Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/130503

A Mathematical Approach to Nonlinear Travelling Waves on Polymeric Jets during Centrifugal Spinning

Abdullah Madhi Alsharif

Mathematics and Statistics Department, Faculty of Science, Taif University, Taif, Saudi Arabia

Received: 23 Feb. 2019 , Revised: 10 Mar. 2019, Accepted: 15 Mar. 2019
Published online: 1 Sep. 2019

Abstract: Centrifugal jet spinning is a novel method that is used to produce nanofibers. In this paper, theoretical and numerical study of the behavior of polymeric liquid jets during centrifugal spinning is investigated. The nonlinear instability of polymeric liquid jets is examined in the presence of centrifugal forces, gravity and surface tension. In addition, an asymptotic analysis and perturbation theory are applied for this polymeric jets during centrifugal spinning to simplify the governing equations into a set of one-dimensional equations. Then, the trajectory of non-Newtonian liquid jets during the centrifugal spinning has been determined. Furthermore, the two-step Lax-Wendroff method is used to determine the nonlinear travelling waves along the polymeric jets. Our results also show that when the rotation rate and gravitational force are high, shear thickening has more break-up lengths than shear thinning jets.

Keywords: Nonlinear waves, centrifugal spinning, polymeric jets, gravity

1 Introduction

The movement of molten polymeric Liquid Jets (LJ) in centrifugal spinning under the influence of gravity has many applications, for example in industrial processes such as ink jets printing, fuel atomisation, agricultural irrigation, spray coating (the reader can refer to Eggers *et al*. [12], Middleman et al. [18], Basaran [5] and Mckinley [16] for more applications). Rayleigh [23] and Weber [34] investigated the capillary instability of liquid jet break-up, such specification was examined by Keller et al. [14]. A similar way of an asymptotic analysis to the dynamics of viscous fluid jets was investigated by Papageorgiou [20].

The prilling process enables the construction of small spherical pellets from the break-up of (Lj), and has multiple uses including the usage in fertilizer manufacture.

Here a liquid flows from a hole on the outer surface of a rapidly rotating cylindrical container forming a liquid jet (surrounded by air) that separates from the edges and falls under gravity.This liquid jet subsequently breaks up because of surface tension-driven instabilities. The result drops falling under gravity and the resulting solidifies forming the required pellets. This was first studied in a mathematical model for inviscid jets in Wallwork et al. [33] but that paper neglected gravity, and was generalized to contain gravity by Decent et al. [6]. Decent et al. [7] then examined the rotating Newtonian viscous jet.

Liquids in real industrial applications are non-Newtonian and the instability of a rotating non-Newtonian liquid jet was examined in Uddin et al. [27], though that paper neglected gravity. Gravity was included for non-Newtonian jets but without rotation in Uddin and Decent [30].

These previous modeling papers have looked at liquid jets where the torsion is zero, with the exception of Decent et al. [6] which incorporated both rotation and gravity to examine prilling when the torsion is $O(1)$. It has recently been shown in Decent et al. [8] that the approach developed in these previous papers to examine slender liquid jets with a curved trajectory is valid so long as the torsion is zero, small or $O(1)$, but this approximation method, which is based upon an assumption that the jet is asymptotically slender, becomes invalid when the torsion becomes asymptotically large. This paper extends Decent et al. [6] to include viscosity, when the viscosity in Newtonian, shear thinning or shear thickening, examining situations when the torsion is $O(1)$ by including both gravity and rotation into the model. This paper is the first time that curved viscous jets have

* Corresponding author e-mail: abu.madhi@hotmail.com

been examined theoretically in a situation when the centreline has $O(1)$ torsion.

Wong et al [35], Hawkins et al. [13], Partridge et al. [22] conducted experimental studies on the nonlinear effects of liquid curved jets. These experiments show a good agreement with theory for droplet sizes produced from the instability mechanism. They also observed that the jets are slender in these experimental papers in agreement with the asymptotic assumption.

The centrifugal jet spinning process is used to produce nanofibers (see Mellado et al. [17] and Padron et al. [19])in which the polymer is placed in a rotating device. In this process, the break-up of the jet increases with increasing the rotation rate. Divvela et al. [9] investigated the behaviour of Newtonian and non-Newtonian centrifugal jet spinning. Taghari and Larson [26] considered centrifugal spinning of viscous (LJ). A perturbation method was investigated by Riahi [25] to study nonlinear polymeric fiber jets during force spinning process.

Various other scenarios have also been examined previously. Renardy [24] numerically described the break-up of Newtonian and non-Newtonian liquid jets. Drop formation of viscoelastic rotation jets was examined by Alsharif [1] and Alsharif et al. [2]. Alsharif et al. [3] examined the influence of torsion on slender curved liquid jets. Lorenz et al. [15] carried out the effect of gravity on a dynamic curved jet. Eggers and Dupont [10] studied drop formation of viscous (LJ). The linear and nonlinear instability of a rotating viscous jet with surfactants was discussed by Uddin [28] and Uddin et al. [29]. Drop formation for non-Newtonian liquid curved jets with surfactant is investigated by Uddin & Decent [31].

