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Architecting a Fully Fuzzy Information Model for Multi-Level Quadratically Constrained Quadratic Programming Problem

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Abstract: An effective model based on the bound and decomposition method and the separable programming method is proposed in this paper for solving Fully Fuzzy Multi-Level Quadratically Constrained Quadratic Programming (FFMLQCQP) problem, where the objective function and the constraints are quadratic, also all the coefficients and variables of both objective functions and constraints are described fuzzily as fuzzy numbers. The bound and decomposition method is recommended to decompose the given (FFMLQCQP) problem into series of crisp Quadratically-Constrained Quadratic Programming (QCQP) problems with bounded variable constraints for each level. Each (QCQP) problem is then solved independently by utilizing the separable programming method, which replaces the quadratic separable functions with linear functions. At last, the fuzzy optimal solution to the given (FFMLQCQP) problem is obtained. The effectiveness of the proposed model is illustrated through an illustrative numerical example.

Keywords: fully fuzzy programming, multi-level programming, quadratic programming, bound and decomposition method, separable programming method

1 Introduction

In most real-world circumstances, the optimization problems includes a lot of parameters whose values are allocated by decision makers. However, those values are frequently imprecisely or ambiguously known to the decision makers. With this observation in mind, it would be more appropriate to represent these parameters as fuzzy number and dealing with them using the concepts of fuzzy set theory, which offers the possibility to construct decision models with vague data. [1,2,3] A fuzzy number is a convex fuzzy subset of the arrangement of real number in the sense that it does not allude to one single value but rather to a connected set of values. The fuzzy programming problems in which all the parameters and the variables of both objective functions and constraints are represented by fuzzy numbers are called Fully Fuzzy Programming (FFP) problems. One of most common (FFP) problems is the model in which all fuzzy parameters are described by triangle numbers. [4,5] Multi-Level Programming (MLP) problems have recently progressively appeared in decentralized management circumstances and have become exceedingly complicated, particularly with the improvement of economic integration and in the age of big data. Multi-level decision making process is based on two key issues; firstly,architecting a multi-level decision model to depict a hierarchical decision making process. For that, the bi-level and tri-level decision making models have been presented. Secondly, obtaining an optimal solution to the decision model. For that, multi-level decision-making techniques, which are closely related to the economic problem of Stackelberg [6] in the field of game theory, have been developed to perceive compromises between the decision makers in a hierarchical organization and make their individual decisions in sequence for optimizing their objectives.[7,8] In many optimization problems, the objective function is non-linear function, or one or more constraints have non-linear relationship or both. This kind of problems is known as a Non-Linear Programming (NLP) problem. Quadratic Programming (QP) problem is one of the

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essential (NLP) problems of both theoretical importance and application interests. Quadratically-Constrained Quadratic Programming (QCQP) problem generalizes the (QP) problem in that some or all the terms in the constraints are quadratic, which include the square of a variable or the product of two variables, instead of linear, in addition to the quadratic objective function. [9,10,11] Kumar et al. in [12] proposed a technique to locate the fuzzy optimal solution of Fully Fuzzy Linear Programming (FFLP) problems by representing all parameters as triangular fuzzy numbers. After that, the ranking function [13] was utilized to change the (FFLP) problem into proportional crisp linear programming problem for solving the problem. Ezzati et al. [14] handled an algorithm to solve the (FFLP) problem by converting it to a Multi-Objective Linear Programming (MOLP) problem with three objective functions. The lexicographic strategy was used to locate a lexicographic optimal solution of (MOLP) problem. Ren developed in [3] a method to deal with the fully fuzzy bi-level linear programming problem by applying interval programming method. The membership grade of fuzzy coefficients and variables was disintegrated into a limited number of alpha-level sets. Then, the fully fuzzy bi-level linear programming problem was transformed into an interval bi-level linear programming problem for every alpha-level set. The acquired interval bi-level linear programming problem was converted into two deterministic sub-problems which are related to the lower and upper bounds of its upper level objective function. Emam et al. in [15] tackled a solution for multi-level large scale quadratic programming problem with stochastic parameters in the objective functions. Taylor's series was solidified with the decomposition algorithm to transform the quadratic into linear objective function. In [16] Loganathan and Lalitha presented a fuzzy multi-objective non-linear programming problem. Every one of the coefficients of the non-linear multi-objective functions and the constraints were fuzzy numbers. Interval arithmetic based on Alpha-cut was utilized to solve the non-linear programming problem. In [17] Youness et al. proposed an algorithm to comprehend Bi-Level Multi-Objective Fractional Integer Programming Problem (BLMOFIPP) including fuzzy numbers in the right-hand side of the constraints. The fuzzy numbers was described by triangle fuzzy membership functions. The algorithm consolidated the strategy of Taylor series with the Kuhn Tucker conditions to solve Fuzzy (BLMOFIPP). In the majority of the previously mentioned strategies, the (FFP) problem is firstly transformed into approximated single crisp programming problem and afterward the acquired crisp problem is solved to locate the optimal solution of the problem. Therefore, the obtained optimal solution is approximate and not accurate, which is not a reliable solution for the decision maker. In this paper, we address the (FFMLQCQP) problem, where the objective function and the constraints are quadratic, also all the coefficients and variables of both objective functions and constraints are fuzzy numbers. The outline of this paper is as follows: In section 2 some important preliminaries are introduced. Section 3 formulates the given (FFMLQCQP) problem. Section 4 presents the proposed model for solving the given (FFMLQCQP) problem. A procedure for the proposed model is suggested in section 5. In section 6, we show the efficiency of the proposed model through an illustrative numerical example. Finally, section 7 finalizes the paper with its conclusion.

2 Preliminaries

Some principal definitions and notions related to the fuzzy set theory and the basic arithmetic operations on fuzzy numbers, which will be used in this paper, are exhibited.

2.1 Basic Definitions [2]

Definition 1. a fuzzy set $\widetilde{A}$ in universe of discourse X ($\widetilde{A} \subset X$) is directly specified by the membership function $\mu_{\widetilde{A}}(x)$ or indirectly by a set of ordered pairs $\left(x,\mu_{\widetilde{A}}(x)\right)$, where $\mu_{\widetilde{A}}(x)$ represents the value of the grade of membership of x in $\widetilde{A}$, as follows:

$$\forall\, x\in X \quad | \quad \widetilde{A}=\left\{\left(x,\mu_{\widetilde{A}}(x)\right)\right\}. \tag{1}$$

Definition 2. A fuzzy set $\widetilde{A}$ is convex if any point located between two other points has a membership degree higher than the minimum membership degree of these two points.

$$\forall\, x_1,x_2\in X,\lambda \in [0,1]\,|\mu_{\widetilde{A}}(\lambda x_1+(1-\lambda)x_2)\geq \quad \min(\mu_{\widetilde{A}}(x_1),\mu_{\widetilde{A}}(x_2)). \tag{2}$$

Definition 3. A fuzzy set $\widetilde{A}$ is said to be normalized if its height (Hgt $(\widetilde{A})$) is unity, i.e., at least one element of X has a membership degree equal to one, as follows:

$$\forall\, x\in X|\mu_{\widetilde{A}}(x)=\text{Hgt}\left(\widetilde{A}\right)=1. \tag{3}$$

Definition 4. A fuzzy number $\widetilde{A}$ is a convex fuzzy set which has a normalized and continuous membership function. The membership function of a fuzzy number $\widetilde{A}$ has the following properties:

1. $\mu_{\widetilde{A}}(x)$ is upper semi-continuous.
2. $\mu_{\widetilde{A}}(x)=0$ outside of the interval [c, d].
3. There are real numbers a and b, such that $c\leq a\leq b\leq d$ and:
   (a) $\mu_{\widetilde{A}}(x)$ is monotone increasing on the interval [c, a].
   (b) $\mu_{\widetilde{A}}(x)$ is monotone decreasing on the interval [b, d].
   (c) $\mu_{\widetilde{A}}(x)$= 1 for each $x\in$ [a, b].

