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Kumaraswamy Exponentiated Chen Distribution for Modelling Lifetime Data

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Abstract: In this paper we introduce the Kumaraswamy exponentiated Chen distribution for modelling a bathtub-shaped hazard rate function. Some structural properties of the Kumaraswamy exponentiated Chen distribution are discussed. The method of maximum likelihood is used for estimating the model parameters. Finally, the flexibility of the proposed distribution is illustrated using reliability data.

Keywords: Exponentiated Chen distribution; moments; entropies, maximum likelihood estimation.

1 Introduction

Models for life testing problems with lifetime distributions occupy an outstanding place in reliability theory. Characteristically, lifetime refers to the span of life of devices, survival times of patients in epidemiology. Recently Chaubey and Zhang [3] introduced and studied the exponentiated Chen distribution having the two parameter Chen [2] lifetime distribution as the special sub-model for modelling lifetime data. The cumulative distribution function (cdf) of the exponentiated Chen distribution is

$$G(x) = \left[1 - \exp\left\{\alpha\left(1 - \exp(x^{\beta})\right)\right\}\right]^{\theta}, \quad x > 0, \quad (1)$$

where $\alpha > 0$ is the scale parameter, $\beta > 0$ and $\theta > 0$ are the shape parameters of the exponentiated Chen distribution. The corresponding probability density function (pdf) is given by

$$g(x) = \alpha\beta\theta x^{\beta-1} exp\left\{x^{\beta} + \alpha\left(1 - \exp(x^{\beta})\right)\right\}\left\{1 - \exp\left\{\alpha\left(1 - \exp(x^{\beta})\right)\right\}\right\}^{\theta-1}, \quad (2)$$

This paper uses the Kumaraswamy G family approach for developing a new distribution called the Kumaraswamy exponentiated Chen distribution. Given a base line distribution this family adds two more shape parameters for making it more flexible.
For an arbitrary baseline cumulative function $G(x)$, Cordeiro and de Castro [6] defined the pdf $f(x)$ and cdf $F(x)$ of the Kw-G distribution by

$$f(x) = abg(x)\, G(x)^{a-1}\left\{1 - G(x)^{a}\right\}^{b-1}, \quad (3)$$

and

$$F(x) = 1 - \left\{1 - G(x)^{a}\right\}^{b}, \quad (4)$$

Various different distributions has been dedicated in the literature to developing the new Kumaraswamy G family of distributions. Correa et al. [4] studied the Kumaraswamy Normal distribution and evaluated the performance of this model using simulation. Nadarajah and Eljbri [11] examined the Kumaraswamy generalized Pareto distribution and obtained the first four moments, the asymptotic distribution of order statistics with L-moments and discussed maximum likelihood estimation using simulation. Elbatal [7] defined the Kumaraswamy linear exponential distribution, derived moments and moment generating function and discussed the method of MLE. Shams [14] proposed the Kumarswamy generalized Lomax distribution. Cordeiro et al [5] proposed the Kumaraswamy modified Weibull distribution with several mathematical properties and showed that it can have bathtub, unimodal, increasing and decreasing hazard rate functions. Gomes et al. [8] proposed the Kumaraswamy generalized Rayleigh distribution with covarites regression modelling for

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analysing lifetime data. More recently Khan et al. [10] proposed the transmuted exponentiated Chen distribution and investigated various structural properties with application. In this article, we consider a generalization of the exponentiated Chen distribution by introducing two extra shape parameters using Kumaraswamy G family generator, which provide greater flexibility in the new extended model. We hope that the Kumaraswamy exponentiated Chen distribution will attract wider applications for modelling real world scenarios.

The rest of the article is organized as follows. In Section 2, we provide the analytical shapes of the probability density and hazard functions of the KEC distribution. The method of moment properties are considered in Section 3, and evaluated the ordinary moments of the new distribution. The Rényi entropy and the q-entropy are derived in Section 4. Maximum likelihood estimates (MLE) of the unknown parameters are discussed in Section 5. We fit the Kumaraswamy exponentiated Chen distribution to fatigue life of 6061-T6 aluminum coupons data in Section 6 which illustrate the usefulness of the proposed model. Finally, concluding remarks are addressed in Section 7.