The aim of this paper is to further explain the work of Uddin et al. [27] to examine the nonlinear instability of slender power law polymeric jets in the presence of gravity. This corresponds to the centrifugal spinning process. The equations which govern the system are simplified to a set of non-dimensional equations by using the slender jet asymptotic approach introduced in Wallwork et al. [33]. We find steady state solutions and then use the Lax-Wendroff method to compute the break-up lengths and drop formation for the polymeric liquid jets in centrifugal spinning with gravity. This provides an overview of the process of producing nanofibers.

2 Formulation of the problem

In this contribution, we study the temporal and spatial instability of shear thinning and shear thickening curved viscous jets by additionally incorporating gravity. We also consider the Newtonian case.

A cylindrical container which has radius s_0 and rotates with angular velocity Ω is considered. The axis of the cylinder is vertical and the cylinder rotates about this axis. On the side of the container, a small orifice (with radius a) is apparent. We define a x,y,z Cartesian coordinate system, which rotates with the container and where the origin is at the centre of the orifice. g is denoted to the acceleration of gravity which is in negative direction of y-axis. The x-axis is normal to the surface of the cylinder and the z-axis is tangential to the surface of the cylinder, and the Cartesian system is right-handed. The functions $(x = X(s,t), y = Y(s,t), z = Z(s,t))$ are used to describe the position of the centreline of the curved liquid jet, where s is the arc-length along the centreline of the jet, and t is the time (see Wallwork [32]). A curvilinear coordinate system (s,n,ϕ) is used in any cross-section of the jet, and (n,ϕ) are the plane polar coordinates, which are the radial and azimuthal coordinates in any cross-section of the jet, and $\mathbf{e_s}$, $\mathbf{e_n}, \mathbf{e}_\phi$ are unit vectors (see Decent *et al*. [6]).

The governing equations are described as follows

$$\nabla \cdot \mathbf{u} = 0,$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p + \mathbf{g} + \nabla\cdot\tau$$
$$-2\,\mathbf{w}\times\mathbf{u} - \mathbf{w}\times(\mathbf{w}\times\mathbf{r}), \qquad (1)$$

where $\mathbf{u} = u\mathbf{e}_x + v\mathbf{e}_n + \omega\mathbf{e}_\phi$ is the liquid velocity, ρ represented the liquid density, p detonated to the pressure, $\mathbf{g}$ denotes the acceleration gravity, $\omega = (0,\omega,0)$ is the angular velocity vector of the container and $\mathbf{r}$ is the position vector.

Here the viscosity is not constant for power law fluids so that we have $\tau = \gamma\eta$, where γ is the rate of strain tensor, $\eta = m\left(\frac{\gamma:\gamma}{2}\right)^{\frac{\alpha-1}{2}}$ is the apparent viscosity, α is the flow index number and m is the fluid consistency number and so that the apparent viscosity is written in terms of the second invariant of the rate of strain tensor. The normal stress condition is given by $\mathbf{n}\cdot\Pi\cdot\mathbf{n} = \sigma\kappa$, where Π is the total stress tensor given by $-p\mathbf{I}+\tau$, σ is the isotropic surface tension and κ is the curvature of the liquid-gas free-surface. The free surface of the jet is described by $n - R(s,t,\phi) = 0$, where $R(s,t,\phi)$ is a function which gives the free-surface position. The tangential stress conditions are $\mathbf{t}_i\cdot\Pi\cdot\mathbf{n} = 0$, where $i = 1,2$ where $\mathbf{n}$, $\mathbf{t}_1$ and $\mathbf{t}_2$ are the normal and tangential vectors to the free-surface. The normal and tangential stress conditions are applied at $n = R(s,\phi,t)$. The kinematic condition is given by $\frac{D}{Dt}(R(s,t,\phi - n) = 0$ on $n = R(s,\phi,t)$, and the arc-length condition is

$$X_s^2 + Y_s^2 + Z_s^2 = 1.$$

The unit vectors are treated as described in Decent et al. [6,8], so that $\mathbf{e_s}$ is assumed to be tangential to the centreline and the other two unit vectors are perpendicular to the centreline. It is shown in Decent et al. [8] which is valid for asymptotically the torsion is $O(1)$, small or zero, but will not be valid if the torsion of the centreline is asymptotically large. This approach is a useful one given the complexity of determining the equations of motion if these assumptions are not made, as is discussed in Decent et al. [8].