Definition 5. A fuzzy set $\widetilde{A}$ over X, is called triangular fuzzy number with a unique maximizing point a, left width $\alpha > 0$ and right width $\beta > 0$, if its membership function has the following form:

$$\mu_{\widetilde{A}}(x) = \left\{ \begin{array}{cc} 1-\frac{a-x}{\alpha} & \text{if } a-\alpha \le x < a \\ 1 & \text{if } x = a \\ 1-\frac{x-a}{\beta} & \text{if } a < x \le a+\beta \\ 0 & \text{otherwise} \end{array} \right\}. \quad (4)$$

2.2 *Fuzzy Arithmetic Operations [18]*

Let $\widetilde{A}=(a_1,a_2,a_3)$ and $\widetilde{B}=(b_1,b_2,b_3)$ are two triangular fuzzy numbers, and k is any real number, then the arithmetic operations between these two triangular fuzzy numbers are presented as follows:

Definition 6. $\widetilde{A}$ and $\widetilde{B}$ are equal if and only if $a_1 = b_1$, $a_2 = b_2$ and $a_3 = b_3$.

Definition 7.
$k\widetilde{A}= k(a_1,a_2,a_3)=(ka_1,ka_2,ka_3)$, for $k \ge 0$.

Definition 8.
$k\widetilde{A}= k(a_1,a_2,a_3)=(ka_3,ka_2,ka_1)$, for $k<0$.

Definition 9.
$\widetilde{A}\oplus\widetilde{B}=(a_1,a_2,a_3)\oplus(b_1,b_2,b_3)=(a_1+b_1,a_2+b_2,a_3+b_3)$.

Definition 10.
$\widetilde{A}\ominus\widetilde{B}=(a_1,a_2,a_3)\ominus(b_1,b_2,b_3)=(a_1-b_3,a_2-b_2,a_3-b_1)$.

Definition 11.

$$\widetilde{A}\otimes\widetilde{B}=\left\{ \begin{array}{cc} (a_1b_1,a_2b_2,a_3b_3) & \text{if } a_1 \ge 0 \\ (a_1b_3,a_2b_2,a_3b_3) & \text{if } a_1 < 0, a_3 \ge 0 \\ (a_1b_3,a_2b_2,a_3b_1) & \text{if } a_3 < 0 \end{array} \right\}.$$

3 Fully Fuzzy Multi-Level Quadratically Constrained Quadratic Programming (FFMLQCQP) Problem

Let $\widetilde{F}_i$: $R^m \to R$, (i = 1, 2, ..., n) are the fully fuzzy first level, second level and n^{th}level objective functions, respectively. $\widetilde{x}$ are fuzzy decision variables indicating the control of each level decision maker. So, the first level decision maker (FLDM) has control over the variable $\widetilde{x}_1$, the second level decision maker (SLDM) has control over the variable $\widetilde{x}_2$, and so on. $(\widetilde{C}_{ij},\widetilde{A}_{hj})$ where $(i = 1, 2, \ldots, n)$, $(j=1, 2, \ldots, m)$, $(h=1, 2, \ldots, o)$, are fuzzy real numbers, which describing the coefficients of the linear terms. $(\widetilde{L}_{ikj},\widetilde{Q}_{hkj})$ where $(i = 1, 2, \ldots, n)$, $(k=1, 2, \ldots, m)$, $(j=1, 2, \ldots, m)$, $(h=1, 2, \ldots, o)$,are fuzzy real matrices, which describe the coefficients of the quadratic terms. G is the quadratically constraint set where, $\widetilde{b}_h$ $(h=1, 2, \ldots, o)$ is a fuzzy vector. Accordingly, the formulation of the (FFMLQCQP) problem can be considered as follows:

[FLDM]

$$\underset{\widetilde{x}_1}{\text{Max}} \quad \widetilde{F}_1(\widetilde{x})=\sum_{j=1}^{m}\widetilde{C}_{1j}\otimes\widetilde{x}_j \bigoplus \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}\widetilde{x}_k\otimes\widetilde{L}_{1kj}\otimes\widetilde{x}_j, \quad (5.a)$$

where $\widetilde{x}_2,\widetilde{x}_3,\ldots,\widetilde{x}_m$ solves;

[SLDM]

$$\underset{\widetilde{x}_2}{\text{Max}} \quad \widetilde{F}_2(\widetilde{x})=\sum_{j=1}^{m}\widetilde{C}_{2j}\otimes\widetilde{x}_j \bigoplus \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}\widetilde{x}_k\otimes\widetilde{L}_{2kj}\otimes\widetilde{x}_j, \quad (5.b)$$

where $\widetilde{x}_3, \ldots, \widetilde{x}_m$ solves;

[nth LDM]

$$\underset{\widetilde{x}_n}{\text{Max}} \quad \widetilde{F}_n(\widetilde{x})=\sum_{j=1}^{m}\widetilde{C}_{nj}\otimes\widetilde{x}_j \bigoplus \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}\widetilde{x}_k\otimes\widetilde{L}_{nkj}\otimes\widetilde{x}_j, \quad (5.c)$$

where $\widetilde{x}_{n+1}, \ldots, \widetilde{x}_m$ solves;

Subject to: $\widetilde{x}_j \in G \quad (j=1, 2, \ldots, m)$, (5.d)

Where (QCQP)

$$mathrmG=\sum_{j=1}^{m}\widetilde{A}_{hj\otimes\widetilde{x}_j}\oplus\tfrac{1}{2}\textstyle\sum_{j=1}^{m}\sum_{k=1}^{m}\widetilde{x}_k\otimes\widetilde{Q}_{hkj}\otimes\widetilde{x}_j\le\widetilde{b}_h$$

$h = 1, 2, \ldots, o$

$\widetilde{x}_j \ge 0 \ (j=1, 2, \ldots, m). \}$

Definition 12. If $\widetilde{x}^* \in R^m$ is a feasible solution of the (FFMLQCQP) problem (5.a) - (5.d); no other feasible solution $\widetilde{x}_j \in G$ exists, where $(j=1, 2, \ldots, m)$, such that $\widetilde{F}_i(\widetilde{x}^*) \le \widetilde{F}_i(\widetilde{x})$; so $\widetilde{x}^*$ is the optimal solution for the (FFMLQCQP) problem.

4 A Proposed Model for Finding the Fuzzy Optimal Solution of (FFMLQCQP) Problem

In what follows, an effective model is proposed for overcoming the intricacy of the (FFMLQCQP) problem using the bound and decomposition method [19,20], and afterward locating the fuzzy optimal solution for the given problem using the separable programming method [21,22].

4.1 Bound and Decomposition Method

In the proposed model, this method [19,20] is used to decompose the (FFMLQCQP) problem into three crisp (QCQP) problems with bounded variable constraints.

