Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/130621

Effect of Initial Stress on the Electromechanical Coupling and the Bleustein-Gulyaev Wave in a Transversely Isotropic Piezoelectric Layered Structure

*Rawan A. Bossly**

Department of Mathematics, Faculty of Science, Jazan University, Jazan, Saudi Arabia

Received: 12 Jun. 2019, Revised: 2 Jul. 2019, Accepted: 12 Oct. 2019.
Published online: 1 Nov. 2019.

Abstract: The present paper investigates the electromechanical coupling and the propagation of Bleustein–Gulyaev (B-G) wave in a transversely isotropic piezoelectric composed half-space under the effect of initial stress. It also addresses the dispersion relation of the Bleustein-Gulyaev wave. The suitable electrical and mechanical boundary conditions are considered for the electric potential, and electric displacements. The dispersion relation is computed numerically and illustrated graphically for the electric open and short cases for different thicknesses of the layer and wave number under the effect initial stress. The results indicate that the Bleustein-Gulyaev wave and electro-mechanical coupling factor are influenced by initial stress and the physical properties of the material. Numerical outcomes are produced employing $LiNbO_3$ as an example of the materials included for clarification. Bleustein-Gulyaev waves under the initial stress have a good deal of practical importance in different signal transmission, micro-machined gyroscopes, sensors, actuators, signal processing and information storage applications.
Keywords: Dispersion relation; Bleustein–Gulyaev wave; Initial stress, Piezoelectric composed structure; Electro-mechanical coupling factor.

Nomenclature	
σ_{ij}	the components of stress
$\sigma_{kj}^{(0)}$	The initial stress tensor
S_{kl}	the strain components
ε_{ik}	the dielectric constants
u_i	the displacement components
D_i	the electric displacement components
$D_i^{(0)}$	the initial elastic displacement components
E_k	the electric field components
n_i	a unit vector representing the direction of the axis of rotation
c_{ijkl}	the elastic stiffness constants
e_{kij}	the piezoelectric constants
ρ	the density
E_i	the electrical intensity components
$\varphi_{,i}$	the electrical potential components

*Corresponding author e-mail: rboolye@jazanu.edu.sa

1 Introduction

Recently, surface acoustic wave propagation in piezoelectric composite frameworks has gained a great attention because of their major achievement in practical manufacturing applications such as surface acoustic wave oscillators, amplifier, sensors, resonators, delay lines, oscillators, filters. Since Bleustein [1] and Gulyaev [2] concurrently found out the existence of a SH electroacoustic surface mode which propagates in piezoelectric materials of class 6 mm, there has been considerable interest in investigated the modeling of this type of surface wave. Furthermore, the results of the B-G have provided a theoretical and practical basis for several studies and applications that address the acoustic surface wave in piezoelectric devices. Thus, B-G waves have become one of the important topics in modern electro-acoustic technology [3-8].
Several authors investigate the surface acoustic waves such as B-G waves with different piezoelectric layers and substrate propagating in the structure under some special conditions and different hypotheses [9-16].
The nonlinear wave propagation in anisotropic elastic materials is a complicated procedure and the solutions for corresponding nonlinear problems are uncommon. The presence of acceleration waves is widely connected with the characteristic of hyperbolic dynamics equations (or elliptic equations of statics). For example, Altenbach [17] extended the problem of acceleration wave propagation analysis to the case of nonlinear thermoelastic micropolar structure. Many studies considering reflection and transmission of fluid streams at discontinuity surfaces of material characteristics in numerous various frames are illustrated in [18-26].

In this study, the propagation attitude of the B-G waves and the elecromechancial coupling factor in a piezoelectric composite structure have been examined the presence of initial stress. The dispersion relation for the B–G wave is given when a layered half-space has identical piezoelectric layer with the substrate. The electromechanical coupling coefficients were also studied in piezoelectric layers under the influence of initial stress. Some of the previous studies can be deduced as special cases of this study. The results of the present study are useful for building SAW devices with high efficiency and quality, but also for estimating the distribution of residual stress in composite materials.

