Z(new) < Z^{CB}$ we term $Z(new)$ as the current best and go to Step(1).
Step (1) Current best (Z^{CB}) $= 790$.

Table 5: Optimal load distribution for lower bound determination

11 **10**	10.8	10 **15**	11.33	0 **20**	25
13	10.2 **5**	11.5	11 **15**	0 **5**	25
12	9.6 **25**	10	10.67	0	25
11	10.8	10 **5**	11.33	0	25
10	30	20	15	25	

Table 6: Row and column labelling of initial solution to apply RPH

Column1	Column 2	Column 3	Column 4	Dummy		A	
14 **5**	11.33	10.67	12	0 **20**	25	47.67	Row1
17	10.5 **5**	12	12 **15**	0 **5**	25	51.5	Row2
15	10 **25**	10.33	12	0 0	25	47.33	Row3
14 **5**	11	10.67 **20**	12	0 0	25	47.67	Row4
10	30	20	15	25			

Table 7: Row selection for perturbation

14 **5**	11.33	10.67	12	47.67	25	Partially utilized
17	10.5 **5**	12	12 **15**	51.5	25	Partially utilized & selected
15	10 **25**	10.33	12	47.33	25	Fully utilized
14 **5**	11	10.67 **20**	12	47.67	25	Fully utilized & selected
10	30	20	15			

Table 8: Load distribution after Applying RPH

14 **5**	11.33	10.67 **20**	12	25
17 **5**	10.5 **5**	12	12 **15**	25
15	10 **25**	10.33	12	25
14	11	10.67	12	25
10	30	20	15	

Step (2a) Variable cost $(Vc) = c_{ij}x_{ij} = 100$

Location or source fixed cost $(Lc) = (F_i y_i) = 550$

Step fixed cost $(SFc) = (\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{2} H_{ijk}z_{ij}) = 140$

Step(2b.1) Therefore since $Lc \gg Vc$ *and* $SFc \rightarrow$ Step (3).

Step(3a.1) Dummy row at Row 4.→ Step(3a.2).

Step(3a.2) Ignore row in calculation of capacity. → Step(3b.1).

Step(3b.1) Check $\sum_{i=1}^{m} S_i y_i - \sum_{j=1}^{n} D_j \geq \min(S_{i=1 \cdots m})$ *i.e* $75 - 75 = 0$ →Step (3b.2).

Step(3b.2) Check $\sum_{i=1}^{m} S_i y_i - \sum_{j=1}^{n} D_j = 0$ i.*e* $75 - 75 = 0$

Stop and exit procedure

Return Z ($Z^{CB}) = 790$

Therefore Z (RPH) = 790

6 Discussion Of Solutions Obtained

For the numerical examples in section 5.1 above, using the recast/ relaxed cost matrix as stated in equation 16(a) and 16(b) Z_{R2} = 835.9. Also, our lower bound calculation from equation (6g) gives $Z_{LB} = 766$. In this example, the relaxed value i.e. Z_{R2} gives an upper bound to the objective function $Z(Z_{R2}) = 935$ obtained by equation (6a). From Figures1 and 2, we note that the ideal relaxed cost matrix is linear in the objective function and should give a lower bound to the SFCLTP objective function. Furthermore, there could be instances where the relaxation type used could give an upper bound at the break point as seen in Figure 2. Our lower-bound Z_{LB} gives the minimum out of Z_{LB} , Z_2, $Z(Z_{LB})$ and $Z(Z_{R2})$. However, the load distribution of Z_{R2} gives better starting solution for our RPH .

The starting solution for our numerical example $Z(Z_{R2})$ is 935. However, our solution heuristic gives an improved objective value Z (RPH) = 790 with all constraints satisfied. Using our RPH solution in section 5.1, location 4 (or Row 4) among the other competing locations with equal supply capacities but different set up costs is closed as unprofitable for shipping through the fixed charges and transportation costs. RPH thus uses a structured combination of fixed-location cost elimination, cheap route variable cost and load consolidation at lower tier route fixed cost to drive towards an improved solution while also ensuring feasibility of all constraints are satisfied.

7 Perspective

An integrated model that combines the fixed location cost and step-fixed charge transportation cost has been proposed in this paper. We have termed it Step-Fixed Charge Location and Transportation Problem (SFCLTP). In this model, the step-fixed charge transportation problem of [18] has been extended. Moreover, the linearization and relaxation method developed by [5] and [1] have been extended using the normal transportation tableau as a starting solution. Through a perturbation technique that uses the variable transportation cost, fixed facility location cost, and step-fixed charge cost along the selected route in deciding the perturbation moves, we progressively obtain better solutions than the optimal solution obtained from the relaxed transportation problem. These solutions are considered good enough, and we have termed the heuristic Row Perturbation Heuristics (RPH).

Future directions on our model could be on applying single solution metaheuristics such as simulated annealing, Tabu search or population metaheuristics such as genetic algorithm, particle swarm optimization to evaluate the relative effectiveness and efficiency of RPH to these metaheuristics. Lastly, initial solutions that do not use the relaxation and linearization which we have employed and better improvement solutions for SFCLTP could be investigated on.

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