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Gravity Effect on The Dual-Phase-Lag Model for Plane Waves of a Fiber-Reinforced Micropolar Thermoelastic Medium in Contact with Newtonian Inviscid Fluid

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Abstract: In this article, we study the general equations of the dual-phase-lag (DPL) model and Lord-Shulman(LS) theory with one relaxation time considering the influence of micro-polar fiber-reinforced on the totally reformed 2D-half-space with gravity. The obtained non-dimensional coupled field equations for the DPL are solved using the harmonic wave analysis technique.Also, the exact expressions for some physical fields and tangential couple stress are obtained under the effect of mechanical forces along the interface of the fluid half space and the fiber-reinforced micro-polar thermoelastic half space. The effect of micro-polarity on the displacement component, force stress, temperature distribution and tangential couple stress, in presence and absence of the gravity field effect, has been depicted graphically.

Keywords: couple stress, gravity, micro-polar thermoelasticity, reinforced composites, micro rotation.

1 Introduction

The dynamical interaction between thermal and mechanical waves in thermo-elastic medium has many important applications in modern physical engineering as aeronautics, astronautics, nuclear reactors and high-energy particle accelerators. Also wave propagation in a reinforced media plays a very interesting role in civil engineering and geophysics. Moreover studying waves propagation, reflection and transmission are very important to seismologists. Such studies give researchers the ability to obtain valuable knowledge about rocks structure, elastic properties and the required information to detect minerals and fluids inside the earth. The model of introducing a continuous self-reinforcement at every point of an elastic solid was first given by Belfied et al. [1]. Later, Verma and Rana [2] applied this model to the rotation of fibre reinforced circular cylindrical tube. Also,Verma [3] discussed the magneto elastic shear waves in self-reinforced bodies. Singh [4] studied the wave propagation in fibre-reinforced anisotropic media and proved that this decoupling cannot be achieved if the displacement potential is introduced. Sengupta and Nath [5] discussed the problem of the surface waves in fibre-reinforced anisotropic elastic media. Many authors have described the elastic moduli for fibre-reinforced materials [6,7,8,9,10] Furthermore, In case of studying the response of materials to external stimuli,the micropolar elastic model gives more realistic results than the purely elastic theory. More significant developments and studies of the general theory of linear micropolar elasticity are achieved by Eringen [11,12,13] and Nowacki [14,15,16] As solids undergo macro-deformations and micro-rotations can completely characterized by the displacement vector u(x, t) and the rotation vector (x, t), in case of classical elasticity; the motion is characterized by the displacement vector only. Nowacki [17], Eringen [13,14,15,16,17] Tauchert [18] and Nowacki and Olszak [19]developed the micropolar theory to include thermal effects (see also [20,21,22]). Because of its dependence on the classical Fourier's law, the classical theory of elasticity fails to describe the behavior of materials with internal structure. Lord and Shulman (LS) [23] generalized the thermoelasticity theory with one relaxation time parameter (single-phase-lag model) by postulating a new law of heat conduction instead of the classical Fourier's law.Green and Lindsay (GL) [24] generalized the theory of

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thermoelasticity involving two thermal relaxation times. Many authors used these theories in their research [25-26]. Considering interactions between photons and electrons on the microscopic level as retarding sources, Tzou [27,28] has developed a new model known as the dual phase-lag model (DPL). The DPL model is more convenient for studying and investigating the micro-structural effect on heat transfer behavior when macroscopic formulation is used. Tzou [28] supported the physical meanings and applications of the DPL model by considering experimental results. The modification of the classical thermoelastic model proposed by Tzou [29] depends on replacing Fourier law by approximate values of the modified Fourier's law with two different times (phase-lag of the heat flux and a phase-lag of temperature gradient). Many authors [30,31,32] used the harmonic wave method with thermal relaxation times to study the wave propagation between two interfaced thermoelastic medium. Abbas et al. [33] applied the generalized thermoelasticity model proposed by dual phase lag (DPL), to study the thermoelastic interactions in an infinite fiber-reinforced anisotropic medium with a circular hole. Hobiny and Abbas [34] used the eigenvalue approach to analytically investigate the solution of a fiber-reinforced anisotropic material under generalized magneto-thermoelastic theory. Studying the interaction of elastic waves with fluid loaded solids has been recognized as a valuable means, since the reflected acoustic field from a fluid solid interface has a wealth of information, which reveals details of many characteristics of solids. This kind of study is conducted for a wide variety of solids extending from the simple isotropic semi-space to the more complicated systems of multilayered anisotropic media [35,36,37]. In this article we study the effect of gravity field on a micropolar fibre-reinforced thermo-elastic medium subjected to mechanical force.Also use the harmonic wave analysis to obtain the physical quantities of the problem. Finally, introduce comparison between the considered variables as calculated from the generalized thermoelasticity based on the influence of gravity,generalized Lord and Shulman (L-S)theory and the dual-phase-lag (DPL).

2 Formulation of the problem basic equations

The constitutive equations for a linear fibre-reinforced elastic anisotropic medium with respect to the reinforcement direction **a** are:

$$\sigma_{ij} = \lambda\, e_{kk}\, \delta_{ij} + 2\,\mu_T\, e_{ij} + \alpha(a_k\, a_m\, e_{km}\, \delta_{ij} + a_i\, a_j\, e_{kk}) + 2(\mu_L - \mu_T)(a_i\, a_k\, e_{kj} + a_j\, a_k\, e_{ki}) + \beta\, a_k\, a_m\, e_{km}\, a_i\, a_j - \beta_{ij}\,(1 + \tau_\theta \frac{\partial}{\partial t})(T - T_0)\delta_{ij}. \quad (1)$$

The strains in terms of the displacements u_i are:

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \quad (2)$$

The heat conduction equation is

$$k(1 + \tau_\theta \frac{\partial}{\partial t})T_{ij} = \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2}\right) p\, C_E\, T + T_0\, \beta_{ij} \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2}\right) u_{ij}, \quad (3)$$

where σ_{ij} are components of the stress; e_{ij} are the components of strain, λ and μ_T are the elastic constants, α, β and $(\mu_L - \mu_T)$ are the reinforcement parameters, and $\mathbf{a} \equiv (a_1, a_2, a_3)$ with $a_1^2 + a_2^2 + a_3^2 = 1$. we choose the fibre-direction **a** as $\mathbf{a} \equiv (1,0,0)$. T and T_0 are the absolute temperature and reference temperature respectively. k is the thermal conductivity of the sample, ρ is the density of the fibre reinforced micropolar solid, C_E is the specific heat at constant strain and δ_{ij} is the well known Kroneker delta. $\tau_\theta, \tau_q (0 \leq \tau_\theta < \tau_q)$ are the phase-lag of temperature gradient and the phase-lag of heat flux respectively. when $\tau_\theta = 0$, the governing equations are reduced to the LS theory.

