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Numerical Solution of Singularly Perturbed Problems via both Galerkin and Subdomain Galerkin methods

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Abstract: Numerical solutions of singularly perturbed problems(SPP) are given by using variants of finite elements method. Both Galerkin and subdomain Galerkin methods based on quadratic B-spline functions are applied to the SPP over the geometrically graded mesh. Results of some text problems are compared with analytical solutions of the SPP.

Keywords: Subdomain Galerkin method, graded mesh, spline, singularly perturbed problem.

1 Introduction

Differential equations including a small parameters ε multiplied with the highest order derivative are known as the singularly perturbed problem and these equations models many real life problems so that it can be used in many areas of science and engineering such as fluid dynamics, quantum mechanics, control theory, chemical science etc. It has the form:

$$-\varepsilon u'' + \rho(x)u' + s(x)u = f(x), \quad x \in [0,1] \tag{1}$$

with boundary conditions

$$u(0) = u_0 \text{ and } u(1) = u_n, \ u_0, \ u_n \in \mathbb{R} \tag{2}$$

where $0 < \varepsilon \leq 1$, $\rho(x), s(x), f(x)$ are sufficiently smooth functions.

When the singular perturbation parameter becomes smaller, the problem possess the boundary layer on the narrow right hand side of the problem region. Thus a small parameter of the mathematical model cause a sudden change of dependent boundaries near boundaries. Occurrence of sharp boundary layers as ε tend to infinity fails to yield analytical solutions of the boundary layer problem. In addition, classical numerical methods leads to happening of the oscillations in the compute solutions. Therefor it is of interest for scientist to be developed new numerical methods to tackle the boundary layer for the singularly perturbed differential equations. In order to model the boundary layer, one way is to use finer mesh on some part of the region on where boundary layer behavior exist. Two of the special meshes known as Shishkin mesh and Bakhvalov meshes have been widely used to have solutions of the singularly perturbed problem. Bakhvalov grid is based on mesh generating functions and used them to develop numerical methods for solving the problems. Shishkin mesh is a special piece-wise uniform mesh in which splitting of the problem domain is conducted according to predetermined boundary layer location and transition of the submesh is achieved with special definition.

B-spline related numerical methods have been constructed to have solution of the singularly perturbed problems in some studies[4,5,6,7,8,9]. Both the cubic B-spline and quadratic B-spline collocation method is set up for the SPP on using the geometrically graded mesh[1, 2]. Also, Galerkin method is set up to find numerical solutions of SPP using cubic B-splines [3]. In this study solution of the SPP is given by using combination of the quadratic B-splines defined over the graded mesh. Quadratic B-spline Galerkin algorithm and quadratic B-spline subdomain algorithm are defined for getting solution of the SPP over the graded mesh. Thus very fine mesh are used over interval on which the rapid change solution exist and larger mesh is employed on the portion of the domain when solution is quite smooth. A mesh parameter is used to arrange the fineness of mesh through the end boundary to have the higher accuracy of the SPP.

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2 Quadratic B-splines

Consider a partition of the problem domain $[0,1]$ with the grid points x_k, such that

$$0 = x_0 < x_1 < \cdots < x_N = 1$$

where $x_{k+1} = x_k + h_k$ and h_k is the size of interval $[x_k, x_{k+1}]$ with relation $h_k = \tau h_{k-1}$. Where τ is mesh ratio constant. Since

$$h_0 + h_1 + \cdots + h_{N-1} = 1$$

it can be concluded that

$$h_0 = \frac{1}{1+\tau+\tau^2+\cdots+\tau^{N-1}}.$$

If the mesh ratio τ is taken unity then the partition will be uniform. To make the mesh size smaller at the right boundary, τ must be selected as $\tau < 1$ Conversely, To get finer mesh at the left boundary, τ must be selected as $\tau > 1$. Mentioned selection of τ will be done by experimentally.