These equations are similar to those found in Uddin et al. [27] but gravity is included in these equations here in equation (2). In order to control the break-up lengths and drop sizes of the centrifugal jet spinning, we use the gravitational force, which can affect the instability theory of liquid jets (see Amini et al. [4]). We can make our governing equations in non-dimensionalization forms by the follows equations

$$\bar{u}=\frac{u}{U},\quad \bar{v}=\frac{v}{U},\quad \bar{w}=\frac{w}{U},\quad \bar{n}=\frac{n}{a},\quad \varepsilon=\frac{a}{s_0},\quad \bar{R}=\frac{R}{a},$$
$$\bar{s}=\frac{s}{s_0},\quad \bar{t}=\frac{U}{s_0}t,\quad \bar{p}=\frac{p}{\rho U^2},\quad \bar{X}=\frac{X}{s_0},\quad \bar{Y}=\frac{Y}{s_0},\quad \bar{Z}=\frac{Z}{s_0},$$

where U is the speed of the jet at the orifice and ε is the aspect ratio of the jet. Therefore we have non-dimensionless parameters, which are the Rossby number $Rb = U/(s_0\Omega)$, the Weber number $We = \rho U^2 a/\sigma$, the generalized Reynolds number $Re_\alpha = (\rho/m)s_0^\alpha U^{2-\alpha}$, the Froude number $Fr = U/(s_0 g)^{1/2}$ and $\hat{Re}_\alpha = \varepsilon Re_\alpha$. As found in Uddin et al. [27] the correct balance for the instability equations is to take $\hat{Re}_\alpha = O(1)$ in this paper.

The jet is assumed to be asymptotically slender in this paper so that $\varepsilon << 1$. The difference then between this paper and Uddin et al. [27] is that in this paper $Fr = O(1)$ and the torsion of the jet's centreline is also $O(1)$, while in Uddin et al. [27] the $Fr = \infty$ and the torsion of the centreline is zero.

3 Steady state solutions

Expanding the variables u, v, w and p in the Taylor series in εn and R, X, Y, Z in ε (see Eggers [11]). We consider that the axial component of the velocity at the leading order is independent of ϕ. Therefore, we have

$$u = u_0(s,t) + (\varepsilon n)u_1(s,\phi,t) + \ldots$$
$$v = (\varepsilon n)v_1(s,\phi,t) + (\varepsilon n)^2 v_2(s,\phi,t) + \ldots$$
$$w = (\varepsilon n)w_1(s,\phi,t) + (\varepsilon n)^2 w_2(s,\phi,t) + \ldots$$
$$p = p_0(s,n,\phi,t) + (\varepsilon n)p_1(s,\phi,t) + \ldots$$
$$R = R_0(s,t) + \varepsilon R_1(s,\phi,t) + \ldots$$
$$X = X_0(s) + \varepsilon X_1(s,t) + \ldots$$
$$Y = Y_0(s) + \varepsilon Y_1(s,t) + \ldots$$
$$Z = Z_0(s) + \varepsilon Z_1(s,t) + \ldots$$

Now by applying these expansions to the steady equations and then follow the method of Decent et al. [6, 7] and Uddin et al. [27] we can fined out the following equations. For simplicity we rewrite X_0, Y_0 and Z_0 here as X, Y and Z since we only need to use the leading-order terms in the above expansions for these quantities in this paper. Then for $\hat{Re}_\alpha = O(1)$, viscosity is found to be absent from the steady equations as in Decent et al. [7] and Uddin et al. [27]. Thus this gives the same steady equations as found in Decent et al. [6,8], which are as follows

$$u_0^2 = 1 - \frac{2Y}{Fr^2} + \frac{(X^2+2X+Z^2)}{Rb^2} + \frac{2}{We}\left(1-\frac{1}{R_0}\right), \quad (2)$$

$$(X_{ss}^2+Y_{ss}^2+Z_{ss}^2)\left(u_0^2-\frac{1}{We R_0}\right) = \frac{2}{Rb}u_0(X_sZ_{ss}-Z_sX_{ss})$$
$$-\frac{Y_{ss}}{F^2} + \frac{(X+1)X_{ss}+ZZ_{ss}}{Rb^2}, \quad (3)$$

$$\frac{Z_sX_{ss}-Z_{ss}X_s}{F^2} - \frac{2Y_{ss}u_0}{Rb} + \frac{(X+1)(Y_{ss}Z_s-Y_sZ_{ss})}{Rb^2}$$
$$+\frac{Z(Y_sX_{ss}-Y_{ss}X_s)}{Rb^2} = 0, \quad (4)$$

and finally the arc-length condition is

$$X_s^2+Y_s^2+Z_s^2 = 1,$$

as well as

$$R_0^2 u_0 = 1.$$

The initial conditions for these equations are $u_0(0) = R_0(0) = X_s(0) = 1$ and $Y(0) = X(0) = Z(0) = Z_s(0) = Y_s(0) = 0$.

The above equations represented a system of four equations in four unknowns variables X, Y, Z, u_0. We solve these equations with the help of MATLAB and using the Rounge-Kutta method for obtaining the trajectory of the jet.

The relationship between the jet's radius and the arc-lengths is displayed in Figs. 1 and 2 for some values of the Froude and Rossby numbers and the remaining parameter ($We = 10$) is fixed. It can be noticed in these two figures that the polymeric jet's radius reduces with the arc-length s that goes on. We can also observe that the gravitational force and centrifugal spinning have an effect on the behavior of the jet's radius. Fig. 3 shows the trajectory of the polymeric jet for two different values of the Froude number ($Fr = 0.5$ and 1) for $Rb = 1$ and $We = 10$. In general the polymeric jet coils is gradually reduced as the Froude number increases and reaches to minimum value after more interval of the Froude number. In addition the polymeric jet coils are increasing when the gravitational force Fr decreases. The same result is obtained in Figs 4 and 5 for changing the centrifugal force and surface tension. In Fig. 6 we plot the torsion κ_2 against the arc-length s, which is same result, with an initially negative value for the torsion at the orifice. For large values of s, the torsion decay gradually and reaches to zero. We pointed that the torsion κ_2 never has a large asymptotic values for any value of s.