2 Kumaraswamy exponentiated Chen distribution

A random variable x has the Kumaraswamy exponentiated Chen distribution defined through equation (3) the KW-G distribution with parameters $a,b,\alpha,\beta,\theta > 0$, then the pdf is

$$f(x) = ab\alpha\beta\theta x^{\beta-1}\frac{exp\left\{x^{\beta}+\alpha\left(1-\exp(x^{\beta})\right)\right\}\left\{1-\exp\left\{\alpha\left(1-\exp(x^{\beta})\right)\right\}\right\}^{a\theta-1}}{\left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x^{\beta})\right)\right\}\right\}^{a\theta}\right\}^{1-b}}, \tag{5}$$

respectively. The cdf corresponding to (5) is

$$F(x) = 1-\left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x^{\beta})\right)\right\}\right\}^{a\theta}\right\}^{b}. \tag{6}$$

where a and b are the additional shape parameters provide extra flexibility in new model which controls the skewness and kurtosis. The parameter α controls the location of the distribution, whereas the parameters β and θ control its shape. We obtain the exponentiated Chen distribution when the shape parameters $a = b = 1$. For $\theta = 1$, in addition to $a = b = 1$ it reduces to the Chen distribution [2]. If X is a random variable with density function (5), then we write this model as X $\sim$ KE-CHEN$(x;a,b,\alpha,\beta,\theta)$.

The plots in Figure 1 show some possible shapes of the Kumaraswamy exponentiated Chen distribution for some selected choice of parameters. Figure 2 shows that the proposed model has the ability to explain bathtub-shaped failure rate functions are very useful in reliability analysis. The reliability and hazard functions of the kumaraswamy exponentiated Chen distribution are

$$R(x) = \left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x^{\beta})\right)\right\}\right\}^{a\theta}\right\}^{b}. \tag{7}$$

and

$$h(x) = \frac{ab\alpha\beta\theta x^{\beta-1}exp\left\{x^{\beta}+\alpha\left(1-\exp(x^{\beta})\right)\right\}\left\{1-\exp\left\{\alpha\left(1-\exp(x^{\beta})\right)\right\}\right\}^{a\theta-1}}{\left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x^{\beta})\right)\right\}\right\}^{a\theta}\right\}^{-1}}, \tag{8}$$

respectively. The qth quantile $F(x_q)$ of the KEC random variable is

$$F^{-1}(u) = \left[ln\left\{1-\frac{1}{\alpha}\ln\left\{1-\left(1-(1-q)^{\frac{1}{b}}\right)^{\frac{1}{a\theta}}\right\}\right\}\right]^{\frac{1}{\beta}}, \tag{9}$$

3 Moments

This section presents the kth moment of the Kumaraswamy exponentiated Chen distribution. We will also evaluate the performance of the ordinary moments for some selected choices of parameters.

Theorem 1. *If X has the KEC$(x;a,b,\alpha,\beta,\theta)$ distribution then the kth moment of X is*

$$\acute{\mu}_k = ab\alpha\theta\sum_{i,j=0}^{\infty}\sum_{m,n=0}^{\infty}\binom{a\theta(i+1)-1}{j}\binom{m}{n}v_{i,j,m}\Gamma\left(\frac{k}{\beta}+1\right).$$

where

$$v_{i,j,m} = \frac{\binom{b-1}{i}(-1)^{i+j+n+\frac{k}{\beta}}\alpha^{m}(j+1)^{m}}{m!\,(n+1)^{\frac{k}{\beta}+1}}.$$

Proof: The kth moment of the KEC *distribution is*

$$\acute{\mu}_k = ab\alpha\beta\theta\int_0^{\infty}x^{k+\beta-1}\frac{exp\left\{x^{\beta}+\alpha\left(1-\exp(x^{\beta})\right)\right\}\left\{1-\exp\left\{\alpha\left(1-\exp(x^{\beta})\right)\right\}\right\}^{a\theta-1}}{\left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x^{\beta})\right)\right\}\right\}^{a\theta}\right\}^{1-b}}dx.$$

The above expression reduces to

$$\acute{\mu}_k = ab\alpha\beta\theta\sum_{i,j=0}^{\infty}\binom{b-1}{i}\binom{a\theta(i+1)-1}{j}(-1)^{i+j}I_k$$

where

$$I_k = \int_0^{\infty}x^{k+\beta-1}\exp(x^{\beta}+\alpha(j+1)\left(1-\exp(x^{\beta})\right))dx$$