2 Basic Equations

The composite piezoelectric structure and the relevant coordinate axis are given in Fig. 1. We consider the normal configuration the substrate is in the region $x_1 > 0$ and the layer is in part $-h < x_1 < 0$. We suppose that the above-mentioned region of the piezoelectric layer ($x_1 = -h$) may be electrically free or shorted. In addition, it will be considered as traction-free. Also, the layer is assumed to be under the effect of steady initial stresses. Therefore, the differential equations of motion in a quasi-static approximation of the hexagonal piezoelectric half-space may be written as follows taking into consideration the effects of the initial stress [4] and [8]:

$$\sigma_{ij,j} + (\sigma_{kj}^{(0)} u_{i,k})_{,j} = \rho \ddot{u}_j , \tag{1}$$

$$D_{i,i} + (D_i^{(0)} u_{i,k})_{,j} = 0. \tag{2}$$

Where the equations of state of the piezoelectric solid may be given as:

$$\sigma_{ij} = c_{ijkl} S_{kl} - e_{kij} E_k , \tag{3}$$

$$D_i = e_{ikl} S_{kl} + \varepsilon_{ik} E_k , \tag{4}$$

$$S_{kl} = \frac{1}{2}(u_{k,l} + u_{l,k}) \tag{5}$$

$$E_i = -\varphi_{,i} \tag{6}$$

Where dot denotes time differentiation, the repeated index in the subscript implies summation, the comma signifies space-coordinate derivative [18].
From Eqs. (3)-(6) into Eqs. (1) and (2), one may obtain [16]:

$$c_{ijkl}\, u_{l,jk} + e_{kij}\varphi_{,jk} + (\sigma_{kj}^{(0)} u_{i,k})_{,j} = \rho \ddot{u}_j \tag{7}$$

$$e_{ikl}\, u_{l,jk} - \varepsilon_{jk}\varphi_{,jk} + (D_i^{(0)} u_{i,k})_{,j} = 0. \tag{8}$$

Where c_{ijkl} and e_{kij} have four and three indices, respectively. These constants may be written in two-index Notations, as shown in [1] and [4].

3 Boundary Conditions and Formulation of the Problem

The motion must satisfy the boundary and continuous conditions which it should be assumed as:
(a) The mechanical condition is:

$$\sigma_{31}(-h, x_2) = 0 \quad , \quad \text{at } x_1 = -h \tag{9}$$

(b) The electrical boundary conditions
(i) For *electrical* open case:

$$\left.\begin{aligned} \varphi_1(-h, x_2) &= \varphi_0(-h, x_2), \\ D_1(-h, x_2)_1 &= D_1(-h, x_2)_0, \end{aligned}\right\} \quad \text{at} \quad x_1 = -h \tag{10}$$

(ii) For electrical short case:

$$\varphi_1(-h, x_2) = 0, \quad \text{at } x_1 = -h \tag{11}$$

(c) The continuity conditions at $x_1 = 0$:

$$\begin{aligned} &\sigma_{31}(0, x_2) = \sigma'_{31}(0, x_2), \quad w_1(0, x_2) = w_2(0, x_2), \\ &\varphi_1(0, x_2) = \varphi_2(0, x_2), \quad D_1(0, x_2)_1 = D_1(0, x_2)_2 . \end{aligned} \tag{12}$$

here σ'_{31} and σ_{31} denote the normal stress components in the substrate and layer, respectively. While, $D_1(\ )_0, D_1(\ )_1$ and $D_1(\ )_2$ indicate the normal components in the vacuum, the layer and the substrate of the electric displacement. Also, φ_1 and φ_0 represent the electric potentials in the substrate and vacuum.

Now, we give concern to transverse surface wave propagation on a hexagonal piezoelectric medium and consider that the propagation of wave has amplitude decays with depth to the x_1-axis.

