Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/130516

New Ostrowski Type Inequalities for Coordinated $(s,m)-$Convex Functions in the Second Sense

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Received: 2 Jul. 2019, Revised: 12 Aug. 2019, Accepted: 21 Aug. 2019
Published online: 1 Sep. 2019

Abstract: In the present work we introduce the class of (s,m)-convex functions on the coordinates and some new Ostrowski-type inequalities are deduced for this kind of generalized convex functions. The results obtained have the absolute value of the second partial derivative with respect to the coordinates $(\partial^2 f/\partial r\partial t)$ in the aforementioned class and bounded, as a necessary condition. This generalizes the results for convex functions of [10]. Also, some corollary is presented.

Keywords: Ostrowski inequality for coordinates, $(s,m)-$convexity in the second sense, generalized convexity

1 Introduction

Let $f : I \subset [0,+\infty) \to \mathbb{R}$ be a mapping differentiable in I°, the interior of the interval I, such that $f' \in \mathscr{L}[a,b]$, where $a,b \in I$ and $a<b$. If $|f'(x)| \le M$, then the following inequality holds

$$\left|f(x) - \frac{1}{b-a}\int_a^b f(u)du\right| \le \frac{M}{b-a}\left[\frac{(x-a)^2+(b-x)^2}{2}\right].$$

This result is known in the literature as the Ostrowski inequality. Recently, many generalizations of the Ostrowski inequality for functions of bounded variation, Lipschitzian, monotone, absolutely continuous, convex functions, s-convex, h-convex and (m,h_1,h_2)-convex among others [1,3,5,4,8] has appeared. In this work we give new Ostrowski-type inequalities for functions coordinated (s,m)-convex.

2 Preliminaries

Let us consider now a bi-dimensional interval $\triangle := [a,b]\times[c,d]$ in $\mathbb{R}^2$ with $a<b$ and $c<d$, a mapping $f : \triangle \to \mathbb{R}$ is said to be convex on $\triangle$ if the inequality

$$f(\lambda x+(1-\lambda)z, \lambda y+(1-\lambda)w) \le \lambda f(x,y)+(1-\lambda)f(z,w),$$

holds for all (x,y), $(z,w) \in \triangle$ and $\lambda \in [0,1]$. The mapping f is said to be concave on the co-ordinates on $\triangle$ if the above inequality holds in reversed direction, for all (x,y), $(z,w) \in \triangle$ and $\lambda \in [0,1]$.

A modification for convex (concave) functions on $\triangle$, which is also known as coordinated convex (concave) functions, was introduced by S. S. Dragomir [6,7] as follows:
A function $f : \triangle \to \mathbb{R}$ is said to be convex (concave) on the co-ordinates on $\triangle$ if the partial mappings $f_y : [a,b] \to \mathbb{R}$, $f_y(u) = f(u,y)$ and $f_x : [c,d] \to \mathbb{R}$, $f_x(v) = f(x,v)$ are convex (concave) where defined for all $x \in [a,b]$, $y \in [c,d]$.

A formal definition for coordinates convex (concave) functions may be stated in:

Definition 1. *[9] A mapping $f : \triangle \to \mathbb{R}$ is said to be convex on the coordinates on $\triangle$ if the inequality*

$$f(tx+(1-t)y, ru+(1-r)w)$$

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$$\leq trf(x,u) + t(1-r)f(x,w) + r(1-t)f(y,u) + (1-t)(1-r)f(y,w), \quad (1)$$

holds for all $t,r \in [0,1]$ *and* $(x,u),\ (y,w) \in \triangle$. *The mapping of* f *is concave on the coordinates on* $\triangle$ *if the inequality* (1.1) *holds in reversed direction.*

Clearly, every convex (concave) mapping $f : \triangle \to \mathbb{R}$ is convex (concave) on the coordinates. Furthermore, there exists coordinated convex (concave) function not convex (concave), (see for instance [6,7]).

The concept of s-convex functions on the coordinates in the second sense was introduced by Alomari and Darus in [2] as a generalization of the coordinates convexity.

Definition 2([2]). *The mapping* $f : \triangle \to \mathbb{R}$ *is s-convex in the second sense on* $\triangle$ *if*

$$f(\lambda x + (1-\lambda)z, \lambda y + (1-\lambda)w) \leq \lambda^s f(x,y) + (1-\lambda)^s f(z,w),$$

holds for all $(x,y),\ (z,w) \in \triangle$, $\lambda \in [0,1]$ *with some fixed* $s \in (0,1]$.

A function $f : \triangle \to \mathbb{R}$ is called s-convex in the second sense on the coordinates on $\triangle$ if the partial mappings $f_y : [a,b] \to \mathbb{R}$, $f_y(u) = f(u,y)$ and $f_x : [c,d] \to \mathbb{R}$, $f_x(v) = f(x,v)$, are s-convex in the second sense for all $y \in [c,d]$, $x \in [a,b]$ and $s \in (0,1]$, i.e., the partial mappings f_y and f_x are s-convex in the second sense with some fixed $s \in (0,1]$.
A formal definition of co-ordinated s-convex function in second sense may be stated as follows:

Definition 3. *A function* $f : \triangle \to \mathbb{R}$ *is called s-convex in the second sense on the co-ordinates on* $\triangle$ *if*

$$f(tx+(1-t)y, ru+(1-r)w) \leq t^s r^s f(x,u) + t^s(1-r)^s f(x,w) + r^s(1-t)^s f(y,u) + (1-t)^s(1-r)^s f(y,w) \quad (2)$$

holds for all $t,r \in [0,1]$ *and* $(x,u),\ (y,w) \in \triangle$, *for some fixed* $s \in (0,1]$. *The mapping* f *is s-concave on the co-ordinates on* $\triangle$ *if the inequality* (1.2) *holds in reversed direction for all* $t,r \in [0,1]$ *and* $(x,y),\ (u,w) \in \triangle$ *with some fixed* $s \in (0,1]$.