In case of plane deformation, where displacement vector $\bar{u} = (u, v, 0)$ and the micro-rotation vector $\bar{\varphi} = (0, 0, \varphi_3)$, the constitutive equations (1)can be written as:

$$\sigma_{xx} = A_{11} u_{,x} + A_1 2 v_{,y} - \beta_{11}(1 + \tau_\theta \frac{\partial}{\partial t})(T - T_0), \quad (4)$$

$$\sigma_{yy} = A_{22} v_{,y} + A_{12} u_x - \beta_{22}(1 + \tau_\theta \frac{\partial}{\partial t})(T - T_0), \quad (5)$$

$$\sigma_{zz} = A_{12} u_, x + \lambda v_{,y} - \beta_{33}(1 + \tau_\theta \frac{\partial}{\partial t})(T - T_0), \quad (6)$$

$$\sigma_{xy} = \mu_L(u_{,y} + v_{,x}) - k\varphi_3, \sigma_{yx} = \mu_L(u_{,y} + v_, x) + k\varphi_3, \sigma_{zx} = \sigma_{zy} = 0. \quad (7)$$

where,

$$A_{11} = \lambda + 2(\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta$$

$$A_{12} = \lambda + \alpha, A_{22} = \lambda + 2\mu_T$$

$$\beta_{11} = (2\lambda + 3\alpha + 4\mu_L - 2\mu_T + \beta)\alpha_1 + (\lambda + \alpha)\alpha_2\ ,$$

$$\beta_{22} = (2\lambda + \alpha)\alpha_1 + (\lambda + 2\mu_T)\alpha_2$$

and α_1 , α_2 are the linear thermal expansion coefficients.

Consider the Cartesian system of coordinates (x, y, z) with origin at $z = 0$. Let a normal force of magnitude P_1 acting along the interface of fibre reinforced micropolar thermoelastic medium (medium I) occupying the region $0 \leq x < \infty$ and a non viscous fluid (medium II) occupying the region $-\infty < x \leq 0$ as shown in figure 1.

According to the generalized thermoelasticity , ignoring gravity effect, the equations of motion are:

$$\rho \left(\frac{\partial^2 u_i}{\partial t^2}\right) = \sigma_{ij,j}, (i, j = 1, 2, 3). \quad (8)$$

Also, the field equations of micropolar generalized thermoelastic medium are:

$$j\rho \frac{\partial^2 \varphi_i}{\partial t^2} = (\alpha_1 + \beta_1 + \gamma_1)\varphi_{j,ji} + \gamma_1 \varphi_{i,jj} + k_1 \varepsilon_{ijr} u_{r,j} - 2k_l \varphi_i. \quad (9)$$

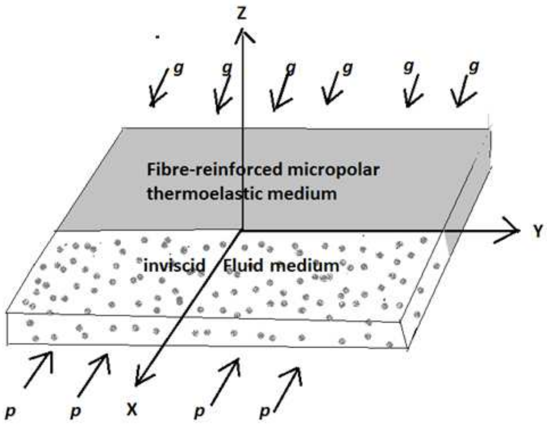

Fig. 1: Physical model and coordinate system.

$$m_{il} = \alpha_1 \varphi_{r,r}\delta_{il} + \gamma \varphi_{l,i}, \tag{10}$$

where k_l, α_l, β_l and γ_l are the material constants, j is micro-inertia and m_{il} is the couple stress tensor. A comma followed by a suffix denotes material derivative and a superposed dot denotes derivative with respect to time. The equations of motion and stress components for an inviscid fluid are:

$$\lambda^f u^f_{j,ji} = \rho^f \frac{\partial^2 u_i^f}{\partial t^2} \tag{11}$$

$$\sigma^f_{ij} = \lambda^f u^f_{r,r}\delta_{ij}, \tag{12}$$

where, u_i^f is the displacement vector components, λ^f is the bulk modulus and ρ^f is fluid density, $\frac{\lambda^f}{\rho^f} = c_f^2$ represents the velocity of acoustic fluid where c_f is the velocity of sound in liquid.

From (4)-(7), we note that the third equation of motion in (8) is identically satisfied and the first two equations with rotation under the influence of gravitational field are:

$$\rho\left(\frac{\partial^2 u}{\partial t^2}\right) = A_{11}\frac{\partial^2 u}{\partial x^2} + B_2\frac{\partial^2 v}{\partial x\partial y} + B_1\frac{\partial^2 u}{\partial y^2} + \rho g\frac{\partial v}{\partial x} + k\frac{\partial \varphi_3}{\partial y} - \beta_{11}\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}, \tag{13}$$

$$\rho\left(\frac{\partial^2 v}{\partial t^2}\right) = A_{22}\frac{\partial^2 v}{\partial y^2} + B_2\frac{\partial^2 u}{\partial x\partial y} + B_1\frac{\partial^2 v}{\partial x^2} - \rho g\frac{\partial u}{\partial x} - k\frac{\partial \varphi_3}{\partial x} - \beta_{22}\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}, \tag{14}$$

where $B_1 = \mu_L$, $B_2 = \alpha + \lambda + \mu_L$.