Furthermore, the components of the plane displacement and electrical potential φ are assumed as:

$$u = v = 0, \quad w = w(x_1, x_2, t), \quad \varphi = \varphi(x_1, x_2, t). \tag{13}$$

It is proposed that the layered piezoelectric structure and the substrate are made of the same transversely isotropic piezoelectric materials but in the opposite direction. Thus, we have:

$$c_{44} = c'_{44}, \quad e_{15} = -e'_{15}, \quad \varepsilon_{11} = \varepsilon'_{11}. \tag{14}$$

Let φ_1 and W_1 individually indicate the electrical potential and mechanical displacement in the layered piezoelectric structure. Therefore, one may obtain the fundamental coupled field equations for B–G wave propagation as:

$$c^*_{44}\nabla^2 w_1 + e_{15}\nabla^2\varphi_1 - \sigma_y^{(0)} w_{1,xx} = \rho\ddot{w}_1,$$
$$e_{15}\nabla^2 w_1 - \varepsilon_{11}\nabla^2\varphi_1 = 0, \qquad x_1 \in [-h, 0] \tag{15}$$

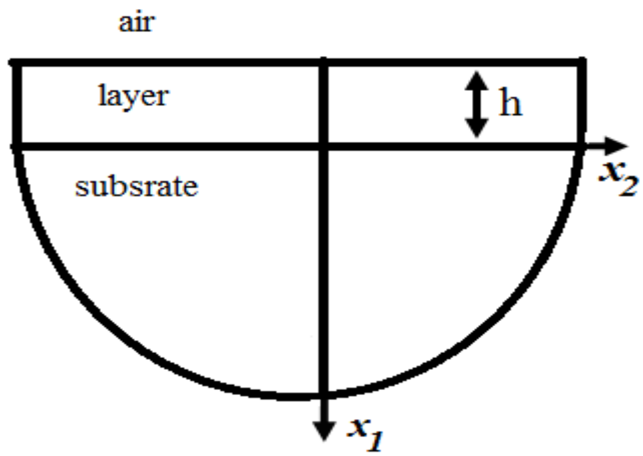

Fig. 1: Geometry of the problem.

$$c_{44}\nabla^2 w_2 + e_{15}\nabla^2\varphi_2 = \rho\ddot{w}_2,$$
$$e_{15}\nabla^2 w_2 + \varepsilon_{11}\nabla^2\varphi_2 = 0, \qquad x \in [0, \infty), \tag{16}$$

where w_1 and φ_1 indicate the displacement component and electric potential function in the layer $-h < x_1 < 0$, while w_2 and φ_2 show the correspondence of them in the substrate $x_1 > 0$, $\nabla^2 = (\partial^2/\partial x_1^2) + (\partial^2/\partial x_2^2)$ and $c^*_{44} = c_{44} + \sigma_y^{(0)}$.

Now, it is clear that Eqs. (15) and Eqs.(16) contain only three constants, i.e., c_{44}, e_{15} and ε_{11} due to the symmetric axis of the hexagonal system of the piezoelectric region which is perpendicular to the $x_1 - x_2$ plane. Let

φ_0 denotes the electrical potential in the air which satisfies the following Laplace equation:

$$\nabla^2\varphi_0(x_1, x_2) = 0. \tag{17}$$

It is clear that the propagation of B-G wave must satisfy and content the boundary conditions and the continuity conditions over the interface between the two composites.

5 Solution of the Problem

5.1 Solution for the Electric Open Condition

The solutions of Eq. (11) may be considered as follows:

$$\{w_1(x_1, x_2, t), \varphi_1(x_1, x_2, t)\}$$
$$= \{W_1(x_1), \phi_1(x_1)\}\exp[ik(x_2 - ct)], \tag{18}$$

where $W_1(x_1)$ and $\Phi_1(x_1)$ are the unknown functions. Substitution of Eq. (18) into (15) presents the following:

$$c^*_{44}[W_1''(x_1) - k^2 W_1(x_1)] - \sigma_y^{(0)} W_1''(x_1) +$$
$$\rho[k^2c^2]W_1(x_1) + e_{15}[\phi_1''(x_1) - k^2\phi_1(x_1)] = 0,$$
$$e_{15}[W_1''(x_1) - k^2 W_1(x_1)] - \varepsilon_{11}[\phi_1''(x_1) - k^2\phi_1(x_1)]$$
$$= 0. \tag{19}$$

From Eq. (20)$_b$, one may obtain:

$$[\phi_1''(x_1) - k^2\phi_1(x_1)] = \frac{e_{15}}{\varepsilon_{11}}[W_1''(x_1) - k^2 W_1(x_1)]. \tag{20}$$