The following lemma can be found in [11].

Lemma 1. *[11] Let* $f : \triangle \to \mathbb{R}$ *be a twice partial differentiable mapping on* $\triangle^\circ$. *If* $\frac{\partial^2 f}{\partial r \partial t} \in \mathscr{L}(\triangle)$, *then the following identity holds:*

$$f(x,y) + \frac{1}{(b-a)(d-c)} \int_a^b \int_c^d f(u,v)dvdu - A$$
$$= \frac{(x-a)^2(y-c)^2}{(b-a)(d-c)} \times \int_0^1 \int_0^1 rt \frac{\partial^2}{\partial r \partial t} f(tx+(1-t)a, ry+(1-r)c)drdt$$
$$- \frac{(x-a)^2(d-y)^2}{(b-a)(d-c)} \times \int_0^1 \int_0^1 rt \frac{\partial^2}{\partial r \partial t} f(tx+(1-t)a, ry+(1-r)d)drdt$$
$$- \frac{(b-x)^2(y-c)^2}{(b-a)(d-c)} \times \int_0^1 \int_0^1 rt \frac{\partial^2}{\partial r \partial t} f(tx+(1-t)b, ry+(1-r)c)drdt$$
$$+ \frac{(b-x)^2(d-y)^2}{(b-a)(d-c)} \times \int_0^1 \int_0^1 rt \frac{\partial^2}{\partial r \partial t} f(tx+(1-t)b, ry+(1-r)d)drdt$$

for all $(x,y) \in \triangle$, *where*

$$A = \frac{1}{d-c} \int_c^d f(x,v)dv + \frac{1}{b-a} \int_a^b f(u,y)du.$$

3 Main Results

In this section we present new Ostrowski types for functions co-ordinates (s,m)-convex.

Definition 4. *A function* $f : \triangle \to \mathbb{R}$ *is called* (s,m)*-convex in the second sense on the co-ordinates on* $\triangle$ *if the inequality*

$$f(tx+(1-t)y, ru+(1-r)w) \leq t^s r^s f(x,u) + mt^s(1-r)^s f(x,w) + mr^s(1-t)^s f(y,u) + m^2(1-t)^s(1-r)^s f(y,w)$$

holds for all $t,r \in [0,1]$ *and* $(x,u),\ (y,w) \in \triangle$, *for some fixed* $s,m \in (0,1]$. *The mapping of* f *is* (s,m)*-concave on the co-ordinates on* $\triangle$ *if the inequality holds in reversed direction for all* $t,r \in [0,1]$ *and* $(x,u),\ (y,w) \in \triangle$.

Theorem 1. *Let* $f : \triangle \to \mathbb{R}$ *be a twice partial differentiable mapping on* $\triangle^\circ$ *such that* $\frac{\partial^2 f}{\partial r \partial t} \in L(\triangle)$. *If* $\left|\frac{\partial^2 f}{\partial r \partial t}\right|$ *is* (s,m)*-convex in the second sense on the co-ordinates on* $\triangle$ *with* $s,m \in (0,1]$ *and* $\left|\frac{\partial f}{\partial r \partial t}(x,y)\right| \leq M$, $(x,y) \in \triangle$, *then the following* *inequality holds*

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|$$
$$\leq \frac{M(s+1+m)^2}{(s+1)^2(s+2)^2}\times$$
$$\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right]$$

for all $(x,y)\in\triangle$*, where A is defined in Lemma 1.*

Proof. By an application of Lemma 1, we have

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|$$
$$\leq \frac{(x-a)^2(y-c)^2}{(b-a)(d-c)}\times$$
$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}(tx+(1-t)a,ry+(1-r)c)\right|drdt$$

$$+\frac{(x-a)^2(y-d)^2}{(b-a)(d-c)}\times$$
$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}(tx+(1-t)a,ry+(1-r)d)\right|drdt$$

$$+\frac{(x-b)^2(y-c)^2}{(b-a)(d-c)}\times$$
$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}(tx+(1-t)b,ry+(1-r)c)\right|drdt$$

$$+\frac{(x-b)^2(y-d)^2}{(b-a)(d-c)}\times$$
$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}(tx+(1-t)b,ry+(1-r)d)\right|drdt$$

$$=\frac{(x-a)^2(y-c)^2}{(b-a)(d-c)}\times$$
$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{a}{m},ry+m(1-r)\frac{c}{m}\right)\right|drdt$$

$$+\frac{(x-a)^2(y-d)^2}{(b-a)(d-c)}\times$$
$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{a}{m},ry+m(1-r)\frac{d}{m}\right)\right|drdt$$

$$+\frac{(x-b)^2(y-c)^2}{(b-a)(d-c)}\times$$
$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{b}{m},ry+m(1-r)\frac{c}{m}\right)\right|drdt$$

$$+\frac{(x-b)^2(y-d)^2}{(b-a)(d-c)}\times$$
$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{b}{m},ry+m(1-r)\frac{d}{m}\right)\right|drdt$$

for all $(x,y)\in\triangle$.