2.1 Quadratic B-spline Galerkin method

The QB-splines over the geometrically graded mesh[10] can be expressed such as:

$$\begin{matrix} Q_{k-1} \\ Q_k \\ Q_{k+1} \end{matrix} = \frac{1}{h_k^2} \begin{cases} (h_k - \xi)^2 \tau, \\ h_k^2 + 2h_k \tau \xi - (1+\tau)\xi^2, \\ \xi^2 \end{cases} \tag{3}$$

where $\xi = x - x_k$ and $0 \le \xi \le h_k$. Each quadratic B-spline Q_k and its derivatives vanish outside of the interval $[x_{k-1}, x_{k+2}]$ and thus an element is covered by three consecutive QB-splines. The set of the QB-splines $\{Q_{-1}, Q_0, \ldots, Q_N\}$ also constitutes a basis for the functions defined in $[a,b]$ [11]. Let $U(x,t)$ be approximate solution to $u(x,t)$ defined as

$$U = \sum_{k=-1}^{N} \gamma_k Q_k \tag{4}$$

where γ_k are unknown parameters. By the substitution of the value of Q_k at the knots x_k in Eq.(4), the nodal value U and its derivative U' are expressed in terms of γ_k by

$$\begin{aligned} U_k = U(x_k) &= \tau \gamma_{k-1} + \gamma_k, \\ U_k' = U'(x_k) &= \frac{2\tau}{h_k}(\gamma_k - \gamma_{k-1}). \end{aligned} \tag{5}$$

To carry on the Galerkin procedure, one can multiply Eq. (1), by the wight function $v(x)$ and integrate over the interval $[x_k, x_{k+1}]$

$$\int_{x_k}^{x_{k+1}} (-\varepsilon v' U'(x) + v\rho(x)U' + vs(x)U(x))dx - \varepsilon v U'(x) \Big|_{x_k}^{x_{k+1}} - \int_{x_k}^{x_{k+1}} vf(x)dx = 0.$$

If weight functions are selected as $Q_j,\ j = k-1, k, k+1$ and (4) is substituted in equation above, we obtain the system of algebraic equations

$$\sum_{j=k-1}^{k+1} [-\varepsilon \int_0^{h_k} (\phi_i' \phi_j' + \rho \phi_i \phi_j' + s\phi_i \phi_j) d\xi] \gamma_j - \varepsilon \phi_i \phi_j' \Big|_0^{h_k} \gamma_j - \int_0^{h_k} \phi_i f(x_k + \xi) d\xi = 0 \tag{6}$$

Then following values can be computed.

$$a_{ij} = \int_0^{h_k} \phi_i' \phi_j' d\xi, \quad r_{ij} = \phi_i \phi_j' \Big|_0^{h_k},$$

$$b_{ij} = \int_0^{h_k} \phi_i \phi_j' d\xi, \quad f_i = \int_0^{h_k} \phi_i f(x_k + \xi) d\xi,$$

$$c_{ij} = \int_0^{h_k} \phi_i \phi_j d\xi,$$

Where $i, j = k-1, k, k+1$ and

$$A^{(k)} = \frac{2}{3h_k} \begin{bmatrix} 2\tau^2 & \tau(1-\tau) & -\tau \\ \tau(1-\tau) & 2(1-\tau+\tau^2) & \tau-2 \\ -\tau & \tau-2 & 2 \end{bmatrix}$$

$$B^{(k)} = \begin{bmatrix} \frac{-\tau^2}{2} & \frac{1}{6}\tau(3\tau-1) & \frac{\tau}{6} \\ -\frac{1}{6}\tau(3\tau+5) & \frac{1}{2}\tau^2 - \frac{1}{2} & \frac{5}{6}\tau + \frac{1}{2} \\ \frac{-\tau}{6} & \frac{1}{6}\tau - \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$C^{(k)} = h_k \begin{bmatrix} \frac{1}{5}\tau^2 h_k & \frac{1}{30}\tau h_k(4\tau+9) & \frac{\tau}{30} \\ \frac{1}{30}\tau h_k(4\tau+9) & \frac{8}{15}\tau^2 + \frac{11}{5}\tau + \frac{8}{15} & \frac{3}{10}\tau + \frac{2}{15} \\ \frac{\tau}{30} & \frac{3}{10}\tau + \frac{2}{15} & \frac{1}{5}h_k \end{bmatrix}$$

$$R^{(k)} = \frac{1}{h_k} \begin{bmatrix} 2\tau^2 & 0 & 0 \\ 2\tau(3\tau+2) & -4\tau & 2\tau \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$F^{(k)} = \frac{1}{h_k^2} \begin{bmatrix} \vartheta_1 & \vartheta_1 & \vartheta_1 \\ \vartheta_2 & \vartheta_2 & \vartheta_2 \\ \vartheta_3 & \vartheta_3 & \vartheta_3 \end{bmatrix}$$