The torsion of the centreline of the curved jet is $\kappa_2 = P/Q$ where

$$P = X'\left(-Z''Y'''+Y''Z'''\right) + Z'\left(-Y''X'''+X''Y'''\right)$$
$$+Y'\left(-X''Z'''+Z''X'''\right) \quad (5)$$

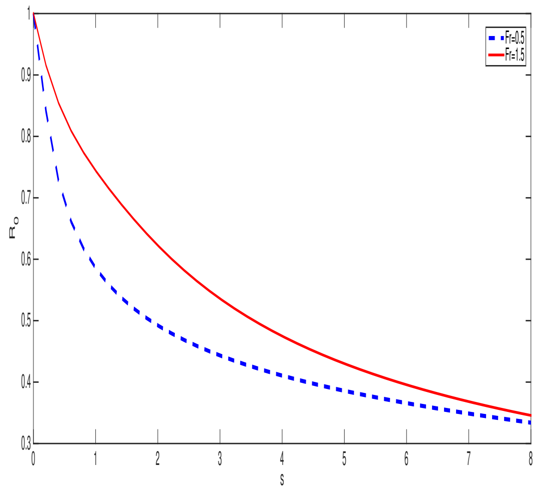

Fig. 1: The jet's radius R_0 versus the arc-length s for varying the values of the Froude number, where $Rb = 1$ and $We = 10$.

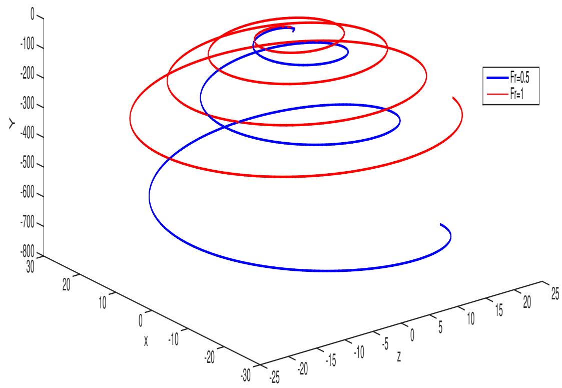

Fig. 3: The jet's trajectory for varying the values of the Froude number, where $Rb = 1$ and $We = 10$.

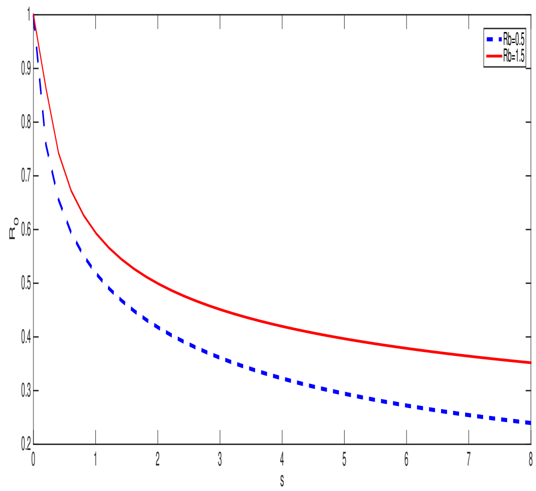

Fig. 2: The jet's radius R_0 versus the arc-length s for varying the values of the Rossby number, where $Fr = 0.5$ and $We = 10$.

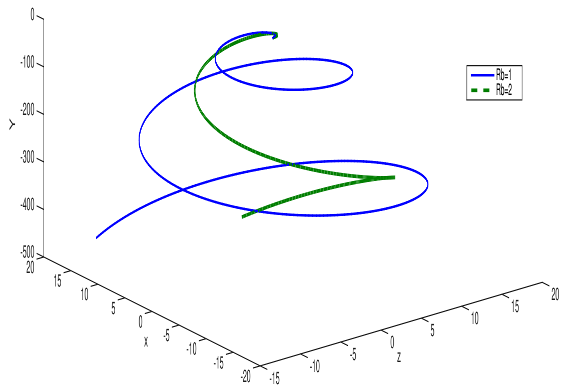

Fig. 4: The jet's trajectory for varying the values of the Rossby number, where $Fr = 0.5$ and $We = 10$.

and

$$Q = \left(-Z'Y'' + Y'Z''\right)^2 + \left(Y'X'' - X'Y''\right)^2 + \left(-X'Z'' + Z'X''\right)^2. \quad (6)$$

(See Decent et al. [8]).