Hence, the above integral reduces to

$$\acute{\mu}_k = ab\alpha\theta\sum_{i,j=0}^{\infty}\sum_{m,n=0}^{\infty}\binom{a\theta(i+1)-1}{j}\binom{m}{n}v_{i,j,m}\Gamma\left(\frac{k}{\beta}+1\right),$$

where

$$v_{i,j,m} = \frac{\binom{b-1}{i}(-1)^{i+j+n+\frac{k}{\beta}}\alpha^{m}(j+1)^{m}}{m!\,(n+1)^{\frac{k}{\beta}+1}}.$$

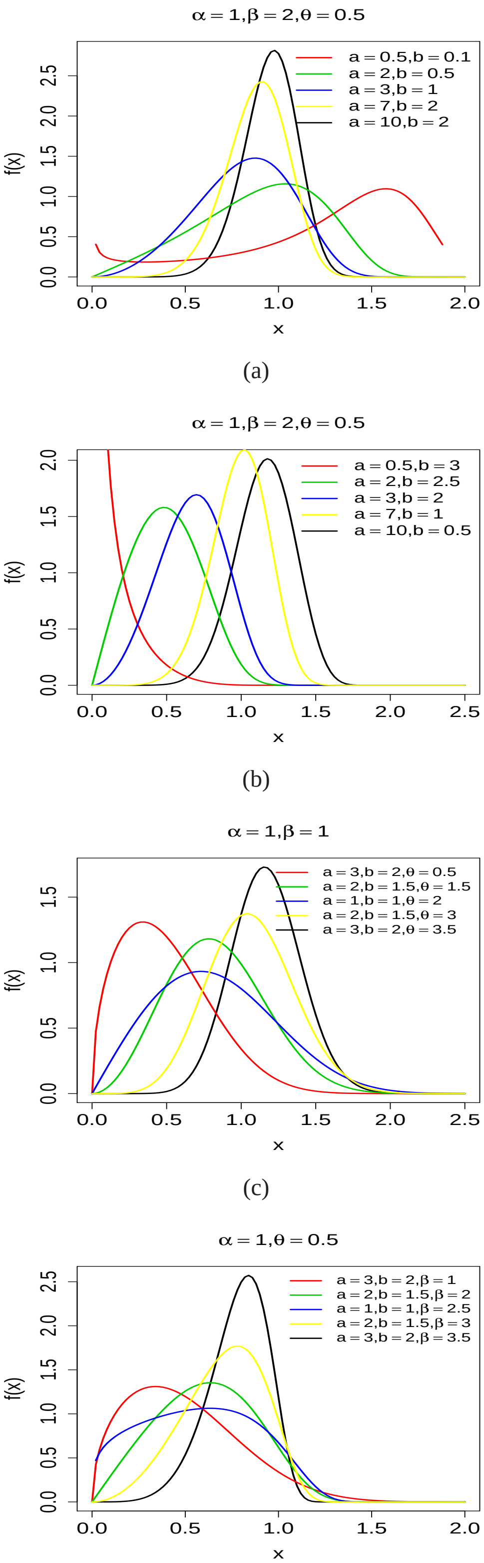


Fig. 1: Plots of the KEC pdf for some parameter values.

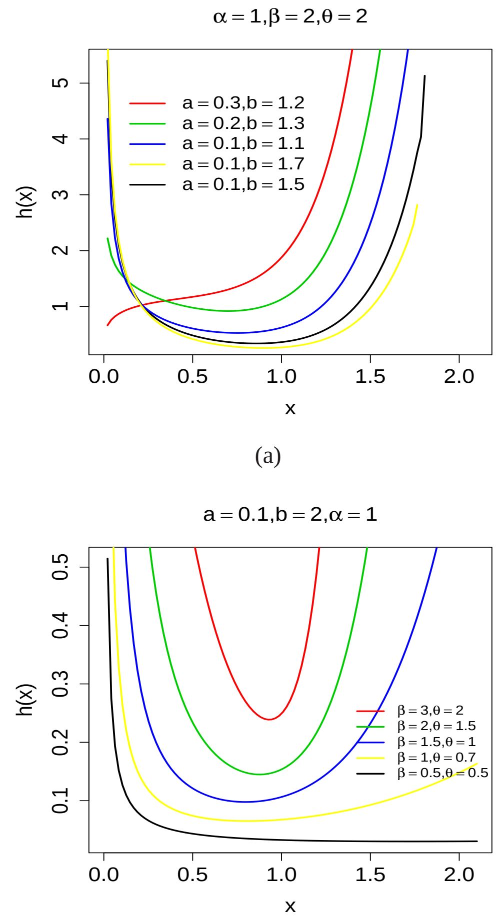


Fig. 2: Plots of the KEC hf for some parameter values.