This system is expressed in notational form:

$$\begin{aligned} \vartheta_1 &= -\tau\left(2h_k - 2e^{h_k} + h_k^2 + 2\right) \\ \vartheta_2 &= 2(\tau+1)(1-e^{h_k}) + 2h_k(\tau + e^{h_k}) - h_k^2(1-\tau e^{h_k}) \\ \vartheta_3 &= \left(e^{h_k}\left(h_k^2 - 2h_k + 2\right) - 2\right) \end{aligned}$$

Using the local matrices $A^{(i)}$, $B^{(i)}$, $C^{(i)}$, $R^{(i)}$ and $F^{(i)}$, equation (6) can be represented as

$$(-\varepsilon A^{(i)} + \rho B^{(i)} + sC^{(i)} - \varepsilon R^{(i)})\gamma^{(i)} = F^{(i)}$$

where

$$\gamma^{(i)} = (\gamma_{k-1}^{(i)}, \gamma_k^{(i)}, \gamma_{k+1}^{(i)}, \gamma_{ki+2}^{(i)}),\ F^{(i)} = (f_{k-1}^{(i)}, f_k^{(i)}, f_{k+1}^{(i)}, f_{k+2}^{(i)})$$

By combining the local matrices which is defined on $[x_i, x_{i+1}]$, $i = 0, \dots, N-1$, the global system in the range of $[x_0, x_N]$ can be defined as follows.

$$(-\varepsilon A + \rho B + sC - \varepsilon R)\gamma = F \qquad (7)$$

The matrix of A is

$$\begin{bmatrix}
\sigma_{0,0}^{(0)} & \sigma_{0,1}^{(0)} & \sigma_{0,2}^{(0)} & & & & \\
\sigma_{1,0}^{(0)} & \sigma_{1,1}^{*(1)} & \sigma_{1,2}^{*(1)} & \sigma_{1,3}^{*(1)} & & & \\
\sigma_{2,0}^{(0)} & \sigma_{2,1}^{*(1)} & \sigma_{2,2}^{*(2)} & \sigma_{2,3}^{*(2)} & \sigma_{2,4}^{*(2)} & & \\
& \ddots & \ddots & \ddots & \ddots & \ddots & \\
& \sigma_{i,i-2}^{*(i-1)} & \sigma_{i,i-1}^{*(i)} & \sigma_{i,i}^{*(i+1)} & \sigma_{i,i+1}^{*(i+1)} & \sigma_{i,i+2}^{*(i+1)} & \\
& \ddots & \ddots & \ddots & \ddots & \ddots & \\
& & \sigma_{n-1,n-3}^{*(n-2)} & \sigma_{n-1,n-2}^{*(n-1)} & \sigma_{n-1,n-1}^{*(n)} & \sigma_{n-1,n}^{*(n)} & \sigma_{n-1,n+1}^{(n)} \\
& & & \sigma_{n,n-2}^{*(n-1)} & \sigma_{n,n-1}^{*(n-1)} & \sigma_{n,n}^{*(n-1)} & \sigma_{n,n+1}^{(n)} \\
& & & & \sigma_{n+1,n-1}^{(n)} & \sigma_{n+1,n}^{(n)} & a_{n+1,n+1}^{(n)}
\end{bmatrix}$$