Firstly, all the fuzzy parameters and the fuzzy variables of both objective functions and constraints for each level $(\widetilde{F}_I, \widetilde{x}_j, \widetilde{x}_k, \widetilde{C}_{ij}, \widetilde{A}_{hj}, \widetilde{L}_{ikj}, \widetilde{Q}_{hkj}$ and $\widetilde{b}_h)$ are represented by the following triangular fuzzy numbers $(Z_{1i}, Z_{2i}, Z_{3i}), (x_j, y_j, t_j), (x_k, y_k, t_k), \left(\varsigma^1_{ij}, \varsigma^2_{ij}, \varsigma^3_{ij}\right), \left(\gamma^1_{hj}, \gamma^2_{hj}, \gamma^3_{hj}\right), \left(\zeta^1_{ikj}, \zeta^2_{ikj}, \zeta^3_{ikj}\right), \left(\varphi^1_{hkj}, \varphi^2_{hkj}, \varphi^3_{hkj}\right)$ and $\left(\beta^1_h, \beta^2_h, \beta^3_h\right)$ Hence, the given (FFMLQCQP) problem can be reformulated as follows:

[ith LDM] where (i = 1, 2, . . . , n)

$$Max\,(Z_{1i}, Z_{2i}, Z_{3i})\,(x, y, t) = (x_i, y_i, t_i)$$

$$\sum_{j=1}^{m}\left(\varsigma^1_{ij}, \varsigma^2_{ij}, \varsigma^3_{ij}\right)\otimes(x_j, y_j, t_j)\oplus\tfrac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}(x_k, y_k, t_k)\otimes\left(\zeta^1_{ikj}, \zeta^2_{ikj}, \zeta^3_{ikj}\right)\otimes(x_j, y_j, t_j), \quad (6.a)$$

where (x_j, y_j, t_j) solves, $(j=i+1, \ldots, m)$;

Subject to:

$$(x_j, y_j, t_j) \in G \quad (j = 1, 2, \ldots, m), \quad (6.b)$$

Where (QCQP)

$$G=\{\sum_{j=1}^{m}\left(\gamma^1_{hj}, \gamma^2_{hj}, \gamma^3_{hj}\right)\otimes(x_j, y_j, t_j)\bigoplus\frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}(x_k, y_k, t_k)\otimes\left(\varphi^1_{hkj}, \varphi^2_{hkj}, \varphi^3_{hkj}\right)\otimes(x_j, y_j, t_j)\le\left(\beta^1_h, \beta^2_h, \beta^3_h\right),$$

$h = 1, 2, \ldots, o$

$$(x_j, y_j, t_j) \ge 0, \ (j = 1, 2, \ldots, m). \}$$

Since (x_j, y_j, t_j) is a triangular fuzzy number, then $x_j \le y_j \le t_j$ $(j = 1, 2, \ldots, m)$. The above relation is called bounded variable constraints. From the definitions of the fuzzy arithmetic operations, which are presented in sub-section 2.2, the above problem (6.a), (6.b) can be converted into three crisp (QCQP) problems with bounded variable constraints for each level, called Middle, Upper and Lower Multi-Level (QCQP) ((MMLQCQP), (UMLQCQP) and (LMLQCQP) respectively), as follows:

[ith LDM] where (i = 1, 2, . . . , n)

(MMLQCQP) Problem:

$$\underset{y_i}{\text{Max}}\ Z_{2i}(y) = \sum_{j=1}^{m}\varsigma^2_{ij}\,y_j + \tfrac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}y_k\,\zeta^2_{ikj}\,y_j, \quad (7.a)$$

where y_j solves, $(j=i+1, \ldots, m)$;

Subject to:

$$y_j \in G_M \quad (j = 1, 2, \ldots, m), \quad (7.b)$$

Where (QCQP)

$$G_M = \{\ \sum_{j=1}^{m}\gamma^2_{hj}\,y_j + \tfrac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}y_k\,\varphi^2_{hkj}\,y_j \le \beta^2_h,$$

$$y_j \ge 0 \quad (j = 1, 2, \ldots, m). \}$$

(UMLQCQP) Problem:

$$\underset{t_i}{\text{Max}}\ Z_{3i}(t) = \sum_{j=1}^{m}\varsigma^3_{ij}\,t_j + \tfrac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}t_k\,\zeta^3_{ikj}\,t_j, \quad (8.a)$$

where t_j solves, $(j=i+1, \ldots, m)$;

Subject to:

$$t_j \in G_U \quad (j = 1, 2, \ldots, m), \quad (8.b)$$

Where (QCQP)

$$G_U = \{\ \sum_{j=1}^{m}\gamma^3_{hj}\,t_j + \tfrac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}t_k\,\varphi^3_{hkj}\,t_j \le \beta^3_h,$$

$$\sum_{j=1}^{m}\varsigma^3_{ij}\,t_j + \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}t_k\,\zeta^3_{ikj}\,t_j \ge {Z_{2i}}^*,$$

$$t_j \ge y_j \quad (j = 1, 2, \ldots, m)$$

$$t_j \ge 0 \quad (j = 1, 2, \ldots, m). \}$$

LMLQCQP Problem:

$$\text{Max}\ Z_{1i}(x) = \sum_{j=1}^{m}\varsigma^1_{ij}\,x_j + \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}x_k\,\zeta^1_{ikj}\,x_j, \quad (9.a)$$

x_i where x_j solves $j = i + 1$

Subject to:

$$x_j \in G_L \quad (j = 1, 2, \ldots, m), \quad (9.b)$$

where **(QCQP)**

$$G_L = \{\ \sum_{j=1}^{m}\gamma^1_{hj}\,x_j + \tfrac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}x_k\,\varphi^1_{hkj}\,x_j \le \beta^1_h,$$

$$\sum_{j=1}^{m}\varsigma^1_{ij}\,x_j + \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}x_k\,\zeta^1_{ikj}\,x_j \le {Z_{2i}}^*,$$

$$x_j \le y_j \quad (j = 1, 2, \ldots, m),$$

$$x_j \ge 0 \quad (j = 1, 2, \ldots, m). \}$$

4.2 Separable Programming Method

Separable programming method [21,22] deals with (QCQP) problems in which both the quadratic objective functions and the quadratic constraints are separable functions. A separable function is a function where each term includes just a solitary variable. Some quadratic functions are not directly separable, but can be made separable by appropriate substitutions. Therefore, each quadratic objective function and quadratic constraint can be expressed as a combination of separable functions with individual variables. After that, each of these separable functions with individual variables can be approximated as nearly as possible to a linear function with a larger number of variables. At last, the optimal solution to the (QCQP) problem can be obtained.

Consider the constructed three crisp (QCQP) problems in sub-section 4.1, which are called (MMLQCQP) (7.a), (7.b), (UMLQCQP) (8.a), (8.b) and (LMLQCQP) (9.a), (9.b) problems. At that point, we express each quadratic objective function $(Z_{1i}(x),\ Z_{2i}(y) \text{ and } Z_{3i}(t))$ as a combination of separable functions with individual variables $(f_{1iw}(x_w),\ f_{2iw}(y_w) \text{ and } f_{3iw}(t_w)$,and each quadratic constraint $G(x,y,t)$ as a combination of separable constraints $(g^d_{1w}(x_w),\ g^d_{2w}(y_w) \text{ and } g^d_{3w}(t_w))$ where $(d = 1,\ 2,\ \ldots,\ D)$.