Substitution of Eq. (20) into (19)$_a$ gives

$$W_1''(x_1) - k^2\left[1 + \left(\frac{\varepsilon_{11}\sigma_y^{(0)} - \rho\varepsilon_{11}c^2}{c_{44}\varepsilon_{11} + e_{15}^2}\right)\right]W_1(x_1) = 0. \tag{21}$$

Where $q = \sqrt{1 + \dfrac{\varepsilon_{11}\sigma_y^{(0)}}{c_{44}\varepsilon_{11} + e_{15}^2} - \dfrac{c^2}{c_{sh}}}$

And $c_{sh} = \sqrt{(c_{44}\varepsilon_{11} + e_{15}^2)/\rho\varepsilon_{11}}$

The solution of Eq. (21) is

$$W_1(x_1) = A_3 e^{-kqx_1} + A_4 e^{kqx_1}, \tag{22}$$

where c_{sh} defines the shear wave velocity in the piezoelectric structure. Equation (20) may be considered as differential equation of $\Phi_1(x_1)$.

It is noticeable that $\Phi_1^{(i)}(x_1) = (e_{15}/\varepsilon_{11})W_1(x_1)$ is a particular solution of Eq. (20). The homogeneous equation (20) has a general solution that can be written as follows

$$\Phi_1^{(h)}(x_1) = B_1 e^{-kx_1} + B_2 e^{kx_1}.$$

Therefore, the complete solution of $\Phi_1(x_1)$ may be considered as

$$\Phi_1^{(h)}(x_1) = B_1 e^{-kx_1} + B_2 e^{kx_1} + A_3\frac{e_{15}}{\varepsilon_{11}}e^{-kqx_1} + A_4\frac{e_{15}}{\varepsilon_{11}}e^{kqx_1}.$$

Substitution the relations $W_1(x_1)$ and $\Phi_1(x_1)$ into (18), one may obtain the following equation:

$$w_1(x_1, x_2, t) = (A_3 e^{-kqx_1} + A_4 e^{kqx_1})\exp[ik(x_2 - ct)],$$
$$\varphi_1(x_1, x_2, t) = (B_1 e^{-kx_1} + B_2 e^{kx_1} + \frac{e_{15}}{\varepsilon_{11}}A_3 e^{-kqx_1} + \frac{e_{15}}{\varepsilon_{11}}A_4 e^{kqx_1})\exp[ik(x_2 - ct)]. \tag{23}$$

Eq. (12) may be solved in a similar way. For $\varphi_2 = -(e_{15}/\varepsilon_{11})w_2$ in the substrate when $x_1 \to \infty$, $w_2 \to 0$ and $\varphi_2 \to 0$; therefore, Eq. (12) has the solution as

$$w_2(x_1, x_2, t) = A_3' e^{-kq_0x_1}\exp[ik(x_2 - ct)],$$
$$\varphi_2(x_1, x_2, t) = (B_1' e^{-kx_1} - \frac{e_{15}}{\varepsilon_{11}}A_3' e^{-kq_0x_1})\exp[ik(x_2 - ct)]. \tag{24}$$

where $q_0 = \sqrt{1 - (c^2/c_{sh}^2)}$. Thus, one may get the solution of Eq. (17) as:

$$\varphi_\circ(x_1, x_2, t) = A_\circ e^{kx_1}\exp[ik(x_2 - ct)]. \tag{25}$$

Substitution (23), (24) and (25) into (9), (10) and (11), provides the linear homogeneous algebraic relations of the arbitrary constants $A_3, A_4, B_1, B_2, A_3', B_1'$ and A_0. Then, the dispersion velocity equation for electrically open case at the free surface may be written as:

$$4\frac{q}{\mu^2} + [\frac{(\varepsilon_r+1)}{\mu^4}q\beta - \frac{(\varepsilon_r+2)}{\mu^2}q - \frac{\beta}{\mu^2} + 1]t^{q+1} + [\frac{(\varepsilon_r-1)}{\mu^2}q - 1]t^{q-1}$$
$$-[(1 + (\frac{\varepsilon_r+1}{\mu^2})q)(1 + \frac{\delta}{\mu^2})]t^{-q+1} + [1 + \frac{(\varepsilon_r-1)}{\mu^2}q]t^{-q-1} = 0, \tag{26}$$