The important features and characteristics of the KEC distribution are studied through moments. The central moment and the cumulants are

$$\mu_n = \Sigma_{i=0}^{n} \binom{n}{i} (-1)^i \acute{\mu}_1^n \acute{\mu_{n-i}} \quad \text{and} \quad k_n = \acute{\mu_n} - \Sigma_{i=1}^{n-1} \binom{n-1}{i-1} k_1 \acute{\mu_{n-i}},$$

respectively, where $k_1 = \acute{\mu_1}$, $k_2 = \acute{\mu_2} - \acute{\mu_1^2}$, $k_3 = \acute{\mu_3} - 3\acute{\mu_2}\acute{\mu_1} + 2\acute{\mu_1^3}$, $k_4 = \acute{\mu_4} - 4\acute{\mu_3}\acute{\mu_1} + 6\acute{\mu_2}\acute{\mu_1^2} - 3\acute{\mu_1^4}$, etc. The skewness and kurtosis measures are calculated using Well-known relationships. The values of the first eight ordinary moments for some selected choices of parameters are shown in Table 1.

4 Entropies

The Rényi [12] introduced the entropy denoted as, $I_R(\rho)$, for X with probability density from the $\text{KEC}(x;a,b,\alpha,\beta,\theta)$ is a measure of dispersion of the

Table 1: *Moments values of the KEC distribution for some selected values of parameters* $\alpha = 1, \beta = 0.5, \theta = 2$.

$\acute{\mu}_k$	$a=0.5, b=1.5$	$a=0.7, b=1.8$	$a=0.9, b=2$	$a=1, b=3$
$\acute{\mu}_1$	0.3144	0.3541	0.4094	0.3107
$\acute{\mu}_2$	0.2819	0.2897	0.3322	0.1829
$\acute{\mu}_3$	0.3931	0.3561	0.3892	0.1544
$\acute{\mu}_4$	0.7195	0.5692	0.5842	0.1671
$\acute{\mu}_5$	1.5968	1.1017	1.0543	0.2181
$\acute{\mu}_6$	4.1085	2.4744	2.2015	0.3310
$\acute{\mu}_7$	11.9067	6.2729	5.1823	0.5690
$\acute{\mu}_8$	38.0982	17.6001	13.4986	1.0875
SD	0.42784	0.4053	0.4056	0.2938
CV	1.3608	1.1447	0.9909	0.9458
CS	2.4179	2.0591	1.7735	1.7298
CK	10.8336	8.7263	7.2588	7.1312

uncertainty and is defined as

$$I_R(\rho) = \frac{1}{1-\rho}\log\left\{\int_0^\infty f(x)^\rho dx\right\}, \qquad (10)$$

where $\rho > 0$ and $\rho \neq 1$. The integral in $I_R(\rho)$ for the $\mathrm{KEC}(x;a,b,\alpha,\beta,\theta)$ can be defined by substituting (5) and (6) in (10) as

$$I_R(\rho) = \frac{1}{1-\rho}\log\left\{(ab\alpha\beta\theta)^\rho\int_0^\infty x^{\rho(\beta-1)}\frac{exp\left\{x^{\rho\beta}\right\}V^\rho\left\{1-V\right\}^{\rho(a\theta-1)}}{\left\{1-\left\{1-V\right\}^{a\theta}\right\}^{\rho(1-b)}}dx\right\},$$

where $V = \exp\left\{\alpha\left(1-\exp(x^\beta)\right)\right\}$, the above integral reduces to

$$I_R(\rho) = \frac{1}{1-\rho}\log\left\{\sum_{i,j=0}^{\infty}\sum_{m=0}^{\infty}z_{i,j,m}\int_0^\infty x^{\rho(\beta-1)}\exp\left\{(\rho+m)x^\beta\right\}dx\right\},$$

where

$$z_{i,j,m} = (ab\alpha\beta\theta)^\rho\binom{\rho(b-1)}{i}\binom{a\theta(\rho+i)-\rho}{j}\binom{k}{m}\frac{(-1)^{i+j+m}}{m!}.$$