Assume that $(a^1_{isw},\ a^2_{isw} \text{ and } a^3_{isw})$ where $(s = 0,\ 1,\ 2,\ \ldots,\ S_w)$ is the breaking points of each separable function $(f_{1iw}(x_w),\ f_{2iw}(y_w) \text{ and } f_{3iw}(t_w))$. $(x_{sw},\ y_{sw} \text{ and } t_{sw})$ are new variables, which representing the increment of each variable $(x_w,\ y_w \text{ and } t_w)$ in the range $\left(\left(a^1_{i(s-1,w)}, a^1_{isw}\right), \left(a^2_{i(s-1,w)}, a^2_{isw}\right), \left(a^3_{i(s-1,w)}, a^3_{isw}\right)\right)$. $(\rho^1_{isw},\ \rho^2_{isw} \text{ and } \rho^3_{isw})$ is the slope of the line segment corresponding to the separable functions $(f_{1iw}(x_w),\ f_{2iw}(y_w) \text{ and } f_{3iw}(t_w))$ in the same range. $(\rho^{1d}_{sw},\ \rho^{2d}_{sw} \text{ and } \rho^{3d}_{sw})$ is the slope of the line segment corresponding to the separable constraints $(g^d_{1w}(x_w),\ g^d_{2w}(y_w) \text{ and } g^d_{3w}(t_w))$. As a result, the approximated linear forms of the constructed three crisp (QCQP) problems can be stated as: [i^{th} LDM] where $(i = 1, 2, \ldots, n)$

MMLQCQP Problem:

$$\underset{y_i}{\text{Max}}\quad Z_{2i}(y) = \sum_{w=1}^{\theta} f_{2iw}(y_w) \tag{10.a}$$

where y_j solves, $(j = i+1, \ldots, m)$;

Subject to:

$$\sum_{w=1}^{\theta} g^d_{2w}(y_w) \le \beta^2_h \quad (d = 1,\ 2,\ \ldots,\ D)\ (h = 1, 2, \ldots, o),$$

$$y_w \ge 0$$

$$(w = 1,\ 2,\ \ldots\theta) \tag{10.b}$$

Where:

$$f_{2iw}(y_w) = \sum_{s=1}^{S_w} \rho^2_{isw}\, y_{sw}$$

$$(w = 1,\ 2,\ \ldots,\ \theta) \tag{10.c}$$

$$\rho^2_{isw} = \frac{f_{2iw}(a^2_{isw}) - f_{2iw}\left(a^2_{i(s-1,w)}\right)}{a^2_{isw} - a^2_{i(s-1,w)}}$$

$$(s = 0,\ 1,\ 2,\ \ldots,\ S_w))$$

$$g^d_{2w}(y_w) = \sum_{s=1}^{S_w} \rho^{2d}_{sw}\, y_{sw} \qquad (d = 1,\ 2,\ \ldots,\ D)$$

$$\rho^{2d}_{sw} = \frac{g^d_{2w}(a^2_{isw}) - g^d_{2w}\left(a^2_{i(s-1,w)}\right)}{a^2_{isw} - a^2_{i(s-1,w)}}$$

$$(s = 0,\ 1,\ 2, \ldots,\ S_w))$$

$$0 \le y_{sw} \le a^2_{isw} - a^2_{i(s-1,w)}$$

$$y_w = \sum_{s=1}^{S_w} y_{sw}$$

UMLQCQP Problem:

$$\underset{t_i}{\text{Max}} Z_{3i}(t) = \sum_{w=1}^{\theta} f_{3iw}(t_w) \tag{11.a}$$

where t_j solves, $(j = i+1, \ldots, m)$;

Subject to:

$$\sum_{w=1}^{\theta} g^d_{3w}(t_w) \le \beta^3_h \quad (d = 1,\ 2,\ \ldots,\ D)\ (h = 1, 2, \ldots, o),$$

$$\sum_{w=1}^{\theta} f_{3iw}(t_w) \ge Z_{2i}{}^*,$$

$$t_w \ge y_w$$

$$(w = 1,\ 2,\ \ldots,\ \theta),$$

$$t_w \ge 0$$

(11.b)

Where:

$$f_{3iw}(t_w) = \sum_{s=1}^{S_w} \rho^3_{isw}\, t_{sw}$$

$$(w = 1,\ 2,\ \ldots,\ \theta)$$

$$\rho^3_{isw} = \frac{f_{3iw}\left(a^3_{isw}\right) - f_{3iw}\left(a^3_{i(s-1,w)}\right)}{a^3_{isw} - a^3_{i(s-1,w)}}$$

$$(s = 0,\ 1,\ 2,\ \ldots,\ S_w))$$

$$g^d_{3w}(t_w) = \sum_{s=1}^{S_w} \rho^{3d}_{sw}\, t_{sw} \qquad (d = 1,\ 2,\ \ldots,\ D)$$

$$\rho^{3d}_{sw} = \frac{g^d_{3w}\left(a^3_{isw}\right) - g^d_{3w}\left(a^3_{i(s-1,w)}\right)}{a^3_{isw} - a^3_{i(s-1,w)}}$$

$$(s = 0,\ 1,\ 2,\ldots,S_w)$$

$$0 \le t_{sw} \le a^3_{isw} - a^3_{i(s-1,w)}$$

$$t_w = \sum_{s=1}^{S_w} t_{sw}$$

$$y_w = \sum_{s=1}^{S_w} y_{sw}$$

(LMLQCQP) Problem:

$$\text{Max} \quad Z_{1i}(x) = \sum_{w=1}^{\theta} f_{1iw}(x_w),$$

$$x_i$$

(12.a)

where x_j solves, $(j=i+1,\ldots,m)$;

Subject to:

$$\sum_{w=1}^{\theta} g^d_{1w}(x_w) \le \beta^1_h \quad (d = 1,\ 2,\ \ldots,\ D)\ (h = 1,\ 2,\ \ldots,\ o),$$

$$\sum_{w=1}^{\theta} f_{1iw}(x_w) \le Z_{2i}{}^*,$$

$$x_w \le y_w \quad (w = 1,\ 2,\ \ldots,\ \theta),$$

$$x_w \ge 0.$$

Where:

$$f_{1iw}(x_w) = \sum_{s=1}^{S_w} \rho^1_{isw} x_{sw}$$

$$(w = 1,\ 2,\ldots,\theta)$$

$$\rho^1_{isw} = \frac{f_{1iw}\left(a^1_{isw}\right) - f_{1iw}\left(a^1_{i(s-1,w)}\right)}{a^1_{isw} - a^1_{i(s-1,w)}}$$

$$(s = 0,\ 1,\ 2,\ \ldots,\ S_w)$$

$$g^d_{1w}(x_w) = \sum_{s=1}^{S_w} \rho^{1d}_{sw}\, x_{sw} \qquad (d = 1,\ 2,\ \ldots,\ D)$$

$$\rho^{1d}_{sw} = \frac{g^d_{1w}\left(a^1_{isw}\right) - g^d_{1w}\left(a^1_{i(s-1,w)}\right)}{a^1_{isw} - a^1_{i(s-1,w)}}$$

$$(s = 0,\ 1,\ 2,\ \ldots)\, S_w)$$

$$0 \le x_{sw} \le a^1_{isw} - a^1_{i(s-1,w)}$$

$$x_w = \sum_{s=1}^{S_w} x_{sw}$$

$$y_w = \sum_{s=1}^{S_w} y_{sw}$$

We can conclude that, if the approximated linear forms of the constructed three crisp (QCQP) problems are solvable, then the given (FFMLQCQP) problem is solvable and the fuzzy optimal solution to the given problem can be obtained.

5 A Procedure for Finding the Fuzzy Optimal Solution of (FFMLQCQP) Problem

Step 1. Formulate the (FFMLQCQP) programming problem (Problem (5.a) - (5.d)).

Step 2. Set i=1, which indicating the i^{th} level decision maker (i^{th} LDM).

Step 3. If the i^{th} LDM obtains the optimal solution then go to Step 20, otherwise go to Step 4.

Step 4. Formulate the i^{th} LDM programming problem.

Step 5. All the fuzzy parameters and variables of both objective functions and constraints for i^{th} LDM are represented by the triangular fuzzy numbers, as problem (6.a), (6.b).

Step 6. Based on the fuzzy arithmetic operations and the bound and decomposition method [19, 20], the fully fuzzy (QCQP) problem for i^{th} LDM is decomposed into three crisp (QCQP) problems for i^{th} LDM, called Middle (7.a), (7.b), Upper (8.a), (8.b) and Lower (9.a), (9.b) i^{th} Level (QCQP) problems.

Step 7. Formulate the Middle i^{th} Level (QCQP) problem (Problem (7.a), (7.b)).

Step 8. Convert the non-separable quadratic objective functions and quadratic constraints into separable forms using appropriate arithmetic substitutions.

