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Linear Instability of Inviscid Jets with Thermo-Capillarity

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Abstract: This study deals with the investigation of the temporal and spatial instability of inviscid jets with the effects of the thermal energy. The variation of surface tension causes Marangoni flow that affects drop formation and break-up of liquid jets. The governing equations are reduced into one-dimensional model using an asymptotic analysis. The dispersion relation for thermo-capillary inviscid liquid jets is derived to examine the behavior of the dimensionless parameter of the linear instability. Moreover, the growth rate and the maximum wavenumber are displayed along the jet.

Keywords: Temporal, Spatial, Instability, Jets.

1 Introduction

The subject of liquid jets has attracted many researchers (e.g. Eggers, 1997; Middleman,1995; Lin, 2003). Instability and break-up of thermo-capillary effects on liquid jets have different disciplines, such as fertilizers and ink-jet printing. Therefore, Rayleigh (1878) investigated the problem of inviscid jets theoretically and reported that surface tension was responsible for the jet's break-up. Weber (1931) applied Rayleigh?s analysis approach to the study of liquid jets in the existence of viscosity and concluded that viscosity increased the jet wavelength . Tomotika (1935) examined the instability of two viscous jets and observed that surface tension as well as viscosity affected both liquids. The study addresses two types of linear stability theory of liquid jets, i.e. temporal and spatial instability, herein we are interested in examining both instabilities, as we will find out in this study.

Papageorgiou (1995) applied the asymptotic analysis to the study of nonlinear solution of viscous jets and indicated its consistency with the theoretical analysis. Grant and Middleman (1965) examined Newtonian jet stability to predict the stability of both turbulent and high-speed laminar jets in stagnant air. Ashgriz and Mashayek (1995) addressed and gave a general overview of studying the subject of liquid jets. One of the many different scenarios of studying the instability of liquid jets is the thermal energy that causes variation of the surface tension. Cheong et al. (2004) examined the effect of gravity on the instability of liquid jets.The study showed the consistency of the experiments with the theoretical investigation (numerical results). Alsharif et al. (2014) explored the instability of a rotating viscoelastic jet. The instability of thermo-capillary viscous liquid jets with surfactants can be examined as done in Alsharif et al. (2014).

The present study adopts the linear stability theory to examine both temporal and spatial of thermo-capillary inviscid jets. The governing equations are presented in cylindrical coordinates and an asymptotic approach is used to involve our equation in a single model. Furthermore, the steady state solutions are obtained for this problem. Moreover, we examined the linear stability analysis of inviscid liquid jets with thermo-capillarity .

2 Problem Formulation

The flow resulting from an incompressible thermo-capillary liquid jet is assumed to emerge from an orifice having radius a. Thus, we use the cylindrical coordinate (r,θ,z) for this problem, where r is the radial direction, θ is the azimuthal direction and z lies along the axis of the jet. We have neglected the influence of the surrounding air. Continuity equation, Navier-stokes equation and nonisothermal equation of our system are as follows:

$$\nabla . u = 0, \quad (1)$$

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$$\rho\frac{Du}{Dt} = -\nabla p, \tag{2}$$

$$\frac{\partial T}{\partial t} + u.\nabla T = k_t \nabla^2 T \tag{3}$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + u.\nabla$ and ρ is the fluid's density. We can write the relationship between the surface tension and temperature as follows:

$$\sigma(T) = \sigma_a - \beta_p (T - T_a)\nabla \tag{4}$$

where

$$\beta_p = -(\frac{d\alpha}{dT})_{T-T_0}, \tag{5}$$

where T is the temperature and σ_a is the surface tension at the ambient temperature T_a respectively.

The normal stress condition is given as

$$P = \sigma k \tag{6}$$

where k is the mean curvature of the free surface

$$k = \frac{1}{r}(-\frac{\partial}{\partial z}(\frac{r}{E}\frac{\partial R}{\partial z}) + \frac{\partial}{\partial R}(\frac{r}{E})) \tag{7}$$

where

$$E = (1 + (\frac{\partial R}{\partial z})^2)^{\frac{1}{2}} \tag{8}$$

and

$$n = \frac{1}{E}(-\frac{\partial R}{\partial z}e_z + e_r) \tag{9}$$

The second boundary condition is the tangential, which is written as follows:

$$(1 - (\frac{\partial R}{\partial z}))(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial t}) + 2\frac{\partial R}{\partial z}(\frac{\partial u}{\partial z} - \frac{\partial v}{\partial \tau}) = 0 \tag{10}$$

and the kinematic condition is

$$\frac{\partial R}{\partial t} - v + u\frac{\partial R}{\partial z} = 0 \tag{11}$$