4 Numerical method

The two-step Lax-Wendroff method has been used to solve nonlinear equations of the form

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{\partial \mathbf{F}(u)}{\partial s} + G(\mathbf{u}, s),$$

where $\mathbf{F}$ and $\mathbf{u}$ are vectors. To apply this method in our nonlinear governing equations, we choose equally spaced

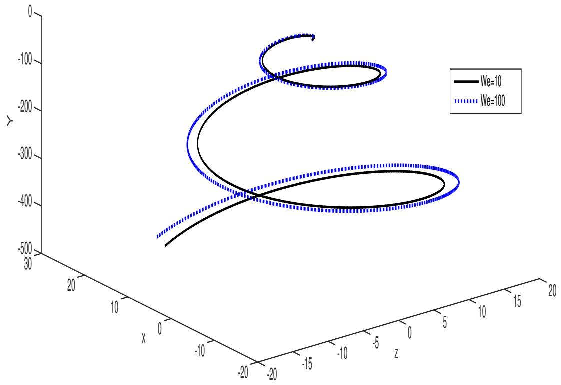

Fig. 5: The jet's trajectory for varying the values of the Weber number, where $Rb = 1$ and $Fr = 0.5$.

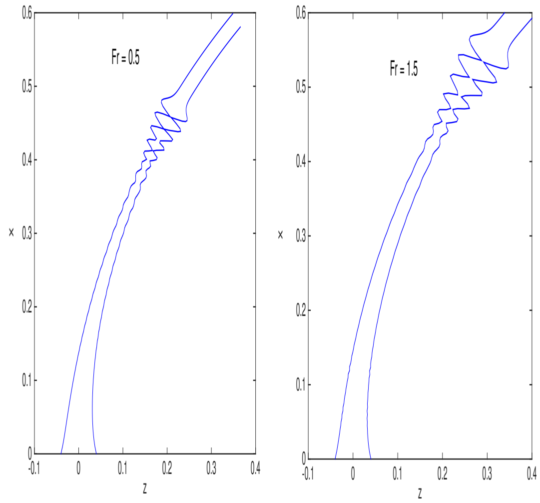

Fig. 7: The trajectory of a rotating polymeric liquid jet for $Rb = 1$, $We = 10$, $Re = 3000$, $\alpha = 1.2$, $\delta = 0.001$ and $\kappa = 0.9$.

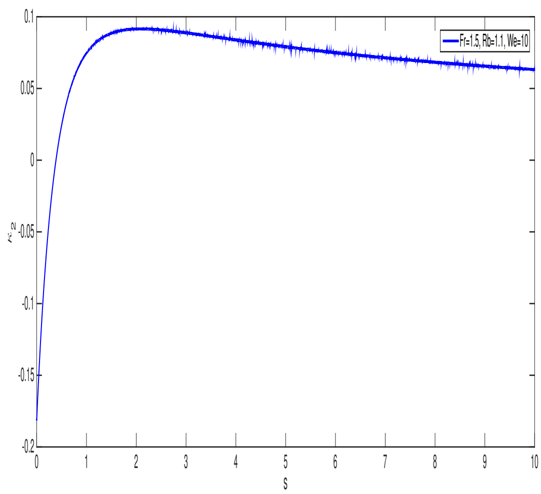

Fig. 6: The torsion $\kappa_2$against the arc-length s for $Fr = 1.5$, $Rb = 1.1$ and $We = 10$.

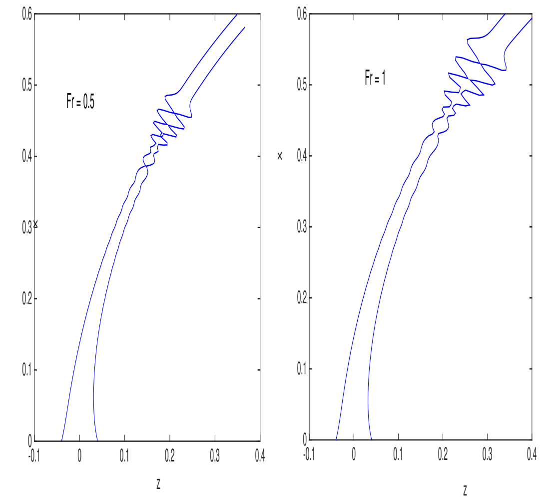

Fig. 8: The trajectory of a rotating polymeric liquid jet for $Rb = 1$, $We = 10$, $Re = 3000$, $\alpha = 0.2$, $\delta = 0.001$ and $\kappa = 0.9$.

points for both space x and t axes, namely

$$s_j = s_0 + jds, \quad t_n = t_0 + ndt,$$

here the space and time intervals are ds and dt respectively, and s_0 and t_0 are the initial values for s and t. For the purpose of making this method stable in terms of the numerical solution, we use half time-steps $t_{j+1/2}$ and half mesh-points $s_{i+1/2}$ as

$$\mathbf{u}_{j+1/2}^{n+1/2} = \frac{1}{2}(\mathbf{u}_{j+1}^{n} + \mathbf{u}_{j}^{n}) - \frac{dt}{2ds}(\mathbf{F}_{j+1}^{n} - \mathbf{F}_{j}^{n}) + 0.5(\Delta)G_j^m.$$