Step 9. Express the quadratic objective function and each quadratic constraint as a combination of separable functions with individual variables.

Step 10. Define the breaking points of each separable function estimated from the bounded variable constraints, and the increment of each individual variable based on the breaking points, which introduce new variables.

Step 11. Compute the slope of the line segment corresponding to each separable function.

Step 12. Each of these separable functions is approximated to a linear function with a combination of larger number of variables, as illustrated in problems (10.a) - (10.c), (11.a) - (11.c) and (12.a) - (12.c).

Step 13. Solve the resulting linear programming problem for obtaining the individual optimal solution of the decomposed i^{th} Level (QCQP) problem.

Step 14. If the individual optimal solution of the Upper i^{th} Level (QCQP) problem is obtained, then go to Step 17, otherwise go to Step 15.

Step 15. All variables in the objective function and constraints of the Upper i^{th} Level (QCQP) problem must satisfy the bounded variable constraints, as problem (8.a), (8.b).

Step 16. Formulate the Upper i^{th} Level (QCQP) problem, then go to Step 8.

Step 17. If the individual optimal solution of the Lower i^{th} Level (QCQP) problem is obtained, then go to Step 3, otherwise go to Step 18.

Step 18. All variables in the objective function and constraints of the Lower i^{th} Level (QCQP) problem must satisfy the bounded variable constraints, as problem (9.a), (9.b).

Step 19. Formulate the Lower i^{th} Level (QCQP) problem, then go to Step 8.

Step 20. If i=n, then go to Step 23, otherwise go to step 21.

Step 21. Set i=i+1.

Step 22. The i^{th} LDM defines his/her problem in point of view of the $(i\text{-}1)^{th}$ LDMs (upper level decision makers) by setting the controlled variables of the $(i\text{-}1)^{th}$ LDMs to the i^{th} LDM constraints., then go to Step 3.

Step 23. The fuzzy optimal solution of the (FFMLQCQP) programming problem is obtained.

Step 24. Stop.

6 Numerical Example

[FLDM]

$$\underset{\tilde{x}_1}{\text{Max}}\ \tilde{F}_1(\tilde{x}) = (7,10,15) \otimes \tilde{x}_1 \oplus (2,4,6) \otimes \tilde{x}_2 \oplus (3,5,9) \otimes \tilde{x}_3 \oplus (\tilde{x}_1,\tilde{x}_2) \otimes \begin{pmatrix} (8,12,17) & (2,6,7) \\ (1,1,3) & (1,2,4) \end{pmatrix} \otimes \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix}),$$

where $\tilde{x}_2, \tilde{x}_3$ solves;

[SLDM]

$$\underset{\tilde{x}_2}{\text{Max}}\ \tilde{F}_2(\tilde{x}) = (4,5,8) \otimes \tilde{x}_1 \oplus (12,16,19) \otimes \tilde{x}_2 \oplus (7,9,11) \otimes \tilde{x}_3 \oplus (\tilde{x}_2,\tilde{x}_3) \otimes \begin{pmatrix} (8,9,11) & (0,4,5) \\ (0,2,3) & (0,0,0) \end{pmatrix} \otimes \begin{pmatrix} \tilde{x}_2 \\ \tilde{x}_3 \end{pmatrix}),$$

where $\tilde{x}_3$ solves;

[TLDM]

$$\underset{\tilde{x}_3}{\text{Max}}\ \tilde{F}_3(\tilde{x}) = (7,8,14) \otimes \tilde{x}_1 \oplus (4,9,9) \otimes \tilde{x}_2 \oplus (13,18,20) \otimes \tilde{x}_3 \oplus (\tilde{x}_1,\tilde{x}_3) \otimes \begin{pmatrix} (1,2,6) & (0,1,3) \\ (3,4,4) & (9,10,14) \end{pmatrix} \otimes \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_3 \end{pmatrix}),$$

Subject to: (QCQP)

$$(1,3,6) \otimes \tilde{x}_1 \oplus (5,7,10) \otimes \tilde{x}_2 \oplus (2,5,9) \otimes \tilde{x}_3 \otimes \tilde{x}_3 \leq (35,60,85),$$

$$(2,10,12)\otimes\widetilde{x}_1\oplus(2,3,5)\otimes\widetilde{x}_2\oplus(6,6,11)\otimes\widetilde{x}_3 \le (10,18,39),$$

$$(3,4,7)\otimes\widetilde{x}_1\oplus(\widetilde{x}_1,\widetilde{x}_2)\otimes\begin{pmatrix}(5,6,7) & (0,2,4)\\(0,1,2) & (1,3,3)\end{pmatrix}\otimes\begin{pmatrix}\widetilde{x}_1\\\widetilde{x}_2\end{pmatrix}\le(40,70,115),$$

$$\widetilde{x}_1,\widetilde{x}_2,\widetilde{x}_3\ge 0.$$

1.FLDM

Assume $\widetilde{x}_1=(x_1,y_1,t_1)$, $\widetilde{x}_2=(x_2,y_2,t_2)$, $\widetilde{x}_3=(x_3,y_3,t_3)$ $\widetilde{F}_1=(Z_{11},Z_{21},Z_{31})$. Therefore, the Fully Fuzzy (QCQP) problem of the FLDM is reformulated as follows:

$$\text{Max }(Z_{11},Z_{21},Z_{31})(x,y,t)=((7,10,15)\otimes(x_1,y_1,t_1)\oplus(2,4,6)\otimes(x_2,y_2,t_2)\oplus(3,5,9)\otimes(x_3,y_3,t_3)\oplus((x_1,y_1,t_1),(x_2,y_2,t_2))\otimes\begin{pmatrix}(8,12,17) & (2,6,7)\\(1,1,3) & (1,2,4)\end{pmatrix}\otimes\begin{pmatrix}(x_1,y_1,t_1)\\(x_2,y_2,t_2)\end{pmatrix}),$$

where

x_2,x_3,y_2,y_3,t_2,t_3 solves;

Subject to: (QCQP)

$$(1,3,6)\otimes(x_1,y_1,t_1)\oplus(5,7,10)\otimes(x_2,y_2,t_2)\oplus(2,5,9)\otimes(x_3,y_3,t_3)\otimes(x_3,y_3,t_3)\le(35,60,85),$$

$$(2,10,12)\otimes(x_1,y_1,t_1)\oplus(2,3,5)\otimes(x_2,y_2,t_2)\oplus(6,6,11)\otimes(x_3,y_3,t_3)\le(10,18,39),$$

$$(3,4,7)\otimes(x_1,y_1,t_1)\oplus((x_1,y_1,t_1),(x_2,y_2,t_2))\otimes\begin{pmatrix}(5,6,7) & (0,2,4)\\(0,1,2) & (1,3,3)\end{pmatrix}\otimes\begin{pmatrix}(x_1,y_1,t_1)\\(x_2,y_2,t_2)\end{pmatrix}\le(40,70,115),$$

$$x_1,x_2,x_3,y_1,y_2,y_3,t_1,t_2,t_3\ge 0.$$

From the fuzzy arithmetic operations, which are presented in sub-section 2.2, and based on the bound and decomposition method [19,20], the Fully Fuzzy (QCQP) problem of the FLDM reduces to the following three crisp (QCQP) problems:

(LLQCQP)
$$\text{Max } Z_{11}(x)=\left(8x_1^2+3x_1x_2+7x_1+x_2^2+2x_2+3x_3\right)$$
$$x_1$$

(MLQCQP)
$$\text{Max } Z_{21}(y)=(12y_1^2+7y_1y_2+10y_1+2y_2^2+4y_2+5y_3)$$
$$y_1$$