To describe the flow in the jet, it is convenient to write our governing equations as follows

$$\frac{\partial v}{\partial \tau} + \frac{\partial u}{\partial z} + \frac{u}{r} = 0 \tag{12}$$

$$\rho(\frac{\partial u}{\partial \tau} + v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}) = -\frac{\partial p}{\partial z}, \tag{13}$$

$$\rho(\frac{\partial v}{\partial \tau} + v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial z}) = -\frac{\partial p}{\partial z}, \tag{14}$$

$$\frac{\partial T}{\partial \tau} + u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r} = k_r(\frac{\partial^2 r}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}) \tag{15}$$

and finally the normal heat flux across the interface is given by

$$k\nabla T.n = -H(T - T_a) \tag{16}$$

where k and H represent the thermal conductivity and the heat transfer coefficient respectively.

We use the following transformation to present our equations in a non-dimensionless form,

$$\bar{z} = \frac{z}{L},\ \bar{r} = \frac{r}{a}, \bar{u} = \frac{u}{U},\ \bar{v} = \frac{v}{U},\ \bar{t} = \frac{U}{L}t,\ \bar{p} = \frac{1}{\rho U^2}p,\ T = \frac{T}{T_a}$$

where a, U, T_α and L are the radii of the orifice, the jet's exit speed, the temperature of the ambient surrounding and the axial length scale respectively. Thus, our governing equations, after dropping over-bars, are given by

$$\varepsilon\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \varepsilon u\frac{\partial u}{\partial z} = -\varepsilon\frac{\partial p}{\partial z}, \tag{17}$$

$$\varepsilon\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \varepsilon v\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial r}, \tag{18}$$

$$\varepsilon\frac{\partial T}{\partial t} + \varepsilon u\frac{\partial T}{\partial z} + v\frac{\partial T}{\partial r} = \frac{1}{Pe}(\varepsilon^2\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}) \tag{19}$$

the normal condition is

$$p = \frac{k}{We}, \tag{20}$$

where

$$k = \frac{1}{r}\left\{-\varepsilon^2\frac{\partial}{\partial z}(\frac{r}{E}\frac{\partial R}{\partial z}) + \frac{\partial}{\partial r}(\frac{r}{E})\right\}, \tag{21}$$

the tangential and normal heat flux conditions are

$$\left\{1 - \varepsilon^2(\frac{\partial R}{\partial z})^2\right\}(\varepsilon\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}) + 2\varepsilon\frac{\partial R}{\partial z}(\varepsilon\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r})$$
$$= \varepsilon\frac{\partial R}{\partial z}\frac{\partial \sigma}{\partial r}\left\{(1 + \varepsilon^2(\frac{\partial R}{\partial z})^2)\right\}^{-\frac{1}{2}}, \tag{22}$$

and

$$\frac{\partial T}{\partial r}\left\{(1 - \varepsilon^2(\frac{\partial R}{\partial z})^2\right\}^{\frac{-1}{2}} - \varepsilon^2\frac{\partial T}{\partial z}\frac{\partial R}{\partial z}\left\{(1 - \varepsilon^2(\frac{\partial R}{\partial z})^2\right\}^{\frac{-1}{2}}$$
$$= -\varepsilon Bi(T - 1), \tag{23}$$

where $We = \frac{\rho L U^2}{\sigma}$ is the Weber number, $Pe = \frac{\rho L U}{M_r}$ is the Peclet number and $Bi = \frac{LH}{K}$ is the Biot number.