Where we have use the following relations:

$$M_1 = -(\frac{c_{44}\varepsilon_{11} + e_{15}^2}{\varepsilon_{11}}),\ M_2 = \frac{e_{15}}{\varepsilon_{11}},$$
$$\delta = (q - q_0)/2,\ \beta = (q + q_0)/2,$$
$$\mu = e_{15}/\sqrt{c_{44}\varepsilon_{11} + e_{15}^2},\ t = e^{kh},\ ,\varepsilon_r = \varepsilon_{11}/\varepsilon_0$$

with ε_r is the relative dielectric constant corresponding to ε_{11}. Eq. (26) represents the equation of the dispersion velocity for the B–G wave in the layered piezoelectric ceramics for the electric open condition when the surface is free from traction. Eq. (26) reveals that the B–G wave in the layered structure is frequency dispersive (see [1]).

5.2 Solution for the Electric Shorted Condition

Based on the previous representation, for the layered piezoelectric half-space structure according to the electric shorted condition at the free surface, the solution may be given by solving Eqs. (15) and Eqs. (16) With the help of the relations (9), (11) and (12). The suitable dispersion velocity equation in this case may be given as:

$$4\frac{q}{\mu^2}+(\frac{q}{\mu^2}-1)(\frac{\beta}{\mu^2}-1)t^{q+1}-(\frac{q}{\mu^2}+1)t^{q-1}-$$
$$(\frac{q}{\mu^2}+1)(\frac{\delta}{\mu^2}+1)t^{-q+1}+(1-\frac{q}{\mu^2})t^{-q-1}=0. \quad (27)$$

The dispersion velocity is associated with influence of the initial stress, layer thickness, wavelength, elastic, dielectric and piezoelectric coefficients.

6 Electromechanical Coupling Factor

A very important factor for the B-G surface wave in many applications is the electrochemical coupling factor K^2 which is known as:

$$K^2 = 2\frac{c_0 - c_s}{c_0} \quad (28)$$

where c_s and c_o are the velocities the B-G wave in the electrically shorted and open conditions, respectively.

The factor of electromechanical coupling is a parameter which straightway connected with the qualification of a transducer in transforming electrical energy to mechanical energy or vice versa. Moreover, it is a significant material operator for building of acoustic surface sensors [9].

7 Numerical Calculations

The $LiNbO_3$ piezoelectric solid ceramic material having hexagonal symmetry (6mm class) is considered for the reason of numerical computation. All the materials data used in the calculation can be seen in [16]. To find the solution of Eqs. (26) and (27) numerically, the values of phase velocity of the B–G wave for the electrically open and electrically shorted cases may be acquired and denoted as $c_\circ$ and c_S, respectively. The change type in values of the propagation velocity of the B–G wave along with the thickness of the layer in the layered structure is illustrated in Figs. 2, 3, 4 and 5, respectively. The important results are defined according to the reality of the graphs as follows:

Figure 2 illustrates the dispersion velocity c for the electrically open condition versus the penetration depth m($= hl\lambda$) for different values of initial stress. Figure (2) illustrates in the period (0.01-0.18) that the phase velocity c dramatically decreases by growing m. Then, it slightly increases in the period (0.19- 0.5). After that, it becomes almost constant. In addition, it increases with the rise of the value of initial stress, especially after the period (0, 0.1).

Figure 3 addresses velocity of the dispersion c for the electric short case versus m for different values of initial Stress. In this case, the dispersion velocity is lower than in the previous case and has a similar behavior with the change of m. It is also affected by the increase of the initial stress as a function of m.

Figure 4 involves the comparison between velocity of the dispersion c for the electric open and short cases versus m for a fixed initial stress. It is confirmed that velocity of the dispersion for the electric open case is greater that of the electric short case.

Figure 5 presents the electromechanical coupling factor K^2 versus m for various values of initial stress. It may be observed that the electromechanical correlation coefficient as a function of m decreases very rapidly in the period (0.01-0.16). Then it gradually increases to reach its maximum value (0.075). After that, it goes down very slowly. Furthermore, the initial stress is obvious in the various curves. It is also noted that there is a significant inverse proportionality between the parameters of electromechanical coupling as a function of initial stress and *m*.