Now, using the coordinates (s,m)-convex $\left|\frac{\partial^2 f}{\partial r\partial t}\right|$, we have

$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{a}{m},ry+m(1-r)\frac{c}{m}\right)\right|drdt$$
$$\leq\left|\frac{\partial^2 f}{\partial r\partial t}(x,y)\right|\int_0^1\int_0^1 r^{s+1}t^{s+1}drdt$$
$$+\left|\frac{\partial^2 f}{\partial r\partial t}(x,\frac{c}{m})\right|\int_0^1\int_0^1 mt^{s+1}r(1-r)^s drdt$$
$$+\left|\frac{\partial^2 f}{\partial r\partial t}(\frac{a}{m},y)\right|\int_0^1\int_0^1 mr^{s+1}t(1-t)^s drdt$$
$$+\left|\frac{\partial^2 f}{\partial r\partial t}(\frac{a}{m},\frac{c}{m})\right|\int_0^1\int_0^1 m^2rt(1-t)^s(1-r)^s drdt. \quad (3)$$

Since

$$\int_0^1\int_0^1 r^{s+1}t^{s+1}drdt=\frac{1}{(s+2)^2}$$

$$\int_0^1\int_0^1 r^{s+1}t(1-t)^s drdt=\int_0^1\int_0^1 t^{s+1}r(1-r)^s drdt$$
$$=\frac{1}{(s+1)(s+2)^2}$$

$$\int_0^1\int_0^1 rt(1-t)^s(1-r)^s drdt=\frac{1}{(s+1)^2(s+2)^2}$$

and we have that $\left|\frac{\partial f}{\partial r\partial t}(x,y)\right|\leq M$ for $(x,y)\in\triangle$, hence from (3), we obtain

$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{a}{m},ry+m(1-r)\frac{c}{m}\right)\right|drdt$$
$$\leq\frac{M}{(s+2)^2}+\frac{2Mm}{(s+1)(s+2)^2}+\frac{Mm^2}{(s+1)^2(s+2)^2}$$
$$=\frac{M(s+1+m)^2}{(s+1)^2(s+2)^2}. \quad (4)$$

Analogously, we also have

$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{a}{m},ry+m(1-r)\frac{d}{m}\right)\right|drdt$$
$$\leq\frac{M(s+1+m)^2}{(s+1)^2(s+2)^2}, \quad (5)$$

$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{b}{m},ry+m(1-r)\frac{c}{m}\right)\right|drdt$$
$$\leq\frac{M(s+1+m)^2}{(s+1)^2(s+2)^2} \quad (6)$$

$$\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{b}{m},ry+m(1-r)\frac{d}{m}\right)\right|drdt \leq \frac{M(s+1+m)^2}{(s+1)^2(s+2)^2} \tag{7}$$

Now using of inequalities (4),(5),(6) and (7) and the fact that

$$(x-a)^2(y-c)^2+(x-a)^2(y-d)^2+(x-b)^2(y-c)^2+(x-b)^2(y-d)^2=[(x-a)^2+(x-b)^2][(y-c)^2+(y-d)^2],$$

it follows that

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right| \leq \frac{M(s+1+m)^2}{(s+1)^2(s+2)^2}\times\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right].$$

The proof is complete.

Theorem 2. *Let $f:\triangle\to\mathbb{R}$ be a twice partial differentiable mapping on $\triangle^\circ$ such that $\frac{\partial^2 f}{\partial r\partial t}\in L(\triangle)$. If $\left|\frac{\partial^2 f}{\partial r\partial t}\right|^q$ is (s,m)-convex in the second sense on the co-ordinates on $\triangle$ with $s,m\in(0,1]$, $p,q>1$, $\frac{1}{p}+\frac{1}{q}=1$ and $\left|\frac{\partial f}{\partial r\partial t}(x,y)\right|\leq M$, $(x,y)\in\triangle$, then the following inequality holds*

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right| \leq \frac{M}{(1+p)^{\frac{2}{q}}}\left(\frac{m+1}{s+1}\right)^{\frac{2}{q}}\times\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right],$$

for all $(x,y)\in\triangle$, where A is defined as in Lemma 1.