Where

$$\begin{aligned}
&\sigma_{1,1}^{*(1)} = \sigma_{1,1}^{(0)} + \sigma_{1,1}^{(1)},\\
&\sigma_{1,2}^{*(1)} = \sigma_{1,2}^{(0)} + \sigma_{1,2}^{(1)},\\
&\sigma_{2,1}^{*(1)} = \sigma_{2,1}^{(0)} + \sigma_{2,1}^{(1)},\\
&\sigma_{2,2}^{*(2)} = \sigma_{2,2}^{(0)} + \sigma_{2,2}^{(1)} + \sigma_{2,2}^{(2)},\\
&\sigma_{2,3}^{*(2)} = \sigma_{2,3}^{(2)} + \sigma_{2,3}^{(1)},\\
&\sigma_{i,i-1}^{*(i)} = \sigma_{i,i-1}^{(i-1)} + \sigma_{i,i-1}^{(i)},\\
&\sigma_{i,i}^{*(i)} = \sigma_{i,i}^{(i-1)} + \sigma_{i,i}^{(i)} + \sigma_{i,i}^{(i+1)},\\
&\sigma_{i,i+1}^{*(i)} = \sigma_{i,i+1}^{(i)} + \sigma_{i,i+1}^{(i+1)},\\
&\sigma_{n-1,n-2}^{*(n-1)} = \sigma_{n-1,n-2}^{(n-2)} + \sigma_{n-1,n-2}^{(n-1)},\\
&\sigma_{n-1,n-1}^{*(n)} = \sigma_{n-1,n-1}^{(n-2)} + \sigma_{n-1,n-1}^{(n-1)} + \sigma_{n-1,n-1}^{(n)},\\
&\sigma_{n-1,n}^{*(n)} = \sigma_{n-1,n}^{(n-1)} + \sigma_{n-1,n}^{(n)},\\
&\sigma_{n,n-1}^{*(n-1)} = \sigma_{n,n-1}^{(n-1)} + \sigma_{n,n-1}^{(n)},
\end{aligned}$$

B, C, R matrices are obtained similarly. Also the matrix of F is computed as follows.

$$F = \begin{bmatrix}
f_0^0 \\
f_1^0 + f_1^1 \\
f_2^0 + f_2^1 + f_2^2 \\
\vdots \\
f_i^{i-1} + f_i^i + f_i^{i+1} \\
\vdots \\
f_{n-2}^{n-1} + f_{n-2}^{n-2} + f_{n-2}^{n-3} \\
f_{n-1}^{n-1} + f_{n-1}^{n-2} \\
f_n^{n-1}
\end{bmatrix}$$

The system (7) consist of $N+1$ linear equation in $N+3$ unknown parameters. To get a unique solution an additional two constraints are needed. The following equations can be written by using boundary conditions

$$\gamma_{-1} = \frac{u_0 - \gamma_0}{\tau}, \quad \gamma_N = u_n - \tau\gamma_{N-1}.$$

From these equations, γ_{-1} and γ_N can be eliminated from the system. This system is solved with Matlab program using five banded Thomas algorithm.

2.2 Quadratic B-spline Subdomain Galerkin method

If equation (1), multiplied by weight function V_n having the form

$$V_n = \begin{cases} 1, & x_k \le x < x_{k+1} \\ 0, & \text{other case} \end{cases}$$

is integrated over the interval $[x_k, x_{k+1}]$ then integrate form of the equation is obtained

$$\int_{x_0}^{x_n} [-\varepsilon U''(x) + \rho U'(x) + sU(x) - f(x)]dx = 0$$

integrating by part is applied to reduce the second order term in equation (1)

$$-\varepsilon U'(x) \Big|_{x_k}^{x_{k+1}} + \rho U(x) \Big|_{x_k}^{x_{k+1}} + s\int_{x_k}^{x_{k+1}} U(x)dx = \int_{x_k}^{x_{k+1}} f(x)dx$$

Substituting Eq. (4) into nodal value and its derivative yield the system of equations

$$\begin{aligned}
&[-\varepsilon(\sum_{J=k-1}^{k+1} \phi_i' \Big|_0^{h_k})\gamma_j) + \rho(x_k+\xi)(\sum_{J=k-1}^{k+1} \phi_i \Big|_0^{h_k} \gamma_j)\\
&+ \int s(x_k+\xi) \sum_{J=k-1}^{k+1} \int_{x_k}^{x_{k+1}} \phi_i\gamma_j dx \qquad (8)\\
&= \int_{x_k}^{x_{k+1}} f(x_k+\xi)dx
\end{aligned}$$

Performing integration an evaluation gives

$$\begin{aligned}
&(-\frac{2\tau\varepsilon}{h_k} + \tau(\frac{1}{3}h_k s(x) - \rho(x)))\gamma_{k-1}\\
&+(\frac{2\varepsilon}{h_k}(1+\tau) + (\tau-1)\rho(x) + \frac{2}{3}h_k(\tau+1)s(x))\gamma_k \qquad (9)\\
&+(-\frac{2\varepsilon}{h_k} + \rho(x) + \frac{1}{3}h_k s(x))\gamma_{k+1} = \int_{x_k}^{x_{k+1}} f(x)dx
\end{aligned}$$