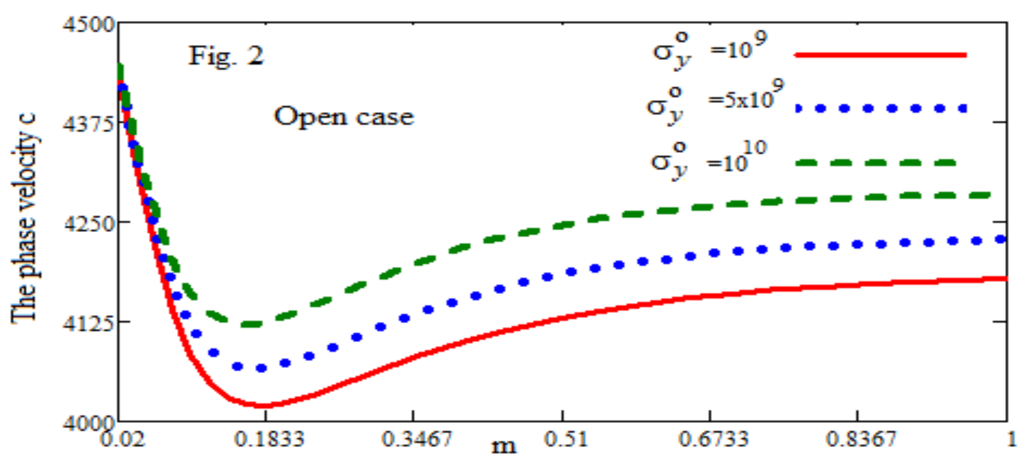

Fig.2: The dispersion velocity c versus m for various values of initial stress for the electrical open case.

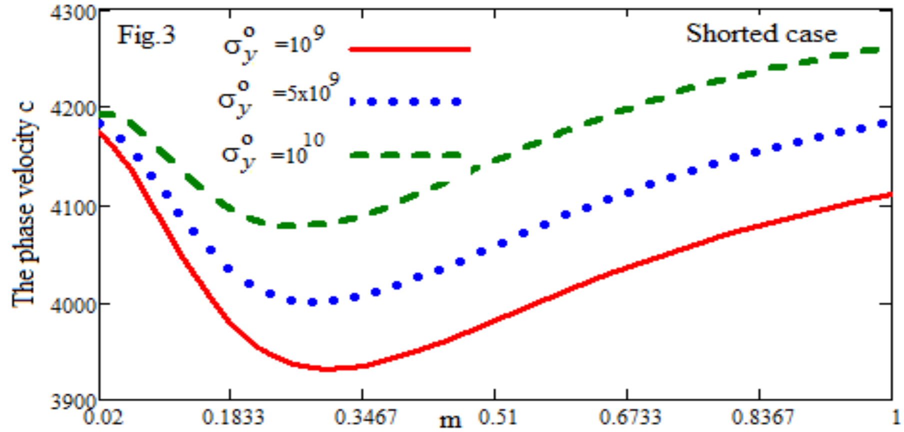

Fig. 3: The dispersion velocity c versus m for various values of initial stress for the electrical shorted case.

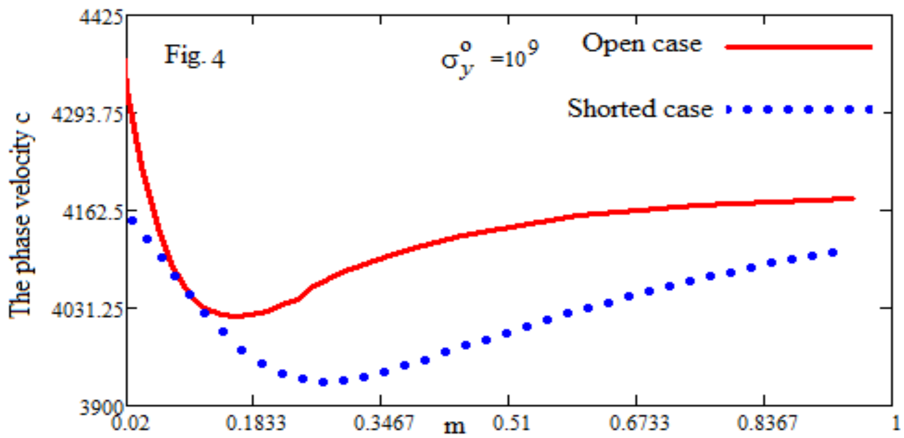

Fig.4: Comparison for the dispersion velocity c versus m for a fixed initial stress.

