Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/120212

Implicit Scheme Solution of Unsteady MHD Flow

A.R. Vijayalakshmi[1,*] *and M. SelvaJayanthi*[2]

[1] Department of Applied Mathematics, Sri Venkateswara College of Engineering, Sriperumbudur- 602105, India.
[2] Department of Mathematics, SKP Engineering College, Tiruvannamalai - 606611, India.

Received: 8 Aug. 2017, Revised: 22 Jan. 2018, Accepted: 28 Jan. 2018
Published online: 1 Mar. 2018

Abstract: We study the unsteady, electrically transmitting fluid past an oscillating semi-infinite vertical plate with uniform temperature and mass diffusion under Chemical reaction(unimolecular). Complete computations are presented and the impact of chemical reaction for the change in the velocity, heat transfer, fluid concentration, skin -friction, the rate of heat and mass diffusion at the plate are exhibited through graphs. The inference from the current observations is that the chemical reaction has a significant effect in controlling the flow. The heat transfer due to viscous dissipation and the induced magnetic field is considered to be negligible. The consequences of cooling of the plate by free convection currents are studied. The heat transfer progress is found to be enhanced with the oscillation frequency, Prandtl number and thermal Grashof numbers.

Keywords: MHD,semi-infinite vertical plate, Chemical reaction, implicit scheme, mass transfer

1 Introduction

MHD study was initiated in 1970 by Hannes Alfven. He has got the Nobel Prize in physics for this study [10]. Magneto hydrodynamic convection plays an important role in various industrial applications. Many transport advancements that have taken place demonstrates the nature and industrial applications in which the simultaneous heat and mass exchange and dissemination of chemical species.The study of fluid motion in Magneto hydrodynamics has arisen due to the fusion of Magnetic field and electrical conductivity [19]. Applications of the MHD principle are found in Medicine, biology, sun spots, semi-conductors and in geophysics.Convective heat and mass transfer along with the first order chemical reaction help in cooling and designing of Nuclear Reactor and in geothermal storage of nuclear waste [7].

Alam et al [1] analyzed the problem of heat absorption and variation in the Prandtl number for a natural convective flow over a semi-infinite vertical plate with the chemical reaction and magnetic field, solved by using a shooting iteration technique along with Runge- Kutta sixth order method. Numerical investigation of the influence of a magnetic field on unsteady combined heat and mass transport by natural convection along a moving semi- infinite vertical plate, with a first order chemical reaction was done by Ai-Odat et al [2], using an implicit finite difference technique. Ananthaswamy et al [3]discussed the suction effects on a porous shrinking sheet under MHD and found a solution using Homotopy analysis.An analysis of first-order chemical reaction in the hydro magnetic flow past a vertical plate with variation in temperature and concentration was done by Bhaben et al [4]. HydroMagnetic flow past an impulsively started infinite isothermal vertical plate with a homogeneous first order chemical reaction has been studied and it was solved by an implicit finite-difference scheme of Crank-Nicolson type by Chandrakala et al [6]. Dakshinamoorthy et al [8] has obtained an approximate solution numerically for MHD flow past a moving plate with viscous dissipation and Joule heat transfer by Runge-Kutta-Gill method along with shooting technique. An explicit finite difference method of DuFort-Fronkel's was applied to find the effects of the unsteady MHD free convective flow over a parabolic starting motion of an infinite vertical plate in a viscous incompressible fluid with viscous dissipation by Girish Kumar et al [9]. The transient flow of a rotating, incompressible, homogeneous, Newtonian fluid past an infinite vertical plate was analyzed for natural convective fluid by Imran et al [11] Solution was established by the Laplace transform technique. Prasad et al [12] have obtained the approximate solution of heat and mass transfer effects on mixed convective, axisymmetric electrically conducting

* Corresponding author e-mail: avijaya@svce.ac.in

flow over a slender cylinder under chemical reaction by using a second order finite difference scheme for different values of the physical parameters.Chemical reaction and radiation on the convective flow of viscous incompressible fluid over an infinite vertical plate under the impact of a uniform transverse magnetic field and in the presence of Newtonian heating, viscous dissipation, and uniform mass diffusion have been studied by Rajesh et al [13]. It was solved by an implicit finite difference method.Rubel Ahmed et al [14] analyzed the effect of radiation and chemical reaction on unsteady free convection flow with temperature dependent viscosity past an oscillating cylinder in the presence of periodic magnetic field and it was solved by using the explicit finite difference technique. The steady, hydromagnetic, mixed convection heat and mass transfer with a convective surface boundary condition over a vertical plate was solved numerically by Makinde et al [15]. The impact of thermal radiation over an infinite vertical plate under the influence of the transverse magnetic field, using the Laplace transform technique was studied for optically thick fluid by Mazumdar et al [16]. An approximate solution by Range-Kutta method was found for the effect of radiation, chemical reaction and MHD on the steady flow over an exponentially stretching sheet by Seini et al [17]. Soundalgekar et al [18] first introduced the natural convective, hydrodynamic flow of viscous incompressible fluid over a moving infinite vertical plate under the action of the transverse magnetic field, using Laplace-transform technique.
The current study investigates a numerical solution of time-dependent MHD flow past a semi-infinite oscillating vertical plate with uniform mass diffusion under the influence of Chemical Reaction by adopting the implicit finite-difference scheme.