We expand all the variables in εr (see Eggers, 1997; Hohman et al., 1984) as follows:

$$u = u_0(t,z) + \varepsilon(rz)u_1(t,z) + ...$$
$$v = (\varepsilon r)v_1(t,z) + (\varepsilon r)^2 v_2(t,z) + ...$$
$$p = p_0(z,t) + (\varepsilon r)p_1(z,t) + ...$$
$$R = R_0(t,z) + \varepsilon R_1(t,z) + ...$$
$$T = T_0(t,z) + (\varepsilon r)^2 T_2(t,z) + ...$$

The equation of motion in the axial direction is written in a leading order

$$\frac{\partial u_0}{\partial t} + u_0 \frac{\partial u_0}{\partial z} = -\frac{\partial p}{\partial z}, \tag{24}$$

from the tangential condition we have

$$u_1 = 0$$
$$u_2 = \frac{1}{4}\frac{\partial^2 u_0}{\partial z^2} + \frac{3}{2R_0}\frac{\partial R_0}{\partial z}\frac{\partial u_0}{\partial z},$$

from the normal stress condition, we get

$$p_0 = \frac{\sigma_0}{WeR_0},$$
$$p_1 = \frac{-R_1}{R_0^2} + \frac{4u_0}{R_0}, \tag{25}$$

p_0 and u_2 are substituted into Eqn. (24) to become in the following form

$$\frac{\partial u_0}{\partial t} + u_0 \frac{\partial u_0}{\partial z} = -\frac{\partial}{\partial z}\left(\frac{\sigma_0}{R_0 We}\right) + \frac{2T_z}{R_0 We}, \tag{26}$$

the energy equation to leading order gives

$$\frac{\partial T_0}{\partial t} + u_0 \frac{\partial T_0}{\partial z} = \frac{1}{Pe}\left(\frac{\partial^2 T_0}{\partial z^2} + 4T_2\right). \tag{27}$$

We use the normal heat flux equation to obtain T_2, so that

$$T_2 = \frac{1}{2R_0}\frac{\partial T_0}{\partial z}\frac{\partial R_0}{\partial z} - \frac{\overline{B_l}}{2R_0}(T_0 - 1), \tag{28}$$

The Biot number is rescaled as $\overline{B_i} = \varepsilon B_i$ with $B_i = 0(1)$. In the leading order, the Kinematic equation becomes

$$\frac{\partial R_0}{\partial t} + u_0 \frac{\partial R_0}{\partial t} + \frac{R_0}{2}\frac{\partial u_0}{\partial z} = 0. \tag{29}$$

We use u_2 and T_2 to get a set of leading order equations as follows

$$u_t + u_0 u_z = -\frac{1}{We}\left(\frac{\sigma}{R}\right) - \frac{2\beta_p T_z}{RWe} \tag{30}$$

$$R_t = -\frac{R_0}{2}u_{0z} - uR_{0z}, \tag{31}$$

where

$$\sigma(T) = 1 - \beta_p(T - 1). \tag{32}$$

These equations are consistent with Furlani (2005).

$$u_0 u_z = -\frac{1}{We}\left(\frac{\sigma}{R}\right)_z - \frac{2\beta_p T_z}{RWe}, \tag{33}$$

$$\frac{1}{2}\frac{\partial u_0}{\partial z}R_o + u_o\frac{\partial R_o}{\partial z} = 0, \tag{34}$$

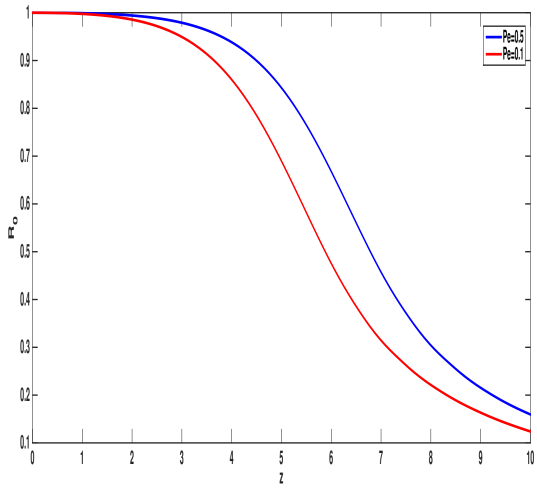

Fig. 1: R_0 versus the axial length z for the Peclet number, where $We = 30$ and $\beta_p = 0.2$.

$$uT_z = \frac{1}{Pe}\frac{1}{R^2}\frac{\partial}{\partial z}\left(R^2\frac{\partial T}{\partial z}\right) - \frac{2\overline{Bi}}{RPe}(T - 1). \tag{35}$$