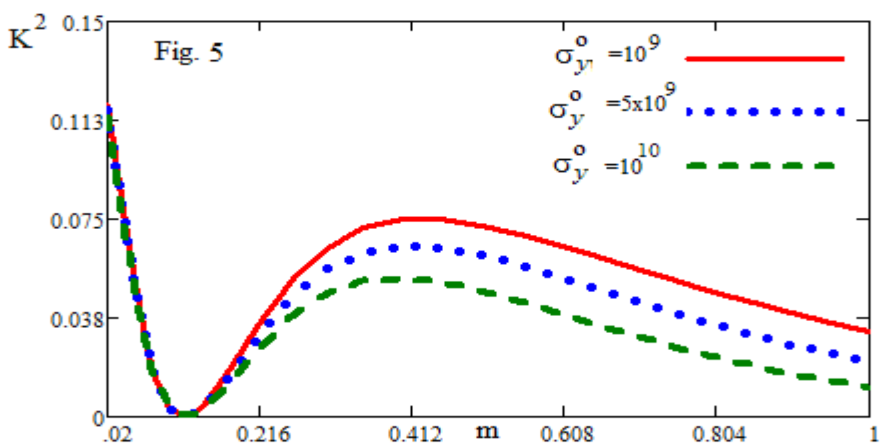

Fig.5: The electromechanical coupling factor K^2 versus m for various values of initial stress.

8 Conclusions

In this research, the computations were achieved to clarify the effect of the initial stress on the dispersion velocities and the electromechanical coupling parameter for Bleustein-Gulyaev surface waves for a piezoelectric structure. The above-mentioned results confirm that the initial stress, the mechanical and electrical conditions on the boundaries have substantial influences

on the dispersion velocities in the composite structure. One may observe that the influence of initial stress on the dispersion relation and electromechanical coupling coefficients is negligible because $\sigma_y^o < 10^8$ Pa. However, the dispersion velocity reduces with the rise of initial stress as $\sigma_y^o > 10^8$ Pa. Furthermore, the significant outcomes of this work not only detecting the complicated behaviour of electromechanical coupling of piezoelectric layered composites under the influence of initial stress but also providing a theoretical basis for shaping high standard electro-acoustic appliances for practical purposes in the microwave devices.

Reference

[1] J.L. Bleustein, A new surface wave in piezoelectric materials, Appl. Phys. Lett., **13**, 412–413 (1968).

[2] Y.V. Gulyaev, Electroacoustic surface waves in solids, Sov. Phys. JETP., 9, 37 (1969).

[3] S.F. Li, The electromagneto-acoustic surface wave in a piezoelectric medium: the Bleustein-Gulyaev mode, J. Appl. Phys., **80**, 5264–5269 (1996).

[4] D. Royer and E. Dieulesaint, Elastic waves in solids I: Free and guided propagation, Springer-Verlag, Berlin, Germany (2000).

[5] M. Ganguly, Propagation of Bleustein-Gulyaev waves in a non-homogeneous piezoelectric half-space of 6 mm hexagonal symmetry, Czechoslovak Journal of Physics B., **34(7)**, 645-654 (1984).

[6] L.P. Zinchuk, A.N. Podlipenets and N.A. Shulga, Construction of dispersion equation for electro-elastic shear wave in layered periodic media, Sov. Appl. Mech., **27**, 84–93 (1990).

[7] L.P. Zinchuk and A.N. Podlipenets, Vibrational modes of a surface shear wave propagating in a regularly layered electro-elastic half-space, Sov. Appl. Mech., **27** 775–779 (1992).

[8] F. Jin., Z. Wang and T. Wang, The Bleustein- Gulyaev (B-G) wave in a piezoelectric layered half-space, International Journal of Engineering Science., **39(1)** 1271-1285 (2001).

[9] X. Cao, F. Jin, Z. Wang and T.J. Lu, Bleustein-Gulyaev waves in a functionally graded piezoelectric material layered structure, Science China Physics Mechanics and Astronomy., **52,** 613- (2009).