Finally we obtain the KEC distribution Rényi entropy as

$$I_R(\rho) = \frac{\rho}{1-\rho}\log a + \frac{\rho}{1-\rho}\log b + \frac{\rho}{1-\rho}\log(\alpha) + \frac{\rho}{1-\rho}\log(\beta) + \frac{\rho}{1-\rho}\log(\theta) + \frac{1}{1-\rho}\log\left\{\sum_{i,j=0}^{\infty}\sum_{m=0}^{\infty}\binom{\rho(b-1)}{i}\binom{a\theta(\rho+i)-\rho}{j}\binom{k}{m}U_{i,j,m}\right\},$$

where

$$U_{i,j,m} = \frac{(-1)^{i+j+m+\rho-\frac{\rho}{\beta}-\beta+1}}{m!\beta(\rho+m)^{\frac{\rho(\beta-1)}{\beta}-\beta+2}}\Gamma\left(\rho-\frac{\rho}{\beta}-\beta+2\right).$$

The q-entropy was introduced by Havrda and Charvat [9], and is defined as

$$I_H(q) = \frac{1}{q-1}\left\{1-\int_0^\infty f(x)^q dx\right\}, \qquad (11)$$

where $q > 0$ and $q \neq 1$. Suppose X has the KEC distribution then by substituting (5) and (6) in (11), we obtain

$$I_H(q) = \frac{1}{q-1}\left\{1-(ab\alpha\beta\theta)^q\int_0^\infty x^{q(\beta-1)}\frac{exp\left\{x^{q\beta}\right\}V^q\left\{1-V\right\}^{q(a\theta-1)}}{\left\{1-\left\{1-V\right\}^{a\theta}\right\}^{q(1-b)}}dx\right\},$$

the above integral yields the KEC distribution q-entropy as

$$I_H(q) = \frac{1}{q-1}\left\{1-\sum_{i,j=0}^{\infty}\sum_{m=0}^{\infty}\frac{z_{i,j,m}(-1)^{i+j+m+q-\frac{q}{\beta}-\beta+1}}{m!\beta(q+m)^{\frac{q(\beta-1)}{\beta}-\beta+2}}\Gamma\left(q-\frac{q}{\beta}-\beta+2\right)\right\},$$

where

$$z_{i,j,m} = (ab\alpha\beta\theta)^q\binom{q(b-1)}{i}\binom{a\theta(q+i)-q}{j}\binom{k}{m}.$$

5 Parameter estimation

Consider the random samples $x_1, x_2, \ldots, x_n$ consisting of n observations from the Kumaraswamy exponentiated Chen distribution. Then the log-likelihood function $\ell(\Theta)$=ln$L$ of (5) is

$$\ell(\Theta) = n\log a + n\log b + n\log\alpha + n\log\beta + n\log\theta + (\beta-1)\sum_{i=1}^{n}logx_i + \sum_{i=1}^{n}x_i^\beta + \alpha\sum_{i=1}^{n}\left(1-\exp(x_i^\beta)\right) + (b-1)\sum_{i=1}^{n}log\left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^\beta)\right)\right\}\right\}^{a\theta}\right\}. \qquad (12)$$

By differentiating (12) with respect to $a, b, \alpha, \beta, \theta$ and then equating it to zero, we obtain the components of score vector $U(\Theta)$:

$$\frac{\partial\ell(\Theta)}{\partial a} = \frac{n}{a} - (b-1)\sum_{i=1}^{n}\frac{\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^\beta)\right)\right\}\right\}^{a\theta}log\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^\beta)\right)\right\}\right\}}{\left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^\beta)\right)\right\}\right\}^{a\theta}\right\}},$$

$$\frac{\partial \ell(\Theta)}{\partial b} = \frac{n}{b} - \sum_{i=1}^{n} log\left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^{\beta})\right)\right\}\right\}^{a\theta}\right\},$$

$$\frac{\partial \ell(\Theta)}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\left(1-\exp(x_i^{\beta})\right) + (b-1)\sum_{i=1}^{n}\frac{a\theta\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^{\beta})\right)\right\}\right\}^{a\theta-1}\exp\left\{\alpha\left(1-\exp(x_i^{\beta})\right)\right\}}{\left(1-\exp(x_i^{\beta})\right)^{-1}\left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^{\beta})\right)\right\}\right\}^{a\theta}\right\}},$$