At $R(0) = u(0) = 1$, Equ. (34) can be written as $R^2 u_o = 1$, which can be substituted into (33) and (35) to get the following equations

$$u_o u_z = -\frac{2\sigma}{We}\left(\frac{u_{oz}}{\sqrt{u}}\right) - \frac{2\beta_p T_z\sqrt{u_0}}{We}, \tag{36}$$

$$u_o T_z = \frac{1}{Pe}u\frac{\partial}{\partial z}\left(\frac{1}{u_o}\frac{\partial T}{\partial z}\right) - \frac{2\overline{Bi}}{RPe}\sqrt{u_0}(T - 1), \tag{37}$$

We solve the equations (36) and (37) using the method of Rung-Kutta with initial $u(0) = T(0) = 1$. Parau et al. (2006, 2007) used the methods of Newton and Rung-kutta to obtain the results of rotating Newtonian viscous jets. They reported that this method provided a good agreement comparing to Newton?s method. Thus, we use the Rung-Kutta method to find the solutions of the steady state of thermo-capillary viscous liquid jets. In Figures (1) and (2), the correlations between the radius and the axial length z have been plotted for the Peclet and Biot numbers, respectively. They also show a correlation between the two numbers and the jet's radius.

3 Temporal Instability Analysis

To see how the perturbations affect the solutions of the steady state, found in the last section, we use the following forms

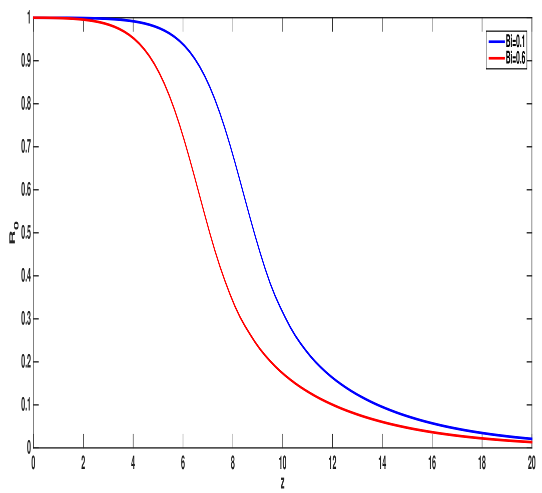

Fig. 2: R_0 against the axial length z for the Biot number, where $We = 30$ and $Pe = 0.2$.

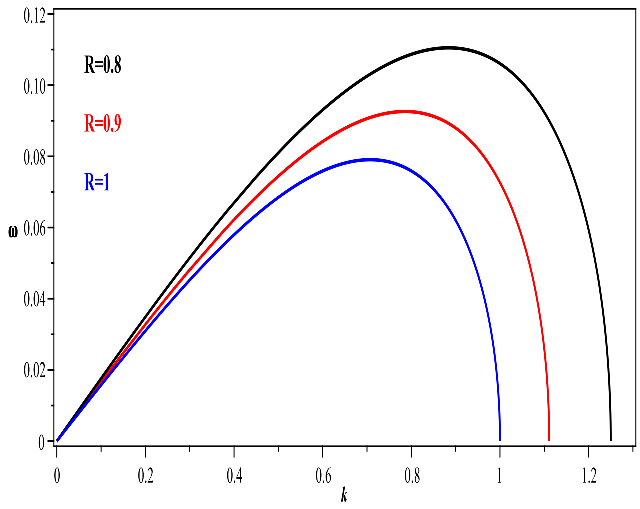

Fig. 3: Behaviour of the growth rate versus the wavenumber for $\overline{Pe} = 2$ at varying R_0, where $We = 10$.

$$u(z,t) = u_o(z) + \delta\hat{u}\exp(ik\bar{z} + \omega\bar{t}), \quad (38)$$
$$R(z,t) = R_o(z) + \delta\hat{R}\exp(ik\bar{z} + \omega\bar{t}), \quad (39)$$
$$T(z,t) = T_o(z) + \delta\hat{T}\exp(ik\bar{z} + \omega\bar{t}), \quad (40)$$
$$P(z,t) = P_o(z) + \delta\hat{P}\exp(ik\bar{z} + \omega\bar{t}), \quad (41)$$

where $k = k(z)$ is the wavenumber of the disturbances $\omega = \omega(z)$ is the frequency of the disturbances and δ is a small constant which is $0 < \delta < \varepsilon^2$ (Uddin (2007)).