2 Mathematical Model of the problem

A laminar, transient MHD flow past an oscillating vertical plate which is assuming at rest and around the plate with temperature T'_∞ and concentration C'_∞ is considered. The geometry of study under consideration is a semi- infinite plate with, the x-axis is assumed along the plate in the vertical orientation and the y-axis is assumed normal to the plate. Initially, it is assumed that the temperature and the concentration level of the plate and ambient fluid are alike. As time advances, the plate begins to oscillate in its own plane with velocity $u = u_0 \cos\omega' t'$, where the constant u_0 is the amplitude of the motion and ω' is the frequency, the temperature of the plate and the concentration level is also raised evenly with time. A magnetic field of uniform strength B_0 is applied normal to the plate. Then, using Boussinesqs approximation, the unsteady MHD flow with chemical reaction is governed by the following set of equations capable of describing the flow of any fluid is taken, the Navier-Stokes equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T'-T'_\infty)+g\beta^*(C'-C'_\infty)$$
$$+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \quad (2)$$

$$\rho C_\rho\left(\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}\right)=\frac{\partial^2 T'}{\partial y^2} \quad (3)$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}-K_1(C'-C'_\infty) \quad (4)$$

The initial and boundary conditions are

$$\begin{aligned}&t'\le 0: u=0, v=0, T'=T'_\infty, C'=C'_\infty\\&t'>0: u=u_0\cos\omega' t', v=0, T'=T'_w, C'=C'_w, \text{at} y=0\\&u=0, v=0, T'=T'_\infty, C'=C'_\infty \text{at } x=0\\&u\to 0, T'\to T'_\infty, C'\to C'_\infty \text{at } y\to\infty\end{aligned} \quad (5)$$

Where, Nomenclature in the above equations are: C' is the concentration mol $[m^{-3}]$, C_p is the specific heat at constant pressure $[J.kg^{-1}.K^{-1}]$, g is the acceleration due to gravity $[m.s^{-2}]$, K_l is the Chemical reaction parameter $[J]$, k is the thermal conductivity $[J.m^{-1}.K^{-1}]$, T is the temperature of the fluid near the plate $[K]$, t' is the Time $[s]$, u & v are the velocity components in x and $y$direction, x is the spatial coordinates along the plate, y is the spatial coordinates normal to the Plate m. β is the volumetric coefficient of thermal expansion $[K^{-1}]$, β^* is the Volumetric coefficient of expansion with concentration $[K^{-1}]$.

On introducing the following non-dimensional quantities

$$X=\frac{xu_0}{\nu}, Y=\frac{yu_0}{\nu}, U=\frac{u}{u_0}, V=\frac{v}{u_0}, \omega=\frac{\omega'\nu}{u_0^2}, M=\frac{\sigma B_0^2}{\rho}u$$
$$t=\frac{t'u_0^2}{\nu},\quad T=\frac{T'-T'_\infty}{T'_w-T'_\infty},\quad Gr=\frac{\nu g\beta(T'_w-T'_\infty)}{u_0^3},\quad Pr=\frac{\nu}{\alpha},$$
$$C=\frac{C'-C'_\infty}{C'_w-C'_\infty},\quad Gc=\frac{\nu g\beta(C'_w-C'_\infty)}{u_0^3},\quad Sc=\frac{\nu}{D},\quad K=\frac{\nu K_1}{u_0^2}, \quad (6)$$

Equations (1) to (4) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (7)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial V}{\partial Y}=GrT+GcC+\frac{\partial^2 U}{\partial Y^2}-MU \quad (8)$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2} \tag{9}$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{SC}\frac{\partial^2 C}{\partial Y^2}-KC \tag{10}$$

The corresponding initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llllll} t\le 0: & U=0, & V=0, & T=0, & C=0 & \\ t>0: & U=\cos\omega t & V=0, & T=1, & C=1 & \text{at } y=0\\ & U=0, & V=0, & T=0, & C=0 & \text{at } x=0\\ & U\to 0, & & T\to 0, & C\to 0 & \text{at } y\to\infty \end{array} \tag{11}$$

The Non-dimensional numbers are:
$Gr=\frac{vg\beta(T'_w-T'_\infty)}{u_0^3}$ is the thermal Grashof number $Gc=\frac{vg\beta^*(C'_w-C'_\infty)}{u_0^3}$ is the mass Grashof number, $Sc=\frac{v}{D}$ is the Schmidt number, $Pr=\frac{v}{\alpha}$ is the Prandtl number, $K=\frac{vK_1}{u_0^2}$ is the non-dimensional chemical reaction parameter, $M=\frac{\sigma B_0^2}{\rho}u$ is the magnetic field, ρ is the Density of the fluid $[kg.m^{-3}]$, v is Kinematic viscosity $[m^2.s^{-1}]$, σ is a scalar electrical conductivity, η is the similarity parameter, θ is the dimensionless temperature μ is the coefficient of viscosity $[Pa.s]$, τ is the dimensionless skin-friction, α is the thermal diffusivity.

3 Numerical Technique

For solving these unsteady, two dimensional coupled equations (7) to (10) under the above said boundary conditions (11), an implicit scheme (Crank-Nicolson type) has been employed. The equations (7) to (10) transformed to corresponding finite difference equations as follows:

$$\frac{1}{4\Delta X}[U_{i,j}^{n+1}-U_{i-1,j}^{n+1}+U_{i,j}^{n}-U_{i-1,j}^{n}+U_{i,j-1}^{n+1}-U_{i-1,j-1}^{n+1} \\ +U_{i,j-1}^{n}-U_{i-1,j-1}^{n}]+\frac{1}{2\Delta Y}[V_{i,j}^{n+1}-V_{i,j-1}^{n+1}+V_{i,j}^{n}-V_{i,j-1}^{n}]=0 \tag{12}$$