The field equations (9),(10) and (3) of micropolar generalized thermoelastic medium become:

$$\gamma_l \nabla^2 \varphi_3 + k_l\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - 2k_l\varphi_3 = j\rho\frac{\partial^2 \varphi_3}{\partial t^2}, \tag{15}$$

$$m_{yz} = \gamma_l \frac{\partial \varphi_3}{\partial y}, \tag{16}$$

$$k\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\nabla^2 T = \rho C_E\left(1+\tau_q\frac{\partial}{\partial t}\right)\dot{T} + \left(1+\tau_q\frac{\partial}{\partial t}\right)\left(\beta_{11}\frac{\partial u}{\partial x} + \beta_{22}\frac{\partial v}{\partial y}\right). \tag{17}$$

In the following we introduce the non-dimensional variables for convenience, where

$$\begin{cases} x' = c_1\eta x, y' = c_1\eta y, u' = \frac{c_1\eta}{\beta_{11}}u, v' = \frac{c_1\eta}{\beta_{11}}v, t' = c_1^2\eta t, \\ \tau_\theta' = c_1^2\eta\tau_\theta, g' = \frac{g}{c_1^3\eta}, \theta = \gamma\frac{(T-T_0)}{\lambda+2\mu_T}, \tau_q' = c_1^2\eta\tau_q, \\ m_{ij}' = \frac{\eta m_{ij}}{\beta_{11}\rho c_1}, \sigma_{ij}' = \frac{\sigma_{ij}}{\beta_{11}\mu_T}, \\ \sigma_{ij}'^{f} = \frac{\sigma_{ij}^f}{\beta_{11}\mu_T}, \varphi_3 = \beta_{11}\varphi_3', \\ i,j = 1,2,3. \end{cases} \tag{18}$$

Here, $\eta = \frac{\rho C_E}{k}$, $c_1^2 = \frac{\lambda+2\mu_T}{\rho}$ and ∇^2 is the Laplace operator. Therefore, the non-dimensional form of the governing equations (15)-(17) becomes (drop the dashed for convenience):

$$\frac{\partial^2 u}{\partial t^2} = h_{11}\frac{\partial^2 u}{\partial x^2} + h_2\frac{\partial^2 v}{\partial x\partial y} + h_1\frac{\partial^2 u}{\partial y^2} + h\frac{\partial \varphi_3}{\partial y} - \left(1+\tau_\theta\frac{\partial}{\partial t}\right)\frac{\partial \theta}{\partial x} + g\frac{\partial v}{\partial x}, \tag{19}$$

$$\frac{\partial^2 v}{\partial t^2} = h_{22}\frac{\partial^2 v}{\partial y^2} + h_2\frac{\partial^2 u}{\partial x\partial y} + h_1\frac{\partial^2 v}{\partial x^2} + h\frac{\partial \varphi_3}{\partial x} + \bar{\beta}\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\frac{\partial \theta}{\partial y} - g\frac{\partial u}{\partial x}, \tag{20}$$

$$\left(1+\tau_\theta\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) = \left(\frac{\partial}{\partial t} + \tau_q\frac{\partial^2}{\partial^2}\right)\theta + \left(\frac{\partial}{\partial t} + \tau_q\frac{\partial^2}{\partial t^2}\right)\left(\varepsilon_1\frac{\partial u}{\partial x} + \varepsilon_2\frac{\partial v}{\partial y}\right), \tag{21}$$

$$\nabla^2\varphi_3 + a_1\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - a_2\varphi_3 = a_3\frac{\partial^2 \varphi_3}{\partial t^2}, \tag{22}$$

$$m_{yz} = a_4\frac{\partial \varphi_3}{\partial y}, \tag{23}$$

where

$$(h_{11}, h_{22}, h_1, h_2, h) = \frac{(A_{11}, A_{22}, B_1, B_2, k)}{\rho c_1^2}, \bar{\beta} = \frac{\beta_{22}}{\beta_{11}},$$

$$\varepsilon_1 = \frac{\beta_{11}^2 T_0}{\rho C_E(\lambda+2\mu_T)}, \varepsilon_2 = \frac{\beta_{11}\beta_{22}T_0}{\rho C_E(\lambda+2\mu_T)}, a_1 = \frac{k_1\beta_{11}}{\gamma_1\eta^2 c_1^2},$$

$$a_2 = 2\frac{k_1}{\gamma_1\eta^2 c_1^2}, a_3 = \frac{j\rho_1^2}{\gamma_1}, a_4 = \frac{\gamma_1\eta^2}{\rho}. \tag{24}$$

Also, the dimensionless forms of equations (4)-(7) are:

$$\mu_T\sigma_{xx} = A_{11}u_{,x} + A_{12}v_{,y} - A_{22}\left(\frac{\partial}{\partial t} + \tau_\theta\frac{\partial}{\partial t}\right)\theta, \tag{25}$$

$$\mu_T\sigma_{yy} = A_{22}v_{,y} + A_{12}u_{,x} - \bar{\beta}A_{22}\left(\frac{\partial}{\partial t} + \tau_\theta\frac{\partial}{\partial t}\right)\theta, \tag{26}$$

$$\mu_T \sigma_{xy} = \mu_L(u_{,y} + v_{,x}) - k\varphi_3, \qquad \sigma_{zx} = \sigma_{zy} = 0, \tag{27}$$

$$\mu_T \sigma_{xy} = \mu_L(u_{,y} + v_{,x}) + k\varphi_3. \tag{28}$$

The dimensionless form of the equations of motion and stress components of medium II is:

$$\frac{\partial^2 u^f}{\partial x^2} + \frac{\partial^2 v^f}{\partial x \partial y} = a_5 \frac{\partial^2 u^f}{\partial t^2}, \tag{29}$$

$$\frac{\partial^2 v^f}{\partial y^2} + \frac{\partial^2 u^f}{\partial x \partial y} = a_5 \frac{\partial^2 v^f}{\partial t^2}, \tag{30}$$

where, $a_5 = \frac{c_1^2}{c_f^2}$.