The equation (9) can be converted the following matrices system;

$$\mathbf{BX} = \mathbf{F} \qquad (10)$$

where

$$B = \begin{bmatrix}
\kappa_{01} & \kappa_{02} & \kappa_{03} & & & & \\
& \kappa_{11} & \kappa_{12} & \kappa_{13} & & & \\
& & \kappa_{21} & \kappa_{22} & \kappa_{23} & & \\
& & & \kappa_{31} & \kappa_{32} & \kappa_{32} & \\
& & & \ddots & \ddots & \ddots & \\
& & & & \kappa_{n1} & \kappa_{n2} & \kappa_{n3}
\end{bmatrix}$$

and

$$\kappa_{k1} = -\frac{2\tau\varepsilon}{h_k} - \tau\rho_k + \frac{1}{3}\tau h_k s_k,$$

$$\kappa_{k2} = \frac{2\varepsilon}{h_k}(1+\tau) + (\tau-1)\rho_k + \frac{2}{3}h_k(\tau+1)s_k$$

$$\kappa_{k3} = -\frac{2\varepsilon}{h_k} + \rho_k + \frac{1}{3}h_k s_k$$

$$X = \left[\,\gamma_{-1},\ \gamma_0,\ \gamma_1,\ \cdots,\ \gamma_{n-1},\ \gamma_n\,\right]^T$$
$$F = \left[\,f_0,\ f_1,\ f_2,\ \cdots,\ f_{n-1},\ f_n\,\right]^T$$
$$f_k = f(x_k),\ k = 0, 1, \cdots, N$$

The system (10) consist of $N+1$ linear equation in $N+3$ unknown parameters $\mathbf{d}^{n+1} = (\gamma_{-1}^{n+1}, \gamma_0^{n+1}, \ldots, \gamma_{N+1}^{n+1})$. To make solvable the system, boundary conditions $u(0) = u_0$ and $u(1) = u_n$ are used to find two additional linear equations:

$$\gamma_{-1} = \frac{u_0 - \gamma_0}{\tau},\quad \gamma_N = u_n - \tau\gamma_{N-1} \tag{11}$$

(11) can be used to eliminate $\delta_{-1}, \delta_{N+1}$ from the system (10) which then becomes the solvable matrix equation. Three banded Thomas algorithm is used to solve the system.

3 Numerical Results

The discrete maximum norm measuring the error between the numerical and the analytical solution is defined as

$$L_\infty = |u - U|_\infty = \max_k |u_k - U_k^n|$$

where u_k^n and U_k^n represent exact and numerical solutions at the n.th time level, respectively. In order to minimize the error, the scan to determine the optimum parameter τ can be searched the interval $(0,1)$, because of the boundary layers are at the right boundary in this example. Our example is

$$-\varepsilon u'' + u' = \exp(x),$$

$$u(0) = u(1) = 0$$

with the exact solution

$$u(x) = \left[e^x - \frac{1 - e^{1-1/\varepsilon} + (e-1)e^{(x-1)/\varepsilon}}{1 - e^{-1/\varepsilon}}\right] / (1-\varepsilon).$$

taken from [12].

Solution profiles are shown in Figs. 1-4 for the example. These figures are depicted for $N = 20$ and $\varepsilon = 0.01, 0.001$. In order to observe the achievement of the proposed methods, analytical solutions and obtained results are illustrated together in all figures. Continuous line represents the exact solutions and the dash lines indicates approximate solutions in Figures .Visual solutions are depicted in Fig. 1-2 for QG method and in Fig. 3-4 for QSG method. Since boundary layer occurs on the right side of solution domain, use of uniform mesh give oscillations on the right side. When the appropriate graded mesh is used, no oscillations is observed in Fig. 2-4.