$$\frac{[U_{i,j}^{n+1}-U_{i,j}^{n}]}{\Delta t}+U_{i,j}^{n}\frac{[U_{i,j}^{n+1}-U_{i-1,j}^{n+1}+U_{i,j}^{n}-U_{i-1,j}^{n}]}{2\Delta X} \\ +V_{i,j}^{n}\frac{[U_{i,j+1}^{n+1}-U_{i,j-1}^{n+1}+U_{i,j+1}^{n}-U_{i,j-1}^{n}]}{4\Delta Y} \\ =\frac{Gr}{2}[T_{i,j}^{n+1}+T_{i,j}^{n}]+\frac{Gc}{2}[C_{i,j}^{n+1}+C_{i,j}^{n}]-\frac{M}{2}[U_{i,j}^{n+1}+U_{i,j}^{n}] \\ +\frac{[U_{i,j-1}^{n+1}-2U_{i,j}^{n+1}+U_{i,j+1}^{n+1}+U_{i,j-1}^{n}-2U_{i,j}^{n}+U_{i,j+1}^{n}]}{2(\Delta Y)^2} \tag{13}$$

$$\frac{[T_{i,j}^{n+1}-T_{i,j}^{n}]}{\Delta t}+U_{i,j}^{n}\frac{[T_{i,j}^{n+1}-T_{i-1,j}^{n+1}+T_{i,j}^{n}-T_{i-1,j}^{n}]}{2\Delta X} \\ +V_{i,j}^{n}\frac{[T_{i,j+1}^{n+1}-T_{i,j-1}^{n+1}+T_{i,j+1}^{n}-T_{i,j-1}^{n}]}{4\Delta Y} \\ =\frac{1}{Pr}\frac{[T_{i,j-1}^{n+1}-2T_{i,j}^{n+1}+T_{i,j+1}^{n+1}+T_{i,j-1}^{n}-2T_{i,j}^{n}+T_{i,j+1}^{n}]}{2(\Delta Y)^2} \tag{14}$$

$$\frac{[C_{i,j}^{n+1}-C_{i,j}^{n}]}{\Delta t}+U_{i,j}^{n}\frac{[C_{i,j}^{n+1}-C_{i-1,j}^{n+1}+C_{i,j}^{n}-C_{i-1,j}^{n}]}{2\Delta X} \\ +V_{i,j}^{n}\frac{[C_{i,j+1}^{n+1}-C_{i,j-1}^{n+1}+C_{i,j+1}^{n}-C_{i,j-1}^{n}]}{4\Delta Y} \\ =\frac{1}{Sc}\frac{[C_{i,j-1}^{n+1}-2C_{i,j}^{n+1}+C_{i,j+1}^{n+1}+C_{i,j-1}^{n}-2C_{i,j}^{n}+C_{i,j+1}^{n}]}{2(\Delta Y)^2} \\ -\frac{K}{2}(C_{i,j}^{n+1}-C_{i,j}^{n}) \tag{15}$$

A rectangular region with sides $Xmax(=1)$ and $Ymax(=14)$,Where $Ymax$ corresponds to $Y=\infty$ which lies outside boundary layers is considered for the study. To satisfy the last two of the boundary conditions (11) within the tolerance limit,the maximum of Y was fixed at 14 with the mesh sizes fixed at the position $\Delta X=0.05$, $\Delta Y=0.25$ with time step $\Delta t=0.01$. The spatial mesh sizes are reduced either in one direction or in both the directions and the results are compared and decided the above mesh sizes as appropriate for the computation. Here i,j denotes the grid point along the X-direction,the Y-direction, respectively, and k denotes the$t$-time. The values of U,V are known at all grid points at $t=0$ from the initial conditions. The calculation of U,V,T and C at the time level $(n+1)$ using the values at previous time level (n) are derived as follows: The finite difference equation (15) at every internal nodal point on a particular i-level constitute a tridiagonal system of equations which is solved by using a Thomas algorithm as disputed in Carnahan et al.[5]. Thus, the values of C are found at every nodal point for a particular i at $(n+1)^{th}$ time level. In an identical manner computation is carried out for the values of C from the previous equation i.e.(14). Using the values of C and T at $(n+1)^{th}$ time level in the previous equation, i.e.(13) , the values of U at $(n+1)^{th}$ time level is found. Using the computations of C,T and U values of a particular i-level, the values of V are calculated explicitly using the Equation (12) at every nodal point on a particular i-level at $(n+1)^{th}$ time level. This procedure is repeated until the Steady -state is arrived at. Whenever the absolute difference between the values of U ,T and C at two consecutive time steps, are less than 0.00001 at all grid points then the steady-state solution has arrived.

4 Stability analysis

The stability part of the implicit scheme for constant mesh sizes, is examined using Von Neumann technique. The

Fourier expansion of U,T and C at an arbitrary time $t=0$ is assumed to be of the form $exp(i\alpha X)\ exp(i\beta Y)$ (here $i=-1$). At a later time t, these terms will become,

$$\begin{aligned} U &= F(t)exp(i\alpha X)exp(i\beta Y) \\ T &= G(t)exp(i\alpha X)exp(i\beta Y) \\ C &= H(t)exp(i\alpha X)exp(i\beta Y) \end{aligned} \tag{16}$$

Substituting (16) in Equations (13) to (15); under the assumption that the coefficients$U$,T and C are constants over any one time step and denoting the values after one time step by F', G', H' . After simplification, we get

$$\begin{aligned} &\frac{(F'-F)}{\Delta t}+\frac{U}{2}\frac{(F'+F)(1-exp(-i\alpha\Delta X))}{\Delta X} \\ &\qquad +\frac{V}{2}\frac{(F'+F)i\sin\beta\Delta Y}{\Delta Y} \\ &=\frac{(G'+G)Gr+(H'+H)Gc-M(F'+F)}{2} \\ &\quad +\frac{(F'+F)(\cos\beta\Delta Y-1)}{(\Delta Y)^2}-\frac{M}{2}(F'+F) \end{aligned} \tag{17}$$

$$\begin{aligned} &\frac{(G'-G)}{\Delta t}+\frac{U}{2}\frac{(G'+G)(1-exp(-i\alpha\Delta X))}{\Delta X} \\ &\qquad +\frac{V}{2}\frac{(G'+G)i\sin\beta\Delta Y}{\Delta Y} \\ &\qquad =\frac{1}{Pr}\frac{(G'+G)(\cos\beta\Delta Y-1)}{(\Delta Y)^2} \end{aligned} \tag{18}$$

$$\begin{aligned} &\frac{(H'-H)}{\Delta t}+\frac{U}{2}\frac{(H'+H)(1-exp(-i\alpha\Delta X))}{\Delta X} \\ &\qquad +\frac{V}{2}\frac{(H'+H)i\sin\beta\Delta Y}{\Delta Y} \\ &\qquad =\frac{1}{Sc}\frac{(H'+H)(\cos\beta\Delta Y-1)}{(\Delta Y)^2} \\ &\qquad\qquad -\frac{G}{2}(H'+H) \end{aligned} \tag{19}$$