$$\frac{\partial \ell(\Theta)}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n}\log x_i + \sum_{i=1}^{n} x_i^{\beta} log x_i + n\alpha - \alpha\sum_{i=1}^{n}\exp(x_i^{\beta})x_i^{\beta} log x_i - (b-1)$$
$$\sum_{i=1}^{n}\frac{a\alpha\theta\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^{\beta})\right)\right\}\right\}^{a\theta-1}\exp(x_i^{\beta})x_i^{\beta} log x_i}{\exp\left\{-\alpha\left(1-\exp(x_i^{\beta})\right)\right\}\left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^{\beta})\right)\right\}\right\}^{a\theta}\right\}},$$

and

$$\frac{\partial \ell(\Theta)}{\partial \theta} = \frac{n}{\theta} - (b-1)$$
$$\sum_{i=1}^{n}\frac{\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^{\beta})\right)\right\}\right\}^{a\theta} log\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^{\beta})\right)\right\}\right\}}{\left\{1-\left\{1-\exp\left\{\alpha\left(1-\exp(x_i^{\beta})\right)\right\}\right\}^{a\theta}\right\}},$$

respectively. The log-likelihood can be maximized by solving the non-linear equations numerically for the parameter vector $\Theta = (a,b,\alpha,\beta,\theta)^T$ is the multivariate normal with the variance covariance matrix. These nonlinear system of equations cannot be solved analytically and statistical software can be used to solve them numerically by using iterative procedures such as Newton Raphson method, BFGS, BHHH and L-BFGS-B through R-package (Adequacy Model). For interval estimation and hypothesis tests on the model parameters, we compute the inverse of the expected information matrix is

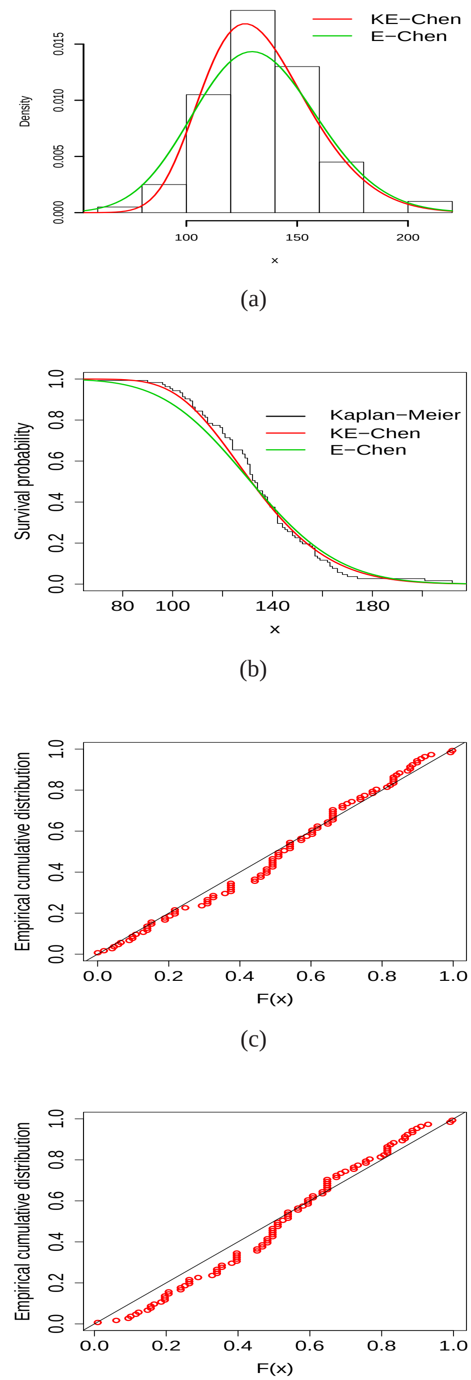
$$\left((\hat{a}-a),(\hat{b}-b),(\hat{\alpha}-\alpha),(\hat{\beta}-\beta),(\hat{\theta}-\theta)\right) \sim N_5\left\{0, I(\Theta)^{-1}\right\},$$

where $I(\Theta)^{-1}$ is the variance covariance matrix of the unknown parameters for the parameter vector $\Theta = (a,b,\alpha,\beta,\theta)^T$. An approximate $100(1-\gamma)\%$ confidence intervals for the parameters a,b,α,β,θ can be determined in the traditional procedure.