(ULQCQP)
$$\text{Max } Z_{31}(t)=(17t_1^2+10t_1t_2+15t_1+4t_2^2+6t_2+9t_3)$$
$$t_1$$

where x_2,x_3,y_2,y_3,t_2,t_3 solves;

Subject to: (QCQP)

$$x_1+5x_2+2x_3^2\le 35,$$
$$2x_1+2x_2+6x_3\le 10,$$
$$5x_1^2+3x_1+x_2^2\le 40,$$
$$3y_1+7y_2+5y_3^2\le 60,$$
$$10y_1+3y_2+6y_3\le 18,$$
$$6y_1^2+3y_1y_2+4y_1+3y_2^2\le 70,$$
$$6t_1+10t_2+9t_3^2\le 85,$$
$$12t_1+5t_2+11t_3\le 39,$$
$$7t_1^2+6t_1t_2+7t_1+3t_2^2\le 115,$$

$$x_1,x_2,x_3,y_1,y_2,y_3,t_1,t_2,t_3\ge 0.$$

1.1. The (MLQCQP) problem of the FLDM

$$\text{Max } Z_{21}(y)=(12y_1^2+7y_1y_2+10y_1+2y_2^2+4y_2+5y_3)$$
$$y_1$$

where

y_2,y_3 solves;

Subject to:
(QCQP)

$$3y_1+7y_2+5y_3^2\le 60,$$
$$10y_1+3y_2+6y_3\le 18,$$
$$6y_1^2+3y_1y_2+4y_1+3y_2^2\le 70,$$
$$y_1,y_2,y_3\ge 0.$$

Now, the separable programming method [21,22] is applied to obtain the individual optimal solution. First, we need to convert the y_1y_2 term into separable form by using the following arithmetic substitution:

$$y_1y_2=\left[\left(\frac{y_1+y_2}{2}\right)^2-\left(\frac{y_1-y_2}{2}\right)^2\right].$$

Assume two new decision variables m_1, m_2, where: $m_1=\frac{y_1+y_2}{2}$, $m_2=\frac{y_1-y_2}{2}$.

So, the (MLQCQP) problem of the FLDM is rewritten as follows:

$$\underset{y_1}{\text{Max}}\ Z_{21}(y)=(12y_1^2+7m_1^2-7m_2^2+10y_1+2y_2^2+4y_2+5y_3)$$

where y_2, y_3 solves;

Subject to: (QCQP)

$$3y_1+7y_2+5y_3^2\leq 60,$$
$$10y_1+3y_2+6y_3\leq 18,$$
$$6y_1^2+3m_1^2-3m_2^2+4y_1+3y_2^2\leq 70,$$
$$\frac{1}{2}y_1+\frac{1}{2}y_2-m_1=0,$$
$$\frac{1}{2}y_1-\frac{1}{2}y_2-m_2=0,$$
$$y_1,y_2,y_3\geq 0.$$

The separable functions of this (MLQCQP) problem for the FLDM are:

$$f_1(y_1)=12y_1^2+10y_1,\ f_2(y_2)=2y_2^2+4y_2,\ f_3(y_3)=5y_3$$
$$,\ f_4(m_1)=7m_1^2,\ f_5(m_2)=-7m_2^2.$$
$$g_1^{\ 1}(y_1)=3y_1,\ g_1^{\ 2}(y_1)=10y_1,\ g_1^{\ 3}(y_1)=6y_1^2+4y_1$$
$$,\ g_1^{\ 4}(y_1)=\frac{1}{2}y_1\ ,\ g_1^{\ 5}(y_1)=-\frac{1}{2}y_1.$$
$$g_2^{\ 1}(y_2)=7y_2,\ g_2^{\ 2}(y_2)=3y_2,\ g_2^{\ 3}(y_2)=3y_2^2$$
$$g_2^4(y_2)=\frac{1}{2},\ g_2^{\ 5}(y_2)=-\frac{1}{2}y_2.$$
$$g_3^{\ 1}(y_3)=5y_3^2,\ g_3^{\ 2}(y_3)=6y_3.$$
$$g_4^{\ 1}(m_1)=3m_1^2,\ g_4^{\ 2}(m_1)=-m_1.$$
$$g_5^{\ 1}(m_2)=-3m_2^2,\ g_5^{\ 2}(m_2)=-m_2.$$

The ranges of the variables y_1,y_2,y_3,m_1,m_2 estimated from the constraints) are:

$0\leq y_1\leq 3.42$, $0\leq y_2\leq 4.83$, $0\leq y_3\leq 3.47$, $0\leq m_1\leq 4.125$

$-2.415\leq m_2\leq 1.71$. Let k_i is the breaking points of the separable functions, where:

$$k_1=4 \text{ for } y_1,\ k_2=5 \text{ for } y_2,\ k_3=4 \text{ for } y_3,\ k_4=5 \text{ for } m_1$$
$$k_5=5 \text{ for } m_2.$$

Based on the equations, which are illustrated in problem (10.c), the slopes of the line segment corresponding to the defined separable functions are computed. The complete resulting linear programming problem then becomes:

$$MaxZ_{21}(y)=(22y_{11}+46y_{21}+70y_{31}+87.04\,y_{41}+7y_{14}+21y_{24}+35y_{34}$$
$$+49y_{44}+56.875\,y_{54}+30.91\,y_{15}+21y_{25}+7y_{35}-7y_{45}-18.97\,y_{55}+6y_{12}+10y_{22}$$
$$+14y_{32}+18y_{42}+21.66y_{52}+5y_{13}+5y_{23}+5y_{33}+5y_{43}$$

Subject to:

$$3y_{11}+3y_{21}+3y_{31}+3y_{41}+7y_{12}+7y_{22}+7y_{32}+7y_{42}+7y_{52}$$
$$+5y_{13}+15y_{23}+25y_{33}+32.35\,y_{43}\leq 60,$$
$$10y_{11}+10y_{21}+10y_{31}+10y_{41}+3y_{12}+3y_{22}+3y_{32}+3y_{42}$$
$$+3y_{52}+6y_{13}+6y_{23}+6y_{33}+6y_{43}\leq 18,$$
$$10y_{11}+22y_{21}+34y_{31}+42.52\,y_{41}+3y_{14}+9y_{24}+15y_{34}$$
$$+21y_{44}+24.375\,y_{54}+13.246\,y_{15}+9y_{25}+3y_{35}-3y_{45}$$
$$-8.13\,y_{55}\ +3y_{12}+9y_{22}+15y_{32}+21y_{42}+26.49\,y_{52}\leq 70,$$
$$\frac{1}{2}y_{11}+\frac{1}{2}y_{21}+\frac{1}{2}y_{31}+\frac{1}{2}y_{41}+\frac{1}{2}y_{12}+\frac{1}{2}y_{22}+\frac{1}{2}y_{32}+\frac{1}{2}y_{42}$$
$$+\frac{1}{2}y_{52}-y_{14}-y_{24}-y_{34}-y_{44}-y_{54}=0,$$
$$\frac{1}{2}y_{11}+\frac{1}{2}y_{21}+\frac{1}{2}y_{31}+\frac{1}{2}y_{41}-\frac{1}{2}y_{12}-\frac{1}{2}y_{22}-\frac{1}{2}y_{32}-\frac{1}{2}y_{42}$$
$$-\frac{1}{2}y_{52}-y_{15}-y_{25}-y_{35}-y_{45}-y_{55}=0,$$
$$y_{k1}\geq 0,\quad k=1,2,3,4,$$
$$y_{k2}\geq 0,\quad k=1,2,3,4,5,$$
$$y_{k3}\geq 0,\quad k=1,2,3,4,$$
$$y_{k4}\geq 0,\quad k=1,2,3,4,5,$$
$$y_{k5}\quad canbepositiveornegative.$$

The optimal solution can translate to:

$$y_1=y_{11}+y_{21}+y_{31}+y_{41}=1.141357.$$
$$y_2=y_{12}+y_{22}+y_{32}+y_{42}+y_{52}=0.$$
$$y_3=y_{13}+y_{23}+y_{33}+y_{43}=1.097738.$$

Hence, the individual optimal solution to the (MLQCQP) problem of the FLDM is: $(y_1,y_2,y_3)=(1.141357,\ 0,\ 1.097738)$, and $Z_{21}=32.5346$.