From (17), (18), (19) modified as,

$$(1+A)F'=(1-A)F+\frac{Gr}{2}(G'+G)\Delta t+\frac{Gc}{2}(H'+H)\Delta t \tag{20}$$

$$(1+B)G'=(1-B)G \tag{21}$$

$$(1+E)H'=(1-E)H \tag{22}$$

Where,

$$\begin{aligned} A &= \frac{U}{2}\frac{\Delta t(1-exp(-i\alpha\Delta X))}{\Delta X}+\frac{V}{2}\frac{\Delta t(i\sin\beta\Delta Y)}{\Delta Y} \\ &\qquad -\frac{(\cos\beta\Delta Y-1)}{(\Delta Y)^2}-\frac{M}{2}\Delta t \\ B &= \frac{U}{2}\frac{\Delta t(1-exp(-i\alpha\Delta X))}{\Delta X}+\frac{V}{2}\frac{\Delta t(i\sin\beta\Delta Y)}{\Delta Y} \\ &\qquad -\frac{1}{Pr}\frac{\Delta t(\cos\beta\Delta Y-1)}{(\Delta Y)^2} \\ E &= \frac{U}{2}\frac{\Delta t(1-exp(-i\alpha\Delta X))}{\Delta X}+\frac{V}{2}\frac{\Delta t(i\sin\beta\Delta Y)}{\Delta Y} \\ &\qquad -\frac{1}{Sc}\frac{\Delta t(\cos\beta\Delta Y-1)}{(\Delta Y)^2}-\frac{G}{2}\Delta t \end{aligned}$$

After eliminating in Equation (20)(21)(22),the resultant equation is given by

$$(1+A)F'=(1-A)F+G\frac{Gr\Delta t}{(1+B)}+H\frac{Gc\Delta t}{(1+E)} \tag{23}$$

Equations (20) to (23) can be written in the matrix form as follows

$$\begin{bmatrix}F'\\G'\\H'\end{bmatrix}=\begin{bmatrix}\frac{1-A}{1+A} & D_1 & D_2\\ 0 & \frac{1-B}{1+B} & 0\\ 0 & 0 & \frac{1-E}{1+E}\end{bmatrix}\begin{bmatrix}F\\G\\H\end{bmatrix} \tag{24}$$

Where $D_1=\frac{Gr\Delta t}{(1+A)(1+B)}$ and $D_2=\frac{Gc\Delta t}{(1+A)(1+E)}$

The Eigen values of the amplification matrix $\frac{1-A}{1+A}$, $\frac{1-B}{1+B}$, $\frac{1-E}{1+E}$ are diagonal elements.

Let us assume that, U is non-negative at all levels and V is non-positive at all levels, we get

$A = 2a\sin^2\left(\frac{\alpha\Delta X}{2}\right)+2c\sin^2\left(\frac{\beta\Delta Y}{2}\right)+i(a\sin\alpha\Delta X - b\sin\beta\Delta Y)+M\frac{\Delta t}{2}$

Where, $a=\frac{U}{2}\frac{\Delta t}{\Delta X}$, $b=\frac{V}{2}\frac{\Delta t}{\Delta Y}$, $c=\frac{\Delta t}{(\Delta Y)^2}$

Since $A\geq 0$,

$|(1-A)/(1+A)|\leq 1$ always,

similarly, $|(1-B)/(1+B)|\leq 1$ and $|(1-E)/(1+E)|\leq 1$

The implicit scheme is unconditionally stable.For the compatible scheme,the Local truncation error is $O(\Delta t^2+\Delta Y^2+\Delta X)$ must tend to zero. As $\Delta t,\Delta X$ and ΔY tend to zero, truncation error tends to zero. When Stability and compatibility are achieved convergence is achieved.

5 Results and discussion

5.1 Validation with published results

To establish the accuracy of the numerical results, the present study compares with the available work in the literature. The velocity profiles for M=2,5,Gr=2, Gc=5,Sc=0. 6,Pr=0. 71 and K=2,10 are compared with the available solution of P.Chandrakala et al (2014) at t = 0.2,ω=0,in Fig.1and Fig.2 and they are found to be in excellent agreement.

Numerical solutions of velocity, temperature and

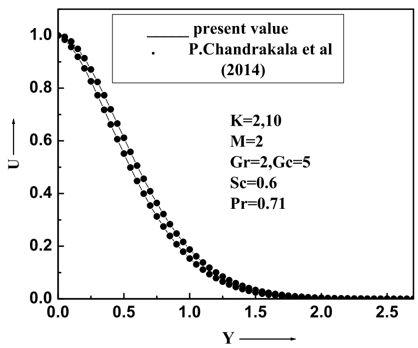


Fig. 1: Comparison of Effects of velocity profiles for different K

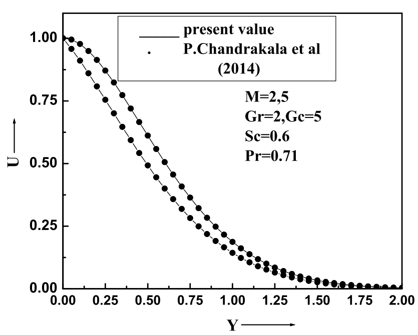


Fig. 2: Comparison of Effects of velocity profiles for different M

Concentration field are discussed graphically. The velocity field is discussed for the chemical reaction parameter, magnetic field parameter, and phase angle, thermal and mass Grashof number in Figures 3-9. The mass diffusion Equation (10) can be adjusted to meet any one of the following three cases (i) $K > 0$ gives the destructive reaction, (ii)$K < 0$gives the generative reaction (iii)$K = 0$gives no reaction.