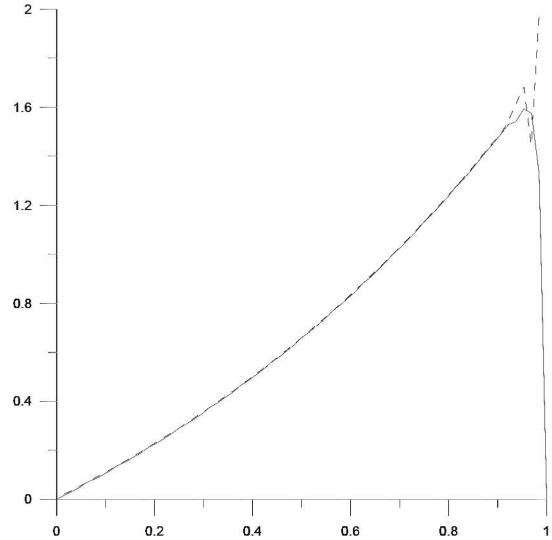

Fig. 1: Uniform mesh for $\varepsilon = 0.01$ Galerkin method

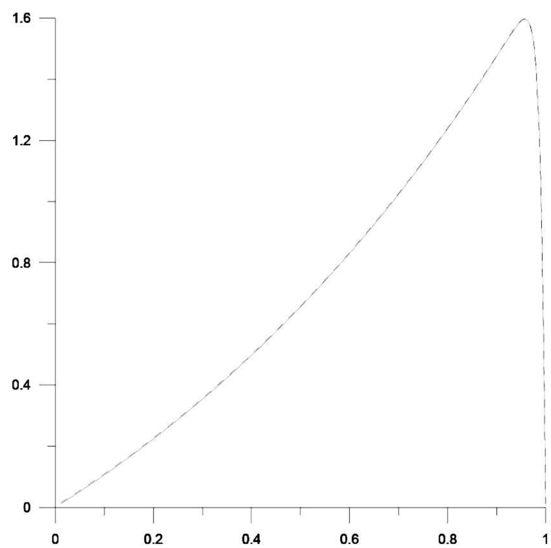

Fig. 2: Graded mesh for $\varepsilon = 0.01$ Galerkin method

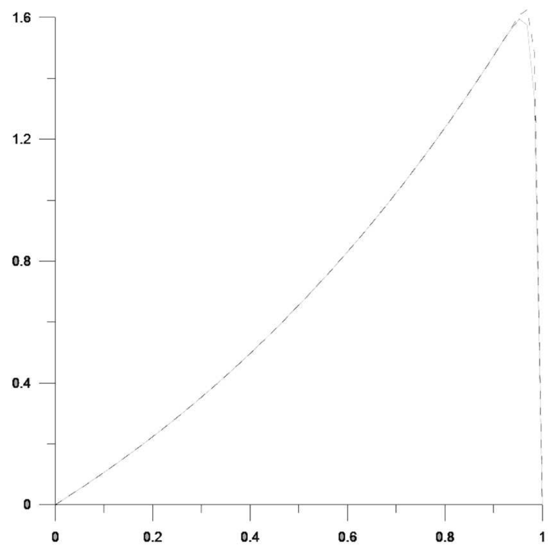

Fig. 3: Uniform mesh for $\varepsilon = 0.01$ subdomain Galerkin method

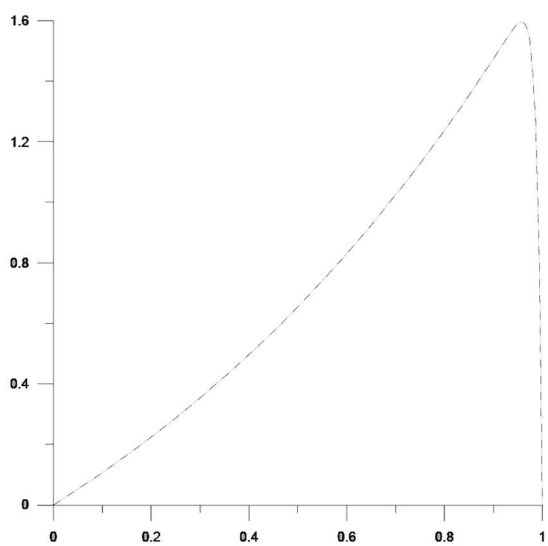

Fig. 4: Graded mesh for $\varepsilon = 0.01$ subdomain Galerkin method

4 Conclusion

The QG and QSG methods based on quadratic B-spline functions are derived for the numerical solutions of the SPP. Difficulties occur from the modelling of the boundary layers in numerical methods are tried to overcome over the geometrically graded mesh via B-splines. We observe that taking suitably free parameters τ for the quadratic B-splines gives the best solutions. We see that QB-splines over the geometrically graded mesh produce slightly better results than the QB-splines over the unit mesh for the SPP. And also the simplicity of the adaptation of B-splines can be noted as advantages of proposed numerical methods. Thus, to obtain the numerical solution of the differential equations having boundary layers, QG and QSG methods are advisable.

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