3 Method of solution

To obtain the physical variables of the considered problem we use the harmonic wave analysis techniques as follows: Let,

$$\{label31[u, v, \theta, \sigma_{ij}, m_{ij}, \varphi_3, u^f, v^f, \sigma_{ij}^f](x, y, t) =$$

$$[u^*, v^*, \theta^*, \sigma_{ij}^*, m_{ij}^*, \varphi_3^*, u^{*f}, v^{*f}, \sigma_{ij}^{*f}](x) exp(\omega t + iay). \tag{31}$$

Where ω is the complex time constant, $i = \sqrt{-1}$; $a$is the wave number in the y- direction and $u^*, v^*, \theta^*, \sigma_{ij}^*, m_{ij}^*, \varphi_3^*, u^{*f}, v^{*f}$ and σ_{ij}^{*f} are the amplitudes of the field quantities. By the help of equation (31) equations (19)-(23) becomes:

$$[h_{11}D^2 - A_1]u^* + iah_2' Dv^* + iah\varphi_3^* = QD\theta^*, \tag{32}$$

$$[h_1 D^2 - A_2]v^* + iah_2'' Du^* - hD\varphi_3^* = ia\bar{\beta} Q\theta^*, \tag{33}$$

$$A_4 Du^* + iaA_5 v^* = [D^2 - A_3]\theta^*, \tag{34}$$

$$a_1 Dv^* - iaa_1 u^* + [D^2 - A_6]\varphi^* = 0, \tag{35}$$

$$m_{yz}^* = ia\, a_4 \varphi_3^*, \tag{36}$$

$$\mu_T \sigma_{xx}^* = A_{11} Du^* + iaA_{12} v^* - A_{22} Q\theta^*, \tag{37}$$

$$\mu_T \sigma_{yy}^* = A_{22} Du^* + iaA_{12} v^* - \bar{\beta} A_{22} Q\theta^*, \tag{38}$$

$$\mu_T \sigma_{xy}^* = \mu_L(iau^* + Dv^*) - k\varphi_3^*, \qquad \sigma_{zx}^* = \sigma_{zy}^* = 0, \tag{39}$$

$$\mu_T \sigma_{yx}^* = \mu_L(iau^* + Dv^*) + k\varphi_3^*, \qquad \sigma_{zx}^* = \sigma_{zy}^* = 0. \tag{40}$$

Where,

$$A_1 = \omega^2 + h_1 a^2, A_2 = \omega^2 + h_{22} a^2, Q = 1 + \omega\tau_\theta,$$

$$Q' = 1 + \omega\tau_q, D = \frac{d}{dx}, A_3 = a^2 + \omega Q'/Q,$$

$$A_4 = \omega\varepsilon_1 Q'/Q, A_5 = \omega\varepsilon_2 Q'/Q,$$

$$A_6 = a^2 + a_2 + a_3\omega^2, h_2' = h_2 + \frac{g}{ia}, h'' = h_2 - \frac{g}{ia}. \tag{41}$$

To obtain a non-zero solution of the system of equation (32)-(35) the determinant

$$\begin{vmatrix} (h_{11}D^2 - A_1) & iah_2' D & iah & -QD \\ iah_2'' D & (h_1 D^2 - A_2) & -hD & -ia\beta Q \\ A_4 D & iaA_5 & 0 & -(D^2 - A_3) \\ -iaa_1 & a_1 D & (D^2 - A_6) & 0 \end{vmatrix}$$

should vanish . Solving the system of equations (32)-(35) by elimination we get the following system of ordinary differential equations of the 8^{th} order in the variables $\theta^*(x)$,$u^*(x)$, $v^*(x)$ and $\varphi^*(x)$:

$$[D^8 - ED^6 + FD^4 - GD^2 + H]\{u^*(x), v^*(x), \theta^*(x), \varphi_3^*(x)\} = 0, \tag{42}$$

where

$$E = \frac{1}{h_1 h_{11}}\{-a_1 h h_{11} + h_1 h_{11}(A_3 + A_6) + A_1 h_1 + h_{11}A_2 + h_1 A_4 Q - a^2 h_2' h_2''\}, \tag{43}$$

$$F = \frac{1}{h_1 h_{11}}\{a^2 a_1 h(h_2' + h_1 - h_2'') + Q(a_1 h + a^2 h_2' A_4 \bar{\beta} + a^2 h_2'' A_5 - a^2 h_{11} - A_2 - A_6 h_1) - a_1 h(A_1 + h_{11}A_3) h_1 h_{11} A_3 A_6 + (A_3 + A_6)(A_2 + h_{11} + A_1 + h_1 - a^2 h_2' h_2'') + A_1 A_2, \tag{44}$$

$$G = \frac{1}{h_1 h_{11}}\{a^2 a_1 h((h_2' + h_1 - h_2'')A_3 + A_2) - a^2 Q(a_1 h\bar{\beta}(A_4 h_2' + A_5 h_2'') - a^2 h_{11} + A_1 + A_5 h_{11}) + A_1 A_2(A_3 + A_6) - a_1 h A_1 A_3 + A_3 A_6(A_2 h_{11} + A_1 h_1 + a^2 h_2' h_2'') - Q\bar{\beta} A_2 A_6 A_2\}, \tag{45}$$

$$H = \frac{1}{h_1 h_{11}}\{a^2 a_1 h(A_2 A_3 h_2' + a^2 A_5 Q\bar{\beta}) + A_1 A_6(A_2 A_3 - a^2 A_4 Q)\}. \tag{46}$$

Equation (43) can be written in the following form:

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(D^2 - k_4^2)\{u^*(x), v^*(x), \theta^*(x), \varphi_3^*(x)\} = 0, \tag{47}$$

where, $k_n^2 (n = 1, 2, 3, 4)$ are the roots of the characteristic equation

$$k^8 - Ek^6 + Fk^- Gk^2 + H = 0 \tag{48}$$

of the homogeneous equation (47). The bounded solutions of equation (48) can be written as:

$$u^*(x) = \sum_{n=1}^{4} M_n(a, \omega) exp(-k_n x), \tag{49}$$

$$v^*(x) = \sum_{n=1}^{4} M_n'(a, \omega) exp(-k_n x), \tag{50}$$

$$\theta^*(x) = \sum_{n=1}^{4} M_n''(a, \omega) exp(-k_n x), \tag{51}$$

$$\varphi_3^*(x) = \sum_{n=1}^{4} M_n'''(a, \omega) exp(-k_n x). \tag{52}$$

Where , M_n, $M_n^{'}$, $M_n^{''}$ and $M_n^{'''}$ are specific functions depending on a and ω. Substituting from equations (51)-(53) into equations (32)-(35), we get

$$M_n^{'}(a,\omega) = H_{1n}M_n(a,\omega), \tag{53}$$

$$M_n^{''}(a,\omega) = H_{2n}M_n(a,\omega), \tag{54}$$

$$M_n^{'''}(a,\omega) = H_{3n}M_n(a,\omega), \tag{55}$$

where

$$H_{1n} = \frac{h_{11}k_n^5 - \alpha_1 k_n^3 + \alpha_2 k_n}{\alpha_3 k_n^4 - \alpha_4 k_n^2 + \alpha_5}, \tag{56}$$