Fig.3 Shows the velocity distribution (steady state) for the different phase angle at $X = 1.0, (\omega t = 0, \pi/6, \pi/3, \pi/4, \pi/2), Pr = 0.71, Gr = 5,$ $Gc = 5$, $Sc = 0.6$.Increasing the phase angle causes a reduction in velocity. Here $\omega t = 0$ represents a vertical plate and$\omega t = \pi/2$ represent a horizontal plate, The numerical values have satisfied the prescribed boundary conditions. It is observed that the time taken to reach steady state is more in the case of the vertical plate than the horizontal plate. The impact of the Magnetic field parameter on the velocity distributions at $M = 0,1,2,3$, $Pr = 0.71$, $Gr = 5$, $Gc = 5$, $Sc = 0.6$, $\omega t = \pi/4$ is shown in Fig.4. In the case of cooling of the plate, an increase in velocity, due to the buoyancy force with decreasing values of the Magnetic field parameter is observed for air.

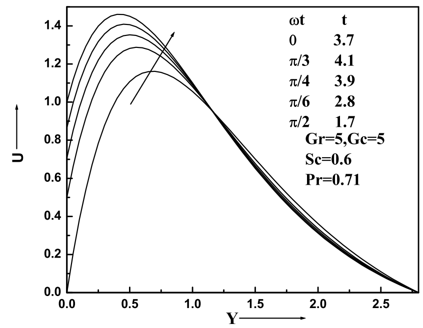


Fig. 3: Effect of Phase angle ωt on Steady State Velocity Profiles

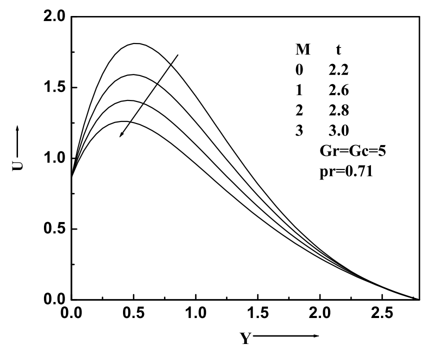


Fig. 4: Effect of Magnetic field Parameter M on Steady State Velocity Profile

The velocity field for different values of the chemical reaction parameter $(K = -5.0, -2.0, 0, 2.0, 5.0)$, $Gr = 5$, $Gc = 5$, $Pr = 0.71$, $Sc = 0.6$,$\omega t = \pi/4$ are shown in Fig.5.This shows the trend that velocity increase with decreasing chemical reaction parameter in the boundary layer. This implied that the velocity increases when the reaction is generative and decreases when it is destructive, showing a fall in the velocity as the result of increasing values of the chemical reaction parameter. Fig.6. Helps to study the effects of velocity profiles for different

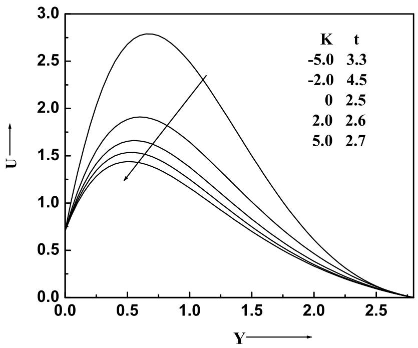


Fig. 5: Effect of Chemical Reaction K on Steady State Velocity Profile

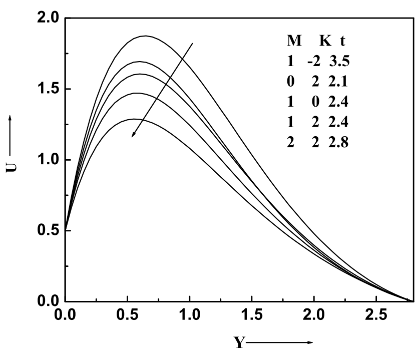


Fig. 6: Effect of M and K on Steady State Velocity profile

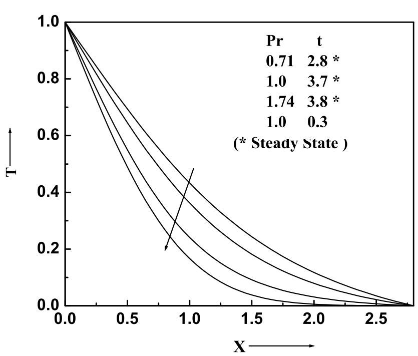


Fig. 7: Influence of Prandtl Number Pr on Transient Temperature Profiles

Magnetic fields $(M = 1,0,1,1,2)$ and chemical reaction

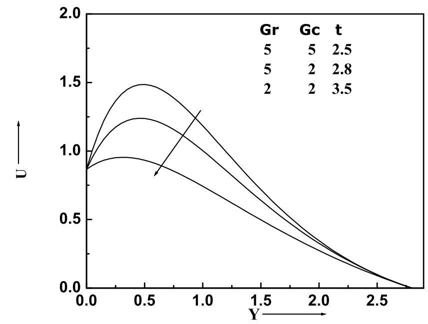


Fig. 8: Impact of Gr and Gc on Steady State Velocity Profiles

parameters$(K = -2,2,0,2,2)$, $Gr = 5$, $Gc = 5$, $Pr = 0.71$,$Sc = 0.6$. It indicates, decrease in velocity along the plate [or away from the wall] with increasing Magnetic field and chemical reaction parameters for air in the case of cooling of the surface. The ratio of the kinematic viscosity to thermal transport of the fluid is Prandtl Number 7. reveals the transient temperature distributions for different values of the Prandtl Number$Pr = 0.71$ (air), $Pr = 1.0, 1.74$ (Water). Usually, the effect of the Prandtl number is studied in the temperature field. It is seen that the temperature increases in the boundary layer with decreasing Prandtl Numbers for air. Physically, it indicates rapid diffusion of heat away from the plate when the Prandtl number is low.