6.1 The (ULQCQP) problem of FLDM with the bounded variable constraints

$$MaxZ_{31}(t) = (17t_1^2+10t_1t_2+15t_1+4t_2^2+6t_2+9t_3)$$

y_1

where t_2, t_3 solves

Subject to: (QCQP)

$$6t_1+10t_2+9t_3^2 \le 85,$$
$$12t_1+5t_2+11t_3 \le 39,$$
$$7t_1^2+6t_1t_2+7t_1+3t_2^2 \le 115,$$
$$17t_1^2+10t_1t_2+15t_1+4t_2^2+6t_2+9t_3 \ge 32.5346,$$
$$t_1 \ge 1.141357,$$
$$t_3 \ge 1.097738,$$
$$t_1, t_2, t_3 \ge 0.$$

Repeat the same action as the (MLQCQP) problem. The (ULQCQP) problem after converting t_1t_2 into separable form is as follows:

$$\text{Max } Z_{31}(t) = (17t_1^2+10m_1^2-10m_2^2+15t_1+4t_2^2+6t_2+9t_3)$$

t_1

where t_2, t_3 solves

Subject to: (QCQP)

$$6t_1+10t_2+9t_3^2 \le 85,$$
$$12t_1+5t_2+11t_3 \le 39,$$
$$7t_1^2+6m_1^2-6m_2^2+7t_1+3t_2^2 \le 115,$$
$$17t_1^2+10m_1^2-10m_2^2+15t_1+4t_2^2+6t_2+9t_3 \ge 32.5346,$$
$$\frac{1}{2}t_1+\frac{1}{2}t_2-m_1=0,$$
$$\frac{1}{2}t_1-\frac{1}{2}t_2-m_2=0,$$
$$t_1 \ge 1.141357,$$
$$t_3 \ge 1.097738,$$
$$t_1, t_2, t_3 \ge 0.$$

The ranges of the variables t_1, t_2, t_3, m_1, m_2 estimated from the constraints are:

$$1.141357 \le t_1 \le 4.06 \ , \quad 0 \le t_2 \le 6.2 \ , \ 1.097738 \le t_3 \le 3.1$$

$$0.571 \le m_1 \le 5.13 \ , \ -2.53 \le m_2 \le 2.03.$$

The slopes of the line segment corresponding to the separable functions are computed based on the equations, which are illustrated in problem (11.c). Hence, the individual optimal solution to the (ULQCQP) problem of the FLDM is:

$$(t_1, t_2, t_3) = (2.052032, \ 0, \ 1.306874), \text{and } Z_{31} = 114.1265$$

6.2 The (LLQCQP) problem of the FLDM with the bounded variable constraints

$$MaxZ_{11}(x) = \left(8x_1^2+3x_1x_2+7x_1+x_2^2+2x_2+3x_3\right)$$

x_1

where

x_2, x_3

solves

Subject to: (QCQP)

$$x_1+5x_2+2x_3^2 \le 35,$$
$$2x_1+2x_2+6x_3 \le 10,$$
$$5x_1^2+3x_1+x_2^2 \le 40,$$
$$8x_1^2+3x_1x_2+7x_1+x_2^2+2x_2+3x_3 \le 32.5346,$$
$$x_1 \le 1.141357,$$
$$x_3 \le 1.097738,$$
$$x_1, x_2, x_3 \ge 0.$$

Repeat the same action as the (MLQCQP) and the (ULQCQP) problems. The problem after converting x_1x_2 into separable form is as follows:

$$\text{Max } Z_{11}(x) = \left(8x_1^2+3m_1^2-3m_2^2+7x_1+x_2^2+2x_2+3x_3\right)$$

x_1

where x_2, x_3 solves;

Subject to:
(QCQP)

$$x_1+5x_2+2x_3^2 < 35,$$
$$2x_1+2x_2+6x_3 \le 10,$$
$$5x_1^2+3x_1+x_2^2 \le 40,$$
$$8x_1^2+3m_1^2-3m_2^2+7x_1+x_2^2+2x_2+3x_3 \le 32.5346,$$
$$\frac{1}{2}x_1+\frac{1}{2}x_2-m_1=0,$$
$$\frac{1}{2}x_1-\frac{1}{2}x_2-m_2=0,$$
$$x_1 \le 1.141357,$$
$$x_2 \le 0,$$
$$x_3 \le 1.097738,$$
$$x_1, x_2, x_3 \ge 0.$$

The ranges of the variables x_1, x_2, x_3, m_1, m_2 (estimated from the constraints) are:

$$0 \le x_1 \le 1.141357 \ , \ x_2 \le 0 \ , \ 0 \le x_3 \le 1.097738$$

$$0 < m_1 < 0.571 \ , \ 0 < m_2 < 0.571.$$

The slopes of the line segment corresponding to the separable functions are computed based on the equations, which are illustrated in problem (12.c). Hence, the individual optimal solution to the (LLQCQP) problem of the FLDM is:

$$(x_1, x_2, x_3) = (1.141357,\ 0,\ 1.097738), \text{and } Z_{11} = 21.7043.$$

Therefore, the optimal solution of the FLDM is obtained as:

$$\tilde{x}_1^F = (1.141357,\ 1.141357,\ 2.052032),\ \tilde{x}_2^F = (0,\ 0,\ 0),$$

$$\tilde{x}_3^F = (1.097738,\ 1.097738,\ 1.306874).$$

[SLDM]
The SLDM defines his/her problem in view of the FLDM by setting $(\tilde{x}_1^F) = (1.141357,\ 1.141357,\ 2.052032)$ to the SLDM constraints. Assume $\tilde{x}_1 = (x_1, y_1, t_1)$, $\tilde{x}_2 = (x_2, y_2, t_2)$, $\tilde{x}_3 = (x_3, y_3, t_3)$, $\tilde{F}_2 = (Z_{12}, Z_{22}, Z_{32})$, as follows:

$$\text{Max } (Z_{12}, Z_{22}, Z_{32})(x, y, t) = ((4,5,8) \otimes (x_1, y_1, t_1) \oplus$$

$$(12,16,19) \otimes (x_2, y_2, t_2) \oplus (7,9,11) \otimes (x_3, y_3, t_3) \oplus$$

$$((x_2, y_2, t_2), (x_3, y_3, t_3)) \otimes$$

$$\begin{pmatrix} (8,9,11) & (0,4,5) \\ (0,2,3 & (0,0,0)) \end{pmatrix} \otimes$$

$$(x_2, y_2, t_2)(x_3, y_3, t_3))$$

$$\text{where } x_3, y_3, t_3$$

solves
subject to: **(QCQP)**

$$(1,3,6) \otimes (x_1, y_1, t_1) \oplus \left(5,7,10 \otimes \left(x_2, y_2, t_2 \oplus\right)\right)$$

$$(2,5,9) \otimes (x_3, y_3, t_3) \otimes (x_3, y_3, t_3) \leq (35, 60, 85),$$

$$(2,10,12) \otimes (x_1, y_1, t_1) \oplus (2,3,5) \otimes (x_2, y_2, t_2) \oplus$$

$$(6,6,11) \otimes (x_3, y_3, t_3) \leq (10, 18, 39),$$

$$(3,4,7) \otimes (x_1, y_1, t_1) \oplus ((x_1, y_1, t_1), (x_2, y_2, t_2)) \otimes$$