[10] Z. Qian F. Jin, P. Li and S. Hirose, Bleustein–Gulyaev waves in 6mm piezoelectric materials loaded with a viscous liquid layer of finite thickness., International Journal of Solids and Structures.,**47 (25-26)** , 3513-3518 (2010).

[11] W. Chen, Surface effect on Bleustein-Gulyaev wave in a piezoelectric half-space, Theoretical and Applied Mechanics Letters 1(4) Solid Mechanics., 410011-410014 (2011).

[12] M. Rousseau and G.A. Maugin, Quasi-particles associated with Bleustein–Gulyaev SAWs: Perturbations by elastic nonlinearities, International Journal of Non-Linear Mechanics., **47(1)**, 67–71 (2012).

[13] P. Li, F. Jin and Z Qian Z, Propagation of the Bleustein–Gulyaev waves in a functionally graded transversely isotropic electro-magneto-elastic half-space, European Journal of Mechanics - A/Solids., **37**, 17–23(2013).

[14] J. Yang, Special topics in the theory of piezoelectricity, Springer (2009).

[15] A.N. Abd-alla, Nonlinear constitutive equations for thermo-electroelastic materials, Mechanics Research Communications., **24(3)**, 335-346 (1999).

[16] A.N. Abd-alla and N. Asker, Numerical simulations for the dispersion velocities and the electromechanical coupling factor of the Bleustein-Gulyaev waves in some piezoelectric smart materials, Mathematics and Mechanics of Solids., **21(5)**, 539 –551(2016).

[17] H. Altenbach, V.A. Eremeyev, L.P. Lebedev and L.A. Rendon, Acceleration waves and ellipticity in thermoelastic micropolar media, Archive of Applied Mechanics., **80(3)**, 217-227, (2010).

[18] I.A. Abbas and A.N. Abd-alla, A study of generalized themoelastic interaction in an infinite fibre-reinforced anisotropic plate containing a circular hole, Acta Physica Polonica A., **119(6)**, 814 – 818 (2011).

[19] I.A. Abbas, A.N. Abd-alla and I.A. Othman, Generalized magneto-thermo-elasticity in a fiber-reinforced anisotropic half-space, Int. J. Thermophsics., **32(5)**, 1071 – 1085(2011).

[20] I.A. Abbas, A.N. Abd-alla, A.F. Alzahrani and M. Spagnuolo, Wave propagation in a generalized eigenvalue approach, Journal of Thermal Stresses., **39 (11)**, 1367 – 1377(2016).

[21] A.N. Abd-alla, N.F. Hasbullah and H.M. Hossen, The frequency equations for shear horizontal waves in semiconductor/piezoelectric structures under the influence of initial stress, Journal of computational and Theoretical Nanoscience ., **13,** 6475-648, (2016)

[22] A.N. Abd-alla, A. Raizah and L. Placidi, The influence of hydrostatic stress on the frequency equation of flexural waves in a magnetoelastic transversely isotropic circular cylinder, Zeitschrift fur Angewandte Mathematik., **96(1),** 53 – 66(2016)

[23] A.N. Abd-alla, F. Alshaikh, I. Giorgio and A. Della Corte, A mathematical model for longitudinal wave propagation in a magnetoelastic hollow circular cylinder of anisotropic material under the influence of initial hydrostatic stress, Mathematics and Mechanics

of Solids., **21(1)**, 104 – 118 (2016).

[24] A.N. Abd-alla, F. Alshaikh, I. Mechai. and I.A. Abass, Influence of initial stresses and piezoelectric constants on the propagation bulk acoustic waves in an anisotropic smart material (Aluminum Nitrite), Journal of computational and Theoretical Nanoscience., **13**, 6488-6494 (2016).

[25] S.M. Abo-Dahab, A. Jahangir and A.N. Abd-alla, Reflection of plane waves in thermoelastic microstructured materials under the influence of gravitation, Continuum Mechanics and Thermo-dynamics, 1-13, (2018), https://doi.org/10.1007/s00161-018-0739-2.

[26] A.N. Abdalla, M.I.A. Othman and N.F. Hasbullah, Shear horizontal waves in composite materials: Behavior under rotation and initial stress, Mathematics and Mechanics of Solid., **24(1)** 85-97, 2019.