Proof. Using Lemma 1 and the Hölder inequality for double integrals, we have

$$\begin{aligned}&\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|\\ &\leq\left(\int_0^1\int_0^1 r^pt^pdrdt\right)^{\frac{1}{p}}\times\Big[\frac{(x-a)^2(y-c)^2}{(b-a)(d-c)}\times\left(\int_0^1\int_0^1\left|\frac{\partial^2 f}{\partial r\partial t}(tx+(1-t)a,ry+(1-r)c)\right|^qdrdt\right)^{\frac{1}{q}}\\ &+\frac{(x-a)^2(y-d)^2}{(b-a)(d-c)}\times\left(\int_0^1\int_0^1\left|\frac{\partial^2 f}{\partial r\partial t}(tx+(1-t)a,ry+(1-r)d)\right|^qdrdt\right)^{\frac{1}{q}}\\ &+\frac{(x-b)^2(y-c)^2}{(b-a)(d-c)}\times\left(\int_0^1\int_0^1\left|\frac{\partial^2 f}{\partial r\partial t}(tx+(1-t)b,ry+(1-r)c)\right|^qdrdt\right)^{\frac{1}{q}}\\ &+\frac{(x-b)^2(y-d)^2}{(b-a)(d-c)}\times\left(\int_0^1\int_0^1\left|\frac{\partial^2 f}{\partial r\partial t}(tx+(1-t)b,ry+(1-r)d)\right|^qdrdt\right)^{\frac{1}{q}}\Big]\\ &=\left(\int_0^1\int_0^1 r^pt^pdrdt\right)^{\frac{1}{p}}\times\Big[\frac{(x-a)^2(y-c)^2}{(b-a)(d-c)}\times\left(\int_0^1\int_0^1\left|\frac{\partial^2 f}{\partial r\partial t}(tx+m(1-t)\frac{a}{m},ry+m(1-r)\frac{c}{m})\right|^qdrdt\right)^{\frac{1}{q}}\\ &+\frac{(x-a)^2(y-d)^2}{(b-a)(d-c)}\left(\int_0^1\int_0^1\left|\frac{\partial^2 f}{\partial r\partial t}(tx+m(1-t)\frac{a}{m},ry+m(1-r)\frac{d}{m})\right|^qdrdt\right)^{\frac{1}{q}}\\ &+\frac{(x-b)^2(y-c)^2}{(b-a)(d-c)}\times\left(\int_0^1\int_0^1\left|\frac{\partial^2 f}{\partial r\partial t}(tx+m(1-t)\frac{b}{m},ry+(1-r)\frac{c}{m})\right|^qdrdt\right)^{\frac{1}{q}}\\ &+\frac{(x-b)^2(y-d)^2}{(b-a)(d-c)}\times\left(\int_0^1\int_0^1\left|\frac{\partial^2 f}{\partial r\partial t}(tx+m(1-t)\frac{b}{m},ry+m(1-r)\frac{d}{m})\right|^qdrdt\right)^{\frac{1}{q}}\Big]\end{aligned}\tag{8}$$

for all $(x,y)\in\triangle$.

Since $\left|\frac{\partial^2 f}{\partial r\partial t}\right|^q$ is (s,m)-convex in the second sense on the co-ordinates on $\triangle$, we have

$$\begin{aligned}&\int_0^1\int_0^1\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{a}{m},ry+m(1-r)\frac{c}{m}\right)\right|^qdrdt\\ &\leq\left|\frac{\partial^2 f}{\partial r\partial t}(x,y)\right|^q\int_0^1\int_0^1 r^st^sdrdt+\left|\frac{\partial^2 f}{\partial r\partial t}\left(x,\frac{c}{m}\right)\right|^q\int_0^1\int_0^1 mt^s(1-r)^sdrdt\\ &\leq\left|\frac{\partial^2 f}{\partial r\partial t}\left(\frac{a}{m},y\right)\right|^q\int_0^1\int_0^1 mr^s(1-t)^sdrdt+\left|\frac{\partial^2 f}{\partial r\partial t}\left(\frac{a}{m},\frac{c}{m}\right)\right|^q\int_0^1\int_0^1 m^2(1-t)^s(1-r)^sdrdt\end{aligned}\tag{9}$$

Since

$$\int_0^1\int_0^1 r^st^sdrdt=\frac{1}{(s+1)^2}$$

$$\int_0^1\int_0^1 t^s(1-r)^sdrdt=\int_0^1\int_0^1 r^s(1-t)^sdrdt=\frac{1}{(s+1)^2}$$

and

$$\int_0^1\int_0^1 (1-r)^s(1-t)^s drdt = \frac{1}{(s+1)^2}.$$

Hence from (8) and since $\left|\frac{\partial^2 f}{\partial r\partial t}\right| \le M$, $(x,y)\in\triangle$, we obtain

$$\int_0^1\int_0^1 \left|\frac{\partial^2 f}{\partial r\partial t}\Big(tx+m(1-t)\frac{a}{m}, ry+m(1-r)\frac{c}{m}\Big)\right|^q drdt \le \frac{M^q}{(s+1)^2}+2\frac{mM^q}{(s+1)^2}+\frac{m^2M^q}{(s+1)^2} = \frac{M^q(m+1)^2}{(s+1)^2}$$

Similarly, we also have the following inequalities

$$\int_0^1\int_0^1 \left|\frac{\partial^2 f}{\partial r\partial t}\Big(tx+m(1-t)\frac{a}{m}, ry+m(1-r)\frac{d}{m}\Big)\right|^q drdt \le \frac{M^q(m+1)^2}{(s+1)^2}$$

$$\int_0^1\int_0^1 \left|\frac{\partial^2 f}{\partial r\partial t}\Big(tx+m(1-t)\frac{b}{m}, ry+m(1-r)\frac{c}{m}\Big)\right|^q drdt \le \frac{M^q(m+1)^2}{(s+1)^2}$$

and

$$\int_0^1\int_0^1 \left|\frac{\partial^2 f}{\partial r\partial t}\Big(tx+m(1-t)\frac{b}{m}, ry+m(1-r)\frac{d}{m}\Big)\right|^q drdt \le \frac{M^q(m+1)^2}{(s+1)^2}.$$