The ratio of the buoyancy to viscous force acting on a fluid is Grashof number. The velocity distribution in the boundary layer for different values of thermal Grashof numbers or mass Grashof number $(Gr = 5,5,2)$, $(Gc = 5,2,2)$, $Pr = 0.71$, $Sc = 0.6$ are shown graphically in Fig.8. This shows an effective increase in velocity due to buoyancy with increasing thermal Grashof numbers or mass Grashof number. The dimensionless Schmidt number is the ratio of momentum diffusivity to the convection mass diffusivity. The concentration profiles for a different Schmidt number and chemical reaction parameter, $(K = -3.0, -2.0, 0, 0.2, 2.0)$, $(Sc = 0.6, 0.3)$, $Gr = 5$, $Gc = 5$, $Pr = 0.71$ are shown in Fig.9. The velocity decreases due to the gases diffusing into the air with increasing chemical reaction parameter and the Schmidt number. This indicates that the destructive reaction and increase in the Schmidt number leading to a fall in the velocity.

The Skin-friction, the rate of heat transfer and the rate of mass transfer in their steady state conditions are studied. The dimensionless local and average values of the skin-friction, the Nusselt number and the Sherwood number is given by the following results.

$$\tau_x = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{25}$$

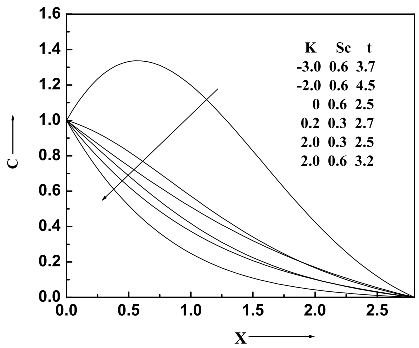


Fig. 9: Influence of chemical reaction K, Schmidt number Sc on Concentration

$$\overline{\tau} = -\int_0^1 \left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX \tag{26}$$

$$NU_x = -X\left[\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}\right] \tag{27}$$

$$\overline{NU} = -\int_0^1 \left[\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}\right] dX \tag{28}$$

$$Sh_X = -X\left(\frac{\partial C}{\partial Y}\right)_{Y=0} \tag{29}$$

$$\overline{Sh} = -\int_0^1 \left(\frac{\partial C}{\partial Y}\right)_{Y=0} dX \tag{30}$$

The derivatives involved in Equation (25) to (30) are evaluated using five point approximation formula and then the integrals are evaluated using Newton-Cotes closed integration formula. The local skin friction values are calculated from Equation (25) and plotted in Fig.10. Skin friction coefficient refers to the ratio of Local wall shear stress to characterize dynamic in the fluid. Local skin friction increases with a decreasing of phase angle.Therefore, it can be concluded that Skin friction appeared to be dependent of phase angle. The local Nusselt number for the different phase angle ωt is shown in Fig.11. The local Nusselt number decreases with increasing phase angle ωt. The Local Sherwood number for different values of the chemical reaction parameter $(K = -3.0, -2.0, 0, 0.2, 2.0)$ and Schmidt number$(Sc = 0.6, 0.3)$, $Gr = 5$, $Gc = 5$, $Pr = 0.71$ are shown in Fig.12. The Local Sherwood number decreases due to mass transfer with increasing chemical reaction parameters and Schmidt number by free convection. Fig.13 Shows decrease in the local skin friction in the momentum boundary layer with increasing Magnetic field and chemical reaction parameter. Hence it is observed from this, skin friction was suppressed due to the Significant presence of Magnetic field and the chemical reaction parameter enabling to aid the fluid flow. The

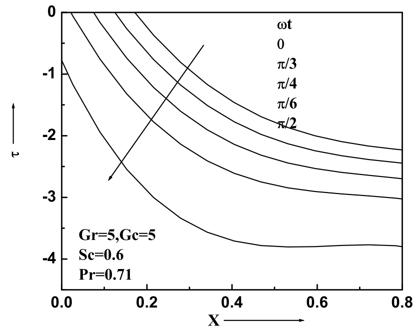


Fig. 10: Effect of Phase Angle ωt on Local Nusselet number

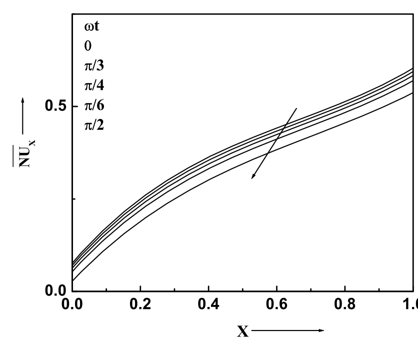


Fig. 11: Effect of Phase Angle ωt on Local Nusselet number

local Nusselt number for different values of $Pr = 0.71$ (air), $Pr = 1.0, 1.74$ (Water) is shown in Fig.14. The local Nusselt number decreases with increasing Pr in the case of cooling of the plate at all points. Usually,the dimensionless Nusselt number is the ratio of free convection heat transfer to conduction heat transfer in the fluid; physically it shows how much heat transferred from the surface to the moving fluid. It is consistent with the fact that Nusselt number decreases with increasing phase angle too. The effects of the phase angles ωt on the