$$H_{2n} = \frac{-A_4 k_n + i\alpha A_5 H_{1n}}{k_n^2 - A_3}, \tag{57}$$

$$H_{3n} = \frac{\alpha_1 k_n H_{1n} + i\alpha A_1}{k_n^2 - A_5}, \tag{58}$$

where

$$\alpha_1 = A_1 h_{11} - i\alpha h_2^{''} + A_4 Q, \alpha_2 = (A_1 - iah_2^{''})A_3 + aA_5\bar{\beta}Q,$$

$$\alpha_3 = h_2^{'} + i\alpha h_2^{''}, \alpha_4 = i\alpha(A_2 + h_1 A_3 + A_4\bar{\beta}Q) + h_2^{'}A_3,$$

$$\alpha_5 = (a^2 A_5 \bar{\beta}Q + A_2 A_3)$$

Thus, we have

$$v^*(x) = \sum_{n=1}^{4} H_{1n}M_n(a,\omega)exp(-k_n x), \tag{59}$$

$$\theta^*(x) = \sum_{n=1}^{4} H_{2n}M_n(a,\omega)exp(-k_n x), \tag{60}$$

$$\varphi_3^*(x) = \sum_{n=1}^{4} H_{3n}M_n(a,\omega)exp(-k_n x). \tag{61}$$

Substitution of equations (49), (60) and (31) into equations (36)-(41), we get the following relations

$$m_{yz}^*(x) = \sum_{n=1}^{4} H_{4n}M_n(a,\omega)exp(-k_n x), \tag{62}$$

$$\sigma_{xx}^*(x) = \sum_{n=1}^{4} H_{5n}M_n(a,\omega)exp(-k_n x), \tag{63}$$

$$\sigma_{yy}^*(x) = \sum_{n=1}^{4} H_{6n}M_n(a,\omega)exp(-k_n x), \tag{64}$$

$$\sigma_{zz}^*(x) = \sum_{n=1}^{4} H_{7n}M_n(a,\omega)exp(-k_n x), \tag{65}$$

$$\sigma_{xy}^*(x) = \sum_{n=1}^{4} H_{8n}M_n(a,\omega)exp(-k_n x), \tag{66}$$

where (for $n = 1,2,3,4$) we have:

$$H_{4n} = iaa_4 H_{3n}, \tag{67}$$

$$H_{5n} = (-A_{11}k_n + iaA_{12}H_{1n} - A_{22}QH_{2n})/\mu_T, \tag{68}$$

$$H_{6n} = (-A_{22}k_n + iaA_{12}H_{1n} - A_{22}QH_{2n})/\mu_T, \tag{69}$$

$$H_{7n} = (-A_{12}k_n + iaA_{12}\lambda H_{1n} - A_{22}Q\bar{\beta}H_{2n})/\mu_T, \tag{70}$$

$$H_{8n} = \mu_T(ia - k_n H_{1n}) - kH_{3n}/\mu_T, \tag{71}$$

$$H_{9n} = \mu_T(ia - k_n H_{1n}) + kH_{3n}/\mu_T,. \tag{72}$$

Also, the variables of medium II are expressed as:

$$u^{*f}(x) = \sum_{n=5}^{6} M_n(a,\omega)exp(-k_n x), \tag{73}$$

$$v^{*f}(x) = \sum_{n=5}^{6} M_n^{'}(a,\omega)exp(-k_{nl} x), \tag{74}$$

where, The parameters $M_n(n = 5,6)$ and $M_n^{'}(n = 5,6)$ depends on α and ω , and $k_n(n = 5,6)$ are the roots of the characteristic equation of $u^{*f}(x)$:

$$(D^4 - AD^2 + B)u^{*f}(x) = 0, \tag{75}$$

where, $A = 2a_5\omega^2 - a^2$, $B = a_5^2\omega^4$. Equation (75) can be factorized as:

$$(D^2 - k^5)(D^2 - k_6^2)u^{*f}(x) = 0, \tag{76}$$

Similarly, $v^{*f}(x)$ satisfies the equation:

$$(D^4 - AD^2 + B)v^{*f}(x) = 0. \tag{77}$$

The relation between the parameters $M_n$and $M_n^{'}$ is:

$$M_n^{'}(a,\omega) = H_{10n}M_n(a,\omega), \quad n = 5,6\ , \tag{78}$$

where

$$H_{10n} = \frac{-iak_n}{k_n^2 - a_5\omega^2}, \quad n = 5,6. \tag{79}$$

The stress components for inviscid fluid can be written in the form

$$\sigma_{xx}^{*f}(x) = \sum_{n=5}^{6} \Re M_n(a,\omega)exp(-k_n x), \tag{80}$$

where, $\Re = \lambda^f \frac{k_n + iaH_{10n}}{\mu_T}$ and $\sigma_{xx}^{*f}(x) = 0$.

4 Special cases

1. As $\tau_\theta, \tau_q (0 \le \tau_\theta < \tau_q)$ represents the phase-lag of temperature gradient and the phase-lag of heat flux respectively, when $tau_\theta = 0$, equation (3) reduced to the equation of the LS theory (the equations of the generalized thermoelasticity with one relaxation time).

2. Neglecting gravity field, we obtain transformed components of displacement, stress forces and temperature distribution in a non-Gravitational generalized thermoelastic medium.

3. Neglecting the micropolarity effect $(i.e. k_l, \alpha_l, \beta_l, \gamma_1, j \to 0)$, we obtain the transformed components of the physical quantities in fibre-reinforced thermoelastic solid medium.