Since

$$\int_0^1\int_0^1 r^pt^p drdt = \frac{1}{(1+p)^2}$$

and the above inequalities (9), we obtain

$$\begin{aligned}&\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv - A\right| \\ &\le \Big(\frac{1}{(1+p)^2}\Big)^{\frac{1}{p}}\Big[\frac{(x-a)^2(y-c)^2}{(b-a)(d-c)}\Big(\frac{M^q(m+1)^2}{(s+1)^2}\Big)^{\frac{1}{q}} + \frac{(x-a)^2(y-d)^2}{(b-a)(d-c)}\Big(\frac{M^q(m+1)^2}{(s+1)^2}\Big)^{\frac{1}{q}} \\ &\quad + \frac{(x-b)^2(y-c)^2}{(b-a)(d-c)}\Big(\frac{M^q(m+1)^2}{(s+1)^2}\Big)^{\frac{1}{q}} + \frac{(x-b)^2(y-d)^2}{(b-a)(d-c)}\Big(\frac{M^q(m+1)^2}{(s+1)^2}\Big)^{\frac{1}{q}}\Big] \\ &= \frac{M}{(1+p)^{\frac{2}{p}}}\Big(\frac{m+1}{s+1}\Big)^{\frac{2}{q}}\Big[\frac{(x-a)^2+(x-b)^2}{b-a}\Big]\Big[\frac{(y-c)^2+(y-d)^2}{d-c}\Big]\end{aligned}$$

The proof is complete.

Theorem 3. *Let $f:\triangle\to\mathbb{R}$ be a twice partial differentiable mapping on $\triangle^\circ$ such that $\frac{\partial^2 f}{\partial r\partial t}\in L(\triangle)$. If $\left|\frac{\partial^2 f}{\partial r\partial t}\right|^q$ is (s,m)-convex in the second sense on the co-ordinates on $\triangle$ with $s,m\in(0,1]$, $q\ge 1$ and $\left|\frac{\partial f}{\partial r\partial t}(x,y)\right|\le M$, $(x,y)\in\triangle$, then the following inequality holds*

$$\begin{aligned}&\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv - A\right| \\ &\le \frac{M}{4}\Big(\frac{2(s+1+m)}{(s+1)(s+2)}\Big)^{\frac{2}{q}}\times \Big[\frac{(x-a)^2+(x-b)^2}{b-a}\Big]\Big[\frac{(y-c)^2+(y-d)^2}{d-c}\Big],\end{aligned}$$

for all $(x,y)\in\triangle$, where A is defined in Lemma 1.

Proof. Suppose $q\ge 1$. From Lemma 1 and using the power mean inequality for double integrals, we have

$$\begin{aligned}&\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv - A\right| \le \Big(\int_0^1\int_0^1 rtdrdt\Big)^{1-\frac{1}{q}}\times \\ &\Big[\frac{(x-a)^2(y-c)^2}{(b-a)(d-c)}\times\Big(\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\Big(tx+m(1-t)\frac{a}{m}, ry+m(1-r)\frac{c}{m}\Big)\right|^q drdt\Big)^{\frac{1}{q}} \\ &+\frac{(x-a)^2(y-d)^2}{(b-a)(d-c)}\times\Big(\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\Big(tx+m(1-t)\frac{a}{m}, ry+m(1-r)\frac{d}{m}\Big)\right|^q drdt\Big)^{\frac{1}{q}} \\ &+\frac{(x-b)^2(y-c)^2}{(b-a)(d-c)}\times\Big(\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\Big(tx+m(1-t)\frac{b}{m}, ry+m(1-r)\frac{c}{m}\Big)\right|^q drdt\Big)^{\frac{1}{q}} \\ &+\frac{(x-b)^2(y-d)^2}{(b-a)(d-c)}\times\Big(\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\Big(tx+m(1-t)\frac{b}{m}, ry+m(1-r)\frac{d}{m}\Big)\right|^q drdt\Big)^{\frac{1}{q}}\Big]\end{aligned} \tag{10}$$

for all $(x,y)\in\triangle$.

Similarly, as in Theorem 2 that $\left|\frac{\partial^2 f}{\partial r\partial t}\right|^q$ is (s,m)-convex in the second sense on the co-ordinates on $\triangle$ and

$\left|\frac{\partial^2 f}{\partial r\partial t}(x,y)\right| \le M$ for all $(x,y)\in\triangle$, we have

$$\begin{aligned}
&\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{a}{m},ry+m(1-r)\frac{c}{m}\right)\right|^q drdt\\
&\le\left|\frac{\partial^2 f}{\partial r\partial t}(x,y)\right|^q\int_0^1\int_0^1 r^{s+1}t^{s+1}drdt\\
&\quad+\left|\frac{\partial^2 f}{\partial r\partial t}\left(x,\frac{c}{m}\right)\right|^q\int_0^1\int_0^1 mt^{s+1}r(1-r)^s drdt\\
&\le\left|\frac{\partial^2 f}{\partial r\partial t}\left(\frac{a}{m},y\right)\right|^q\int_0^1\int_0^1 mt(1-t)^s r^{s+1}drdt\\
&\quad+\left|\frac{\partial^2 f}{\partial r\partial t}\left(\frac{a}{m},\frac{c}{m}\right)\right|^q\int_0^1\int_0^1 m^2t(1-t)^s r(1-r)^s drdt\\
&\le\frac{M^q}{(s+2)^2}+\frac{mM^q}{(s+1)(s+2)^2}\\
&\qquad+\frac{mM^q}{(s+1)(s+2)^2}+\frac{m^2M^q}{(s+1)^2(s+2)^2}\\
&=\frac{M^q(s+1+m)^2}{(s+1)^2(s+2)^2}.
\end{aligned}$$