In Eq. (30), the full expression of the mean curvature is used instead of the leading order pressure, which is in the form

$$\frac{1}{We}\left(\frac{1}{R(1+\varepsilon^2 R^2 z)^{\frac{1}{z}}} - \frac{\varepsilon^2}{R(1+\varepsilon^2 R^2 z)^{\frac{1}{z}}}\right) \quad (42)$$

Several authors, including Lee (1974) and Eggers (1997), adopt this idea. Consequently, the motion in the axial direction is represented as follows:

$$u_t + u_o u_z = -\frac{\sigma}{We}\frac{\partial}{\partial z}\left(\frac{1}{R(1+\varepsilon^2 R^2 z)^{\frac{1}{z}}} - \frac{\varepsilon^2}{R(1+\varepsilon^2 R^2 z)^{\frac{1}{z}}}\right) - \frac{2\beta_p T_z}{RWe}, \quad (43)$$

with

$$T_t + uT_z = \frac{1}{\overline{Pe}\, R^2}\frac{\partial}{\partial z}(R^2\frac{\partial T}{\partial z}) - \frac{2\beta_p}{R\,\overline{Pe}}\sqrt{u}(T-1). \quad (44)$$

We substitute the equations (38)-(41) into (43), (31) and (44), so the dispersion relationship is

$$(\omega + iku_0)^3 + \frac{k^2}{\overline{Pe}}(\omega + iku_0)^2 - \frac{k^2\sigma}{2We}R_o\left(\frac{1}{R^2} - k^2\right) \times (\omega + iku_0) - \frac{k^4}{2We\,\overline{Pe}}R_o\left(\frac{1}{R^2} - k^2\right) = 0 \quad (45)$$

Peclet number is recalled as follows:

$$\overline{Pe} = \varepsilon Pe \Rightarrow Pe = \frac{\overline{Pe}}{\varepsilon},$$

Growth rate occurs when $0 < kR_o < 1$. Our dispersion relation reveals that the same dispersion relation for invscid liquid jets is obtained when $\overline{Pe} \to \infty$ (see Rayleigh, 1878).

4 Spatial Instability

In order to investigate spatial instability of the dispersion relation for viscous liquid jets (45), we consider that the wave number k is in a complex form ($k = k_r + ik_i$) and the growth rate is an imaginary complex number ($l = -i\omega$). See Keller et al. (1973) for further details concerning this type of instability. Spatial instability occurs when ($k < 0$).

5 Results and Discussion

To examine the linear instability, the behavior of the dispersion relation was investigated (45). The present study investigated the temporal and spatial instability theory for thermo-capillary viscous jets. In addition, we

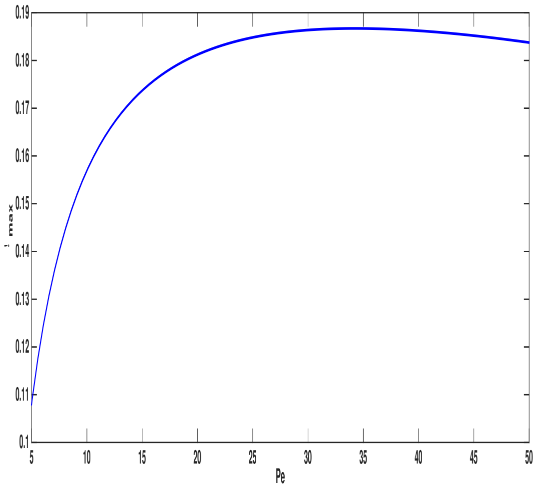

Fig. 4: ω_{max} versus the Peclet number, where $We = 10$.