6 Application

In this section, we present analysis for illustrative purpose using the fatigue life of 6061-T6 aluminum coupons cut parallel with the direction of rolling and oscillated at 18 cycles per second data, reported by Birnbaum and Saunders [1]. The data are

70, 90, 96, 97, 99, 100, 103, 104, 104, 105, 107, 108, 108, 108, 109, 109, 112, 112, 113, 114, 114, 114, 116, 119, 120, 120, 120, 121, 121, 123, 124, 124, 124, 124, 124, 128, 128, 129, 129, 130, 130,130, 131, 131, 131, 131, 131, 132, 132, 132, 133, 134, 134, 134, 134, 136, 136, 137, 138, 138, 138, 139, 139, 141, 141, 142,142, 142, 142, 142, 142, 144, 144, 145, 146, 148, 148, 149, 151, 151, 152, 155, 156, 157, 157, 157, 157, 158, 159, 162, 163,163, 164, 166, 166, 168, 170, 174, 201, 212.



(d)

Fig. 3: Estimated Fitted KE-CHEN and E-CHEN distributions with histogram (top left), Estimated survival function for KE-CHEN and E-CHEN distributions and the empirical survival curve (top right), pp-plot to assess the fit of the model for KE-CHEN distribution (bottom left) and pp-plot to assess the fit of the model for E-CHEN distribution(bottom right) for fatigue life of 6061-T6 aluminum coupons data.

Table 2: *MLEs of the Parameters for fatigue life of 6061-T6 aluminum coupons data, the Corresponding SE (given in parentheses)with AIC, CAIC and BIC goodness of-fit measures*

Model	*a*	*b*	α	β	θ	AIC	CAIC	BIC
KE-Chen	3.119	0.468	0.006	0.384	5.376	918.19	918.83	931.22
	(1.467)	(0.183)	(0.001)	(0.009)	(2.528)			
E-Chen	-	-	0.004	0.375	5.693	920.56	920.81	928.38
			(0.001)	(0.004)	(0.901)			
NGW	-	2E+2	0.004	0.001	1.216	925.28	925.70	935.70
		(1E+2)	(0.003)	(0.008)	(0.187)			
EW	-	-	29.848	0.005	1.340	927.18	927.43	934.99
			(5.824)	(0.001)	(0.019)			

We examine the use of the KE-CHEN distribution for modeling the fatigue life of aluminum data. We fit the Kumaraswamy exponentiated Chen (KE-CHEN), exponentiated Chen (E-CHEN), New generalized Weibull (NGW) and exponentiated Weibull (EW) distributions. For each fitted distribution, we provide the maximum likelihood estimates their standard errors (in parentheses) of the parameters and the values of the Akaike information criteria (AIC), Corrected Akaike information criteria (CAIC), BIC (Bayesian Information Criterion), for four distributions are displayed in Table 2. The preferred model based on these goodness of fit measures is the (KE-CHEN) distribution among the four fitted lifetime distributions.
The lower values of the AIC, CAIC and BIC goodness of fit statistics indicate that the (KE-CHEN) distribution could be chosen as the best model for fatigue life of aluminum data. All calculations were performed using R language [13]. To assess whether the (KE-CHEN) distribution is an appropriate model, Figure 3(top left) plots the histogram of the fatigue life of aluminum and fitted the (KE-CHEN) and (E-CHEN) density functions. Furthermore, Figure 3 also plots the empirical and estimated survival function (top right), pp-plot (bottom left) for the (KE-CHEN) distribution and pp-plot for (E-CHEN) distribution(bottom right). All these plots indicate that the (E-CHEN) distribution provides a superior fit for fatigue life of aluminum data.

7 Perspective

We have proposed a new distribution called the Kumaraswamy exponentiated Chen distribution, which is an extension of the exponentiated Chen distribution. The proposed distribution could have increasing, decreasing, bathtub shaped hazard rate functions. Some statistical properties of the Kumaraswamy exponentiated Chen distribution are discussed including moments and entropies. We have considered the method of maximum likelihood for estimating the model parameters. We have illustrated the use of the new distribution using fatigue life of aluminum data. We have shown that the (KE-CHEN) distribution performs better than other three lifetime distributions in terms of AIC, CAIC and BIC goodness of fit measures. In conclusion, the (KE-CHEN) distribution could be chosen as the best model for fitting real world applications.

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