In a similar way, we have the following inequalities

$$\begin{aligned}
&\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{a}{m},ry+m(1-r)\frac{d}{m}\right)\right|^q drdt\\
&\qquad\le\frac{M^q(s+1+m)^2}{(s+1)^2(s+2)^2},
\end{aligned}$$

$$\begin{aligned}
&\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{b}{m},ry+m(1-r)\frac{c}{m}\right)\right|^q drdt\\
&\qquad\le\frac{M^q(s+1+m)^2}{(s+1)^2(s+2)^2}
\end{aligned}$$

and

$$\begin{aligned}
&\int_0^1\int_0^1 rt\left|\frac{\partial^2 f}{\partial r\partial t}\left(tx+m(1-t)\frac{b}{m},ry+m(1-r)\frac{d}{m}\right)\right|^q drdt\\
&\qquad\le\frac{M^q(s+1+m)^2}{(s+1)^2(s+2)^2}.
\end{aligned}$$

Now using the above inequalities and

$$\int_0^1\int_0^1 rt\,drdt=\frac{1}{4},$$

in (10), we obtain

$$\begin{aligned}
&\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|\\
&\le\left(\frac{1}{4}\right)^{1-\frac{1}{q}}\Bigg[\frac{(x-a)^2(y-c)^2}{(b-a)(d-c)}\left(\frac{M^q(s+1+m)^2}{(s+1)^2(s+2)^2}\right)^{\frac{1}{q}}\\
&\quad+\frac{(x-a)^2(y-d)^2}{(b-a)(d-c)}\left(\frac{M^q(s+1+m)^2}{(s+1)^2(s+2)^2}\right)^{\frac{1}{q}}\\
&\quad+\frac{(x-b)^2(y-c)^2}{(b-a)(d-c)}\left(\frac{M^q(s+1+m)^2}{(s+1)^2(s+2)^2}\right)^{\frac{1}{q}}\\
&\quad+\frac{(x-b)^2(y-d)^2}{(b-a)(d-c)}\left(\frac{M^q(s+1+m)^2}{(s+1)^2(s+2)^2}\right)^{\frac{1}{q}}\Bigg]\\
&=\frac{M}{4}\left(\frac{2(s+1+m)}{(s+1)(s+2)}\right)^{\frac{2}{q}}\times\\
&\qquad\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right].
\end{aligned}$$

The proof is complete.

4 Some applications obtained.

The Theorem 2.2 in [10] is obtained from Theorem 1 as a corollary.

Corollary 1. *Let $f:\triangle\to\mathbb{R}$ be a twice partial differentiable mapping on $\triangle^\circ$ such that $\frac{\partial^2 f}{\partial r\partial t}\in L(\triangle)$. If $\left|\frac{\partial^2 f}{\partial r\partial t}\right|$ is s-convex in the second sense on the co-ordinates on $\triangle$ with $s\in(0,1]$ and $\left|\frac{\partial f}{\partial r\partial t}(x,y)\right|\le M$, $(x,y)\in\triangle$, then the following inequality holds*

$$\begin{aligned}
&\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|\\
&\le\frac{M}{(s+1)^2}\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right]
\end{aligned}$$

for all $(x,y)\in\triangle$, where A is defined in Lemma 1.

Proof. Letting $m=1$ in Theorem 1 we get the desired result.

From Theorem 2 we get the Theorem 2.3 in [10].

Corollary 2.*Let $f:\triangle\to\mathbb{R}$ be a twice partial differentiable mapping on $\triangle^\circ$ such that $\frac{\partial^2 f}{\partial r\partial t}\in L(\triangle)$. If $\left|\frac{\partial^2 f}{\partial r\partial t}\right|^q$ is s-convex in the second sense on the co-ordinates on $\triangle$ with $s\in(0,1]$, $p,q>1$, $\frac{1}{p}+\frac{1}{q}=1$ and $\left|\frac{\partial f}{\partial r\partial t}(x,y)\right|\le M$, $(x,y)\in\triangle$, then the following* *inequality holds*

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|$$
$$\leq \frac{M}{(1+p)^{\frac{2}{q}}}\left(\frac{2}{s+1}\right)^{\frac{2}{q}}\times$$
$$\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right],$$

for all $(x,y)\in\triangle$*, where A is defined as in Lemma 1.*

Proof. Letting $m=1$ in Theorem 2 we get the desired result.

From Theorem 3 we obtain the Theorem 2.4 in [10].