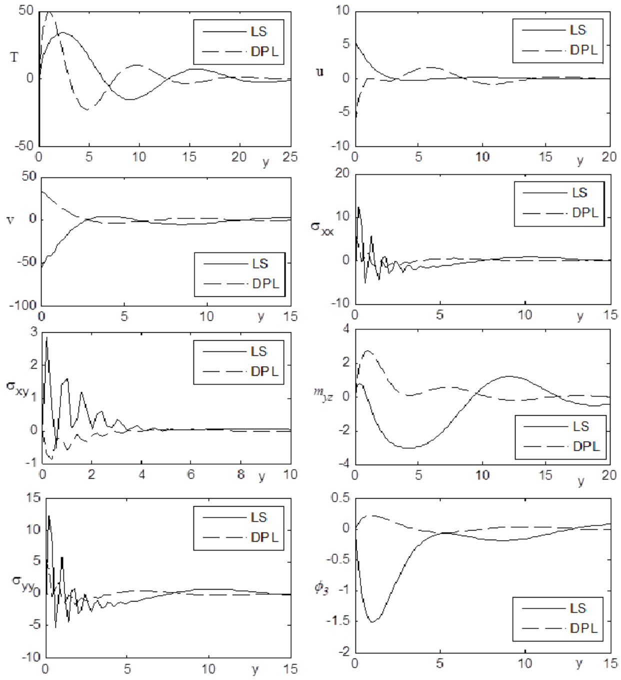

Fig. 2: Variation of some physical quantities with distance under LS theory and DPL model when $g = 9.8m/sec^2$.

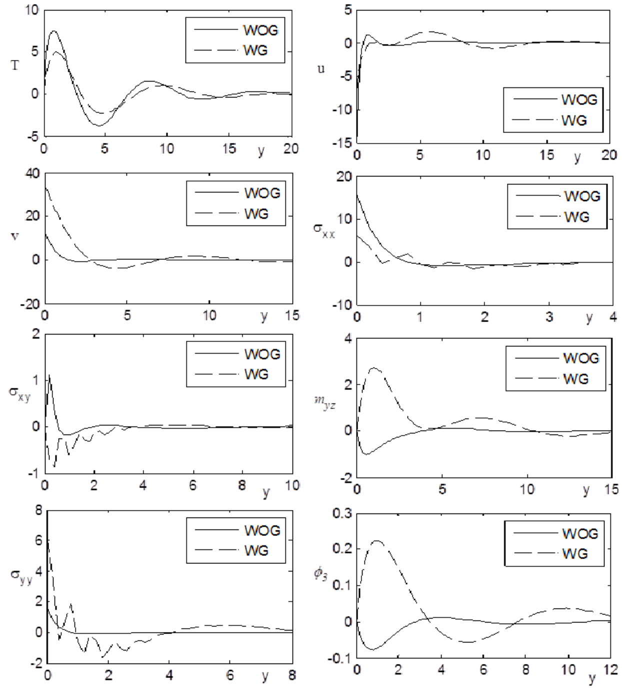

Fig. 3: The physical quantities with different values of gravity(WOG and WG)with distance under DPL model.

4. When, $\mu_l = \mu_T$, we obtain transformed components of displacement, stress forces and temperature distribution in a non-fibrereinforced generalized thermoelasticity with micropolar effect .

5. In case of $\mu_l = \mu_T$ and neglecting the micropolarity effect $(i.e.k_l, \alpha_l, \beta_l, \gamma_1 \to 0)$ we obtain the expressions for the generalized thermoelastic solid.

5 Applications

The pressure punches across the surface of the semi-infinite thermo-elastic and the non-viscous fluid half space are time-dependent. In the physical problem, we should suppress the positive exponential that are unbounded at infinity. The constants $M_n(n = 1,2,3,4,5,6)$ must be chosen such that the boundary conditions at the surface of the solid-liquid interface are:

1. The thermal boundary condition:

$$\frac{\partial \theta(x,y,t)}{\partial x} = 0, \quad at \quad x = 0 \tag{81}$$

2. The magnitude of the tangential component of the stress at the surface of the fibre- reinforced must be equal to the tangential component of the stress for fluid:

$$\sigma_{xy}(x,y,t) = \sigma_{xy}^{f}(x,y,t), \quad at \quad x = 0 \tag{82}$$

3. The magnitude of the normal component of the stress vector of the plate and the magnitude of the normal component of the

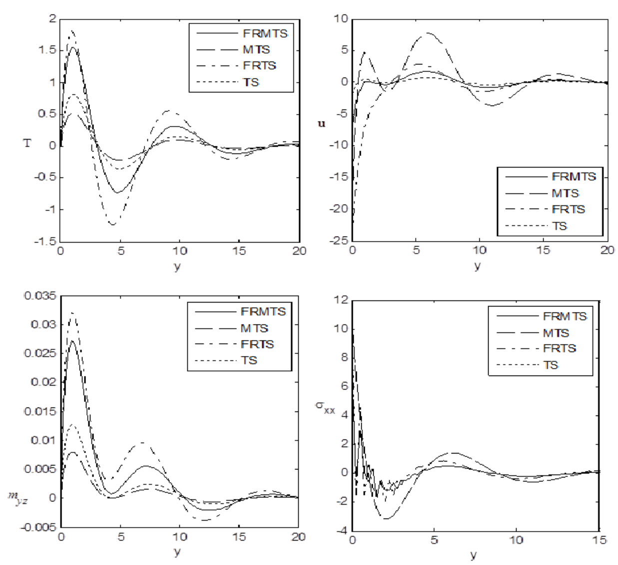

Fig. 4: Some physical quantities with different values of Fibre-einforced,micropolar and thermal parameters with distance y under DPL model and $g = 9.8m/sec^2$.

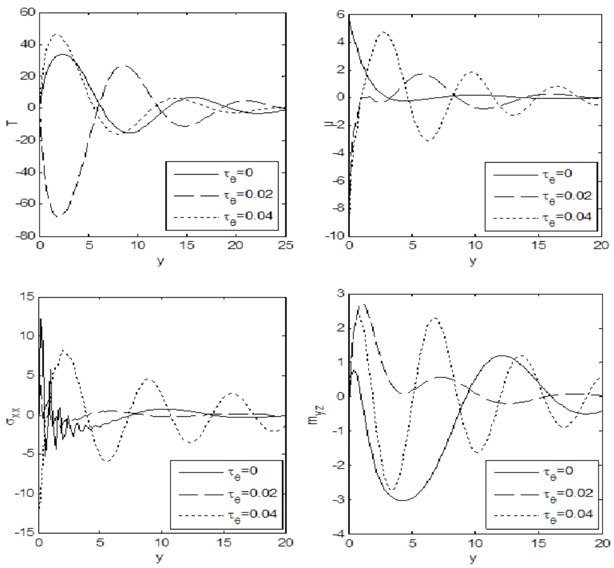

Fig. 5: Variation of some physical quantities with distance y with different values of relaxation time τ_θ and $g = 9.8m/sec^2$.