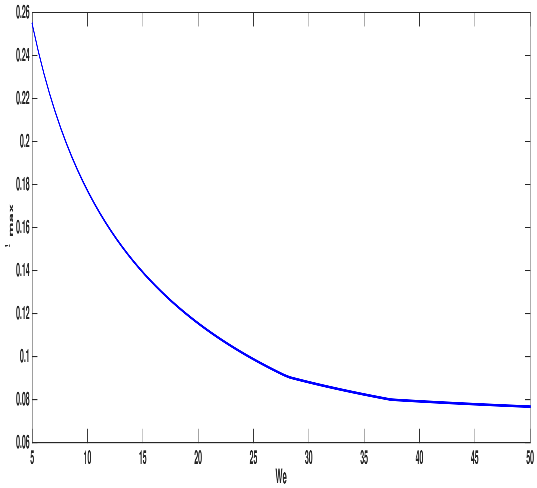

Fig. 6: $\omega_{\max}$ versus the Weber number, where $\overline{Pe} = 20$

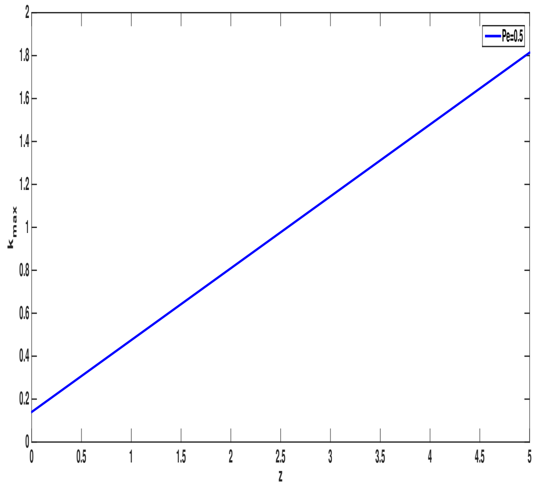

Fig. 5: k_{max} versus the axial length z, where $We = 20$

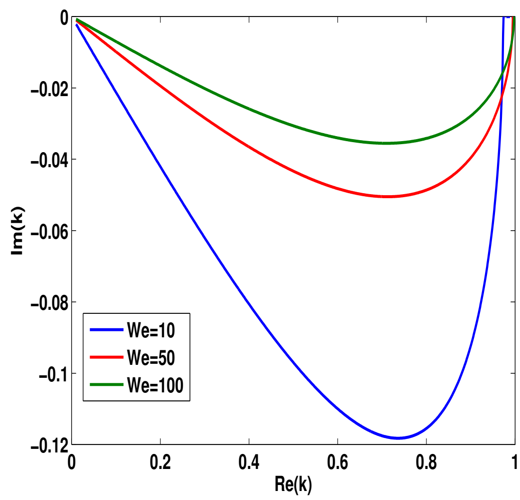

Fig. 7: $Im(k)$ versus $Re(k)$ for three values of the Weber number, where $\overline{Pe} = 2$.

numerically solved the dispersion relation (45) to interpret the results obtained for this phenomenon. We also identified the wavenumber of most unstable modes, termed k_{max} and the maximum growth rate, termed K_{max} along the axial length z. The wavenumber of the most unstable mode changed along the jet. Figure (3) demonstrated that the effect of the increasing Peclet number on the behavior of the growth rate, and by observing this figure we can see that the increase in the Peclet number led to a decrease in the growth rate. Moreover, the numerical results in figure (4) indicated that the ω_{max} increased gradually as the the Peclet number reduced and after a short interval the ω_{max} approached to the maximum value as shown in figure (3). However, the ω_{max} decayed as the Peclet number increased and reached to the minimum values after a shot interval (see figure 5). It is clear that the k_{max} is proportional the axial length z; therefore, the k_{max} is very sensitive to the axial length z as shown in figure (6). Figure (7) shows that the highest value of the Peclet number at different values of the jet's radius gives the most negative growth rate value. In other words, when we increase the Peclet number, we obtain the most negative growth rate. We also illustrate the relationship between the real and imaginary parts of the wavenumber. A simple relationship exists between the real and imaginary parts of the wavenumber. The graph indicates that the increase in the Peclet number leads to an increase in the most negative growth rate (see Figure 7). However, Figure (8) reveals that enhancing the Weber number results in a higher growth rate.

6 Conclusion

Temporal and spatial instability analysis have been examined for thermo-capillary inviscid liquid jets. An asymptotic approach has been used to reduce the governing system equations to one-dimensional equations a leading order. Moreover, the temporal instability was conducted to derive the dispersion relation for thermo-capillary viscoelastic jets. Dispersion relation helped identify the growth rate of different values of the dimensionless parameters and the linear instability enabled us to estimate the break-up lengths and time of thermo-capillary viscous liquid jets. Liquid jets are very crucial in terms of their applications. Therefore, we recommend addressing compound liquid jet, which have practical applications such as exploring liquid CO2 sequestration in the deep ocean and deep ocean oil spills like what Tang experimented in 2004. Gao et al. (2017) examined an underwater horizontal oil jet experimentally and numerically using Navier-Stokes equations. Therefore, our linear instability results can be used to predict the droplet size distribution in ocean oil spills. We can also apply our linear theory results to temperature influence on the analysis of an underwater horizontal injected oil jet. Theoretically, the oil jet with high temperature must rise faster compared to isothermal one.

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