Then, the new flux is worked out from $\mathbf{u}_{j+1/2}^{n+1/2}$, thus the final step is

$$\mathbf{u}_j^{n+1} = \mathbf{u}_j^n - \frac{dt}{ds}(\mathbf{F}_{j+1/2}^{n+1/2} - \mathbf{F}_{j-1/2}^{n+1/2}) + \Delta G_j^m.$$

5 Nonlinear Temporal Solutions and Results

Here we focus on investigating the break-up lengths and drop formation, which can be found from the nonlinear

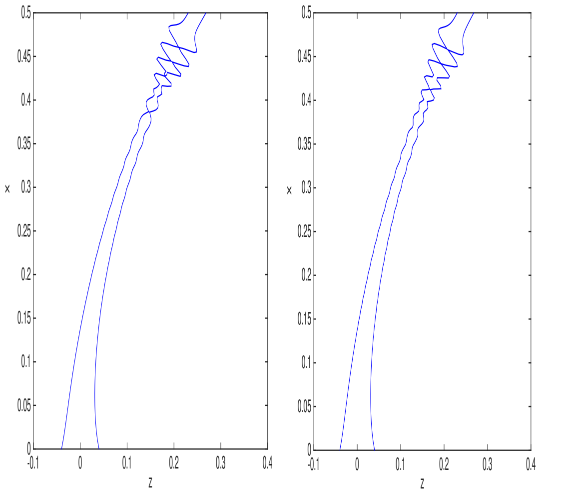

Fig. 9: The trajectory of a rotating polymeric liquid jet for $Rb = 1$, $We = 10$, $Re = 3000$, $\alpha = 0.2$ and 1.2, $\delta = 0.001$ and $\kappa = 0.9$.

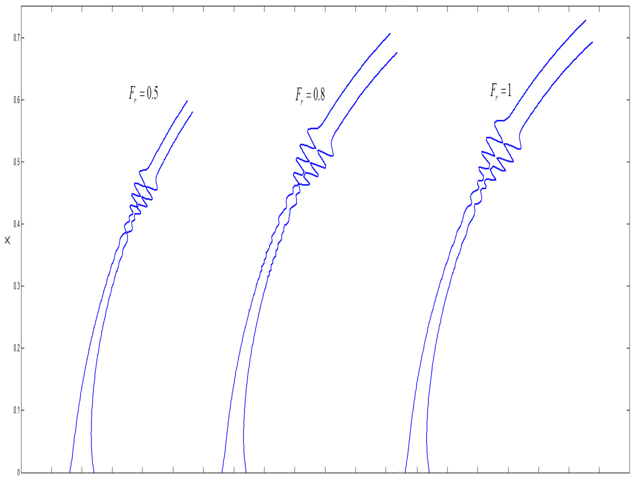

Fig. 10: The trajectory of a rotating polymeric liquid jet for $Rb = 1$, $We = 10$, $Re = 3000$, $\alpha = 0.2$, $\delta = 0.001$ and $\kappa = 0.9$.

instability theory for our problem. This is because typically there are many small satellites wasted in the process of producing nanofibers. Therefore, we replace the leading order pressure term $p_0 = \frac{1}{We}\frac{1}{R_0}$ in Eqn. (3) by the following expression as follows

$$p = \frac{1}{We}\left[\frac{1}{R_0(1+\varepsilon^2 R_{0s}^2)^{1/2}} - \frac{\varepsilon^2 R_{0ss}}{(1+\varepsilon^2 R_{0s}^2)^{3/2}}\right]. \quad (7)$$

Multiple authors have carried out the same approach (see Papageorgiou [20] and Eggers [11]). Here we denote $\hat{\eta} = |\sqrt{3}u_s|^{\alpha-1}$ and $A = A(s,t)$, where $A(s,t) = R^2(s,t)$, thus we can write our nonlinear system of equations as follows

$$\frac{\partial u}{\partial t} = -\left(\frac{u^2}{2}\right)_s - \frac{1}{We}\frac{\partial}{\partial s}\frac{4\sigma\left(2A + (\varepsilon A_s)^2 - \varepsilon^2 A A_{ss}\right)}{\left(4A + (\varepsilon A_s)^2\right)^{3/2}} + \frac{(X+1)X_s + ZZ_s}{Rb^2} - \frac{Y_s}{Fr} + \frac{3}{Re}\frac{(\hat{\eta} A u_s)_s}{A}, \quad (8)$$

$$\frac{\partial A}{\partial t} = -\frac{\partial}{\partial s}(Au), \quad (9)$$

Here we use the initial conditions at $t = 0$ which are $A(s,t=0) = R_0^2(s)$, $u(s,t=0) = u_0(s)$ as discussed earlier for the steady state solutions (see section 3). We use upstream boundary conditions at the nozzle as follows

$$u(0,t) = \delta \sin\left(\frac{\kappa t}{\varepsilon}\right) + 1, \; A(0,t) = 1, \; A(0,1) = 1.$$

where κ is the disturbance frequency, and δ is the amplitude of the initial non-dimensional velocity disturbance.