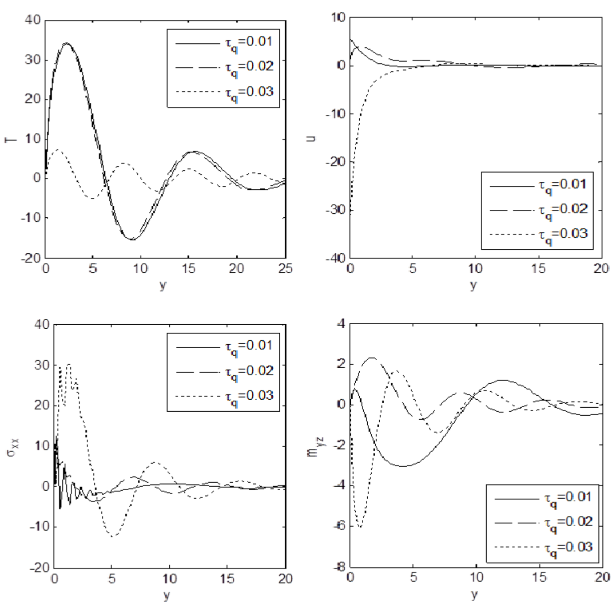

Fig. 6: Variation of some physical quantities with distance y with different values of relaxation time τ_q and $g = 9.8m/sec^2$.

stress vector for the fluid must be equal:

$$\sigma_{xx}(x,y,t) = \sigma^f_{xx}(x,y,t) - P_1, \quad at \quad x = 0 \tag{83}$$

where $P_1$is the magnitude of the applied mechanical force. **4**. The magnitude of the stress components of the plate and the liquid must be equal.

$$\sigma_{yy}(x,y,t) = \sigma^f_{yy}(x,y,t), \quad at \quad x = 0 \tag{84}$$

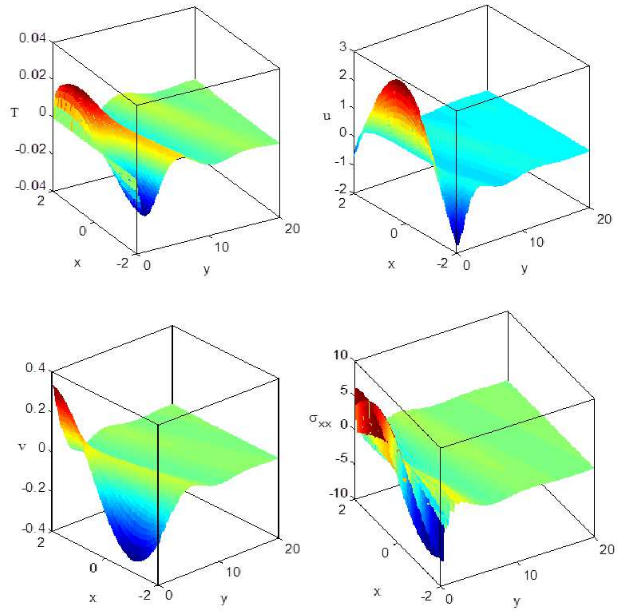

Fig. 7: Variation of the temperature distribution T, displacement distribution u , v and stress force distribution σ_{xx} in 3D with distance x, y and $g = 9.8m/sec^2$ under DPL theory .

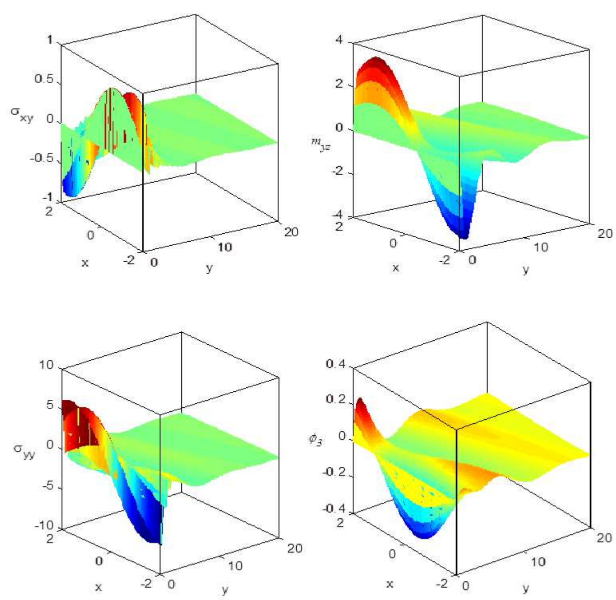

Fig. 8: Variation of the tangential coupled stress m_{yz}, stress force distribution σ_{xy}, σ_{yy} and micro-rotation ϕ_3 in3D with distance x, y and $g = 9.8m/sec^2$ under DPL theory.

5. The vertical velocity component of the fibre-reinforced must be equal to that of the liquid.

$$\dot{v}(x,y,t) = \dot{v}^f(x,y,t), \quad at \quad x = 0. \tag{85}$$

6. The magnitude of the tangential component of the couple stress at the surface must vanish.

$$m_{yz}(x,y,t) = 0 \quad atx = 0. \tag{86}$$

Using the boundary conditions (82)-(87), we get:

$$\sum_{n=1}^{4} H_{2n}k_n M_n(a,\omega) = 0, \tag{87}$$

$$\sum_{n=1}^{4} H_{8n} M_n(a,\omega) = 0, \tag{88}$$

$$\sum_{n=1}^{4} H_{5n} M_n(a,\omega) - \sum_{n=5}^{6} \Re_n M_n(a,\omega) = -P_1, \tag{89}$$

$$\sum_{n=1}^{4} H_{6n} M_n(a,\omega) - \sum_{n=5}^{6} \Re_n M_n(a,\omega) = 0, \tag{90}$$

$$\sum_{n=1}^{4} H_{ln} M_n(a,\omega) - \sum_{n=5}^{6} H_{10} M_n(a,\omega) = 0, \tag{91}$$

$$\sum_{n=1}^{4} H_{4n} M_n(a,\omega) = 0. \tag{92}$$

Using the inverse matrix method we can solve the system of algebraic equations (87)-(92) and get the values of the constants $M_n, n = 1,2,3,4,5,6$. Hence, the expressions of displacements, temperature distribution and the other physical quantities of the plate at the interface of fibre reinforced micropolar thermoelastic half space as well as the fluid half space quantities under the influence of gravity field can be obtained.