Corollary 3. *Let* $f:\triangle\to\mathbb{R}$ *be a twice partial differentiable mapping on* $\triangle^\circ$ *such that* $\frac{\partial^2 f}{\partial r\partial t}\in L(\triangle)$*. If* $\left|\frac{\partial^2 f}{\partial r\partial t}\right|^q$ *is s-convex in the second sense on the co-ordinates on* $\triangle$ *with* $s,m\in(0,1]$*,* $q\geq 1$ *and* $\left|\frac{\partial f}{\partial r\partial t}(x,y)\right|\leq M$*,* $(x,y)\in\triangle$*, then the following inequality holds*

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|$$
$$\leq \frac{M}{4}\left(\frac{2}{s+1}\right)^{\frac{2}{q}}\times$$
$$\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right],$$

for all $(x,y)\in\triangle$*, where A is defined in Lemma 1.*

Proof. Letting $m=1$ in Theorem 3 we get the desired result.

For the $m-$convexity of $\frac{\partial^2 f}{\partial r\partial t}$ we have the following inequalities.

Corollary 4. *Let* $f:\triangle\to\mathbb{R}$ *be a twice partial differentiable mapping on* $\triangle^\circ$ *such that* $\frac{\partial^2 f}{\partial r\partial t}\in L(\triangle)$*. If* $\left|\frac{\partial^2 f}{\partial r\partial t}\right|$ *is m-convex on the co-ordinates on* $\triangle$ *with* $m\in(0,1]$ *and* $\left|\frac{\partial f}{\partial r\partial t}(x,y)\right|\leq M$*,* $(x,y)\in\triangle$*, then the following inequality holds*

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|$$
$$\leq \frac{M(2+m)^2}{36}\times$$
$$\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right]$$

for all $(x,y)\in\triangle$*, where A is defined in Lemma 1.*

Corollary 5. *Let* $f:\triangle\to\mathbb{R}$ *be a twice partial differentiable mapping on* $\triangle^\circ$ *such that* $\frac{\partial^2 f}{\partial r\partial t}\in L(\triangle)$*. If* $\left|\frac{\partial^2 f}{\partial r\partial t}\right|^q$ *is m-convex on the co-ordinates on* $\triangle$ *with* $m\in(0,1]$*,* $p,q>1$*,* $\frac{1}{p}+\frac{1}{q}=1$ *and* $\left|\frac{\partial f}{\partial r\partial t}(x,y)\right|\leq M$*,* $(x,y)\in\triangle$*, then the following inequality holds*

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|$$
$$\leq \frac{M}{(1+p)^{\frac{2}{q}}}\left(\frac{m+1}{2}\right)^{\frac{2}{q}}\times$$
$$\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right],$$

for all $(x,y)\in\triangle$*, where A is defined as Lemma 1.*

Corollary 6. *Let* $f:\triangle\to\mathbb{R}$ *be a twice partial differentiable mapping on* $\triangle^\circ$ *such that* $\frac{\partial^2 f}{\partial r\partial t}\in L(\triangle)$*. If* $\left|\frac{\partial^2 f}{\partial r\partial t}\right|^q$ *is m-convex on the co-ordinates on* $\triangle$ *with* $s,m\in(0,1]$*,* $q\geq 1$ *and* $\left|\frac{\partial f}{\partial r\partial t}(x,y)\right|\leq M$*,* $(x,y)\in\triangle$*, then the following inequality holds*

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|$$
$$\leq \frac{M}{4}\left(\frac{m+2}{3}\right)^{\frac{2}{q}}\times$$
$$\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right],$$

for all $(x,y)\in\triangle$*, where A is defined in Lemma 1.*

When $\frac{\partial^2 f}{\partial r\partial t}$ is convex we have the following inequalities whose proofs follows the same method of the above results.

Corollary 7. *Let* $f:\triangle\to\mathbb{R}$ *be a twice partial differentiable mapping on* $\triangle^\circ$ *such that* $\frac{\partial^2 f}{\partial r\partial t}\in L(\triangle)$*. If* $\left|\frac{\partial^2 f}{\partial r\partial t}\right|$ *is convex on the co-ordinates on* $\triangle$ *and* $\left|\frac{\partial f}{\partial r\partial t}(x,y)\right|\leq M$*,* $(x,y)\in\triangle$*, then the following inequality holds*

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv-A\right|$$
$$\leq \frac{M}{4}\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right]$$

for all $(x,y)\in\triangle$*, where A is defined in Lemma 1.*

Corollary 8. *Let* $f : \triangle \to \mathbb{R}$ *be a twice partial differentiable mapping on* $\triangle^\circ$ *such that* $\frac{\partial^2 f}{\partial r \partial t} \in L(\triangle)$. *If* $\left|\frac{\partial^2 f}{\partial r \partial t}\right|^q$ *is convex on the co-ordinates on* $\triangle$, $p,q > 1$, $\frac{1}{p}+\frac{1}{q} = 1$ *and* $\left|\frac{\partial f}{\partial r \partial t}(x,y)\right| \leq M$, $(x,y) \in \triangle$, *then the following inequality holds*