Now we use the experimental value of $\varepsilon (= \frac{a}{s_0})$ namely is 0.01 (see Wong *et al.* [35]). The second-order finite difference method is based on an explicit scheme, and then we use the Lax-Wendroff two-stage scheme to point out the break-up lengths and small satellites of polymeric (LJ) in centrifugal spinning with gravity.

To determine drop formation and break-up lengths of polymeric jets in centrifugal spinning, we use the nonlinear instability theory that can be examined using the two-step Lax-Wendroff method. Fig. 7 represents the numerical simulation of the polymeric jet in the $x-z$ plane with the effect of the gravitational force Fr. When $Fr = 0.5$, we see that the thickening polymeric liquid jets gradually decreases followed by oscillations with small amplitude and again gradually increases. By increasing the Fr parameter due to increases of thickening polymeric liquid jets and the amplitude of oscillation increases as observed in Fig. (7). This shows that when Fr increases, the break-up lengths of thickening polymeric liquid jets ($\alpha = 1.2$) increases. We have found the same result in Fig. 8 for shear polymeric liquid jet ($\alpha = 0.2$), where the rest of the parameters is $Rb = 1, We = 10, Re = 3000, \delta = 0.001$ and $\kappa = 0.9$.

We have compared the break-up lengths of shear and thickening liquid jets in the presence of gravity. From this comparison, we have found that thickening polymeric liquid jets has longer break-up times than shear polymeric liquid jet (see Figure 9). A comparison is also made in Fig. 10 to see the effects of gravitational force on the break-up lengths of polymeric liquid jets for three different values of Froude Fr. This figure shows that the increase in the number of Froude gives us a longer break-up length for polymeric jets in centrifugal spinning, which means that the force of gravity has an affected on this jet.

6 Conclusions

In this paper, the problem of polymer processing examined in Uddin et al. [27] is extended to include gravity. Similar to the analysis in Uddin et al. [27], the asymptotic and perturbation theory are applied for this polymeric jets during centrifugal spinning. Then, we have followed the method of Decent et al. [6,7] and Uddin et al. [27] to find the steady state solutions of non-Newtonian liquid jets during the centrifugal spinning in the presence of gravity. We have also considered the nonlinear travelling waves of polymeric liquid jets in the centrifugal spinning with gravity to determine the break-up lengths for different flow index numbers. We have found that the gravitational force has an effect on the break-up lengths and drop sizes of polymeric jets in centrifugal spinning. Our results also confirm that the polymeric jet bends more when the gravitational force is small which means that the Froude number Fr decreases. The same result is obtained in Figs 4 and 5 for changing the centrifugal force and surface tension the Rossby Rb and Weber We numbers respectively.

Acknowledgment

Abdullah M. Alsharif would like to express his heartfelt gratitude for Taif University for their financial support without which this study would not have finished.