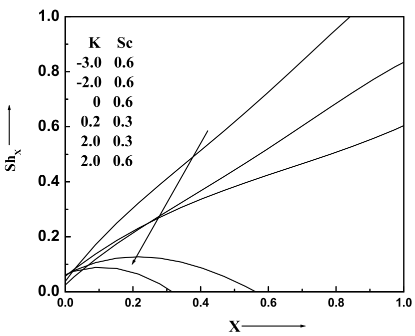


Fig. 12: Influence of K, Sc on Local Sherwood Number

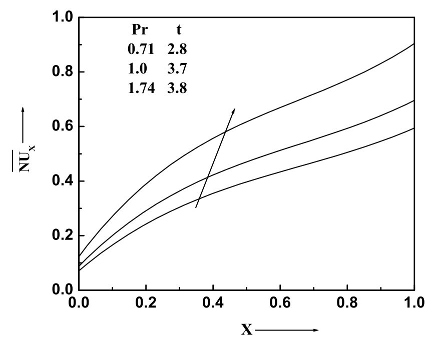


Fig. 14: Effect of Prandtl Number Pr on Local Nusselet number

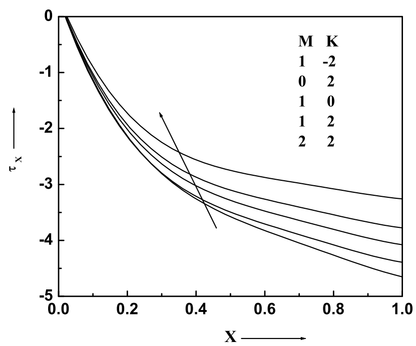


Fig. 13: Influence of M and K on Local Skin-Friction

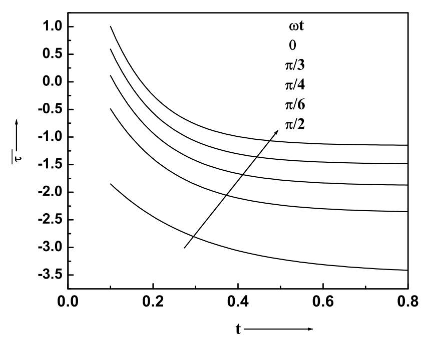


Fig. 15: Influence of Phase Angle ωt on Average Skin-Friction

average values of the skin friction are shown in Fig.15. This figure illustrates the skin friction for $Gr = 5 > 0$, $Gc = 5 > 0$,$Sc = 0.6$(water vapor), $Pr = 0.71$(air). The average skin friction decelerates with increasing phase angle in the boundary layer and reaches a steady state at $t = 1.7$.The effects of the phase angle ωt on the average values of the Nusselt number are shown in Fig.16. The average Nusselt numbers diminish with increasing phase angles ωt. It is perceived that at initial times, there is no heat transfer, this is because the heat transfer is only by original conduction. The effect of the chemical reaction parameter and Magnetic field on the average Skin friction is shown in Fig.17. The average skin friction decelerates with escalating Magnetic field and chemical reaction parameter in the case of cooling of the plate. It is found that the Local Skin friction too increases for similar values of the Magnetic field and chemical reaction parameter. The influence of Prandtl numbers on the average values of the Nusselt number is shown in Fig.18. The average Nusselt number decreases with the increasing Prandtl number is due to increase in the viscosity of the fluid, which makes the fluid thick. It is

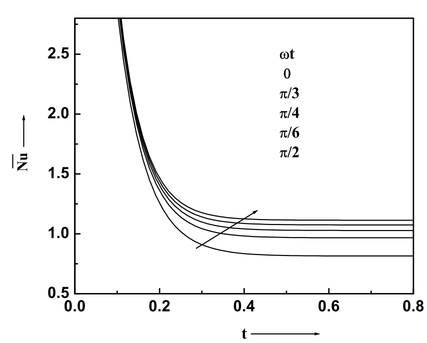


Fig. 16: Effect of Phase Angle ωt on Average Nusselet Number

detected that an increase in the prandlt number decreases the temperature of the flow field.Chemical reaction effects on the average Value of the Sherwood number is shown Fig.19. The Sherwood number increases as a result of mass transfer with increasing values of the chemical

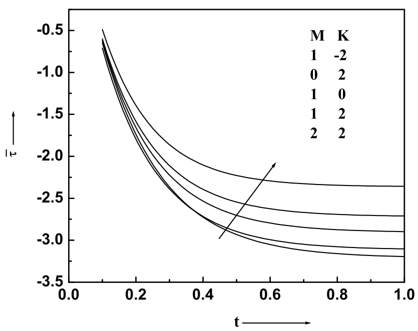


Fig. 17: Influence of M and K on Average Skin-friction

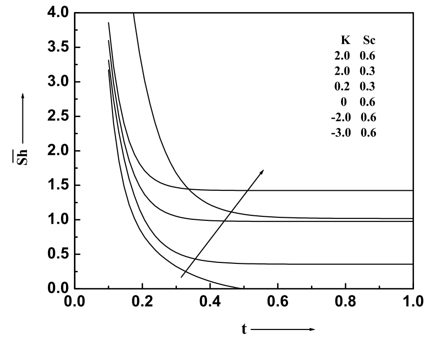


Fig. 19: Effect of K and Sc on Average Sherwood Number

reaction parameter.It is detected that Sherwood number increases with an increase in K and Sc.