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv - A\right| \leq \frac{M}{(1+p)^{\frac{2}{q}}}\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right],$$

for all $(x,y) \in \triangle$, *where A is defined as in Lemma 1.*

Corollary 9. *Let* $f : \triangle \to \mathbb{R}$ *be a twice partial differentiable mapping on* $\triangle^\circ$ *such that* $\frac{\partial^2 f}{\partial r \partial t} \in L(\triangle)$. *If* $\left|\frac{\partial^2 f}{\partial r \partial t}\right|^q$ *is convex on the co-ordinates on* $\triangle$, $q \geq 1$ *and* $\left|\frac{\partial f}{\partial r \partial t}(x,y)\right| \leq M$, $(x,y) \in \triangle$, *then the following inequality holds*

$$\left|f(x,y)+\frac{1}{(b-a)(d-c)}\int_a^b\int_c^d f(u,v)dudv - A\right| \leq \frac{M}{4}\left[\frac{(x-a)^2+(x-b)^2}{b-a}\right]\left[\frac{(y-c)^2+(y-d)^2}{d-c}\right],$$

for all $(x,y) \in \triangle$, *where A is defined in Lemma 1.*

5 Conclusions

In this work the class of $(s,m)-$convex functions in the second sense on the coordinates has been introduced, and some Ostrowski-type inequalities for this kind of functions has been established. From Theorems 1, 2 and 3 some corollary, as applications to $s-$convexity in the second sense, $m-$convexity and the classical convexity on the coordinates, has been found,also, a generalization of the results presented by M. A. Latif and S.S. Dragomir [10].

Acknowledgement

Dr. Miguel J. Vivas-Cortez wants to thank Dirección de Investigación from Pontificia Universidad Católica del Ecuador (PUCE) for the financial support under the project: Fuciones convexas generalizadas y algunas desigualdades. Also M.Sc. Carlos García and M.Sc. Jorge E. Hernández Hernández wants to thank Consejo de Desarrollo Científico, Humanístico y Tecnológico from Universidad Centroccidental Lisandro Alvarado (Venezuela) for the technical support given in the development of this work.

The authors are grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

References

[1] M. Alomari, M. Darus, Some Otrowski type inequalities for quasi-convex functions with applications to special means, RGMIA Res. Rep. Coll., **13**(1), (2010), preprint.

[2] M. Alomari, M. Darus, The Hadamards Hadamards inequality for s-convex function of 2-variables on the co-ordinates. , Int. Journal of Math. Anal., **2** (13), 629-638, (2008).

[3] M. Alomari, M. Darus, S.S. Dragomir, P. Cerone, Otrowski type inequalities for functions whose derivatives are s-convex in the second sense. , Appl. Math. Lett. , **23**, 1071-1076 ,(2010).

[4] M. Alomari, Several Inequalities of Hermite-Hadamard, Ostrowski and Simpson type for s-convex, Quasi-convex and r-convex mappings and applications. Thesis Submitted in Fulfilment for the degree of Doctor of Philosophy. Faculty of Science and Technology University Kebangsaan Malaysia, Bangi, (2008).

[5] P. Cerone, S.S. Dragomir, Ostrowski type inequalities for functions whose derivative satisfy certain convexity assumptions, Demonstr. Math., **37**(2), 299-308, (2004).

[6] S.S. Dragomir, On Hadamard's inequatity for convex functions on the co-ordinates in a rectangle from the plane, Taiwanese Journal of Mathematics , **5**, 775-788, 2001.

[7] S.S Dragomir , C.E.M Pearse, Selected Topics on Hermite-Hadamard Inequalities and Applications, RGMIA Monographs, Victoria University, Online: $http : //www.staff.vu.edu.au/RGMIA/monographs/hermite_hadamard.html$,(2002).

[8] J.E. Hernández Hernández, Ostrowski Type Fractional Integral Operator Inequalities for $(m,h_1,h_2)-$Convex Functions, Mayfeb Journal of Mathematics, **4**, 13-28, (2017).

[9] M.A. Latif, M. Alomari, Hadamard-type inequalities for product two convex functions on the co-ordinates. International Mathematical Forum, **4**(47), 2327-2338, (2009).

[10] M. A. Latif , S.S. Dragomir, New Ostrowski type inequalities for co-ordinated s-covex functions in the second sense., Le Matematiche., **LXVII** (1), 57-72, (2012).

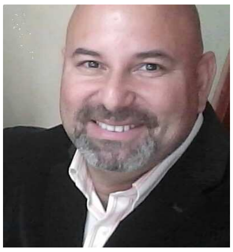
[11] M.A. Latif, S. Hussain, S.S. Dragomir, New Ostrowski type inequalities for co-ordinated convex functions. RGMIA Research Report Collection, **14**, (2011).

[12] M. A. Noor, K. I. Noor , M. U. Awan, Generalized Convexity and Integral Inequalities, Appl. Math. & Inf. Sci., **1**, 233-243, (2015).

[13] M. A. Noor, On some characterizations of non-convex functions, Nonlinear Anal. For., **12** (2), 193-201, (2007).

[14] A. Ostrowski, die Absolutabweichung einer differentiebaren Funk-tion von ihrem Integralmittelwert, Comment. Math. Helv., **10**, 226227, (1938).

[15] M. J. Vivas-Cortez, C. García, Ostrowski Type Inequalities for Function Whose Derivatives are (m,h_1,h_2)-Convex. Appl. Math. Inf. Sci., **11** (1), 79-86,(2017).



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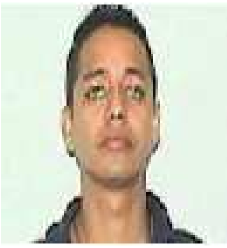

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