References

[1] A. M. Alsharif. *Instability of non-uniform of viscoelastic liquid jets*, PhD Thesis, University of Birmingham, UK, (2014).
[2] A. M. Alsharif, J. Uddin and M. F. Afzaal. Instability of viscoelastic curved jets, *Appl. Math. Modell*, **39**, 3924-3938 (2015).
[3] A.M. Alsharif, S. P. Decent, E. I. Parau, M. J. H. Simmons and J. Uddin. The influence of torsion on slender curved liquid jets, *IMA Journal of Applied Mathematics*, **00**, 1-22 (2018).
[4] G. Amini, M. Ihme and Doblatabadi. Effect of gravity on capillary instability of liquid jets, *Phys. Rev. E*, **87**, 053017 (2013).
[5] O. A. Basaran. Small-scaled free surface flows with breakup: drop formation and emerging applications A. *I.Ch. E. Journal* , **48**, (9), 1842-1848 (2013).
[6] S. P. Decent, A. C. King and I. M. Wallwork. Free jets spun from a prilling tower, *Journal of Engineering Mathematics*, **42**, 265-282 (2002).
[7] S. P. Decent, A. C. King, M. H. Simmons, E. I. Pără u, D. C. Y. Wong, I. M. Wallwork, C. Gurney and J. Uddin. The trajectory and stability of a spiralling liquid jet: Part II. Viscous Theory, *Appl. Math. Modelling*, **33**, 12, 4283-4302 (2009).
[8] S. P. Decent, E. I. Părău, M. J .H. Simmons and J. Uddin. On mathematical approaches to modelling slender liquid jets with a curved trajectory, *J. Fluid Mech.*, **844**, 905-916 (2018).
[9] M. J. Divvela, A. Ruo, Y. Zhmayev and Y. L. Joo. Discretized modeling for centrifugal spinning of viscoelastic liquids, *Journal of Non-Newtonian Fluid Mechanics*, **247**, 62-77 (2017).
[10] J. Eggers and T. F. Dupont. Drop formation in a one-dimensional approximation of the Navier Stokes equation. *J. Fluid Mech.*, **262**, 205-221 (1994).
[11] J. Eggers. Nonlinear dynamics and breakup of free surface flows, *Rev. Mod. Physis*, **69** , 3, 865-929 (1997).
[12] J. Eggers and E. Villermaux. Physics of liquid jets, *Rep. Prog. Phys.*, **71**, 036601(79 pp), (2008).
[13] V. L. Hawkins, C. J. Gurney, S. P. Decent, M. J. H. Simmon and J. Uddin. Unstable waves on a curved non-Newtonian liquid jet, *J. Phys. A: Math. Theor.*, **43**, 055501 (2010).
[14] J. B. Keller, S. J. Runbinow and Y. O. Tu. Spatial instability of a jet, *Phys. Fluids*, **16**, 2052 (1973).
[15] M. Lorenz, N. Marheineke and R. Wegner. On simulation of spinning processes with a stationary on dimensional upper convected Maxwell model, *Journal of Mathematics in industry*, **4**, 2 (2014).
[16] G. H. Mckinley and A. Tripathi. How to extract the Newtonian viscosity from capillary break-up experiments in a filament rheometer , *J. Rheol.* **44**, 653 (2000).
[17] P. Mellado, H. A. Mcllwee, M. R. Badrossamay, J. A. Goss, L. Mahadevan, and K. K. Parker. A simple model for nanofiber formation by rotary jet-spinning, *Appl. Phys. Lett.*, **99**, 203107 (2011).
[18] S. Middleman. *Modeling Axisymmetric flows; dynamics of films, jets and drops*, Academic Press (1995).
[19] S. Padron, A. Fuentes, D. Caruntu, K. Lozano. Experimental study of nanofiber production through forcespinning, *J. Appl. Phys.* , **113**, 024318 (2013).
[20] D. T. Papageorgiou. On the breakup of viscous liquid threads, *Phys. Fluids*, **7**, 1529 (1995).
[21] E. I. Părău, S. P. Decent, M. J. H. Simmons, D. C. Y. Wong and A. C. King. Nonlinear viscous liquid jets from a rotating orifice, *J. of Eng. Maths.*, **57**, 159-179 (2007).
[22] L. Partridge, D. C. Y. Wong, M. J. H. Simmons, E. I. Pă rău and S. P. Decent. Experimental and theoretical description of the break-up of curved liquid jets in the prilling process, *Chem. Eng. Res. Des.*, **83(A11)**, 1267-1275 (2005).
[23] W. S. Rayleigh. On the instability of jets, *Proc. Lond. Math. Soc.*, **10**, 4 (1878).
[24] M Renardy. A numerical study of the asymptotic evolution and breakup of Newtonian and viscoelastic jets, *J. Non-Newtonian Fluid Mech.*, **59**, 267-282 (1995).
[25] D. N. Riahi. Modeling and computation of nonlinear rotating polymeric jets during force spinning process, *International Journal of Nonlinear Mechanics*, **92**, 1-7 (2017).
[26] S. M. Taghavi and R. G. Larson. Regularized thin-fiber model for nanofiber formation by centrifugal spinning, *Physical Review E* , **89 (2)**, 023011 (2014).
[27] J. Uddin, S. P. Decent and M. H. Simmons. The instability of shear thinning and shear thickening liquid jets: linear theory, *ASME J. of Fluids Eng.*, **128**, 968-975 (2006).

[28] J. Uddin. *An investigation into methods to control breakup and droplet formation in single and compound liquid jets*, PhD Thesis, University of Birmingham,UK, (2007).
[29] J. Uddin, S. P. Decent and M. J. H. Simmons. The effect of surfactants on the instability of a rotating liquid jet, *Fluid Dyn. Res.*, **40**, 827851 (2008).
[30] J. Uddin and S. P. Decent. Instability of non-Newtonian liquid jets curved by gravity, *Mathematics in industry*, **15**, 597-602 (2010).
[31] J. Uddin and S. P. Decent. Drop formation in rotating non-Newtonian jets with surfactants, *IMA Journal of Applied Mathematics*, **77**, 86-96 (2012).
[32] I. M. Wallwork. *The trajectory and stability of a spiralling liquid jet*, PhD Thesis, University of Birmingham, UK, (2002a).
[33] I. M. Wallwork, S. P. Decent, A. C. King and R. M. Schulkes. The trajectory and stability of a spiralling liquid jet Part 1, Inviscid Theory , *J. Fluid Mech.*, **459**, 43-65 (2002b).
[34] C. Weber. Zum Zerfall eines Flussigkeitsstrahles , *Z. Angew. Math. Mech.*, **11**, 136-154 (1931).
[35] D. C. Y. Wong, M. J. H. Simmons, S. P. Decent, E. I. Parau and A. C. King. Break-up dynamics and drop sizes distributions created from spiralling liquid jets, *International Journal of Multiphase Flow*, **30**, 499-520 (2004).

Abdullah Madhi Alsharif received the PhD degree in Mathematics for Engineering Science at the University of Birmingham, UK. His research interests are in the areas of applied mathematics and mathematical physics including the mathematical methods and models for complex systems, Newtonian and non-Newtonian fluids, mathematical engineering, and numerical methods for rheological fluids. He has published research articles in reputed international journals of mathematical and engineering sciences. He is referee of mathematical journals.