$$\begin{pmatrix} (5,6,7) & (0,2,4) \\ (0,1,2) & (1,3,3) \end{pmatrix} \otimes \begin{pmatrix} (x_1, y_1, t_1) \\ (x_2, y_2, t_2) \end{pmatrix} \leq (40, 70, 115),$$

$$(x_1, y_1, t_1) = (1.141357,\ 1.141357,\ 2.052032),$$

$$x_1, x_2, x_3, y_1, y_2, y_3, t_1, t_2, t_3 \geq 0.$$

The Fully Fuzzy (QCQP) problem of the SLDM reduces to the following three-crisp (QCQP) problems:
(LLQCQP)

$$\text{Max} Z_{12}(x) = \left(8x_2^2 + 12x_2 + 7x_3 + 4.565428\right)$$

$$x_2$$

(MLQCQP)

$$\text{Max} \quad Z_{22}(y) = (9y_2^2 + 6y_2y_3 + 16y_2 + 9y_3 + 5.706785)$$

$$y_2$$

(ULQCQP)

$$\text{Max} Z_{32}(t) = (11t_2^2 + 8t_2t_3 + 19t_2 + 11t_3 + 16.416256)$$

$$t_2$$

where x_3, y_3, t_3 solves;
subject to: **(QCQP)**

$$5x_2 + 2x_3^2 \leq 33.85643,$$

$$2x_2 + 6x_3 \leq 7.717286,$$

$$x_2^2 \leq 30.06245,$$

$$7y_2 + 5y_3^2 \leq 56.575929,$$

$$3y_2 + 6y_3 \leq 6.58643,$$

$$3y_2^2 + 3.424071\, y_2 \leq 57.6184,$$

$$10t_2 + 9t_3^2 \leq 72.687808,$$

$$5t_2 + 11t_3 \leq 14.375616,$$

$$3t_2^2 + 12.312192 t_2 \leq 71.1599287,$$

$$x_2, x_3, y_2, y_3, t_2, t_3 \geq 0.$$

The SLDM repeats the same procedure as the FLDM by applying the separable programming method on the three crisp (QCQP) problems of the SLDM with the bounded variable constraints till obtaining the optimal solution, as follows:

$$\tilde{x}_2^S = (2.1954759,\ 2.1954764,\ 2.8751236),$$

$$\tilde{x}_3^S = (0,\ 0,\ 0).$$

3. [TLDM]
The TLDM defines his/her problem in view of the FLDM and SLDM by setting $(\tilde{x}_1^F) = (1.141357,\ 1.141357,\ 2.052032)$,

$$(\tilde{x}_2^S) = (2.1954759,\ 2.1954764,\ 2.8751236)$$

to the TLDM constraints.

Assume $\widetilde{x}_1 = (x_1, y_1, t_1)$, $\widetilde{x}_2 = (x_2, y_2, t_2)$, $\widetilde{x}_3 = (x_3, y_3, t_3)$, $\widetilde{F}_3 = (Z_{13}, Z_{23}, Z_{33})$, as follows:

$$\text{Max} \quad (Z_{13}, Z_{23}, Z_{33})(x, y, t) = ((7,8,14) \otimes (x_1, y_1, t_1) \oplus (4,9,9) \otimes (x_2, y_2, t_2) \oplus (13,18,20) \otimes (x_3, y_3, t_3) \oplus ((x_1, y_1, t_1), (x_3, y_3, t_3)) \otimes \begin{pmatrix} (1,2,6) & (0,1,3) \\ (3,4,4) & (9,10,14) \end{pmatrix} \otimes \begin{pmatrix} (x_1, y_1, t_1) \\ (x_3, y_3, t_3) \end{pmatrix}),$$

Subject to:
(QCQP)

$$(1,3,6) \otimes (x_1, y_1, t_1) \oplus (5,7,10) \otimes (x_2, y_2, t_2) \oplus (2,5,9) \otimes (x_3, y_3, t_3) \otimes (x_3, y_3, t_3) \leq (35,60,85),$$

$$(2,10,12) \otimes (x_1, y_1, t_1) \oplus (2,3,5) \otimes (x_2, y_2, t_2) \oplus (6,6,11) \otimes (x_3, y_3, t_3) \leq (10,18,39),$$

$$(3,4,7) \otimes (x_1, y_1, t_1) \oplus ((x_1, y_1, t_1), (x_2, y_2, t_2)) \otimes \begin{pmatrix} (5,6,7) & (0,2,4) \\ (0,1,2) & (1,3,3) \end{pmatrix} \otimes \begin{pmatrix} (x_1, y_1, t_1) \\ (x_2, y_2, t_2) \end{pmatrix} \leq (40,70,115),$$

$$(x_1, y_1, t_1) = (1.141357, \quad 1.141357, \quad 2.052032),$$
$$(x_2, y_2, t_2) = (2.1954759, \quad 2.1954764, \quad 2.8751236),$$
$$x_1, x_2, x_3, y_1, y_2, y_3, t_1, t_2, t_3 \geq 0.$$

The Fully Fuzzy (QCQP) problem of the TLDM reduces to the following three crisp (QCQP) problems:

LLQCQP $\underset{x_3}{Max} Z_{13}(x) = (9x_3^2 + 16.424071\, x_3 + 18.0741)$

(MLQCQP)
$\underset{y_3}{\text{Max}} \quad Z_{23}(y) = (10y_3^2 + 23.706785\, y_3 + 31.4955)$

(ULQCQP)
$\underset{t_3}{\text{Max}} \quad Z_{33}(t) = (14t_3^2 + 34.364224\, t_3 + 79.86956)$

Subject to: (QCQP)

$$2x_3^2 \leq 22.88126,$$
$$6x_3 \leq 3.32633,$$
$$5y_3^2 \leq 41.2076,$$
$$6y_3 \leq 0.0001,$$
$$9t_3^2 \leq 43.93657,$$
$$11t_3 \leq 1,$$
$$x_3, y_3, t_3 \geq 0.$$

The TLDM repeats the same procedure as the FLDM and SLDM by applying the separable programming method on the three crisp (QCQP) problems of the TLDM with the bounded variable constraints till obtaining the optimal solution, as follows:

$$\widetilde{x}_3^T = (0.00001666667, \quad 0.00001666667, \quad 0.09090909).$$

Finally, the fuzzy optimal solution for the (FFMLQCQP) problem is as follows:

$$\widetilde{x}_1^F = (1.14136, \quad 1.14136, \quad 2.05203),$$
$$\widetilde{x}_2^S = (2.19548, \quad 2.19548, \quad 2.87512),$$
$$\widetilde{x}_3^T = (0.0000167, \quad 0.0000167, \quad 0.090909).$$

Where:

$$\widetilde{\mathbf{F}}_{\mathbf{1}} = (35.1396, \quad 63.0089, \quad 212.4974),$$
$$\widetilde{\mathbf{F}}_{\mathbf{2}} = (69.4722, \quad 84.2158, \quad 165.0643),$$
$$\widetilde{\mathbf{F}}_{\mathbf{3}} = (18.07437, \quad 31.49589, \quad 83.10928).$$

7 Perspective

An effort has been made in this paper for finding the fuzzy optimal solution of (FFMLQCQP) problem. We have firstly decomposed the (FFMLQCQP) problem into series of crisp (QCQP) problems with bounded variable constraints. For each (QCQP) problem, the problem is solved by utilizing the separable programming method. The proposed model combines the techniques of bound and decomposition and separable programming without ignoring any part of solution area.

The future scope of the mentioned idea is that this model can be extended to fully fuzzy multi objective multi-level quadratic or fractional programming. Further the model can be also extended to fully fuzzy fully rough multi-level quadratic or fractional programming. Application of the proposed method to the real world decision making situations is required in the near future.

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