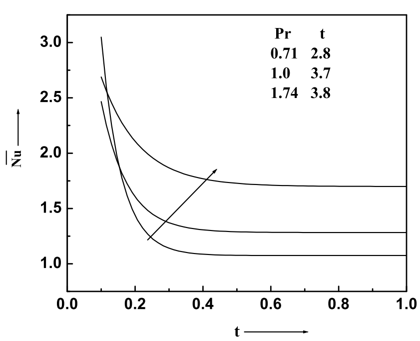


Fig. 18: Effect of Prandtl Number Pr on Average Nusselet Number

5.2 Conclusion

The Numerical study has been carried out for the unsteady MHD and chemical reaction flow past an oscillating semi-infinite vertical plate. The dimensionless governing equations are solved through an implicit scheme (Crank-Nicolson type). The effect of velocity, temperature, and concentration of the different parameter is studied. The observations are

- The velocity decreases with increasing phase angle with respect to Pr, Gr, Gc, and Sc.
- The velocity decreases with increasing Magnetic field parameter and Chemical reaction parameter for both air and water vapor.
- The velocity decreases due to the diffusion of gases into the air with increasing Chemical reaction parameter and the Schmidt number.
- Prandtl numbers have a significant effect on the local as well as average heat transfer rates (Nusselt number)
- The local and average skin friction decelerates with increasing phase angle, Magnetic field and chemical reaction which aids the fluid flow and this property helps in designing equipments.
- The local and average rate of mass transfer (Sherwood Numbers) is enhanced in the significant presence of chemical reaction parameter and Schmidt number.

5.3 Application

- It is believed that the obtained results from MHD effect, Flow Past an oscillating semi-infinite vertical plate, electrically transmitting fluid and the chemical reaction will provide useful information for applications in information science.
- The study of heat and mass transfer problems with chemical reaction is of extensive practical significance to engineers and scientists, but its universal occurrence in so many branches of science and engineering.
- MHD flows play an indispensable quota in determining the efficiency of movement in the field of power engineering devices, metallurgical and electrical technologies where staunch electromagnetic fields exist. Such kind of this problem has more applications in administering media and transmission switches.
- In many chemical processes, Mass transfer plays a key role. For the elimination of sulfur dioxide from the flue gas or hydrogen and oxygen from water (a chemical process).
- MHD is imperative in electrolysis, particularly in those electrolysis cells cashed to decreased aluminum oxide

to aluminum. This system is highly energy intensive. The electrolyte is high electrical resisting, while due to this process, MHD is used in the military arena as a propulsion mechanism for the submarine.

References

[1] M.S.Alam, M. Ali and M.A. Alim, Journal of.Scientific.Research**8**, 41-48(2016).

[2] M.Q. Ai-odat and T.A Ai-Azab,Emirates Journal for Engineering Research**12**,15-21(2007).

[3] V.Ananthaswamy and M.A Kalaivani, American Journal of Engineering Science and Technology Research**4**,1-17(2016).

[4] Bhaben,Ch.Neog and Rudra Kr.Das, International Journal of Scientific and Engineering Research **3**,1-5,(.2012).

[5] B.Carnahan, H.A.Luther and J.O.Wilkes, Applied Numerical Methods New York: John Wiley and Sons (1969).

[6] P.Chandrakala and P.Narayanabhaskar, International Journal of Applied Mechanics and Engineering**19**,419-426 (2014).

[7] Carlos O. Maidana, Thermo-Magnetic Systems for Space Nuclear Reactors, Springer, 2014.

[8] M.Dakshinamoorthy,P.Geetha and M.B.K Moorthy, International Journal of Mathematical Analysis**8**, 1859 1872 (2014).

[9] J.Girish Kumar and R.Mohana Ramana, International Journal of Scientific and Engineering Research**6**,57,(2015).

[10] Hannes Olof Gosa Alfven, Biographical Memoirs of Fellows of the Royal Society **44**,1998.

[11] M.A.Imran,Shakila Sarwar.Vieru.D.and M. Nazar, American Journal of Applied Mathematics**3**,6-13, (2015).

[12] K.V.Prasad, H. Vaidyak, H. VAjravelu, P.S.Datti and V.Umesh,International Journal of Applied Mechanics and Engineering**21**, 121-141,(2016).

[13] V.Rajesh, A.J.Chamkha,D.Bhanumath and S.Vijaya Kumar Varma, Computational Thermal Sciences**5**, 355367(2013).

[14] Rubel Ahmed, Reaz Uddin, B.M.Jewel Rana, Rafrafin Selim and Sarder Firoz Ahmmed, World Journal of Mechanics**6**,419-440(2016).

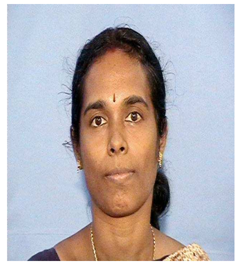
[15] O.D.Makinde, International Journal of the Physical Sciences**5**,700-710,(2010).

[16] M.K.Mazumdarand R.K. Deka, Romanian Journal Physics**52**,565573(2007).

[17] Y.I.Seini and O.D. Makinde, Hindawi Publishing Corporation Mathematical Problems in Engineering**2013**,7,(2013).

[18] V.M.Soundalgekar, S.K. Gupta and R.N. Aranake, Nuclear Engineering and Design**51**,403-407(1979).

[19] N.O. Weiss, M.R.E. Proctor, Proctor Magnetoconvection, University Printing House, Cambridge, United Kingdom,2014



A. R. Vijayalakshmi is known Mathematician obtained her PhD from Algappa University,has got a 20 years of teaching experience in handling UG and PG students and published around 24 papers invarious national and international journals.Currently,three research scholar are pursuing research and one has completed PhD under her guidance.she is specialized in computational fluid dynamics and differential equations.she is a hard worker and striven hard to mould a society with an energetic Mathematicians.

M. Selvajayanthi is known Mathematician obtained her M.phil degree from Madurai Kamaraj University,has got a 15 years of teaching experience in handling UG and PG students and mould them as a good students and published Two international Journal,now currently working as Assistant professor at SKP.Engineering College in the department of Mathematics.She is interested in computational fluid dynamics .