6 Numerical results

To illustrate the analytical procedure of the problem and illustrate the effect of gravity field on wave propagation at the interface between fiber-reinforcement and Liquid, we shall introduce some numerical example for which computational results are given[4].

(i) Fibre reinforced micropolar parameters

$$\lambda = 9.4x10^9 N/m^2, \mu_T = 1.89x10N/m^2, \mu_L = 2.45x10^9 N/m^2,$$

$$\alpha = -1.28x10^9 N/m^2, \beta = 0.32x10^9 N/m^2, \rho = 7800kg/m^3,$$

$$k_L = 10^{11} N/m^2, \mu = 3.86x10^{11} N/m^2, j = 0.2x10^{11} m^2,$$

$$\gamma_l = 0.779x10^{-1} N.$$

(ii) Thermal parameters for the medium:

$$T_0 = 293K, C_E = 383.1\, j/kg, \tau_\theta = 0.02s, \tau_q = 0.03s,$$

$$k = 1.7 \times 10^2 Nsec^{-1}, \omega = \omega_0 + i\xi, \omega_0 = 2,$$

$$\xi = 1, a = 1, \alpha_t = 7.4033 \times 10^{-7} k^{-1}$$

(iii) Fluid parameters:Consider water (as Newtonian fluid): The physical constants for water are:

$$\lambda^f = 2.14 \times 10^9 N/m^2, \rho^f = 10^3 kg/m^3$$

All calculations are carried out at time $t = 0.008$ and mechanical force pressure with magnitude $P_{1.0}$. The numerical technique is used to obtain the values of the real part of the thermal temperature T , the displacement components u and v , the tangential coupled stress m_{yz} , the micro-rotation ϕ_3 , the stresses σ_{xx} , σ_{yy} and σ_{xy} distribution for the problem in dimensionless forms:

6.1 Comparison between LS theory and DPL model

The first group of figure (2): shows the predicted curves of the two different theories of thermoelasticity when $g = 9.8m/sec^2$. In this figure, the solid lines represent the solution in the LS theory, and the dashed lines represent the solution derived using the DPL model. The physical field quantities temperature, displacement components, tangential coupled stress, micro-rotation and stress components depend not only on space x and time t but also on the thermal relaxation time τ_θ and τ_q . It is noticed when the relaxation time is included in the heat equation the obtained results are different from those when the relaxation time is not included (phase-lag of temperature gradient τ_θ). This clarifies the difference between thermoelasticity theory with one relaxation time (LS) and DPL model.

6.2 Influence of gravitational field:

For two thermal relaxation times (τ_θ and τ_q) considering two different gravity values $g = 0.0$(WOG-solid line) and $g = 9.8m/sec^2$ (WG- dashed line). The results in the second group of figure (3) show the variations of some physical variables with distance .It is found that the presence of gravity has caused both decreasing and increasing effects on the displacement and stress distributions. On the other hand, temperature, tangential coupled stress m_{xy}, and micro-rotation distributions are significantly increased due gravity effect.

6.3 Effect of fibre reinforced micropolar parameters and thermal parameters:

The third group in figure (4),represents the effects of fibre reinforced micropolar and thermal parameters ,under DPL model with the effect of gravitational field, has significant effect on all physical fields where the variations for some physical fields are shown graphically taken into consideration the following:

(i) Fibre-Reinforced Micropolar Thermo elastic Solid (FRMTS) by solid line.
(ii) Micropolar Thermoelastic Solid (MTS) by dashed-dashed line
(iii) Fibre-Reinforced Thermoelastic Solid (FRTS) by dashed-dot line
(iv) Thermoelastic Solid (TS) with dot-dot line.
From this group, we notice that, all field quantities increases as distance$x$ decreases. The field quantities in most cases increases at the beginning and start to decrease and reach zero value at infinity as the distance $x$increases. These trends obey elastic and thermoelastic properties of the solid under investigation. In addition, all waves propagation lines begin to coincide when the horizontal distance$x$ is increased to reach the reference temperature of the solid. These results are in complete agreement. with the physical behavior of fibre materials as a polycrystalline solid.

6.4 Effect of relaxation times parameters:

In this group, figures (5) and (6) shows the effect of relaxation times on some physical quantities with the effect of gravitational field. This parameters have a significant effect on all physical fields.

6.5 Effect of all parameters in 3D-plots:

Figures(7,8),show the 3D distribution of the temperature$T$, displacements(u,v),stresses σ_{ij},the tangential coupled stress and the micro-rotation distributions with respect to$x$ and y under the influence of gravity with DPL model. Moreover the fiber reinforced, micropolar and thermal parameters are found.We notice that:

As variable varies from zero to infinity, all values increases from their initial values and then decreases to have zero value at infinity.
With any relatively small increase in y- direction all physical field quantities decreases
The temperature, displacement, micro-rotation and shear stress component start from zero and decrease with the smaller values of x-axis and return to increase to tend zero as x tends to infinity.
The normal stress component having zero value at the beginning, increases with the smaller values of x-axis then decreases and increases periodically to have zero value as x tends to infinity.
The elastic wave propagation between two interface in 3D overlapping and damping when and increases to reach the equilibrium state for the particles.

These figures are very important to study the dependence of the physical fields on the vertical component of distance. The surfaces obtained are highly depending on the vertical distance from origin, most physical quantities are moving in wave propagation.this phenomenon has more applications in modern physical engineering as aeronautics, astronautics, nuclear reactors and high-energy particle accelerators.

7 Perspective

The physical quantities, depend not only on space x and time t but also on the thermal relaxation time τ_θ and τ_q. The thermal relaxation times have significant effect on all field quantities.

1. The values of the field quantities decrease as the horizontal distance increases, continuously and approach to zero at infinity. So the anisotropy and micropolarity have significant effects on all physical quantities of the problem, these results agree with the physical behavior of fibre materials as a polycrystalline solid.

2. The used method is applicable to a wide range of thermodynamics and thermoelasticity problems.

3. The Gravity filed has significant effect on all physical quantities of the problem as their amplitudes decrease with gravity change.

4. The elastic waves between the two faces depend on the nature of the applied force as well as the type of boundary conditions.

5. The obtained results of this article are of great interest for material science and designers of new materials researchers. Moreover, the study of relaxation time and gravity phenomenon are useful to improve the conditions of oil extractions and drilling.

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